Experimental modal models for exterior vibroacoustic state estimation

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Abstract
This paper explores the application of experimentally derived vibroacoustic state-space models in virtual sensor schemes. These models are obtained through Experimental modal analysis (EMA) by considering both in-band and out-of-band modal contributions. Additionally, the study illustrates how various measurement types, such as acceleration or acoustic pressures, can be derived from the temporal evolution of the experimental state space model. This allows the state estimator, specifically a Kalman filter, to effectively merge data from both the acoustical and structural domains with the numerical process model. To validate the framework, an industrial test setup is utilized, where the complexity prohibits the use of first-principle techniques for modelling. The results indicate that the state estimation outperforms forward simulation for excitation signals that were not initially incorporated into the model.

1 Introduction

For the reduction of noise pollution in addition to passive solutions, active noise reduction is becoming an indispensable tool in achieving this goal. Because reference measurements of the quantity that needs to be reduced are not always possible virtual sensing techniques have emerged in recent years to estimate vibroacoustic quantities using limited measurement information and numerical process models. Virtual sensing for vibroacoustic systems has been applied to academic problems that have been modelled using the Finite Element Method (FEM) [1, 2], yielding promising initial results. However, model updating has to be performed to obtain an accurate model. Therefore, for vibroacoustic problems, a data-driven modelling approach can be adopted, relying on measured data to construct a numerical representation of the system. One approach to deriving physically inspired experimental state space models is to utilize the results of an EMA.

2 Experimental state-space model identification for vibroacoustics

The vibroacoustic transfer function relating $\mathbf{H}^s(\omega)$ and $\mathbf{H}^a(\omega)$ the structural and acoustic Frequency response function (FRF)s respectively is given as:

$$\mathbf{H}^{va}(\omega) = \begin{bmatrix} \mathbf{H}^s(\omega) \\ \mathbf{H}^a(\omega) \end{bmatrix} = \begin{bmatrix} \mathbf{H}^s(\omega) \\ -\omega^2 \rho \mathscr{V}^e \mathbf{H}^s(\omega) \end{bmatrix}, \tag{1}$$

with ρ and $\mathscr{V}^e$ are the acoustic medium density and a coupling term respectively. Hereof, a state-space representation can be found from modal parameters combining in- $\mathbf{\Psi}_{ib}$ and out-of-band $\mathbf{\Psi}_{RCM}$ mode shape contributions [3]:

$$\mathbf{H}^{va}(\omega) = \begin{bmatrix} \mathbf{\Psi}_{ib} & \mathbf{\Psi}_{ib}^H & \mathbf{\Psi}_{RCM} & \mathbf{\Psi}_{RCM}^H \end{bmatrix} \left(i\omega\mathbf{I} - \begin{bmatrix} \mathbf{\Lambda}_{ib} & 0 & 0 & 0 \\ 0 & \mathbf{\Lambda}_{ib}^H & 0 & 0 \\ 0 & 0 & \mathbf{\Lambda}_{RCM} & 0 \\ 0 & 0 & 0 & \mathbf{\Lambda}_{RCM}^H \end{bmatrix} \right)^{-1} \begin{bmatrix} \mathbf{L}_{ib} \\ \mathbf{L}_{ib}^H \\ \mathbf{L}_{RCM} \\ \mathbf{L}_{RCM}^H \end{bmatrix}. \tag{2}$$

This *compliance* modal state-space model relates the acting force to nodal displacements in the structural and nodal sound pressures in the acoustical domain. Eq. (2) can be written in matrix notation:

$$\mathbf{H}^{va}(\omega) = \tilde{\mathbf{C}}\left(i\omega\mathbf{I} - \tilde{\mathbf{A}}\right)^{-1}\tilde{\mathbf{B}}. \tag{3}$$

Differentiating twice yields the *accelerance* state-space model relating the acting force to nodal accelerations in the structural domain.

$$\frac{\mathrm{d}^2\mathbf{H}^{va}(\omega)}{\mathrm{d}\omega^2} = -(\omega)^2\mathbf{H}^{va}(\omega) = \underbrace{\tilde{\mathbf{C}}\tilde{\mathbf{A}}\tilde{\mathbf{A}}}_{\tilde{\mathbf{C}}_a}\left(i\omega\mathbf{I} - \tilde{\mathbf{A}}\right)^{-1}\tilde{\mathbf{B}} + \underbrace{\tilde{\mathbf{C}}\tilde{\mathbf{A}}\tilde{\mathbf{B}}}_{\tilde{\mathbf{D}}_a} + \underbrace{\omega\tilde{\mathbf{C}}\tilde{\mathbf{B}}}_{\mathbf{0}}. \tag{4}$$

As was found by Dias et al. the additional term $\omega\tilde{\mathbf{C}}\tilde{\mathbf{B}}$ is null for the assumption of proportionally damped modes [4]. A common state-space model can be formulated accommodating all possible types of remeasurements obtained from Eq. (3) and (4) and some boolean selection matrices $\mathbf{B}$ as:

$$\dot{\tilde{\mathbf{x}}}(t) = \tilde{\mathbf{A}}\tilde{\mathbf{x}}(t) + \tilde{\mathbf{B}}\mathbf{u}(t), \tag{5}$$

$$\mathbf{y}(t) = \begin{bmatrix} \mathbf{B}_d\tilde{\mathbf{C}} \\ \mathbf{B}_a\tilde{\mathbf{C}}\tilde{\mathbf{A}}\tilde{\mathbf{A}} \end{bmatrix}\tilde{\mathbf{x}}(t) + \begin{bmatrix} \mathbf{0} \\ \mathbf{B}_a\tilde{\mathbf{C}}\tilde{\mathbf{A}}\tilde{\mathbf{B}} \end{bmatrix}\mathbf{u}(t), \tag{6}$$

3 Experimental validation on an industrial example

The steel core of an automotive car door is considered as industrial validation case as pictured in Fig. 1. A

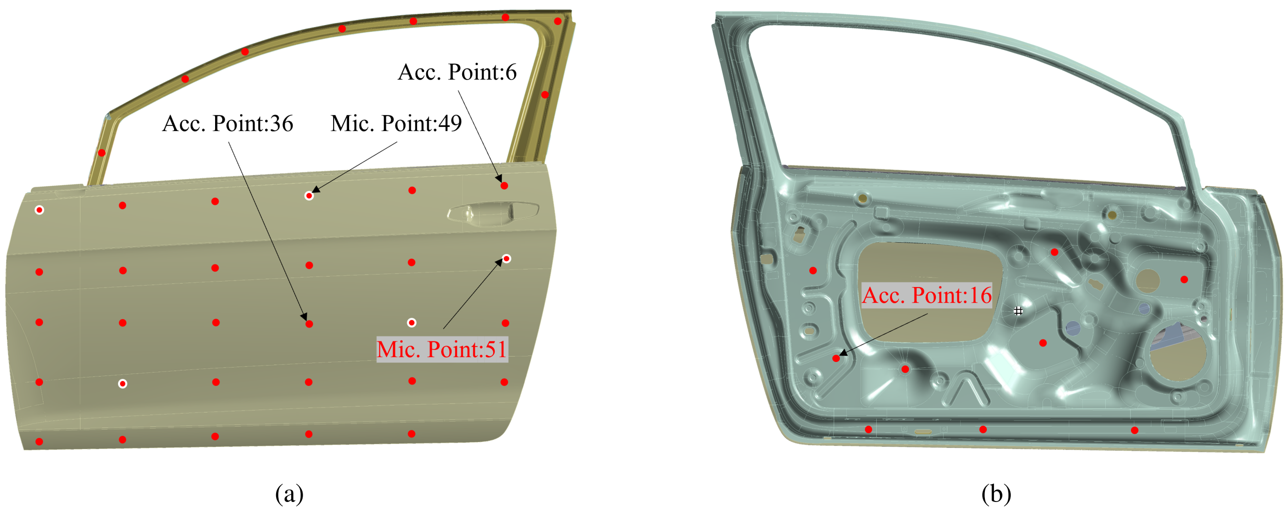

Figure 1: Test object. Measurement locations of accelerometers (red) and microphones (white outline). Force expatiation point (white/black checked). Red and black annotations indicate the virtual sensor locations and used sensors respectively

sweep excitation ($0 - 200Hz$) is applied for validation. The estimation utilized structural acceleration at positions 6 and 36, as well as the acoustic response at location 49. Comparing the results in Fig. 2 shows differences between the forward simulation and estimation, with better correspondence observed in the estimated signal compared to the measured signal. This improvement is more prominent in the acoustic domain and is further supported by the overall reconstruction results using the GoF metric stated in the figures.

4 Conclusion

This paper demonstrates the use of an EMA to create a first-order state space model for state estimation in vibroacoustics. Using the experimental modal state-space model circumvents the cumbersome model updating step and the state estimation can be performed on a complex structure over a wide frequency. A formulation of how to combine the measurement of acceleration and sound pressure employing the respective measurement

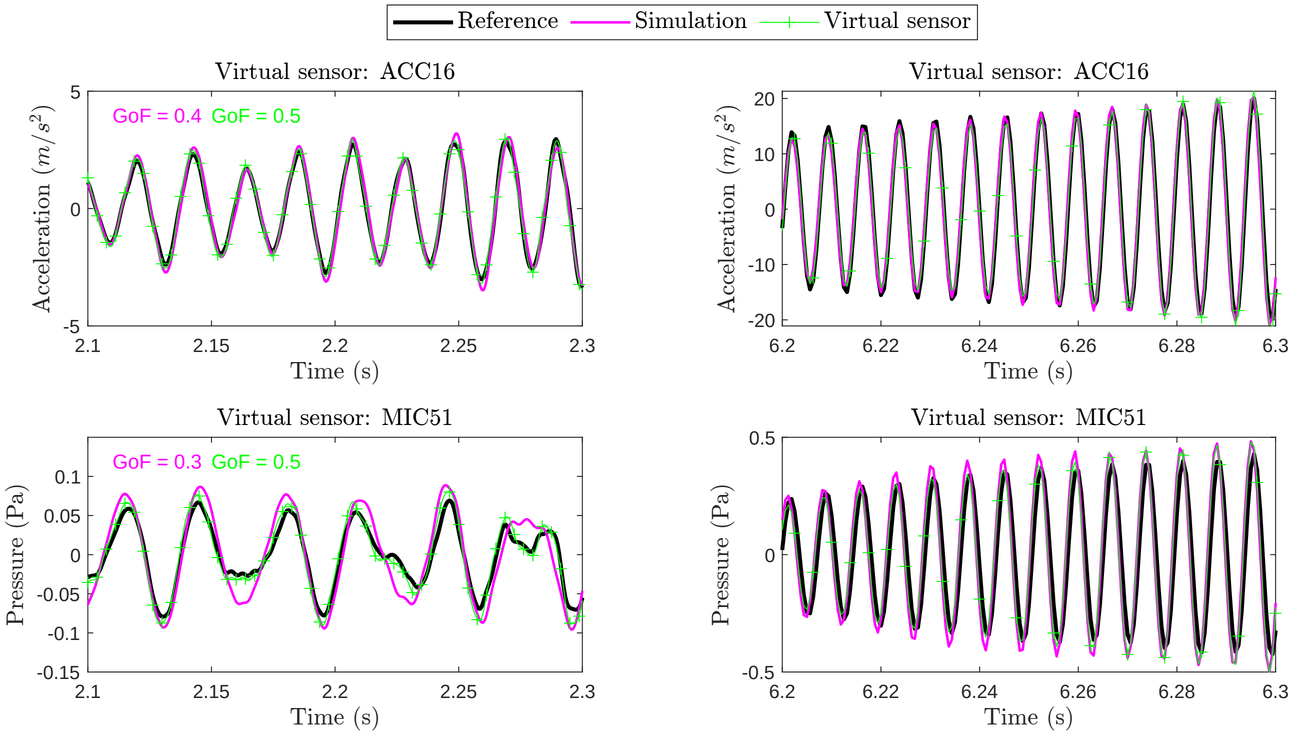

Figure 2: State estimation results. Simulation and virtual sensors.

and throughput matrices is derived which can be used directly with the system and input matrices in a Kalman filter operating on the complex matrices. This enables vibroacoustic state estimation by combining structural acceleration measurements and free-field pressure microphones. The validation of the method using a different input as was used for the model identification yields promising results and the estimation results outperform forward simulations. Moreover, it was found that the potential accuracy gains are higher in the acoustic domain which can be attributed to the better model fit of the structural FRFs and the higher number of used measurement information in the structural domain. Overall the approach of using experimental state space models is promising in situations where first principle models can not be obtained, are of too high dimension or the model updating is too cumbersome.

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