KU LEUVEN

CAMPUS BRUSSEL

A brief history of the international modern mathematics/New Math movement

Toeplitz-Kolloquium zur Didaktik und Geschichte der Mathematik

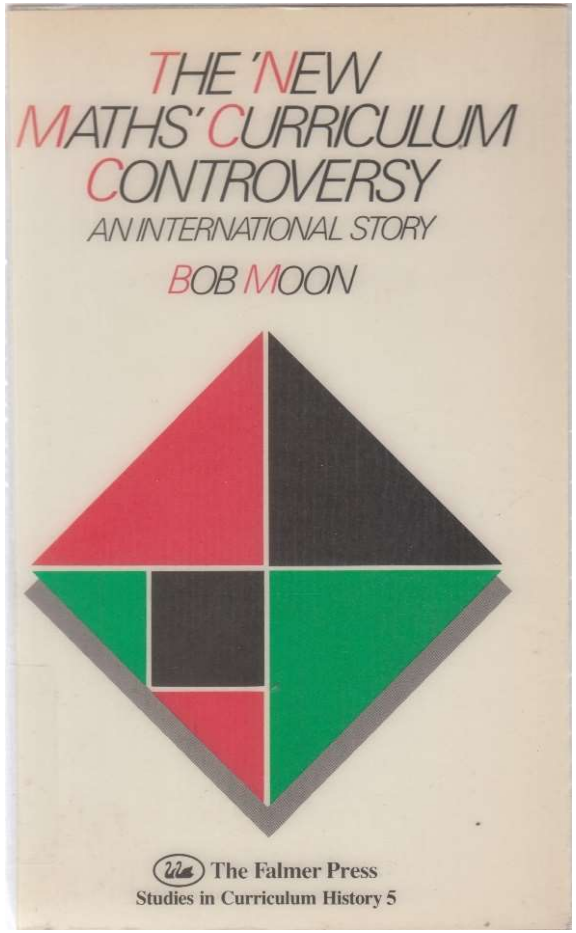
Bonn, November 13, 2023

Dirk De Bock

KU Leuven, Belgium

Introduction

European MM movement vs. American New Math movement



1986

A 'wave' of development in the USA 'crossed over' to Europe, although it is oft repeated, may be too simplistic a picture

A pattern of 'parallel' innovation would be a more appropriate characterization

BOB MOON

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Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques (CIEAEM)

- *Study* in order to *improve* (in that order)
- Focus on psycho-pedagogical and foundational issues

  Differences resulting from cultures are less important than similarities resulting from the structure of science and mathematical thought
  (Le Bureau, 1955)

- Founder and actual animator: Caleb Gattegno (at that time working at the University of London)
- Official foundation in1952 in La Rochette par Melin, France

CALEB GATTEGNO

Some CIEAEM members in La Rochette, 1952

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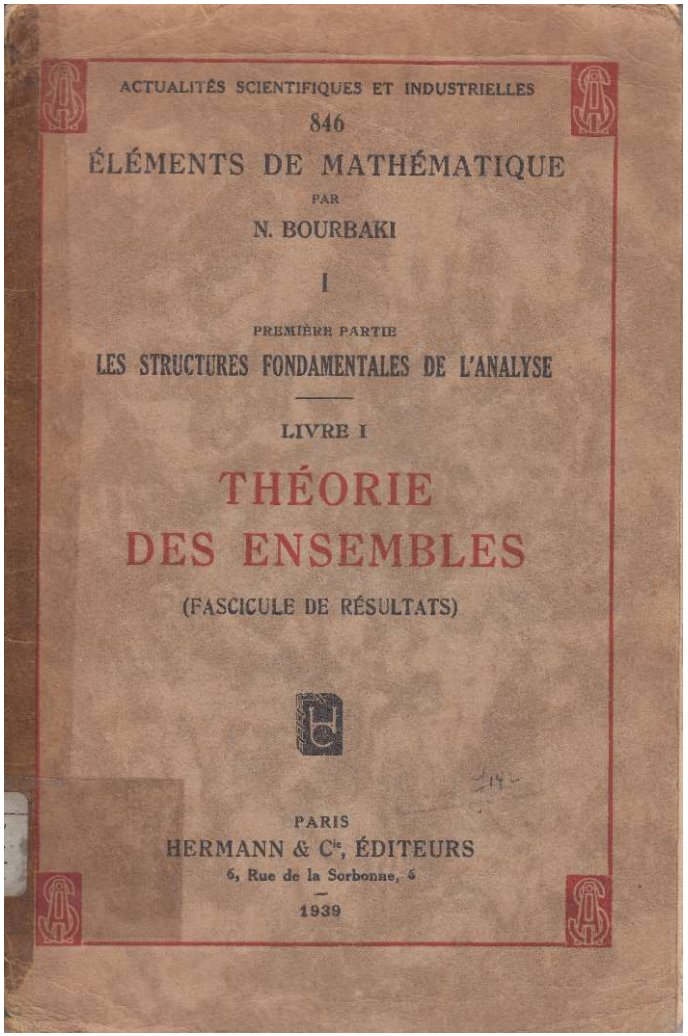
Bourbaki

- Qua methode geïnspireerd op Euclides
- Vanuit de zgn. “moederstructuren”
- Strikt axiomatisch-deductief
- Formele taal, rigoureuze stijl
- Geen didactische of heuristische concessies
- 1ste deel verschenen in 1939
- *L’Architecture des Mathématiques* (1948)



Role of structures according to Bourbaki

"Structures" are *TOOLS* for the mathematician; as soon as he has recognized among the elements, which he is studying, relations which satisfy the axioms of a known type, he has at his disposal immediately the entire arsenal of general theorems which belong to the structures of that type. Previously, on the other hand, he was obliged to forge for himself the means of attack on his problems; their power depended on his personal talents and they were often loaded down with restrictive hypotheses, resulting from the peculiarities of the problem that was being studied.

(L'Architecture des Mathématiques, BOURBAKI, 1948)

Link between mathematical and mental structures

If we retrace to its roots the psychological development of the arithmetic and geometric operations of the child, […] *we find, at every stage, a fundamental tendency to organize wholes or systems,* […] *according to three kinds of properties which precisely correspond to those of algebraic structures, order structures, and topological structures.*

(PIAGET, 1955)

A model for the science of mathematics became a model for mathematics education…

If the building of mathematics is based on 'structures,' which moreover correspond to the structures of intelligence, then it is on the gradual organization of these operational structures that the didactics of mathematics must be based (PIAGET, 1955)

From theory to practice…

In the light of the discussions, Servais already foresaw the direction he had to bring into his teaching. On his return to Belgium, in his class, he made tests; they were conclusive and students told him: "Why were we not told earlier about the structures? We would have seen more clearly." …

I [Félix] *decided to try the initiation at a very basic level with my students aged 13–14 and, as with Servais, it was received with relief thanks mainly to the simplicity of the language.*

(FÉLIX, 1986)

LUCIENNE FÉLIX AND WILLY SERVAIS

Two additional quotes

Modern mathematics appeared as the daughter of Bourbaki and Piaget, and she inherited from them—explicitly or more diffusely—structure, formalism and active pedagogy.

(CHARLOT, 1985)

The moment was propitious for the renewal of the teaching of mathematics. There was, on the one hand, the object of a global and structured reconstruction under the impetus of the Bourbaki group and, on the other hand, its learning was studied from a psycho-genetical perspective by the school of Piaget.

(SERVAIS looking back at La Rochette, 2000)

CIEAEM, 1954

Theme: The modern mathematics at school

Modern mathematics … is the result of an awareness of structures and the relationships between these structures. The thinking of the modern mathematician is relational. And it seems to me that relational mathematical thinking can be recreated by a child or adolescent if the teacher is aware of it and if he presents the appropriate situations.

(LENGER, 1954-55)

LENGER

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1955: UNESCO enters the scene

International survey (1955) and conference (1956) on secondary school mathematics

To what extent does the evolution of modern mathematics affect secondary education?

To what extent can the more abstract … mathematical structures, discovered within classical mathematics and developed worldwide by today's mathematicians, have a beneficial impact on secondary education? This is a very recent question to which pioneers in many countries are seeking an answer. The results obtained so far hold the promise of a pedagogical innovation. . (SERVAIS, 1956-57)

UNESCO enters the scene

Recommendations:

It is desirable to draw up curricula so as to base the teaching of mathematics on fundamental topics which bring out the general notions while coordinating the separate branches.

In this connection, it is also desirable to determine, by careful experiment, to what extent the multiform structure of modern mathematics can be made to enrich secondary education.

(UNESCO, 1956)

Some early developments in France

[Mathematics teachers are]

guards of a museum, which show dusty objects, most of which are of no interest

These windows, we must break them!

CHOQUET (Sèvres, 1955)

Some early developments in France

The important thing now is to help our colleagues ... those who were educated 20 years ago

CHOQUET quoted in FÉLIX, 1986

→ Series of 17 lectures at the Sorbonne University in 1956-57 by research mathematicians

Some early developments in France

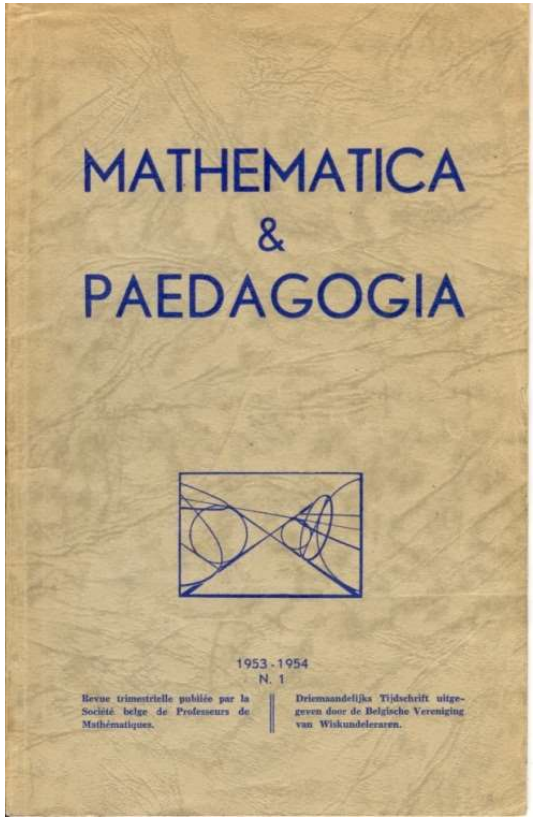
Between the mathematics we teach in our classrooms and the books for researchers, such as the Éléments of Bourbaki, the distance is enormous. But, like modern mathematicians, by means of axiomatics, return to the roots and ask to be aware of structures, starting from the simplest ones, the teacher who, like a student, takes the effort to see the usual facts in a new light, will also see his teaching in a new light. Because his habits are challenged, he will better understand children who do not have our habits. (Félix, 1955)

Some early developments in Belgium

- From the mid-1950s: The reform of mathematics programs is subject of debate in *Mathematica & Paedagogia*

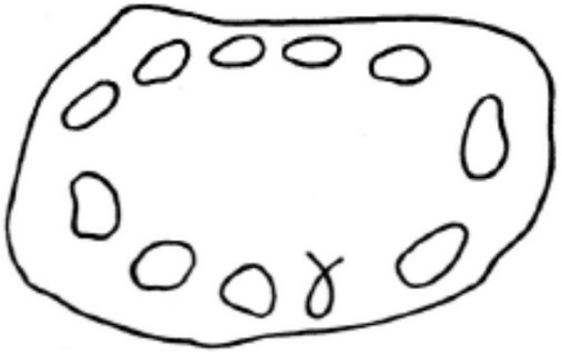
- August 1958: A concrete program for the teaching of modern mathematics

- 1958-59: First experiment with modern mathematics in two schools for kindergarten teachers…



The 1958-59 Belgian experiment

- Main ingredients: elements of
  - set theory (genesis of natural numbers) and
  - topology (study of geometry)
- "These notions are *innate*, very young children are naturally inclined toward abstraction…"



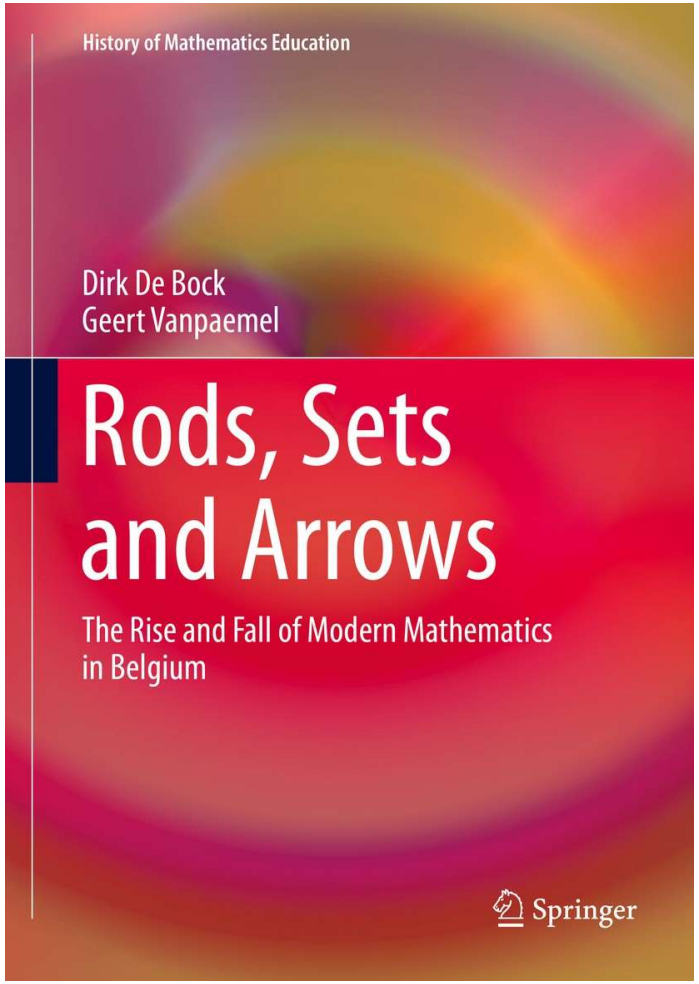
The 1958-59 Belgian experiment

- "This experiment will be of interest for all mathematics teachers, because it is likely to influence, in the relatively near future, mathematics teaching in all sections of secondary education" (LENGER & LEPROPRE, 1958-59)

- "Education was provided in these classes in an atmosphere of happiness. The hostility of the students toward mathematics had completely disappeared. We saw vividly that today's children are in resonance with the mathematics that is currently in use" (LENGER, 1968)

Servais at Royaumont

The syllabus proposed has not so far been tried out on any extensive scale, but isolated tests have shown that it is by no means utopian and that young people when faced with modern ideas show much greater aptitude than their reactions to traditional teaching might lead teachers to suppose. (SERVAIS, 1959)

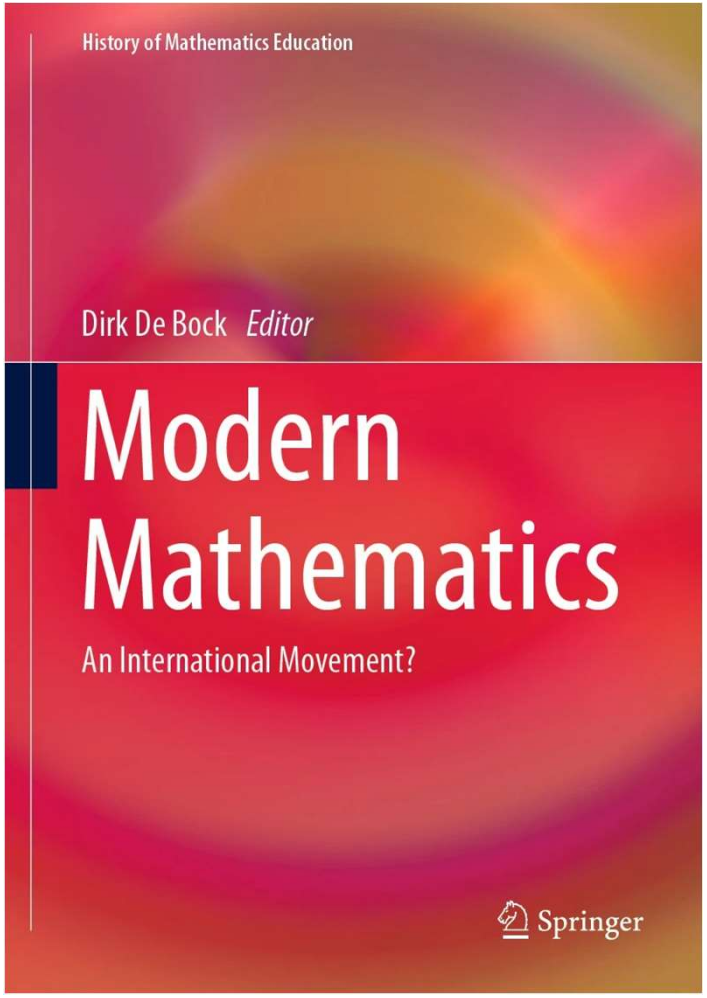


Germany?

- Before Royaumont: Modern mathematics at school was not a real issue

- (West) Germany was represented at Royaumont
  - Heinrich Schoene (Ministry)
  - Herman Athen
  - Otto Botch (“*Bewegungsgeometrie*“)

- More information:
  Chapter by Ysette Weiss in:

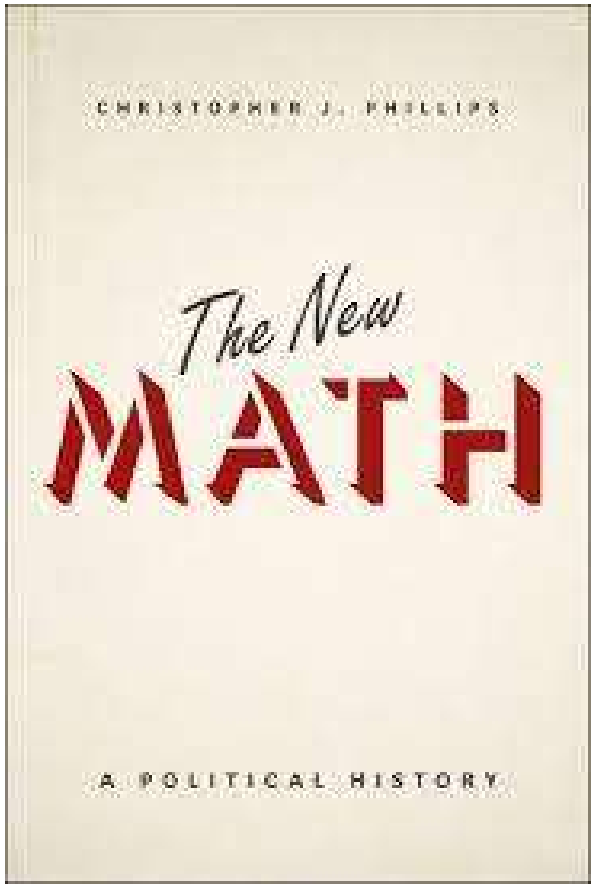


An anti-didactic inversion

The general, unifying structures of modern mathematics cannot be dictated a priori, but must be conquered. An antihistorical approach to the teaching of mathematics “ignores the time needed for these conquests from a superstition that the ‘last show’ of mathematics must be taught at all costs” (BARBIN, 2013)

New Math in the USA

- Not a coherent thing, rather a collective name for a number of more or less related curricular projects.
- More than in Europe: driven by political motives
- Recommend book:



The University of Illinois Committee on School Mathematics

- Established in 1951; first experiments in 1952-1953
- Max Beberman
- Unambiguous/precise language; careful/thorough explanations of basic concepts (numbers vs. numerals; “5” and “V”)
- Involvement of research mathematicians

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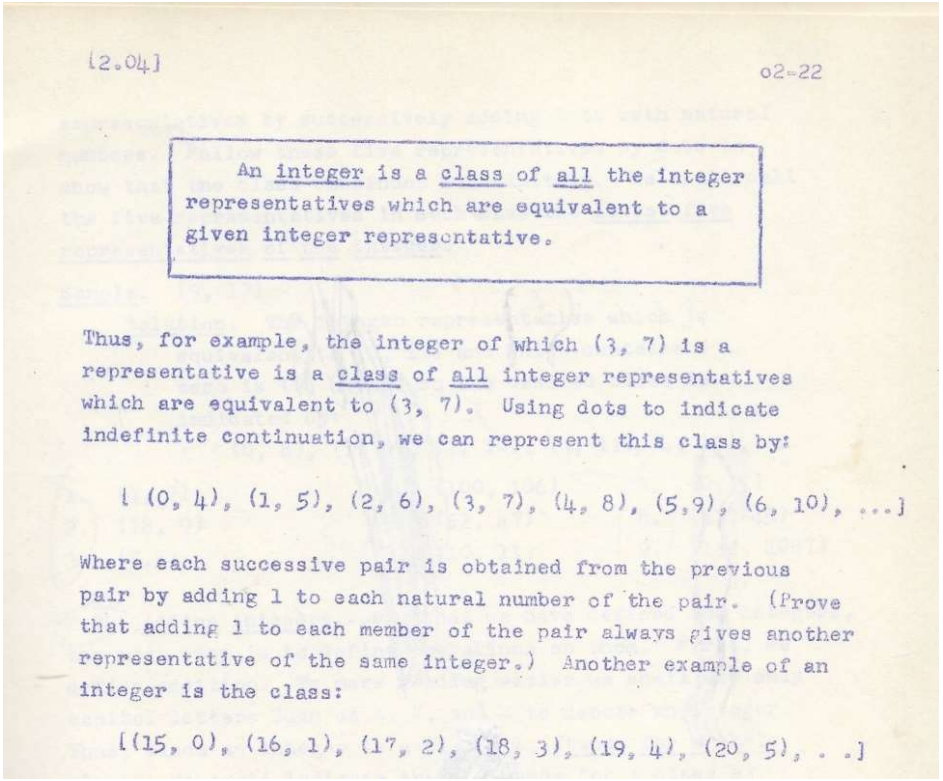
The University of Illinois Committee on School Mathematics

- Integers were defined equivalence classes of ordered pairs of natural numbers
  - positive integers were represented by pairs (a, b) with $a > b$
  - Negative integers were represented by pairs (a, b) with $a < b$
  - The class of all pairs (a, a) was defined as zero.
- "There are many properties of the integers that you have taken for granted in the past that can now be proved"



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An integer is a class of all the integer representatives which are equivalent to a given integer representative.

Thus, for example, the integer of which (3, 7) is a representative is a class of all integer representatives which are equivalent to (3, 7). Using dots to indicate indefinite continuation, we can represent this class by:

[(0, 4), (1, 5), (2, 6), (3, 7), (4, 8), (5,9), (6, 10), ...]

Where each successive pair is obtained from the previous pair by adding 1 to each natural number of the pair. (Prove that adding 1 to each member of the pair always gives another representative of the same integer.) Another example of an integer is the class:

[(15, 0), (16, 1), (17, 2), (18, 3), (19, 4), (20, 5), . .]

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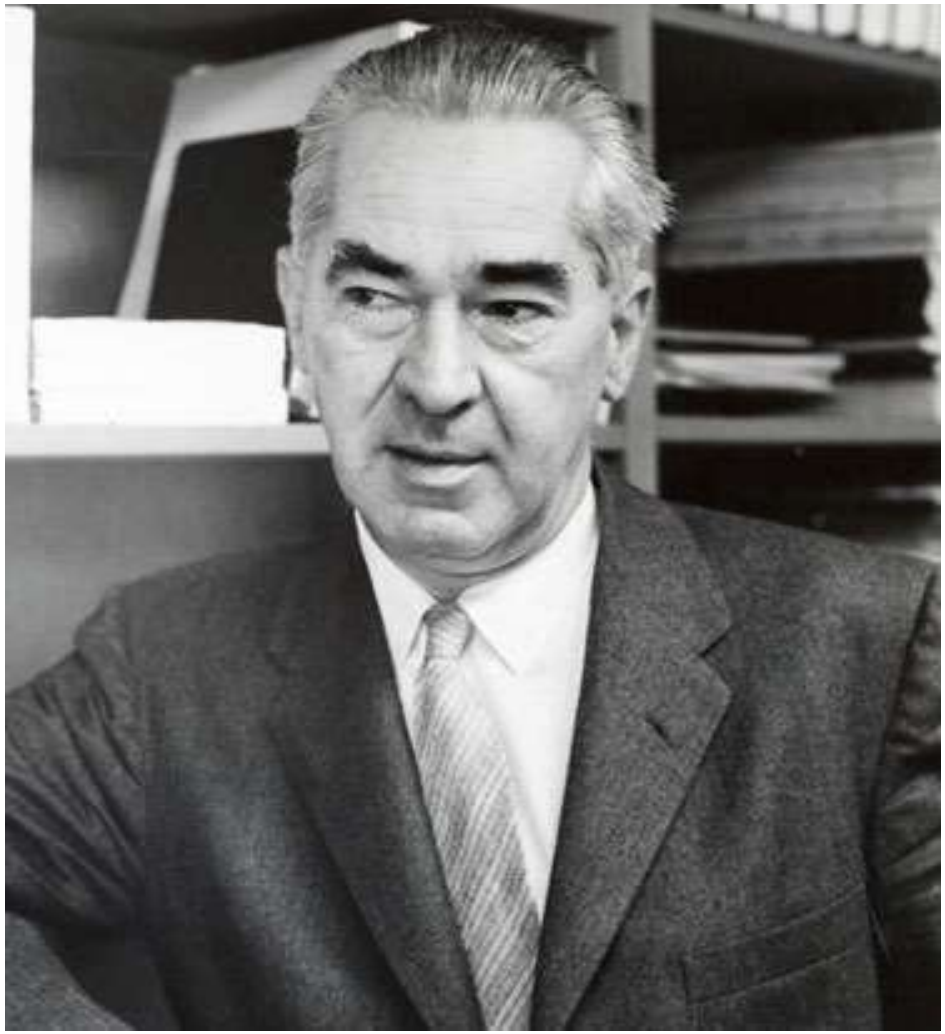
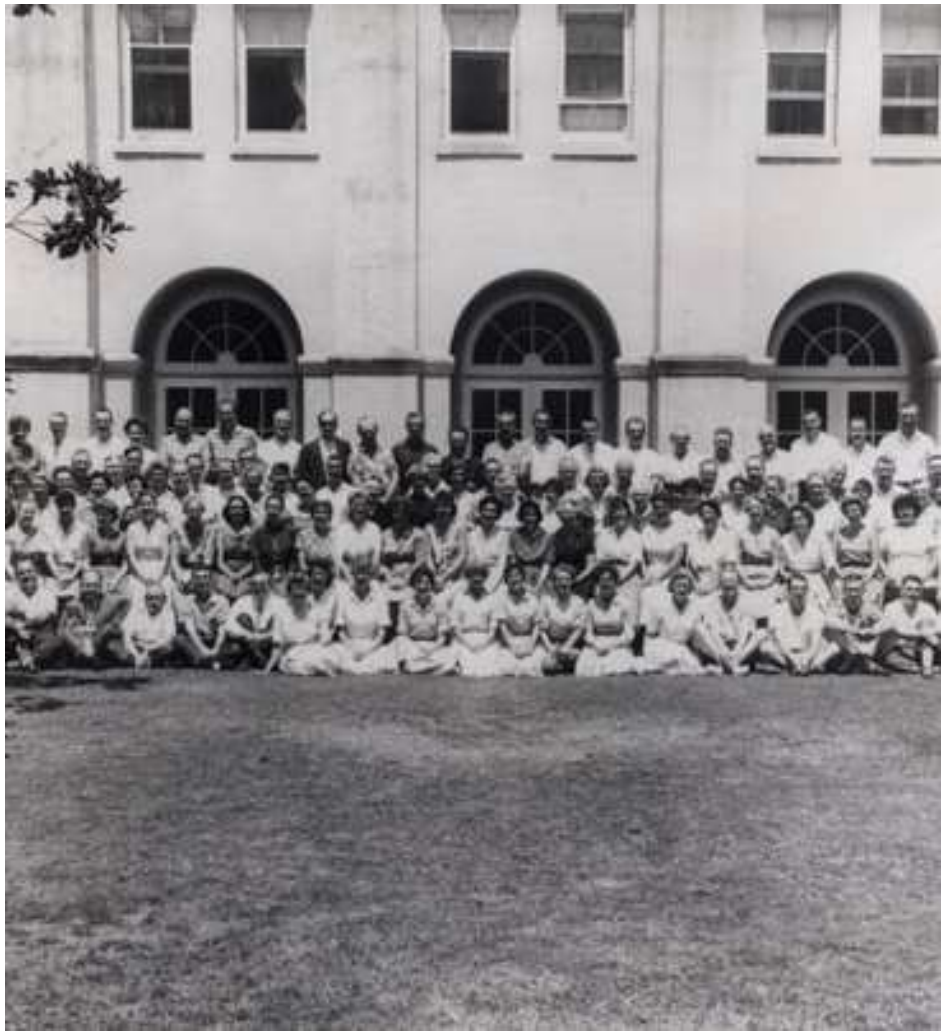
The School Mathematics Study Group

- Founded in 1958 in the wake of the Sputnik shock
- Ed Begle
- Massively funded by the National Science Foundation
- Revised curriculum + sample textbooks



The School Mathematics Study Group

- Writing groups (high school teachers and college mathematicians)
- “New” view on mathematics inspired by Bourbaki
- Mathematicians do not calculate, instead their job is “logical reasoning”
- Treatment of arithmetic as an example



How then can it be explained that similar reform movements emerged more or less simultaneously on both sides of the Atlantic?

- Post-WWII *Zeitgeist*
- Structuralism as a dominant paradigm in science
- Bourbaki…
- Marshall Stone

A modern mathematician would prefer the positive characterization of his subject as the study of general abstract systems, each one of which is an edifice built of specified abstract elements and structured by the presence of arbitrary but unambiguously specified relations among them.

An attempt to characterize the reform

Naïve set theory became the new framework for a unified presentation of mathematics and a starting point for teaching.

While the space of Euclid could for a long time serve as the framework for a unified presentation of basic mathematics, it can no longer today, but its role can now be fulfilled by the universe of sets. Moreover, as it has been proved by experiments carried out in America, England, Russia, Poland and in our country, the teaching of the basic notions of set theory fascinates young students. It therefore seems inevitable to propose that topic as starting point in secondary education. (PAPY, 1961)

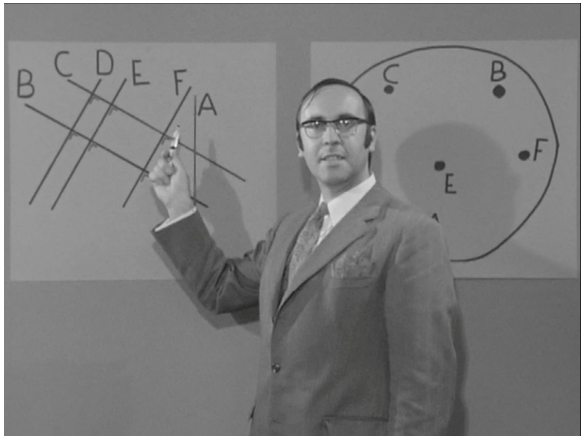
An attempt to characterize the reform

It sometimes led to pervasive abstraction and counter-intuitive situations…

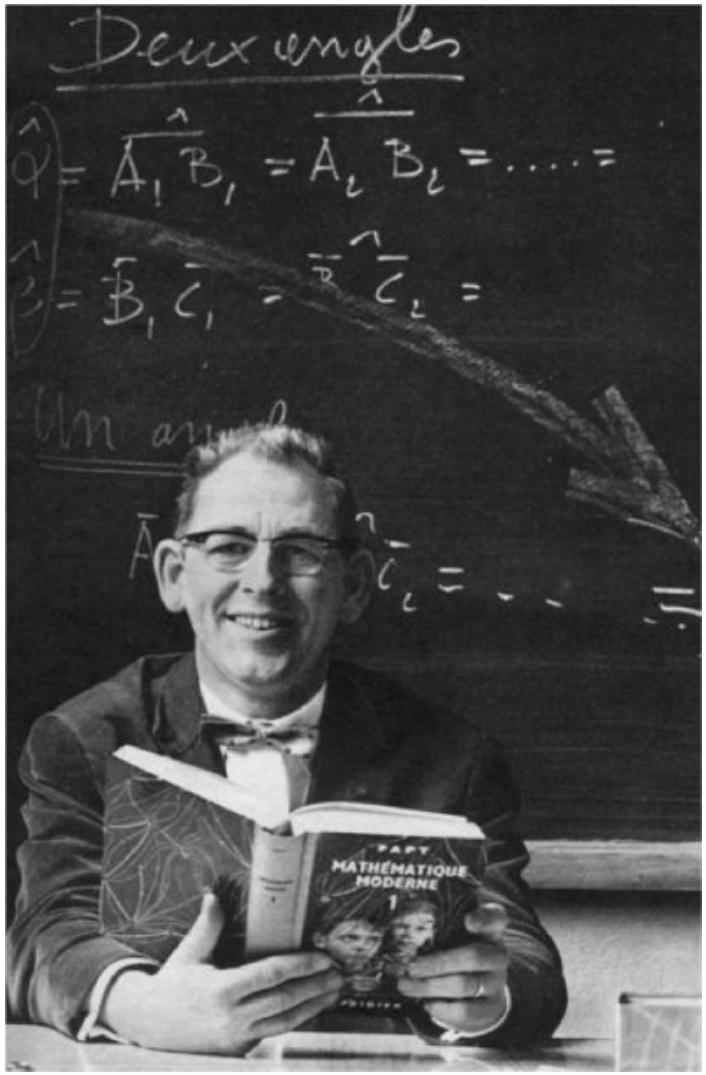
Two examples:

- Papy's reformulation of Euclid's parallel postulate: "*Every direction is a partition of the plane*"

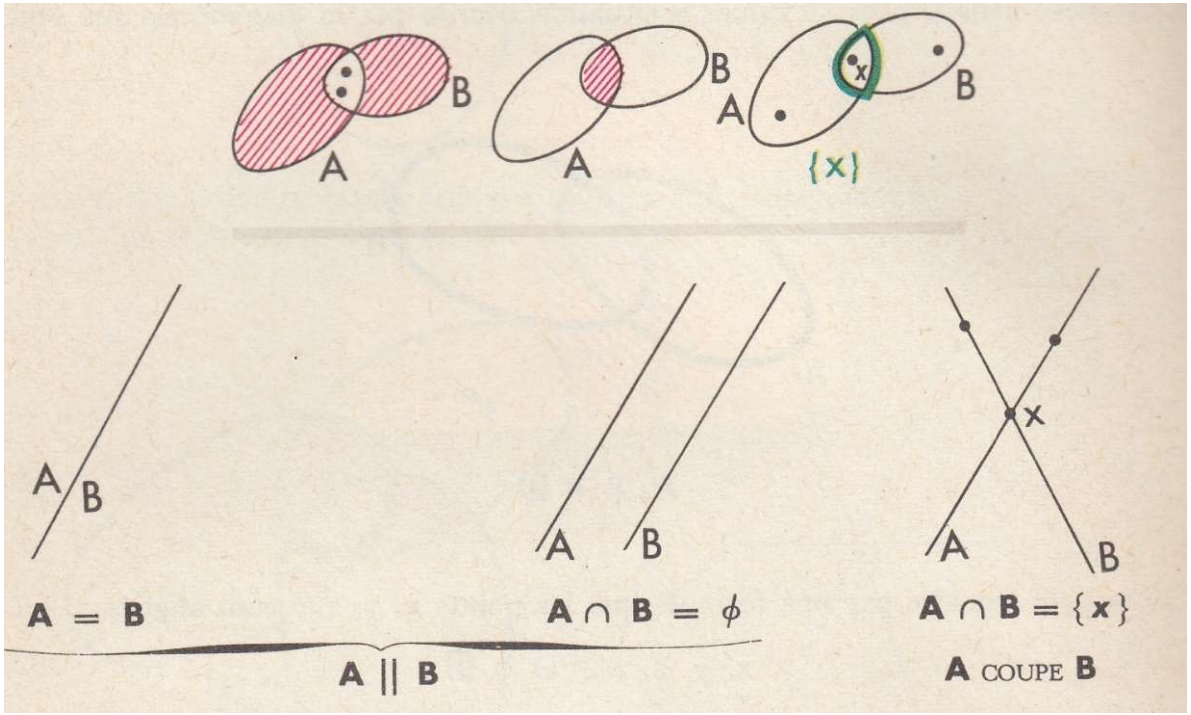
- In a set of straight lines, specific lines were represented with dots....



Venn diagrams to deliberately avoid relying on intuition…



The set diagram provide intuitive support for the logical structure of the theory (PAPY, 1963)



An attempt to characterize the reform

- Shaping mathematics education from the standpoint of **structures**.
- From poor to rich structures, from abstract situations to more concrete ones
- Prioritization of the affine standpoint in geometry
- Geometric transformations received increased attention; underlying algebraic structures were highlighted
- Algebraization of geometry ("the royal road to geometry is linear algebra"; CHOQUET, 1964)
- Problem solving received decreased attention…

An attempt to characterize the reform

- In Belgium, already in the final year of primary school, an introduction to structures was provided…

  "Movements" to illustrate the cyclic group of order 4… (Zoltan Dienes)

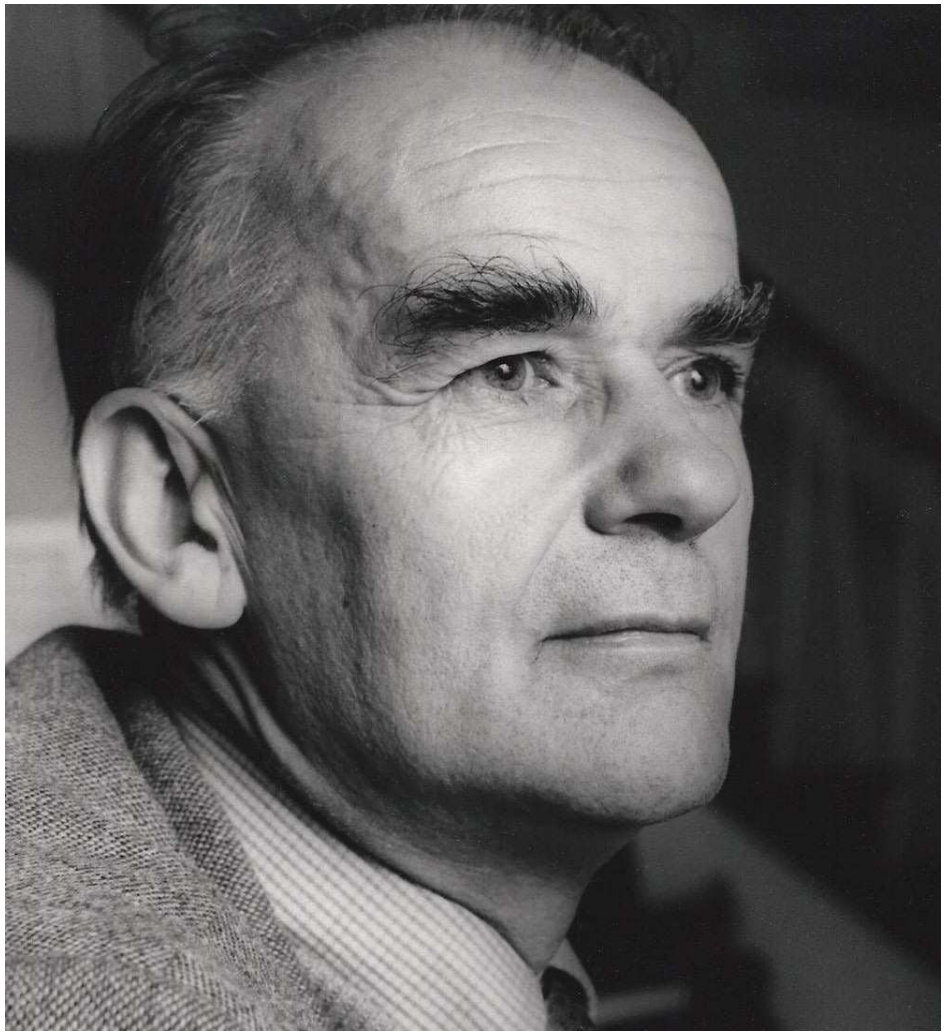


Illustration vs. application

A situation constitutes an illustration of a mathematical theory (or a mathematical concept) if this situation clarifies this theory (or concept).

A situation constitutes an application of a mathematical theory (or concept), if that theory (or concept) clarifies the situation.

PETER HILTON, 1973

In modern mathematics education, “illustrations” were often provided; “applications” rarely, if ever…

Lowering the age…

Any subject can be learned in some intellectually honest form to any child at any developmental stage, as long as instruction is organized appropriately (BRUNER, 1960).

Prof. Papy had taught sets to children from eight to twenty-five years old, and it was more difficult with the twenty-five year olds! Undergraduates were conditioned by the bad habits of traditional mathematics […]. Children of eight or ten were not so conditioned, and most success transpired with some fifteen-year-olds so poor in mathematics that they were uninfluenced by previous courses! (FIELKER, 1961)

Rejection of “drill and practice”

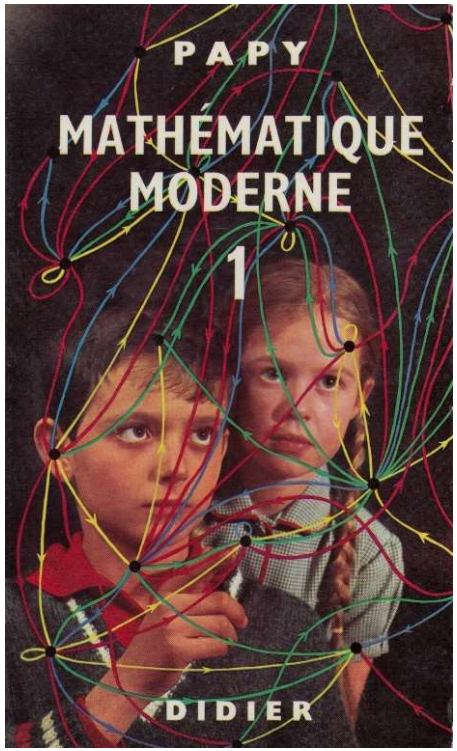
Focus on conceptual mathematical understanding rather than on computational fluency and number facts

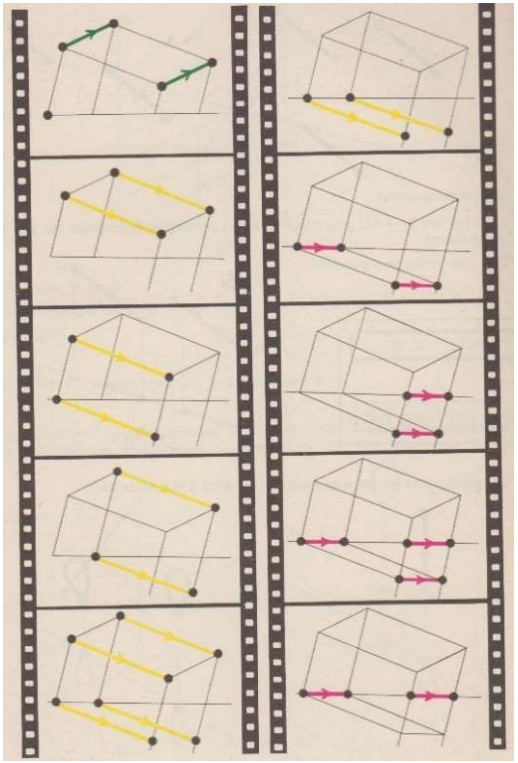
- Belief that thorough conceptual understanding would make drill unnecessary

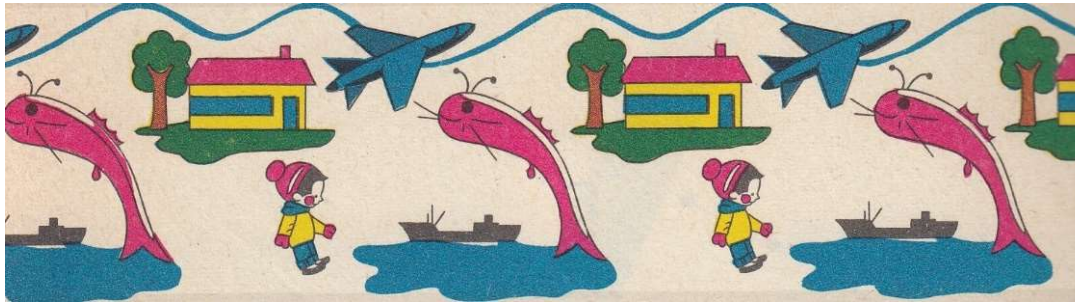
This method [drill and practice] *certainly contributes to train children to become respectful citizens, disciplined soldiers, obedient employees … . We suggest a diametrically opposite view on education, a method that is aimed at the development of personal creative freedom.* (PAPY, 1976)

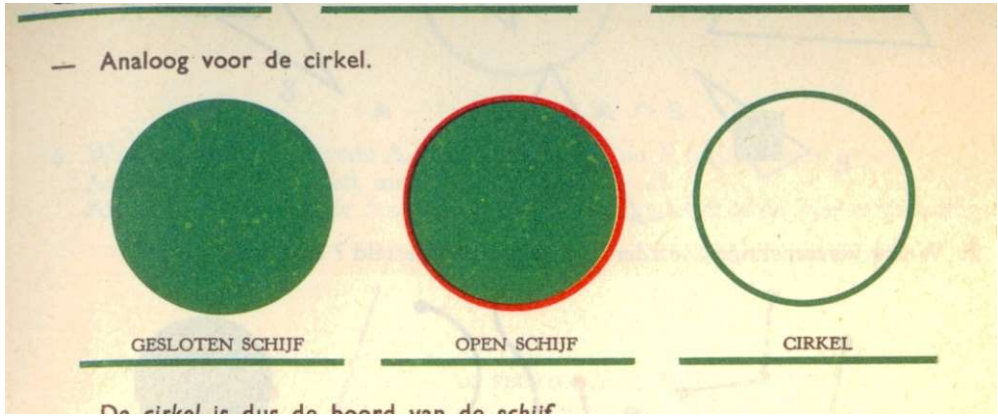
Updating both content and methodology

New language, new symbols, new didactical tools (use of color, Venn diagrams, arrow representations, drawings, proving with film strips, red/green color convention, ...)









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The fall of modern mathematics

René Thom in *American Scientist* (1971):

Geometry is a natural and possibly irreplaceable intermediary between ordinary language and mathematical formalism

Opening of Thom's lecture at ICME-2 (1972):

The future historian of mathematics will not fail to be amazed by the extent of the movement of the 1960s known as Modern Mathematics. … Only dogmatic spirits (and they are not lacking among "modernists") can believe that there is in these questions a truth capable of being logically established and before which one needs must bow. … "Modern Mathematics" has a very complex origin and composition. (Proceedings published in 1973)

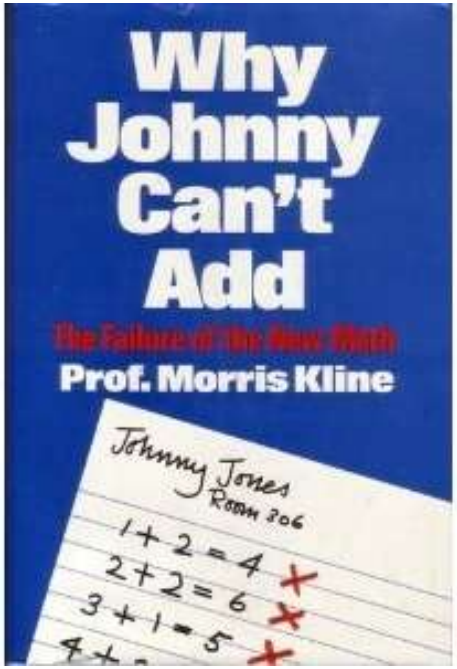
Erich Wittmann (e-mail to me, dated August 6, 2023):

Thom's plenary lecture at ICME-2 was immediately translated into German and played an important role in the discussions. (e-mail from Erich Wittmann to me, dated August 6, 2023)

The fall of modern mathematics

Morris Kline (1973): “Why Johnny can’t add”, subtitled “The failure of new math” became a best seller

A severe critique on the teaching practices characteristic of the New Math for school teaching



The fall of modern mathematics

At national levels, critique was often provided in press articles, often with telling headings…

Some examples:

France: *Le cauchemar des maths modernes* [The nightmare of modern mathematics] (in *L'Express*, 1972)

Belgium: *Les désastres de la mathématique moderne* [The disasters of modern mathematics] (in *La Libre Belgique*, 1980)

Germany: *Macht Mengenlehre krank?* (in *Der Spiegel*, 1974)

Did modern mathematics fail?

Basically, it did, but…

It enhanced collaboration between all stakeholders in mathematics education, nationally and internationally.
This did not stop along with modern mathematics…

It gave room to new developments

- The recognition of mathematics education as an autonomous scientific discipline with
    - Disciplinary research-oriented journals (ESM and other)
    - International congresses (the ICMEs)
- Alternatives were developed (e.g., realistic mathematics education)

Thank you for your attention!

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