Recursive relations for block Hankel and Toeplitz systems

Part II : dual recursions

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Report TW 58, August 1982
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Part II : dual recursions

Abstract

The recursive relations given in part 1 of this report can be interpreted as recursions for the denominators of matrix-Padé approximants. In this part we shall give dual relations for the corresponding numerators and residuals.

Motivation and notations

All the notations and conventions used in part one are maintained in this second part. Part I will be referred to by (I). It was explained in (I) how the recursions were related to denominator recursions of Padé approximants. In [1] we showed how these same relations could be expressed in terms of the corresponding numerators and residuals. In this part we shall give these dual formulations for all the recursions of (I). Let us introduce the notation with an example, viz.F2 of (I) treating the step $x \to x \to 0$

a) Suppose that the elements of T_{n+1} are part of the longer block sequence $\{t_k\}_{-M}^{N}$, $M,N \geqslant n+1$, then we shall denote by $\hat{T}_k$ an extended matrix

$$\hat{T}_k = \left[\begin{array}{c} \begin{matrix} t_{-M+k} & \cdots & t_{-M} \\ \vdots & \ddots & \vdots \\ t_{-1} & \cdots & t_{-(k+1)} \end{matrix} \\ \hline T_k \\ \hline \begin{matrix} t_{k+1} & \cdots & t_{1} \\ \vdots & \ddots & \vdots \\ t_{N} & \cdots & t_{N-k} \end{matrix} \end{array}\right]$$

and define E_k by $\hat{T}_k Q_k = E_k$ where Q_k is as in F2a. E_k has the following form

$$E_k = [\overbrace{\ldots\ s'_k\ s_k}^{M-k}\ \overbrace{0\ \ldots\ 0}^{k}\ \overbrace{r_k\ r'_k\ \ldots}^{N+1-k}]^t \qquad (^t \text{ denotes block-transpose})$$

and consists of two nonzero parts

$$A_k = [s_k\ s'_k\ \ldots]^t \quad \text{and} \quad B_k = [r_k\ r'_k\ \ldots]^t.$$

A_k contains numerator coëfficients in descending order and B_k contains the residual coëfficients in ascending order. If $B_n(k)$ denotes the k-th block of B_n, then $B_n(0) = r_n$ and similarly $A_n(0) = s_n$. Thus if A_n, B_n, A_{n-1} and B_{n-1} are known, then we know also a_{n+1} and b_{n+1}, so that the recursion

$$Q_{n+1} = \begin{bmatrix} 0 \\ \\ Q_n \end{bmatrix} + \begin{bmatrix} 0 \\ Q_{n-1} \\ 0 \end{bmatrix} a_{n+1} + \begin{bmatrix} Q_n \\ \\ 0 \end{bmatrix} b_{n+1}$$

can be evaluated. If we multiply this with $\hat{T}_{n+1}$, we obtain

$$E_{n+1} = \lrcorner E_n + \lrcorner \urcorner E_{n-1}\, a_{n+1} + \urcorner E_n\, b_{n+1}$$

Because $E_n = [\tilde{A}_n^t \; 0^t \; B_n^t]^t$, with $\tilde{A}_n = (J_{M-n} \otimes I_p)A_n$

we can split this recursion into two parts :

$$B_{n+1} = \lrcorner B_n + \lrcorner \urcorner B_{n-1}\, a_{n+1} + \urcorner B_n\, b_{n+1}$$

$$A_{n+1} = \urcorner A_n + \urcorner \lrcorner A_{n-1}\, a_{n+1} + \lrcorner A_n\, b_{n+1}$$

with $a_{n+1} = -s_{n-1}^{-1}\, s_n$

$$b_{n+1} = -r_n^{-1}\, r_{n-1}\, a_{n+1}$$

where $r_k = B_k(0)$, $s_k = A_k(0)$, $k = n-1, n$.

These are the dual relations of F2a).

b) In case b) we suppose that the elements of H_{n+1} are part of the longer sequence $\{h_k\}_{-M}^{N}$ with $M \geqslant 0$ and $N \geqslant 2(n+1)$ and we define $\hat{H}_k$ by

$$\hat{H}_k = \left[\begin{array}{ccc} h_{-M} & \cdots & h_{m+k} \\ \vdots & \iddots & \vdots \\ h_{-1} & \cdots & h_{k-1} \\ \hline & H_k & \\ \hline h_{k+1} & \cdots & h_{2k+1} \\ \vdots & \iddots & \vdots \\ h_{N-k} & \cdots & h_N \end{array}\right]$$

and E_k is defined by $\hat{H}_k Q_k = E_k$ with Q_k as in F2b).
E_k has the form

$$E_k = [\underbrace{\ldots\ s'_k\ s_k}_{M}\ \underbrace{0\ \ldots\ 0}_{k}\ \underbrace{r_k\ r'_k\ \ldots}_{N-2k+1}]^t = [\tilde{A}_k^t\ 0^t\ B_k^t]^t$$

with $\tilde{A}_k = (J_M \otimes I_p)A_k$.

Multiplying the recursion

$$Q_{n+1} = \begin{bmatrix} Q_n \\ \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ Q_{n-1} \\ 0 \end{bmatrix} a_{n+1} + \begin{bmatrix} 0 \\ \\ Q_n \end{bmatrix} b_{n+1}$$

from the left with $\hat{H}_{n+1}$ gives exactly the same formulas as under a).
Thus there is no difference between the a) and b) case conserning the formal form of the recursions.

c) Take now for $\{t_k\}$ the elements of T_{n+1} in F2c). Extend this sequence and form the matrices $\hat{T}_k$ as we did under case a) and define

$$\hat{T}_k Q_k = E_k = [\underbrace{\ldots\ r'_k\ r_k}_{M-k}\ \underbrace{0\ \ldots\ 0}_{k}\ \underbrace{s_k\ s'_k\ \ldots}_{N+1-k}]^t \quad \text{and}$$

set $B_k = [r_k\ r'_k\ \ldots]^t$ and $A_k = [s_k\ s'_k\ \ldots]^t$.

Note that now B_k is the top part and A_k the lower part of the right hand side, as opposed to case a) and b). The reason for this interchange of A and B is that thus the formulas of a) are unaltered when passing from a) to c).

d) As c) was derived from a), we can derive d) from b). Again, the right hand sides E_k are of the form

$$E_k = [\tilde{B}_k^t \; 0^t \; A_k^t]^t \quad ,$$

and again the same recursion is valid.

As a conclusion we can give the same recursion formulas for the four different cases. Note however that, although the recursions are the same, the vectors A_k, B_k are different e.g. in dimension.

Note also that if M and $N \to \infty$, then we can drop all the $\sqcup$ operations in the recursions for A_k and B_k.

The extensions for other nestings of matrices is understood in an obvious way. As in (I) the $\{t_k\}$ refer to to the elements of the Toeplitz matrices on the third line (following?). The extension of the other matrices is then such that the sequence $\{t_k\}_{-M}^{N}$ is exhausted completely e.g. in F3a) $\hat{T}_n$ means

$$\hat{T}_n = \left[\begin{array}{ccc} t_{-M+n} & \cdots & t_{-M} \\ \vdots & & \\ t_{-2} & \cdots & t_{-M-2} \\ \hline & T_n & \\ \hline t_n & \cdots & t_0 \\ \vdots & & \\ t_N & \cdots & t_{N-n} \end{array}\right]$$

and E_n is defined correspondingly by $\hat{T}_n \, Q_n = E_n$. For Hankel matrices, the extensions $\hat{H}_k$ are understood in a similar way.

The recursions F11 - F23 of (I) involve denominator coëfficients, i.e. coëfficients of Q_k and/or P_k. These can not be found in the right hand sides E_k or F_k. Thus we need the Q_k and P_k vectors also.

The scheme of all possible steps is as in (I).

F2	F17	F6	F4
F14	X	X	F2
F21	F17	F6	F4

F22	F12	F8
F18	X	F5
F18	X	F5
F22	F12	F8

	F23	F11	F1
F15	F19	X	F9
F16	X	F7	F3
F13	F20	F10	

F13	F20	F10	
F16	X	F7	F3
F15	F19	X	F9
	F23	F11	F1

References

1. A. Bultheel : Recursive algorithms for the matrix Padé table. Math. Comp. 35 (151), 875-892, 1980.

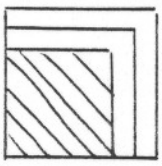
2. A. Bultheel : Recursive relations for block Hankel and Toeplitz systems, Part I : Direct recursion. Report TW56, Dept. Computer Science, K.U.Leuven, March 1982.

a

$$!\hat{T}_{n-1}\ Q_{n-1} = [\tilde{A}^t_{n-1}\ 0^t\ B^t_{n-1}]^t$$

$$\hat{T}_n\ Q_n = [\tilde{A}^t_n\ 0^t\ B^t_n]^t$$

$$?\hat{T}_{n+1}\ Q_{n+1} = [\tilde{A}^t_{n+1}\ 0^t\ B^t_{n+1}]^t$$



$$T_n = \urcorner\, T_{n+1}\, \llcorner^t$$

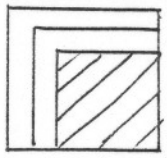
$$T_{n-1} = \urcorner\, T_n\, \llcorner^t$$

b

$$!\hat{H}_{n-1}\ Q_{n-1} = [\tilde{A}^t_{n-1}\ 0^t\ B^t_{n-1}]^t$$

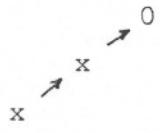
$$\hat{H}_n\ Q_n = [\tilde{A}^t_n\ 0^t\ B^t_n]^t$$

$$?\hat{H}_{n+1}\ Q_{n+1} = [\tilde{A}^t_{n+1}\ 0^t\ B^t_{n+1}]^t$$



$$H_n = \urcorner\, H_{n+1}\, \urcorner^t$$

$$H_{n-1} = \urcorner\, H_n\, \urcorner^t$$



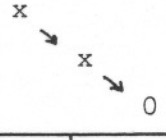
$$B_{n+1} = \begin{bmatrix} 0 \\ \\ \llcorner\, B_n \end{bmatrix} + B_{n-1}\, a_{n+1} + B_n\, b_{n+1}$$

$$A_{n+1} = \urcorner\,(\urcorner\,(A_n + \llcorner\,\llcorner\, A_{n-1}\, a_{n+1}) + \llcorner\, A_n\, b_{n+1})$$

$$a_{n+1} = -s_{n-1}^{-1}\, s_n$$

$$b_{n+1} = -s_n^{-1}(s'_n + s'_{n-1}\, a_{n+1})$$

$$s_k = A_k(0)\ ,\ s'_k = A_k(1) \quad ,\quad k = n-1,n$$

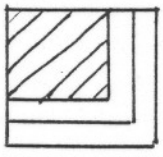


d

$$!\hat{H}_{n-1}\ Q_{n-1} = [\tilde{B}^t_{n-1}\ 0^t\ A^t_{n-1}]^t$$

$$\hat{H}_n\ Q_n = [\tilde{B}^t_n\ 0^t\ A^t_n]^t$$

$$?\hat{H}_{n+1}\ Q_{n+1} = [\tilde{B}^t_{n+1}\ 0^t\ A^t_{n+1}]^t$$



$$H_n = \llcorner\, H_{n+1}\, \llcorner^t$$

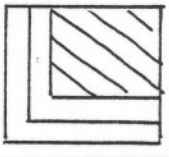
$$H_{n-1} = \llcorner\, H_n\, \llcorner^t$$

c

$$!\hat{T}_{n-1}\ Q_{n-1} = [\tilde{B}^t_{n-1}\ 0^t\ A^t_{n-1}]^t$$

$$\hat{T}_n\ Q_n = [\tilde{B}^t_n\ 0^t\ A^t_n]^t$$

$$?\hat{T}_{n+1}\ Q_{n+1} = [\tilde{B}^t_{n+1}\ 0^t\ A^t_{n+1}]^t$$



$$T_n = \llcorner\, T_{n+1}\, \urcorner^t$$

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a

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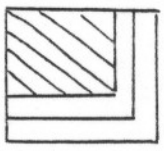

$$T_n = \lrcorner\, T_{n+1}\, \lrcorner^t$$

$$T_{n-1} = \lrcorner\, T_n\, \lrcorner^t$$

b

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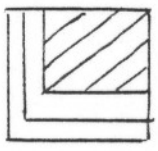


$$H_n = \lrcorner\, H_{n+1}\, \urcorner^t$$

$$H_{n-1} = \lrcorner\, H_n\, \urcorner^t$$

$x \to x \to 0$

$$B_{n+1} = \lrcorner\, B_n + \urcorner\,(\lrcorner\, B_{n-1}\ a_{n+1} + B_n\ b_{n+1})$$

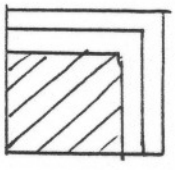
$$A_{n+1} = \urcorner\,(A_n + \lrcorner\, A_{n-1}\ a_{n+1}) + \lrcorner\, A_n\ b_{n+1}$$

$$a_{n+1} = -s^{-1}_{n-1}\ s_n$$

$$b_{n+1} = -r^{-1}_n\ r_{n-1}\ a_{n+1}$$

$$r_k = A_k(0),\ s_k = B_k(0) \quad , \quad k = n-1,n$$

$x \to x \to 0$



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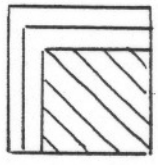

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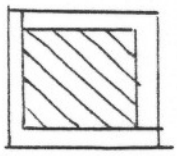

$$T_n = \lrcorner\, T_{n+1}\, \urcorner^t$$

$$T_{n-1} = \urcorner\, T_n\, \lrcorner^t$$

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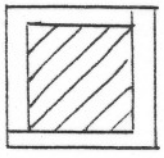


$$H_n = \lrcorner\, H_{n+1}\, \lrcorner^t$$

$$H_{n-1} = \urcorner\, H_n\, \urcorner^t$$

x → x, x → 0

$$B_{n+1} = \urcorner\,(\,\lrcorner\, B_n + \urcorner\,(B_{n-1}\ a_{n+1} + B_n\ b_{n+1})$$

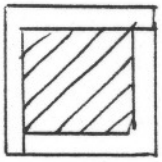
$$A_{n+1} = A_n + \lrcorner\,\lrcorner\, A_{n-1}\ a_{n+1} + \begin{bmatrix} 0 \\ \lrcorner\, A_n \end{bmatrix} b_{n+1}$$

$$a_{n+1} = -r^{-1}_{n-1}(r'_{n-1}\ r^{-1}_{n-1} - r'_n\ r^{-1}_n)^{-1}\ r_n$$

$$b_{n+1} = r^{-1}_n(r'_{n-1}\ r^{-1}_{n-1} - r'_n\ r^{-1}_n)^{-1}\ r_n$$

$$r_k = B_k(0)\ ,\ r'_k = B_k(1) \quad ,\quad k = n-1, n$$

x → x, 0 ←



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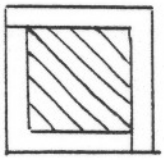

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c

a

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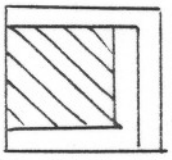

$$T_n = \urcorner\, T_{n+1}\, \llcorner^t$$

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b

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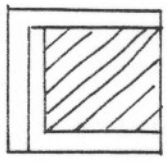

$$H_n = \urcorner\, H_{n+1}\, \urcorner^t$$

$$H_{n-1} = \llcorner\, H_n\, \urcorner^t$$

$$x \to x \nearrow 0$$

$$B_{n+1} = \begin{bmatrix} 0 \\ \llcorner\, B_n \end{bmatrix} + \llcorner\, B_{n-1}\ a_{n+1} + B_n\ b_{n+1}$$

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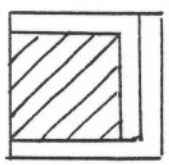
$$b_{n+1} = -s^{-1}_n (s'_n + s'_{n-1}\ a_{n+1})$$

$$s_k = A_k(0)\ ,\ s'_k = A_k(1) \quad ,\quad k = n-1,n$$

$$x \to x \searrow 0$$



$$H_n = \llcorner\, H_{n+1}\, \llcorner^t$$

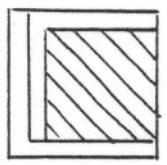
$$H_{n-1} = \urcorner\, H_n\, \llcorner^t$$

$$!\hat{H}_{n-1}\ Q_{n-1} = [\tilde{B}^t_{n-1}\ 0^t\ A^t_{n-1}]^t$$

$$\hat{H}_n\ Q_n = [\tilde{B}^t_n\ 0^t\ A^t_n]^t$$

$$?\hat{H}_{n+1}\ Q_{n+1} = [\tilde{B}^t_{n+1}\ 0^t\ A^t_{n+1}]^t$$

d



$$T_n = \llcorner\, T_{n+1}\, \urcorner^t$$

$$T_{n-1} = \urcorner\, T_n\, \urcorner^t$$

$$!\hat{T}_{n-1}\ Q_{n-1} = [\tilde{B}^t_{n-1}\ 0^t\ A^t_{n-1}]^t$$

$$\hat{T}_n\ Q_n = [\tilde{B}^t_n\ 0^t\ A^t_n]^t$$

$$?\hat{T}_{n+1}\ Q_{n+1} = [\tilde{B}^t_{n+1}\ 0^t\ A^t_{n+1}]^t$$

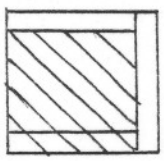
c

a

$$!\hat{S}_n\ P_n = [\tilde{C}_n^t\ 0^t\ D_n^t]^t$$

$$\hat{T}_n\ Q_n = [\tilde{A}_n^t\ 0^t\ B_n^t]^t$$

$$?\hat{T}_{n+1}\ Q_{n+1} = [\tilde{A}_{n+1}^t\ 0^t\ B_{n+1}^t]^t$$



$$S_n = \downarrow T_n$$

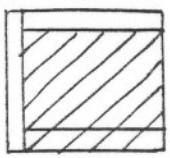
$$T_n = \lrcorner\, T_{n+1}\, \lrcorner^t$$

b

$$!\hat{K}_n\ P_n = [\tilde{C}_n^t\ 0^t\ D_n^t]^t$$

$$\hat{H}_n\ Q_n = [\tilde{A}_n^t\ 0^t\ B_n^t]^t$$

$$?\hat{H}_{n+1}\ Q_{n+1} = [\tilde{A}_{n+1}^t\ 0^t\ B_{n+1}^t]^t$$



$$K_n = \downarrow H_n$$

$$H_n = \lrcorner\, H_{n+1}\, \urcorner^t$$

$$\begin{array}{ll} x \to 0 & x \quad 0 \\ \uparrow & \downarrow \nearrow \\ x & x \end{array}$$

$$B_{n+1} = \lrcorner\, B_n + D_n\, a_{n+1}$$

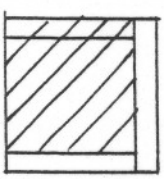
$$A_{n+1} = \urcorner\, (A_n + \lrcorner\, C_n\, a_{n+1})$$

$$a_{n+1} = -w_n^{-1}\, s_n$$

$$w_n = C_n(0)\ ,\ s_n = A_n(0)$$

$$\begin{array}{ll} x & x \\ \downarrow & \uparrow \searrow \\ x \to 0 & x \quad 0 \end{array}$$

d



$$K_n = \uparrow H_n$$

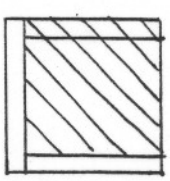
$$H_n = \urcorner\, H_{n+1}\, \lrcorner^t$$

$$!\hat{K}_n\ P_n = [\tilde{D}_n^t\ 0^t\ C_n^t]^t$$

$$\hat{H}_n\ Q_n = [\tilde{B}_n^t\ 0^t\ A_n^t]^t$$

$$?\hat{H}_{n+1}\ Q_{n+1} = [\tilde{B}_{n+1}^t\ 0^t\ A_{n+1}^t]^t$$

c



$$S_n = \uparrow T_n$$

$$T_n = \urcorner\, H_{n+1}\, \urcorner^t$$

$$!\hat{S}_n\ P_n = [\tilde{D}_n^t\ 0^t\ C_n^t]^t$$

$$\hat{T}_n\ Q_n = [\tilde{B}_n^t\ 0^t\ A_n^t]^t$$

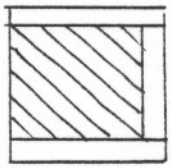
$$?\hat{T}_{n+1}\ Q_{n+1} = [\tilde{B}_{n+1}^t\ 0^t\ A_{n+1}^t]^t$$

a

$$!\hat{S}_n\ P_n = [\tilde{C}_n^t\ 0_n^t\ D_n^t]^t$$

$$\hat{S}_{n+1}\ P_{n+1} = [\tilde{C}_{n+1}^t\ 0^t\ D_{n+1}^t]^t$$

$$?\hat{T}_{n+1}\ Q_{n+1} = [\tilde{A}_{n+1}^t\ 0^t\ B_{n+1}^t]^t$$



$$S_n = \lrcorner\, S_{n+1}\, \lrcorner^t$$

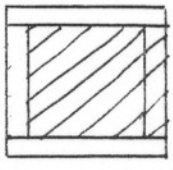
$$S_{n+1} = \downarrow T_{n+1}$$

b

$$!\hat{K}_n\ P_n = [\tilde{C}_n^t\ 0^t\ D_n^t]^t$$

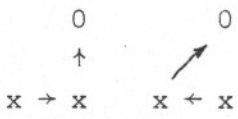
$$\hat{K}_{n+1}\ P_{n+1} = [\tilde{C}_{n+1}^t\ 0^t\ D_{n+1}^t]^t$$

$$?\hat{H}_{n+1}\ Q_{n+1} = [\tilde{A}_{n+1}^t\ 0^t\ B_{n+1}^t]^t$$



$$K_n = \lrcorner\, K_{n+1}\, \urcorner^t$$

$$K_{n+1} = \downarrow H_{n+1}$$

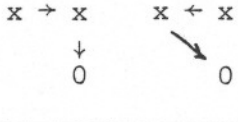


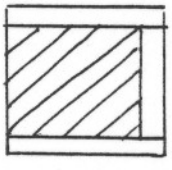
$$B_{n+1} = \begin{bmatrix} 0 \\ D_{n+1} \end{bmatrix} + D_n\ a_{n+1}$$

$$A_{n+1} = \urcorner\, (C_{n+1} + \lrcorner\, C_n\ a_{n+1})$$

$$a_{n+1} = -w_n^{-1}\ w_{n+1}$$

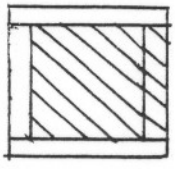
$$w_k = C_k(0) \quad , \quad k = n, n+1$$





$$K_n = \urcorner\, K_{n+1}\, \lrcorner^t$$

$$K_{n+1} = \uparrow H_{n+1}$$



$$S_n = \urcorner\, S_{n+1}\, \urcorner^t$$

$$S_{n+1} = \uparrow T_{n+1}$$

$$!\hat{K}_n\ P_n = [\tilde{D}_n^t\ 0^t\ C_n^t]^t$$

$$\hat{K}_{n+1}\ P_{n+1} = [\tilde{D}_{n+1}^t\ 0^t\ C_{n+1}^t]^t$$

$$?\hat{H}_{n+1}\ Q_{n+1} = [\tilde{B}_{n+1}^t\ 0^t\ A_{n+1}^t]^t$$

d

$$!\hat{S}_n\ P_n = [\tilde{D}_n^t\ 0^t\ C_n^t]^t$$

$$\hat{S}_{n+1}\ P_{n+1} = [\tilde{D}_{n+1}^t\ 0^t\ C_{n+1}^t]^t$$

$$?\hat{T}_{n+1}\ Q_{n+1} = [\tilde{B}_{n+1}^t\ 0^t\ A_{n+1}^t]^t$$

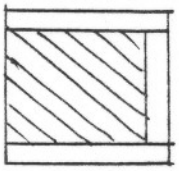
c

a

$$!\hat{S}_{n+1}\ P_{n+1} = [\tilde{C}^t_{n+1}\ 0^t\ D^t_{n+1}]^t$$

$$\hat{T}_n\ Q_n = [\tilde{A}^t_n\ 0^t\ B^t_n]^t$$

$$?\hat{T}_{n+1}\ Q_{n+1} = [\tilde{A}^t_{n+1}\ 0^t\ B^t_{n+1}]^t$$



$$S_{n+1} = \uparrow T_{n+1}$$

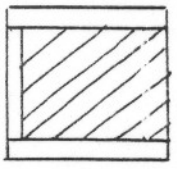
$$T_n = \lrcorner\, T_{n+1}\, \lrcorner^t$$

b

$$!\hat{K}_{n+1}\ P_{n+1} = [\tilde{C}^t_{n+1}\ 0^t\ D^t_{n+1}]^t$$

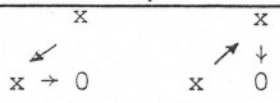
$$\hat{H}_n\ Q_n = [\tilde{A}^t_n\ 0^t\ B^t_n]^t$$

$$?\hat{H}_{n+1}\ Q_{n+1} = [\tilde{A}^t_{n+1}\ 0^t\ B^t_{n+1}]^t$$



$$K_{n+1} = \uparrow H_{n+1}$$

$$H_n = \lrcorner\, H_{n+1}\, \urcorner^t$$



$$B_{n+1} = \urcorner\,(D_{n+1} + B_n\, a_{n+1})$$

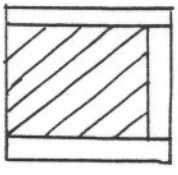
$$A_{n+1} = \begin{bmatrix} 0 \\ C_{n+1} \end{bmatrix} + \lrcorner\, A_n\, a_{n+1}$$

$$a_{n+1} = -r_n^{-1}\, v_{n+1}$$

$$r_n = B_n(0)\ ,\ v_{n+1} = D_{n+1}(0)$$



d



$$K_{n+1} = \downarrow H_{n+1}$$

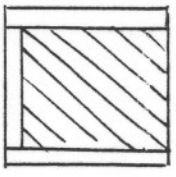
$$H_n = \urcorner\, H_{n+1}\, \lrcorner^t$$

$$!\hat{K}_{n+1}\ P_{n+1} = [\tilde{D}^t_{n+1}\ 0^t\ C^t_{n+1}]^t$$

$$\hat{H}_n\ Q_n = [\tilde{B}^t_n\ 0^t\ A^t_n]^t$$

$$?\hat{H}_{n+1}\ Q_{n+1} = [\tilde{B}^t_{n+1}\ 0^t\ A^t_{n+1}]^t$$

c



$$S_{n+1} = \downarrow T_{n+1}$$

$$T_n = \urcorner\, T_{n+1}\, \urcorner^t$$

$$!\hat{S}_{n+1}\ P_{n+1} = [\tilde{D}^t_{n+1}\ 0^t\ C^t_{n+1}]^t$$

$$\hat{T}_n\ Q_n = [\tilde{B}^t_n\ 0^t\ A^t_n]^t$$

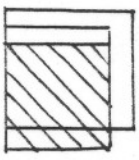
$$?\hat{T}_{n+1}\ Q_{n+1} = [\tilde{B}^t_{n+1}\ 0^t\ A^t_{n+1}]^t$$

a

$$!\hat{T}_n\ Q_n = [\tilde{A}_n^t\ 0^t\ B_n^t]^t$$

$$\hat{S}_n\ P_n = [\tilde{C}_n^t\ 0^t\ D_n^t]^t$$

$$?\hat{T}_{n+1}\ Q_{n+1} = [\tilde{A}_{n+1}^t\ 0^t\ B_{n+1}^t]^t$$



$$T_n = \urcorner\, T_{n+1}\, \llcorner^t$$

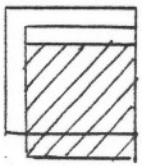
$$S_n = \downarrow T_n$$

b

$$!\hat{H}_n\ Q_n = [\tilde{A}_n^t\ 0^t\ B_n^t]^t$$

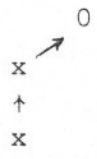
$$\hat{K}_n\ P_n = [\tilde{C}_n^t\ 0^t\ D_n^t]^t$$

$$?\hat{H}_{n+1}\ Q_{n+1} = [\tilde{A}_{n+1}^t\ 0^t\ B_{n+1}^t]^t$$



$$H_n = \urcorner\, H_{n+1}\, \urcorner^t$$

$$K_n = \downarrow H_n$$



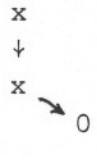
$$B_{n+1} = \begin{bmatrix} 0 \\ \llcorner\, B_n \end{bmatrix} + \begin{bmatrix} 0 \\ D_n \end{bmatrix} a_{n+1} + B_n\, b_{n+1}$$

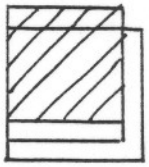
$$A_{n+1} = \urcorner\,(\,\urcorner\,(B_n + \llcorner\, C_n\, a_{n+1}) + \llcorner\, A_n\, b_{n+1})$$

$$a_{n+1} = -w_n^{-1}\, s_n$$

$$b_{n+1} = -s_n^{-1}(s'_n + w'_n\, a_{n+1})$$

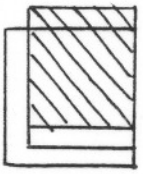
$$w_n = C_n(0)\ ,\ w'_n = C_n(1)\ ,\ s_n = A_n(0)\ ,\ s'_n = A_n(1)$$





$$H_n = \llcorner\, H_{n+1}\, \llcorner^t$$

$$K_n = \uparrow H_n$$



$$T_n = \llcorner\, T_{n+1}\, \urcorner^t$$

$$S_n = \uparrow T_n$$

d

$$!\hat{H}_n\ Q_n = [\tilde{B}_n^t\ 0^t\ A_n^t]^t$$

$$\hat{K}_n\ P_n = [\tilde{D}_n^t\ 0^t\ C_n^t]^t$$

$$?\hat{H}_{n+1}\ Q_{n+1} = [\tilde{B}_{n+1}^t\ 0^t\ A_{n+1}^t]^t$$

c

$$!\hat{T}_n\ Q_n = [\tilde{B}_n^t\ 0^t\ A_n^t]^t$$

$$\hat{S}_n\ P_n = [\tilde{D}_n^t\ 0^t\ C_n^t]^t$$

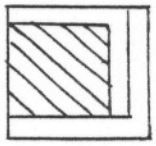
$$?\hat{T}_{n+1}\ Q_{n+1} = [\tilde{B}_{n+1}^t\ 0^t\ A_{n+1}^t]^t$$

a

$$!\hat{T}_{n-1}\ Q_{n-1} = [\tilde{A}{}^{t}_{n-1}\ 0^{t}\ B^{t}_{n-1}]^{t}$$

$$\hat{T}_{n}\ Q_{n} = [\tilde{A}{}^{t}_{n}\ 0^{t}\ B^{t}_{n}]^{t}$$

$$?\hat{T}_{n+1}\ Q_{n+1} = [\tilde{A}{}^{t}_{n+1}\ 0^{t}\ B^{t}_{n+1}]^{t}$$



$$T_n = \lrcorner\, T_{n+1}\, \lrcorner^{t}$$

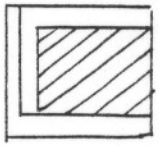
$$T_{n-1} = \urcorner\, T_n\, \lrcorner^{t}$$

b

$$!\hat{H}_{n-1}\ Q_{n-1} = [\tilde{A}{}^{t}_{n-1}\ 0^{t}\ B^{t}_{n-1}]^{t}$$

$$\hat{H}_{n}\ Q_{n} = [\tilde{A}{}^{t}_{n}\ 0^{t}\ B^{t}_{n}]^{t}$$

$$?\hat{H}_{n+1}\ Q_{n+1} = [\tilde{A}{}^{t}_{n+1}\ 0^{t}\ B^{t}_{n+1}]^{t}$$



$$H_n = \lrcorner\, H_{n+1}\, \urcorner^{t}$$

$$H_{n-1} = \urcorner\, H_n\, \urcorner^{t}$$

$x \to 0$: x

$$B_{n+1} = \lrcorner\, B_n + \urcorner\, (B_{n-1}\ a_{n+1} + B_n\ b_{n+1})$$

$$A_{n+1} = \urcorner\, (A_n + \lrcorner\lrcorner\, A_{n-1}\ a_{n+1}) + \lrcorner\, A_n\ b_{n+1}$$

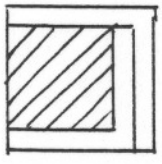
$$a_{n+1} = -s^{-1}_{n-1}\ s_n$$

$$b_{n+1} = -r^{-1}_{n}\ r_{n-1}\ a_{n+1}$$

$$r_k = B_k(0)\ ,\ s_k = A_k(0) \quad ,\quad k = n-1, n$$

x : $x \to 0$

d



$$H_n = \urcorner\, H_{n+1}\, \lrcorner^{t}$$

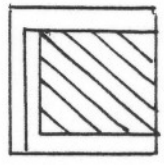
$$H_{n-1} = \lrcorner\, H_n\, \lrcorner^{t}$$

$$!\hat{H}_{n-1}\ Q_{n-1} = [\tilde{B}{}^{t}_{n-1}\ 0^{t}\ A^{t}_{n-1}]^{t}$$

$$\hat{H}_{n}\ Q_{n} = [\tilde{B}{}^{t}_{n}\ 0^{t}\ A^{t}_{n}]^{t}$$

$$?\hat{H}_{n+1}\ Q_{n+1} = [\tilde{B}{}^{t}_{n+1}\ 0^{t}\ A^{t}_{n+1}]^{t}$$

c



$$T_n = \urcorner\, T_{n+1}\, \urcorner^{t}$$

$$T_{n-1} = \lrcorner\, T_n\, \urcorner^{t}$$

$$!\hat{T}_{n-1}\ Q_{n-1} = [\tilde{B}{}^{t}_{n-1}\ 0^{t}\ A^{t}_{n-1}]^{t}$$

$$\hat{T}_{n}\ Q_{n} = [\tilde{B}{}^{t}_{n}\ 0^{t}\ A^{t}_{n}]^{t}$$

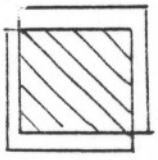
$$?\hat{T}_{n+1}\ Q_{n+1} = [\tilde{B}{}^{t}_{n+1}\ 0^{t}\ A^{t}_{n+1}]^{t}$$

a

$$!\hat{S}_n\ P_n = [\tilde{C}_n^t\ 0^t\ D_n^t]^t$$

$$\hat{S}_{n+1}\ P_{n+1} = [\tilde{C}_{n+1}^t\ 0^t\ D_{n+1}^t]^t$$

$$?\hat{T}_{n+1}\ Q_{n+1} = [\tilde{A}_{n+1}^t\ 0^t\ B_{n+1}^t]^t$$



$$S_n = \urcorner\, T_{n+1}\, \llcorner^t$$

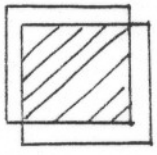
$$S_{n+1} = \downarrow\downarrow T_{n+1}$$

b

$$!\hat{K}_n\ P_n = [\tilde{C}_n^t\ 0^t\ D_n^t]^t$$

$$\hat{K}_{n+1}\ P_{n+1} = [\tilde{C}_{n+1}^t\ 0^t\ D_{n+1}^t]^t$$

$$?\hat{H}_{n+1}\ Q_{n+1} = [\tilde{A}_{n+1}^t\ 0^t\ B_{n+1}^t]^t$$



$$K_n = \urcorner\, H_{n+1}\, \urcorner^t$$

$$K_{n+1} = \downarrow\downarrow H_{n+1}$$



$$B_{n+1} = \left(\begin{bmatrix} 0 \\ 0 \\ D_{n+1} \end{bmatrix} + \begin{bmatrix} 0 \\ \\ \llcorner\, D_n \end{bmatrix} a_{n+1} + D_n\, b_{n+1} \right) \Lambda_{n+1}$$

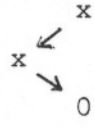
$$A_{n+1} = \urcorner\, (\, \urcorner\, (C_{n+1} + C_n\, a_{n+1}) + \llcorner\, C_n\, b_{n+1}) \Lambda_{n+1}$$

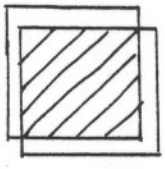
$$a_{n+1} = -w_n^{-1}\, w_{n+1}$$

$$b_{n+1} = -w_n^{-1} (w'_{n+1}\ w'_n\ a_{n+1})$$

$$\Lambda_{n+1} = (I + a_{n+1})^{-1}$$

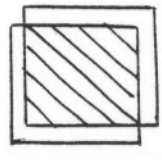
$$w_k = C_k(0)\ ,\ w'_k = C_k(1) \quad ,\quad k = n, n+1$$





$$K_n = \llcorner\, H_{n+1}\, \llcorner^t$$

$$K_{n+1} = \uparrow\uparrow H_{n+1}$$



$$S_n = \llcorner\, T_{n+1}\, \urcorner^t$$

$$S_{n+1} = \uparrow\uparrow T_{n+1}$$

d

$$!\hat{K}_n\ P_n = [\tilde{D}_n^t\ 0^t\ C_n^t]^t$$

$$\hat{K}_{n+1}\ P_{n+1} = [\tilde{D}_{n+1}^t\ 0^t\ C_{n+1}^t]^t$$

$$?\hat{H}_{n+1}\ Q_{n+1} = [\tilde{B}_{n+1}^t\ 0^t\ A_{n+1}^t]^t$$

c

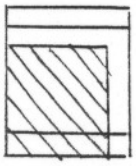
$$!\hat{S}_n\ P_n = [\tilde{D}_n^t\ 0^t\ C_n^t]^t$$

$$\hat{S}_{n+1}\ P_{n+1} = [\tilde{D}_{n+1}^t\ 0^t\ C_{n+1}^t]^t$$

$$?\hat{T}_{n+1}\ Q_{n+1} = [\tilde{B}_{n+1}^t\ 0^t\ A_{n+1}^t]^t$$

a

$$!\hat{S}_n\, P_n = [\tilde{C}_n^t\ 0^t\ D_n^t]^t$$
$$\hat{S}_{n+1}\, P_{n+1} = [\tilde{C}_{n+1}^t\ 0^t\ D_{n+1}^t]^t$$
$$?\hat{T}_{n+1}\, Q_{n+1} = [\tilde{A}_{n+1}^t\ 0^t\ B_{n+1}^t]^t$$

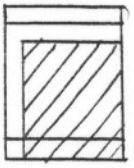


$$S_{n+1} = \downarrow T_{n+1}$$
$$S_n = \urcorner\, S_{n+1}\, \lrcorner^t$$

$$b_{n+1} = -(p_0^{n+1})^{-1}\ (p_0^n)\ a_{n+1}$$
$$d_k = [t_N \ldots t_{N-k+1}]\ \urcorner\, P_k\ ,\ k = n, n+1$$

b

$$!\hat{K}_n\, P_n = [\tilde{C}_n^t\ 0^t\ D_n^t]^t$$
$$\hat{K}_{n+1}\, P_{n+1} = [\tilde{C}_{n+1}^t\ 0^t\ D_{n+1}^t]^t$$
$$?\hat{H}_{n+1}\, Q_{n+1} = [\tilde{A}_{n+1}^t\ 0^t\ B_{n+1}^t]^t$$



$$K_{n+1} = \downarrow H_{n+1}$$
$$K_n = \urcorner\, K_{n+1}\, \urcorner^t$$

$$b_{n+1} = -(p_{n+1}^{n+1})^{-1}\ (p_n^n)\ a_{n+1}$$
$$d_k = [h_{N-k+1} \ldots h_N]\ \lrcorner\, P_k\ ,\ k = n, n+1$$

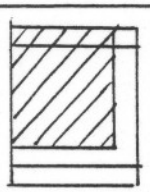
$$\begin{array}{c} 0 \\ \uparrow \\ x \\ x \nearrow \end{array}$$

$$B_{n+1} = \begin{bmatrix} 0 \\ D_{n+1} \end{bmatrix} + \begin{bmatrix} D_n \\ d_n \end{bmatrix} a_{n+1} + \begin{bmatrix} D_{n+1} \\ d_{n+1} \end{bmatrix} b_{n+1}$$
$$A_{n+1} = \urcorner\, (C_{n+1} + \lrcorner\, \lrcorner\, C_n\, a_{n+1}) + \lrcorner\, C_{n+1}\, b_{n+1}$$
$$a_{n+1} = -w_n^{-1}\ w_{n+1}$$
$$w_k = C_k(0)\quad ,\quad k = n, n+1$$

$$b_{n+1} = -(p_0^{n+1})^{-1}\ (p_0^n)\ a_{n+1}$$
$$d_k = [h_{-M} \ldots h_{-M+k-1}]\ \urcorner\, P_k\ ,\ k = n, n+1$$

$$\begin{array}{c} x \searrow \\ x \\ \downarrow \\ 0 \end{array}$$

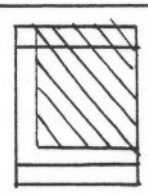
$$b_{n+1} = -(p_{n+1}^{n+1})^{-1}\ (p_n^n)\ a_{n+1}$$
$$d_k = [t_{-M+k-1} \ldots t_{-M}]\ \lrcorner\, P_k\ ,\ k = n, n+1$$



$$K_{n+1} = \uparrow H_{n+1}$$
$$K_n = \lrcorner\, K_{n+1}\, \lrcorner^t$$

$$!\hat{K}_n\, P_n = [\tilde{D}_n^t\ 0^t\ C_n^t]^t$$
$$\hat{K}_{n+1}\, P_{n+1} = [\tilde{D}_{n+1}^t\ 0^t\ C_{n+1}^t]^t$$
$$?\hat{H}_{n+1}\, Q_{n+1} = [\tilde{B}_{n+1}^t\ 0^t\ A_{n+1}^t]^t$$

d



$$S_{n+1} = \uparrow T_{n+1}$$
$$S_n = \lrcorner\, S_{n+1}\, \urcorner^t$$

$$!\hat{S}_n\, P_n = [\tilde{D}_n^t\ 0^t\ C_n^t]^t$$
$$\hat{S}_{n+1}\, P_{n+1} = [\tilde{D}_{n+1}^t\ 0^t\ C_{n+1}^t]^t$$
$$?\hat{T}_{n+1}\, Q_{n+1} = [\tilde{B}_{n+1}^t\ 0^t\ A_{n+1}^t]^t$$

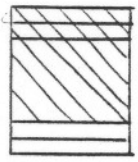
c

a

$$!\hat{\underline{S}}_n\ P_n = [\tilde{\underline{C}}_n^t\ 0^t\ \underline{D}_n^t]^t$$

$$\hat{S}_n\ P_n = [\tilde{C}_n^t\ 0^t\ D_n^t]^t$$

$$?\hat{T}_n\ Q_n = [\tilde{A}_n^t\ 0^t\ B_n^t]^t$$



$$S_n = \uparrow T_n$$

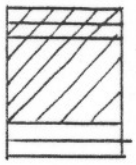
$$\underline{S}_n = \uparrow S_n$$

b

$$!\hat{\underline{K}}_n\ P_n = [\tilde{\underline{C}}_n^t\ 0^t\ \underline{D}_n^t]^t$$

$$\hat{K}_n\ P_n = [\tilde{C}_n^t\ 0^t\ D_n^t]^t$$

$$?\hat{H}_n\ Q_n = [\tilde{A}_n^t\ 0^t\ B_n^t]^t$$



$$K_n = \uparrow H_n$$

$$\underline{K}_n = \uparrow K_n$$

$$[\underline{c}_{n+1} c_{n+1}] = [t_{-M+n-1} \ldots t_{-M}][\lrcorner \underline{P}_n \mid \lrcorner P_n]$$

$$\Lambda_{n+1} = (I + (p_{n-1}^n - p_{n-1}^n)\, a_{n+1})^{-1}$$

x
↓
x
↓
0

$$[\underline{c}_{n+1} c_{n+1}] = [h_{-M} \ldots h_{-M+n-1}][\urcorner \underline{P}_n \mid \urcorner P_n]$$

$$\Lambda_{n+1} = (I + (p_1^n - p_1^n)\, a_{n+1})^{-1}$$

$$B_{n+1} = (\urcorner D_n + (\urcorner \lrcorner \underline{D}_n - \lrcorner D_n)\, a_{n+1})\, \Lambda_{n+1}$$

$$A_{n+1} = \left(\begin{bmatrix} 0 \\ C_n \end{bmatrix} + \left(\begin{bmatrix} 0 \\ \underline{C}_n \\ \underline{c}_{n+1} \end{bmatrix} - \begin{bmatrix} C_n \\ c_{n+1} \end{bmatrix}\right) a_{n+1}\right) \Lambda_{n+1}$$

$$a_{n+1} = -\underline{v}_n\, v_n$$

$$\underline{v}_n = \underline{D}_n(0)\ ,\ v_n = D_n(0)$$

$$[\underline{c}_{n+1} c_{n+1}] = [h_{N-n+1} \ldots h_N][\lrcorner \underline{P}_n \mid \lrcorner P_n]$$

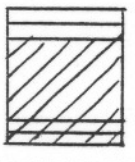
$$\Lambda_{n+1} = (I + (p_{n-1}^n - p_{n-1}^n)\, a_{n+1})^{-1}$$

0
↑
x
↑
x

$$[\underline{c}_{n+1} c_{n+1}] = [t_N \ldots t_{N-n+1}][\urcorner \underline{P}_n \mid \urcorner P_n]$$

$$\Lambda_{n+1} = (I + (p_1^n - p_1^n)\, a_{n+1})^{-1}$$

d



$$K_n = \downarrow H_n$$

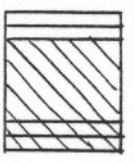
$$\underline{K}_n = \downarrow K_n$$

$$!\hat{\underline{K}}_n\ P_n = [\tilde{\underline{D}}_n^t\ 0^t\ \underline{C}_n^t]^t$$

$$\hat{K}_n\ P_n = [\tilde{D}_n^t\ 0^t\ C_n^t]^t$$

$$?\hat{H}_n\ Q_n = [\tilde{B}_n^t\ 0^t\ A_n^t]^t$$

c



$$S_n = \downarrow T_n$$

$$\underline{S}_n = \downarrow S_n$$

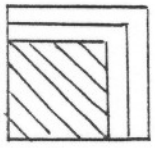
$$!\hat{\underline{S}}_n\ P_n = [\tilde{\underline{D}}_n^t\ 0^t\ \underline{C}_n^t]^t$$

$$\hat{S}_n\ P_n = [\tilde{D}_n^t\ 0^t\ C_n^t]^t$$

$$?\hat{T}_n\ Q_n = [\tilde{B}_n^t\ 0^t\ A_n^t]^t$$

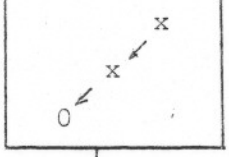
a

$$!\hat{T}_{n+1}\ Q_{n+1} = [\tilde{A}^t_{n+1}\ 0^t\ B^t_{n+1}]^t$$
$$\hat{T}_n\ Q_n = [\tilde{A}^t_n\ 0^t\ B^t_n]^t$$
$$?\hat{T}_{n-1}\ Q_{n-1} = [\tilde{A}^t_{n-1}\ 0^t\ B^t_{n-1}]^t$$



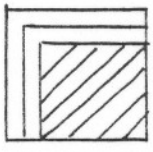
$$T_n = \urcorner\, T_{n+1}\, \llcorner^t$$
$$T_{n-1} = \urcorner\, T_n\, \llcorner^t$$

$$a_{n+1} = -(q^{n+1}_n - q^n_{n-1})^{-1}$$
$$\Lambda_{n+1} = (q^{n+1}_{n-1} - q^n_{n-2} + q^n_{n-1}\ a_{n+1})^{-1}$$
$$\begin{bmatrix} b_k \\ b'_k \end{bmatrix} = \begin{bmatrix} t_{-M+k-2} \cdots t_{-M} \\ t_{-M+k-1} \cdots t_{-M+1} \end{bmatrix} \llcorner\llcorner\, Q_k, k=n, n+1$$
$$c_n = [\, t_{-M+n-1} \cdots t_{-M}\,] \llcorner\, Q_n$$



b

$$!\hat{H}_{n+1}\ Q_{n+1} = [\tilde{A}^t_{n+1}\ 0^t\ B^t_{n+1}]^t$$
$$\hat{H}_n\ Q_n = [\tilde{A}^t_n\ 0^t\ B^t_n]^t$$
$$?\hat{H}_{n-1}\ Q_{n-1} = [\tilde{A}^t_{n-1}\ 0^t\ B^t_{n-1}]^t$$

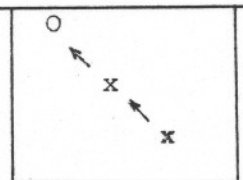


$$H_n = \urcorner\, H_{n+1}\, \urcorner^t$$
$$H_{n-1} = \urcorner\, H_n\, \urcorner^t$$

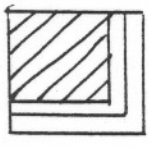
$$a_{n+1} = -(q^{n+1}_1 - q^n_1)^{-1}$$
$$\Lambda_{n+1} = (q^{n+1}_2 - q^n_2 + q^n_1\ a_{n+1})^{-1}$$
$$\begin{bmatrix} b_k \\ b'_k \end{bmatrix} = \begin{bmatrix} h_{-M} \cdots h_{-M+k-2} \\ h_{-M+1} \cdots h_{-M+k-1} \end{bmatrix} \urcorner\,\urcorner\, Q_k, k=n, n+1$$
$$c_n = [\, h_{-M} \cdots h_{-M+n-1}\,] \urcorner\, Q_n$$

$$B_{n-1} = (B_{n+1} - \begin{bmatrix} 0 \\ \llcorner\, B_n \end{bmatrix} + B_n\ a_{n+1})\Lambda_{n+1}$$
$$A_{n-1} = (\begin{bmatrix} 0 \\ 0 \\ A_{n+1} \\ b_{n+1} \\ b'_{n+1} \end{bmatrix} - \begin{bmatrix} A_n \\ b_n \\ b'_n \end{bmatrix} + \begin{bmatrix} 0 \\ A_n \\ c_n \end{bmatrix} a_{n+1})\Lambda_{n+1}$$

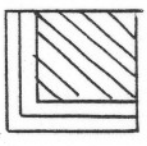
$$a_{n+1} = -(q^{n+1}_n - q^n_{n-1})^{-1}$$
$$\Lambda_{n+1} = (q^{n+1}_{n-1} - q^n_{n-2} + q^n_{n-1}\ a_{n+1})^{-1}$$
$$\begin{bmatrix} b_k \\ b'_k \end{bmatrix} = \begin{bmatrix} h_{N-k+2} \cdots h_N \\ h_{N-k+1} \cdots h_{N-1} \end{bmatrix} \llcorner\llcorner\, Q_k, k=n, n+1$$
$$c_n = [\, h_{N-n+1} \cdots h_N\,] \llcorner\, Q_n$$



$$a_{n+1} = -(q^{n+1}_1 - q^n_1)^{-1}$$
$$\Lambda_{n+1} = (q^{n+1}_2 - q^n_2 + q^n_1\ a_{n+1})^{-1}$$
$$\begin{bmatrix} b_k \\ b'_k \end{bmatrix} = \begin{bmatrix} t_N \cdots t_{N-k+2} \\ t_{N-1} \cdots t_{N-k+1} \end{bmatrix} \urcorner\,\urcorner\, Q_k, k=n, n+1$$
$$c_k = [\, t_N \cdots t_{N-n+1}\,] \urcorner\, Q_n$$



$$H_n = \llcorner\, H_{n+1}\, \llcorner^t$$
$$H_{n-1} = \llcorner\, H_n\, \llcorner^t$$



$$T_n = \llcorner\, T_{n+1}\, \urcorner^t$$
$$T_{n-1} = \llcorner\, T_n\, \urcorner^t$$

$$!\hat{H}_{n+1}\ Q_{n+1} = [\tilde{B}^t_{n+1}\ 0^t\ A^t_{n+1}]^t$$
$$\hat{H}_n\ Q_n = [\tilde{B}^t_n\ 0^t\ A^t_n]^t$$
$$?\hat{H}_{n-1}\ Q_{n-1} = [\tilde{B}^t_{n-1}\ 0^t\ A^t_{n-1}]^t$$

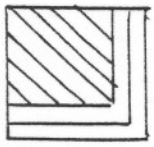
d

$$!\hat{T}_{n+1}\ Q_{n+1} = [\tilde{B}^t_{n+1}\ 0^t\ A^t_{n+1}]^t$$
$$\hat{T}_n\ Q_n = [\tilde{B}^t_n\ 0^t\ A^t_n]^t$$
$$?\hat{T}_{n-1}\ Q_{n-1} = [\tilde{B}^t_{n-1}\ 0^t\ A^t_{n-1}]^t$$

c

a

$$!\hat{T}_n\ Q_n = [\tilde{A}_n^t\ 0^t\ B_n^t]^t$$
$$\hat{T}_{n+1}\ Q_{n+1} = [\tilde{A}_{n+1}^t\ 0^t\ B_{n+1}^t]^t$$
$$?\hat{T}_{n-1}\ Q_{n-1} = [\tilde{A}_{n-1}^t\ 0^t\ B_{n-1}^t]^t$$



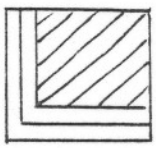
$$T_n = \lrcorner\, T_{n+1}\, \lrcorner^t$$
$$T_{n-1} = \lrcorner\, T_n\, \lrcorner^t$$

$$a_{n+1} = -(q_0^n)^{-1}\ q_0^{n+1}$$
$$\Lambda_{n+1} = (q_n^{n+1} - q_{n-1}^n + a_{n+1})^{-1}$$
$$[c_{n+1}\ d_{n+1}] = [t_N \cdots t_{N-n+1}][\urcorner\lrcorner Q_{n+1} \mid \urcorner Q_n]$$
$$[c'_{n+1}\ d'_{n+1}] = [t_{-M+n-1} \cdots t_{-M}][\urcorner\lrcorner Q_{n+1} \mid \lrcorner Q_n]$$

$0 \leftarrow x \leftarrow x$

b

$$!\hat{H}_n\ Q_n = [\tilde{A}_n^t\ 0^t\ B_n^t]^t$$
$$\hat{H}_{n+1}\ Q_{n+1} = [\tilde{A}_{n+1}^t\ 0^t\ B_{n+1}^t]^t$$
$$?\hat{H}_{n-1}\ Q_{n-1} = [\tilde{A}_{n-1}^t\ 0^t\ B_{n-1}^t]^t$$



$$H_n = \lrcorner\, H_{n+1}\, \urcorner^t$$
$$H_{n-1} = \lrcorner\, H_n\, \urcorner^t$$

$$a_{n+1} = -(q_n^n)^{-1}\ q_{n+1}^{n+1}$$
$$\Lambda_{n+1} = (q_1^{n+1} - q_1^n + a_{n+1})^{-1}$$
$$[c_{n+1}\ d_{n+1}] = [h_{N-n+1} \cdots h_N][\lrcorner\urcorner Q_{n+1} \mid \lrcorner Q_n]$$
$$[c'_{n+1}\ d'_{n+1}] = [h_{-M} \cdots h_{-M+n-1}][\lrcorner\urcorner Q_{n+1} \mid \urcorner Q_n]$$

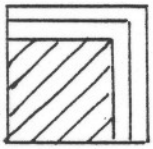
$$B_{n-1} = \left(\begin{bmatrix} 0 \\ B_{n+1} \\ c_{n+1} \end{bmatrix} - \begin{bmatrix} 0 \\ \\ B_n \end{bmatrix} + \begin{bmatrix} B_n \\ \\ d_{n+1} \end{bmatrix} a_{n+1} \right) \Lambda_{n+1}$$

$$A_{n+1} = \left(\begin{bmatrix} 0 \\ A_{n+1} \\ c'_{n+1} \end{bmatrix} - \begin{bmatrix} A_n \\ \\ d'_{n+1} \end{bmatrix} + \begin{bmatrix} 0 \\ \\ A_n \end{bmatrix} a_{n+1} \right) \Lambda_{n+1}$$

$$a_{n+1} = -(q_0^n)^{-1}\ q_0^{n+1}$$
$$\Lambda_{n+1} = (q_n^{n+1} - q_{n-1}^n + a_{n+1})^{-1}$$
$$[c_{n+1}\ d_{n+1}] = [h_{-M} \cdots h_{-M+n-1}][\urcorner\lrcorner Q_{n+1} \mid \urcorner Q_n]$$
$$[c'_{n+1}\ d'_{n+1}] = [h_{N-n+1} \cdots h_N][\urcorner\lrcorner Q_{n+1} \mid \lrcorner Q_n]$$

$0 \leftarrow x \leftarrow x$

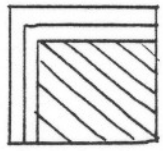
$$a_{n+1} = -(q_n^n)^{-1}\ q_{n+1}^{n+1}$$
$$\Lambda_{n+1} = (q_1^{n+1} - q_1^n + a_{n+1})^{-1}$$
$$[c_{n+1}\ d_{n+1}] = [t_{-M+n-1} \cdots t_{-M}][\lrcorner\urcorner Q_{n+1} \mid \lrcorner Q_n]$$
$$[c'_{n+1}\ d'_{n+1}] = [t_N \cdots t_{N-n+1}][\lrcorner\urcorner Q_{n+1} \mid \urcorner Q_n]$$



$$H_n = \urcorner\, H_{n+1}\, \lrcorner^t$$
$$H_{n-1} = \urcorner\, H_n\, \lrcorner^t$$

$$!\hat{H}_n\ Q_n = [\tilde{B}_n^t\ 0^t\ A_n^t]^t$$
$$\hat{H}_{n+1}\ Q_{n+1} = [\tilde{B}_{n+1}^t\ 0^t\ A_{n+1}^t]^t$$
$$?\hat{H}_{n-1}\ Q_{n-1} = [\tilde{B}_{n-1}^t\ 0^t\ A_{n-1}^t]^t$$

d



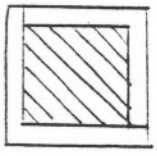
$$T_n = \urcorner\, T_{n+1}\, \urcorner^t$$
$$T_{n-1} = \urcorner\, T_n\, \urcorner^t$$

$$!\hat{T}_n\ Q_n = [\tilde{B}_n^t\ 0^t\ A_n^t]^t$$
$$\hat{T}_{n+1}\ Q_{n+1} = [\tilde{B}_{n+1}^t\ 0^t\ A_{n+1}^t]^t$$
$$?\hat{T}_{n-1}\ Q_{n-1} = [\tilde{B}_{n-1}^t\ 0^t\ A_{n-1}^t]^t$$

c

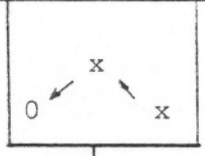
a

$$!\hat{T}_{n+1}\ Q_{n+1} = [\tilde{A}^t_{n+1}\ 0^t\ B^t_{n+1}]^t$$
$$\hat{T}_n\ Q_n = [\tilde{A}^t_n\ 0^t\ B^t_n]^t$$
$$?\hat{T}_{n-1}\ Q_{n-1} = [\tilde{A}^t_{n-1}\ 0^t\ B^t_{n-1}]^t$$



$$T_n = \lrcorner\, T_{n+1}\, \urcorner^t$$
$$T_{n-1} = \urcorner\, T_n\, \urcorner^t$$

$$a_{n+1} = q^n_{n-1} - q^{n+1}_n$$
$$\Lambda_{n+1} = (q^{n+1}_{n-1} - q^n_{n-2} + q^n_{n-1}\ a_{n+1})^{-1}$$

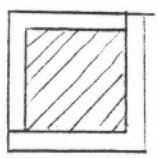


$$\begin{bmatrix} b_k \\ b'_k \end{bmatrix} = \begin{bmatrix} t_{-M+k-1} \cdots t_{-M+1} \\ t_{-M+k-2} \cdots t_{-M} \end{bmatrix} \lrcorner\lrcorner\, Q_k, k=n,n+1$$

$$d'_{n+1} = [\, t_{-M+n-1} \cdots t_{-M}\,] \lrcorner\, Q_n$$

b

$$!\hat{H}_{n+1}\ Q_{n+1} = [\tilde{A}^t_{n+1}\ 0^t\ B^t_{n+1}]^t$$
$$\hat{H}_n\ Q_n = [\tilde{A}^t_n\ 0^t\ B^t_n]^t$$
$$?\hat{H}_{n-1}\ Q_{n-1} = [\tilde{A}^t_{n-1}\ 0^t\ B^t_{n-1}]^t$$



$$H_n = \lrcorner\, H_{n+1}\, \lrcorner^t$$
$$H_{n-1} = \urcorner\, H_n\, \lrcorner^t$$

$$a_{n+1} = q^n_1 - q^{n+1}_1$$
$$\Lambda_{n+1} = (q^{n+1}_2 - q^n_2 + q^n_1\ a_{n+1})^{-1}$$

$$\begin{bmatrix} b_k \\ b'_k \end{bmatrix} = \begin{bmatrix} h_{-M+1} \cdots h_{-M+k-1} \\ h_{-M} \cdots h_{-M+k-2} \end{bmatrix} \urcorner\urcorner\, Q_k, k=n,n+1$$

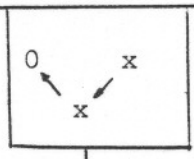
$$d'_{n+1} = [\, h_{-M} \cdots h_{-M+n-1}\,] \urcorner\, Q_n$$

$$B_{n-1} = \left(\begin{bmatrix} 0 \\ 0 \\ B_{n+1} \end{bmatrix} - \begin{bmatrix} 0 \\ \\ \lrcorner\, B_n \end{bmatrix} + B_n\ a_{n+1} \right) \Lambda_{n+1}$$

$$A_{n-1} = \left(\begin{bmatrix} A_{n+1} \\ b_{n+1} \\ b'_{n+1} \end{bmatrix} - \begin{bmatrix} A_n \\ b_n \\ b'_n \end{bmatrix} + \begin{bmatrix} 0 \\ A_n \\ d'_{n+1} \end{bmatrix} a_{n+1} \right) \Lambda_{n+1}$$

d

$$a_{n+1} = q^n_{n-1} - q^{n+1}_n$$
$$\Lambda_{n+1} = (q^{n+1}_{n-1} - q^n_{n-2} + q^n_{n-1}\ a_{n+1})^{-1}$$



$$\begin{bmatrix} b_k \\ b'_k \end{bmatrix} = \begin{bmatrix} h_{N-k+1} \cdots h_{N-1} \\ h_{N-k+2} \cdots h_N \end{bmatrix} \lrcorner\lrcorner\, Q_k\ ,\ k=n,n+1$$

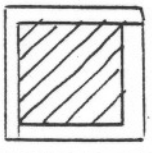
$$d'_{n+1} = [\, h_{N-n+1} \cdots h_N\,] \lrcorner\, Q_n$$

c

$$a_{n+1} = q^n_1 - q^{n+1}_1$$
$$\Lambda_{n+1} = (q^{n+1}_2 - q^n_2 + q^n_1\ a_{n+1})^{-1}$$

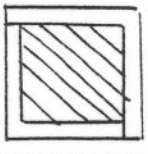
$$\begin{bmatrix} b_k \\ b'_k \end{bmatrix} = \begin{bmatrix} t_{N-1} \cdots t_{N-k+1} \\ t_N \cdots t_{N-k+2} \end{bmatrix} \urcorner\urcorner\, Q_k\ ,\ k=n,n+1$$

$$d'_{n+1} = [\, t_N \cdots t_{N-n+1}\,] \urcorner\, Q_n$$



$$H_n = \urcorner\, H_{n+1}\, \urcorner^t$$
$$H_{n-1} = \lrcorner\, H_n\, \lrcorner^t$$

$$!\hat{H}_{n+1}\ Q_{n+1} = [\tilde{B}^t_{n+1}\ 0^t\ A^t_{n+1}]^t$$
$$\hat{H}_n\ Q_n = [\tilde{B}^t_n\ 0^t\ A^t_n]^t$$
$$?\hat{H}_{n-1}\ Q_{n-1} = [\tilde{B}^t_{n-1}\ 0^t\ A^t_{n-1}]^t$$

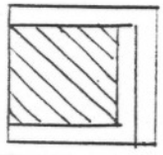


$$T_n = \urcorner\, T_{n+1}\, \lrcorner^t$$
$$T_{n-1} = \lrcorner\, T_n\, \urcorner^t$$

$$!\hat{T}_{n+1}\ Q_{n+1} = [\tilde{B}^t_{n+1}\ 0^t\ A^t_{n+1}]^t$$
$$\hat{T}_n\ Q_n = [\tilde{B}^t_n\ 0^t\ A^t_n]^t$$
$$?\hat{T}_{n-1}\ Q_{n-1} = [\tilde{B}^t_{n-1}\ 0^t\ A^t_{n-1}]^t$$

a

$$!\hat{T}_{n+1}\ Q_{n+1} = [\tilde{A}^t_{n+1}\ 0^t\ B^t_{n+1}]^t$$
$$\hat{T}_n\ Q_n = [\tilde{A}^t_n\ 0^t\ B^t_n]^t$$
$$?\hat{T}_{n-1}\ Q_{n-1} = [\tilde{A}^t_{n-1}\ 0^t\ B^t_{n-1}]^t$$

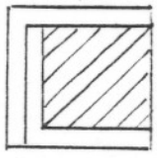


$$T_n = \urcorner T_{n+1} \llcorner^t$$
$$T_{n-1} = \llcorner T_n \llcorner^t$$

$$a_{n+1} = -(q^n_0)^{-1}\ q^{n+1}_0$$
$$\Lambda_{n+1} = (q^{n+1}_n - q^n_{n-1} + a_{n+1})^{-1}$$
$$[b_{n+1}b'_{n+1}] = [t_N \cdots t_{N-n+1}]\,[\urcorner\llcorner Q_{n+1} \mid \urcorner Q_n]$$
$$[c_{n+1}c'_{n+1}] = [t_{-M+n-1} \cdots t_{-M}]\,[\urcorner\llcorner Q_{n+1} \mid \llcorner Q_n]$$

b

$$!\hat{H}_{n+1}\ Q_{n+1} = [\tilde{A}^t_{n+1}\ 0^t\ B^t_{n+1}]^t$$
$$\hat{H}_n\ Q_n = [\tilde{A}^t_n\ 0^t\ B^t_n]^t$$
$$?\hat{H}_{n-1}\ Q_{n-1} = [\tilde{A}^t_{n-1}\ 0^t\ B^t_{n-1}]^t$$



$$H_n = \urcorner H_{n+1} \urcorner^t$$
$$H_{n-1} = \llcorner H_n \urcorner^t$$

$0 \leftarrow x \nwarrow x$

$$a_{n+1} = -(q^n_n)^{-1}\ q^{n+1}_{n+1}$$
$$\Lambda_{n+1} = (q^{n+1}_1 - q^n_1 + a_{n+1})^{-1}$$
$$[b_{n+1}b'_{n+1}] = [h_{N-n+1} \cdots h_N]\,[\llcorner\urcorner Q_{n+1} \mid \llcorner Q_n]$$
$$[c_{n+1}c'_{n+1}] = [h_{-M} \cdots h_{-M+n-1}]\,[\llcorner\urcorner Q_{n+1} \mid \urcorner Q_n]$$

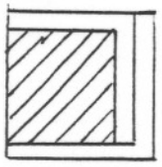
$$B_{n+1} = \left(\begin{bmatrix} B_{n+1} \\ b_{n+1} \end{bmatrix} - \begin{bmatrix} 0 \\ B_n \end{bmatrix} + \begin{bmatrix} B_n \\ b'_{n+1} \end{bmatrix} a_{n+1} \right) \Lambda_{n+1}$$

$$A_{n+1} = \left(\begin{bmatrix} 0 \\ 0 \\ A_{n+1} \\ c_{n+1} \end{bmatrix} - \begin{bmatrix} A_n \\ c'_{n+1} \end{bmatrix} + \begin{bmatrix} 0 \\ A_n \end{bmatrix} a_{n+1} \right) \Lambda_{n+1}$$

$$a_{n+1} = -(q^n_0)^{-1}\ q^{n+1}_0$$
$$\Lambda_{n+1} = (q^{n+1}_n - q^n_{n-1} + a_{n+1})^{-1}$$
$$[b_{n+1}b'_{n+1}] = [h_{-M} \cdots h_{-M+n-1}]\,[\urcorner\llcorner Q_{n+1} \mid \urcorner Q_n]$$
$$[c_{n+1}c'_{n+1}] = [h_{N-n+1} \cdots h_N]\,[\urcorner\llcorner Q_{n+1} \mid \llcorner Q_n]$$

$0 \leftarrow x \swarrow x$

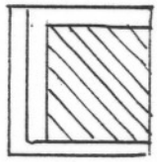
$$a_{n+1} = -(q^n_n)^{-1}\ q^{n+1}_{n+1}$$
$$\Lambda_{n+1} = (q^{n+1}_1 - q^n_1 + a_{n+1})^{-1}$$
$$[b_{n+1}b'_{n+1}] = [t_{-M+n-1} \cdots t_{-M}]\,[\llcorner\urcorner Q_{n+1} \mid \llcorner Q_n]$$
$$[c_{n+1}c'_{n+1}] = [t_N \cdots t_{N-n+1}]\,[\llcorner\urcorner Q_{n+1} \mid \urcorner Q_n]$$



$$H_n = \llcorner H_{n+1} \llcorner^t$$
$$H_{n-1} = \urcorner H_n \llcorner^t$$

$$!\hat{H}_{n+1}\ Q_{n+1} = [\tilde{B}^t_{n+1}\ 0^t\ A^t_{n+1}]^t$$
$$\hat{H}_n\ Q_n = [\tilde{B}^t_n\ 0^t\ A^t_n]^t$$
$$?\hat{H}_{n-1}\ Q_{n-1} = [\tilde{B}^t_{n-1}\ 0^t\ A^t_{n-1}]^t$$

d



$$T_n = \llcorner T_{n+1} \urcorner^t$$
$$T_{n-1} = \urcorner T_n \urcorner^t$$

$$!\hat{T}_{n+1}\ Q_{n+1} = [\tilde{B}^t_{n+1}\ 0^t\ A^t_{n+1}]^t$$
$$\hat{T}_n\ Q_n = [\tilde{B}^t_n\ 0^t\ A^t_n]^t$$
$$?\hat{T}_{n-1}\ Q_{n-1} = [\tilde{B}^t_{n-1}\ 0^t\ A^t_{n-1}]^t$$

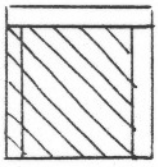
c

a

$$!\hat{S}_n\ P_n = [\tilde{C}_n^t\ 0^t\ D_n^t]^t$$

$$\hat{T}_{n+1}\ Q_{n+1} = [\tilde{A}_{n+1}^t\ 0^t\ B_{n+1}^t]^t$$

$$?\hat{T}_n\ Q_n = [\tilde{A}_n^t\ 0^t\ B_n^t]^t$$



$$S_n = \lceil T_{n+1} \rceil^t$$

$$T_n = \lceil T_{n+1} \rfloor^t$$

$$\Lambda_{n+1} = (p_{n-1}^n - q_n^{n+1})^{-1}$$

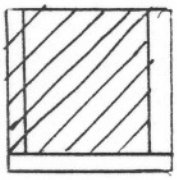
$$[c_{n+1}\ a_{n+1}] = [t_{-M+n} \ldots t_{-M}] \begin{bmatrix} 0 & \vert & \rfloor Q_{n+1} \\ \rfloor P_n & \vert & \end{bmatrix}$$

$$B_n = (\rfloor D_n - B_{n+1})\Lambda_{n+1}$$

$$A_n = \left(\begin{bmatrix} C_n \\ \\ c_{n+1} \end{bmatrix} - \begin{bmatrix} 0 \\ A_{n+1} \\ a_{n+1} \end{bmatrix} \right) \Lambda_{n+1}$$

$$\Lambda_{n+1} = (p_{n-1}^n - q_n^{n+1})^{-1}$$

$$[c_{n+1}\ a_{n+1}] = [h_{N-n} \ldots h_N] \begin{bmatrix} 0 & \vert & \rfloor Q_{n+1} \\ \rfloor P_n & \vert & \end{bmatrix}$$



$$K_n = \rfloor H_{n+1} \lceil^t$$

$$H_n = \rfloor H_{n+1} \rfloor^t$$

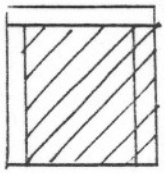
d

b

$$!\hat{K}_n\ P_n = [\tilde{C}_n^t\ 0^t\ D_n^t]^t$$

$$\hat{H}_{n+1}\ Q_{n+1} = [\tilde{A}_{n+1}^t\ 0^t\ B_{n+1}^t]^t$$

$$?\hat{H}_n\ Q_n = [\tilde{A}_n^t\ 0^t\ B_n^t]^t$$



$$K_n = \lceil H_{n+1} \rfloor^t$$

$$H_n = \lceil H_{n+1} \rceil^t$$

$$\Lambda_{n+1} = (p_1^n - q_1^{n+1})^{-1}$$

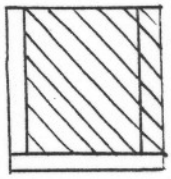
$$[c_{n+1}\ a_{n+1}] = [h_{-M} \ldots h_{-M+n}] \begin{bmatrix} \lceil P_n & \vert & \lceil Q_{n+1} \\ 0 & \vert & \end{bmatrix}$$

$$B_n = (\rfloor D_n - B_{n+1})\Lambda_{n+1}$$

$$A_n = \left(\begin{bmatrix} C_n \\ \\ c_{n+1} \end{bmatrix} - \begin{bmatrix} 0 \\ A_{n+1} \\ a_{n+1} \end{bmatrix} \right) \Lambda_{n+1}$$

$$\Lambda_{n+1} = (p_1^n - q_1^{n+1})^{-1}$$

$$[c_{n+1}\ a_{n+1}] = [t_N \ldots t_{N-n}] \begin{bmatrix} \lceil P_n & \vert & \lceil Q_{n+1} \\ 0 & \vert & \end{bmatrix}$$



$$S_n = \rfloor T_{n+1} \rfloor^t$$

$$T_n = \lceil T_{n+1} \rceil^t$$

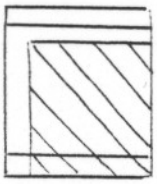
c

a

$$!\hat{S}_{n+1}\ P_{n+1} = [\tilde{C}{}^{t}_{n+1}\ 0^{t}\ D^{t}_{n+1}]^{t}$$

$$\hat{T}_{n+1}\ Q_{n+1} = [\tilde{A}{}^{t}_{n+1}\ 0^{t}\ B^{t}_{n+1}]^{t}$$

$$?\hat{T}_{n}\ Q_{n} = [\tilde{A}{}^{t}_{n}\ 0^{t}\ B^{t}_{n}]^{t}$$



$$T_n = \urcorner\, T_{n+1} \,\urcorner^{t}$$

$$S_{n+1} = \uparrow T_{n+1}$$

$$\Lambda_{n+1} = (p^{n+1}_{n} - q^{n+1}_{n})^{-1}$$

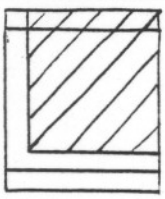
$$[c_{n+1}\, a_{n+1}] = [t_{-M+n} \ldots t_{-M}][\llcorner P_{n+1} \mid \llcorner Q_{n+1}]$$

$$B_n = (D_{n+1} - \begin{bmatrix} 0 \\ \\ B_{n+1} \end{bmatrix})\Lambda_{n+1}$$

$$A_n = (\begin{bmatrix} 0 \\ C_{n+1} \\ c_{n+1} \end{bmatrix} - \begin{bmatrix} A_{n+1} \\ \\ a_{n+1} \end{bmatrix})\Lambda_{n+1}$$

$$\Lambda_{n+1} = (p^{n+1}_{n} - q^{n+1}_{n})^{-1}$$

$$[c_{n+1}\, a_{n+1}] = [h_{N-n} \ldots h_{N}][\llcorner P_{n+1} \mid \llcorner Q_{n+1}]$$



$$H_n = \llcorner\, H_{n+1} \,\urcorner^{t}$$

$$K_{n+1} = \downarrow H_{n+1}$$

$$!\hat{K}_{n+1}\ P_{n+1} = [\tilde{D}{}^{t}_{n+1}\ 0^{t}\ C^{t}_{n+1}]^{t}$$

$$\hat{H}_{n+1}\ Q_{n+1} = [\tilde{B}{}^{t}_{n+1}\ 0^{t}\ A^{t}_{n+1}]^{t}$$

$$?\hat{H}_{n}\ Q_{n} = [\tilde{B}{}^{t}_{n}\ 0^{t}\ A^{t}_{n}]^{t}$$

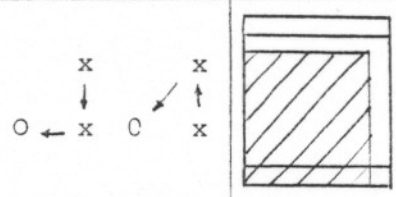
d

b

$$!\hat{K}_{n+1}\ P_{n+1} = [\tilde{C}{}^{t}_{n+1}\ 0^{t}\ D^{t}_{n+1}]^{t}$$

$$\hat{H}_{n+1}\ Q_{n+1} = [\tilde{A}{}^{t}_{n+1}\ 0^{t}\ B^{t}_{n+1}]^{t}$$

$$?\hat{H}_{n}\ Q_{n} = [\tilde{A}{}^{t}_{n}\ 0^{t}\ B^{t}_{n}]^{t}$$



$$H_n = \urcorner\, H_{n+1} \,\llcorner^{t}$$

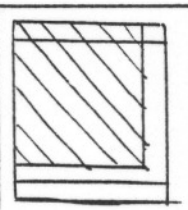
$$K_{n+1} = \uparrow H_{n+1}$$

$$\Lambda_{n+1} = (p^{n+1}_{1} - q^{n+1}_{1})^{-1}$$

$$[c_{n+1}\, a_{n+1}] = [h_{-M} \ldots h_{-M+n}][\urcorner P_{n+1} \mid \urcorner Q_{n+1}]$$

$$\Lambda_{n+1} = (p^{n+1}_{1} - q^{n+1}_{1})^{-1}$$

$$[c_{n+1}\, a_{n+1}] = [t_{N} \ldots t_{N-n}][\urcorner P_{n+1} \mid \urcorner Q_{n+1}]$$



$$T_n = \llcorner\, T_{n+1} \,\llcorner^{t}$$

$$S_{n+1} = \downarrow T_{n+1}$$

$$!\hat{S}_{n+1}\ P_{n+1} = [\tilde{D}{}^{t}_{n+1}\ 0^{t}\ C^{t}_{n+1}]^{t}$$

$$\hat{T}_{n+1}\ Q_{n+1} = [\tilde{B}{}^{t}_{n+1}\ 0^{t}\ A^{t}_{n+1}]^{t}$$

$$?\hat{T}_{n}\ Q_{n} = [\tilde{B}{}^{t}_{n}\ 0^{t}\ A^{t}_{n}]^{t}$$

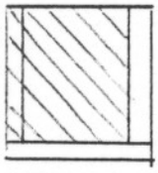
c

a

$$!\hat{S}_n\ P_n = [\tilde{C}{}^t_n\ 0^t\ D^t_n]^t$$

$$\hat{T}_{n+1}\ Q_{n+1} = [\tilde{A}{}^t_{n+1}\ 0^t\ B^t_{n+1}]^t$$

$$?\hat{T}_n\ Q_n = [\tilde{A}{}^t_n\ 0^t\ B^t_n]^t$$



$$T_n = \lrcorner\, T_{n+1}\, \lrcorner^t$$

$$S_n = \lrcorner\, T_{n+1}\, \urcorner^t$$

$$\Lambda_{n+1} = (p^n_{n-1} - q^{n+1}_n)^{-1}$$

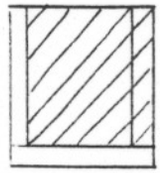
$$[c_{n+1}\ a_{n+1}] = [t_{-M+n} \cdots t_{-M}] \begin{bmatrix} 0 & \\ \lrcorner P_n & \lrcorner Q_{n+1} \end{bmatrix}$$

b

$$!\hat{K}_n\ P_n = [\tilde{C}{}^t_n\ 0^t\ D^t_n]^t$$

$$\hat{H}_{n+1}\ Q_{n+1} = [\tilde{A}{}^t_{n+1}\ 0^t\ B^t_{n+1}]^t$$

$$?\hat{H}_n\ Q_n = [\tilde{A}{}^t_n\ 0^t\ B^t_n]^t$$



$$H_n = \lrcorner\, H_{n+1}\, \urcorner^t$$

$$K_n = \lrcorner\, H_{n+1}\, \lrcorner^t$$

$$\Lambda_{n+1} = (p^n_1 - q^{n+1}_1)^{-1}$$

$$[c_{n+1}\ a_{n+1}] = [h_{-M} \cdots h_{-M+n}] \begin{bmatrix} \urcorner P_n & \\ 0 & \urcorner Q_{n+1} \end{bmatrix}$$

$$B_n = \left(\lrcorner\, D_n - \begin{bmatrix} 0 \\ \\ B_{n+1} \end{bmatrix} \right) \Lambda_{n+1}$$

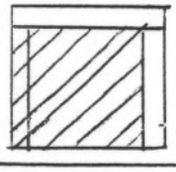
$$A_n = \left(\begin{bmatrix} c_{n+1} \\ \\ C_n \end{bmatrix} - \begin{bmatrix} a_{n+1} \\ \\ A_{n+1} \end{bmatrix} \right) \Lambda_{n+1}$$

$$\Lambda_{n+1} = (p^n_{n-1} - q^{n+1}_n)^{-1}$$

$$[c_{n+1}\ a_{n+1}] = [h_{N-n} \cdots h_N] \begin{bmatrix} 0 & \\ \lrcorner P_n & \lrcorner Q_{n+1} \end{bmatrix}$$

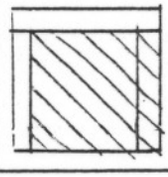
$$\Lambda_{n+1} = (p^n_1 - q^{n+1}_1)^{-1}$$

$$[c_{n+1}\ a_{n+1}] = [t_N \cdots t_{N-n}] \begin{bmatrix} \urcorner P_n & \\ 0 & \urcorner Q_{n+1} \end{bmatrix}$$



$$H_n = \urcorner\, H_{n+1}\, \lrcorner^t$$

$$K_n = \urcorner\, H_{n+1}\, \urcorner^t$$



$$T_n = \urcorner\, T_{n+1}\, \urcorner^t$$

$$S_n = \urcorner\, T_{n+1}\, \lrcorner^t$$

d

$$!\hat{K}_n\ P_n = [\tilde{D}{}^t_n\ 0^t\ C^t_n]^t$$

$$\hat{H}_{n+1}\ Q_{n+1} = [\tilde{B}{}^t_{n+1}\ 0^t\ A^t_{n+1}]^t$$

$$?\hat{H}_n\ Q_n = [\tilde{B}{}^t_n\ 0^t\ D^t_n]^t$$

c

$$!\hat{S}_n\ P_n = [\tilde{D}{}^t_n\ 0^t\ C^t_n]^t$$

$$\hat{T}_{n+1}\ Q_{n+1} = [\tilde{B}{}^t_{n+1}\ 0^t\ A^t_{n+1}]^t$$

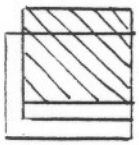
$$?\hat{T}_n\ Q_n = [\tilde{B}{}^t_n\ 0^t\ A^t_n]^t$$

a

$$!\hat{S}_n\ P_n = [\tilde{C}_n^t\ 0^t\ D_n^t]^t$$

$$\hat{S}_{n+1}\ P_{n+1} = [\tilde{C}_{n+1}^t\ 0^t\ D_{n+1}^t]^t$$

$$?\hat{T}_n\ Q_n = [\tilde{A}_n^t\ 0^t\ B_n^t]^t$$



$$T_n = \uparrow S_n$$

$$S_n = \lrcorner\, S_{n+1}\, \urcorner^t$$

$$b_{n+1} = -(p_0^n)^{-1}\ p_0^{n+1}$$

$$d_k = [\,t_N \cdots t_{N-k+1}\,]\ \urcorner P_k,\quad k=n,n+1$$

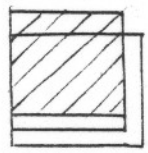
0
↑
x ← x

b

$$!\hat{K}_n\ P_n = [\tilde{C}_n^t\ 0^t\ D_n^t]^t$$

$$\hat{K}_{n+1}\ P_{n+1} = [\tilde{C}_{n+1}^t\ 0^t\ D_{n+1}^t]^t$$

$$?\hat{H}_n\ Q_n = [\tilde{A}_n^t\ 0^t\ B_n^t]^t$$



$$H_n = \uparrow K_n$$

$$K_n = \lrcorner\, K_{n+1}\, \lrcorner^t$$

$$b_{n+1} = -(p_n^n)^{-1}\ p_{n+1}^{n+1}$$

$$d_k = [\,h_{N-k+1} \cdots h_N\,]\ \lrcorner P_k,\quad k=n,n+1$$

$$B_n = \left(\begin{bmatrix} 0 \\ 0 \\ D_{n+1} \\ d_{n+1} \end{bmatrix} + \begin{bmatrix} 0 \\ \\ D_n \end{bmatrix} a_{n+1} + \begin{bmatrix} D_n \\ d_n \end{bmatrix} b_{n+1} \right) \Lambda_{n+1}$$

$$A_n = \left(\urcorner\, (C_{n+1} + C_n\, a_{n+1}) + \lrcorner\, C_n\, b_{n+1} \right) \Lambda_{n+1}$$

$$a_{n+1} = -w_n^{-1}\ w_{n+1}\ ,\quad \Lambda_{n+1} = (I + a_{n+1})^{-1}$$

$$w_k = C_k(0)\ ,\quad k = n,n+1$$

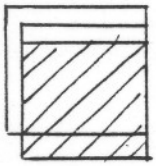
$$b_{n+1} = -(p_0^n)^{-1}\ p_0^{n+1}$$

$$d_k = [\,h_{-M} \cdots h_{-M+k-1}\,]\ \urcorner P_k,\quad k=n,n+1$$

x ↙ x
↓
0

$$b_{n+1} = -(p_n^n)^{-1}\ p_{n+1}^{n+1}$$

$$d_k = [\,t_{-M+k-1} \cdots t_{-M}\,]\ \lrcorner P_k,\quad k=n,n+1$$



$$H_n = \downarrow K_n$$

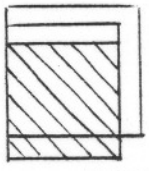
$$K_n = \urcorner\, K_{n+1}\, \urcorner^t$$

$$!\hat{K}_n\ P_n = [\tilde{D}_n^t\ 0^t\ C_n^t]^t$$

$$\hat{K}_{n+1}\ P_{n+1} = [\tilde{D}_{n+1}^t\ 0^t\ C_{n+1}^t]^t$$

$$?\hat{H}_n\ Q_n = [\tilde{B}_n^t\ 0^t\ A_n^t]^t$$

d



$$T_n = \downarrow S_n$$

$$S_n = \urcorner\, S_{n+1}\, \lrcorner$$

$$!\hat{S}_n\ P_n = [\tilde{D}_n^t\ 0^t\ C_n^t]^t$$

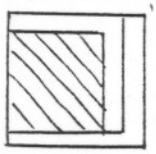
$$\hat{S}_{n+1}\ P_{n+1} = [\tilde{D}_{n+1}^t\ 0^t\ C_{n+1}^t]^t$$

$$?\hat{T}_n\ Q_n = [\tilde{B}_n^t\ 0^t\ A_n^t]^t$$

c

a

$$!\hat{T}_n\, Q_n = [\tilde{A}_n^t\ 0^t\ B_n^t]^t$$
$$\hat{T}_{n+1}\, Q_{n+1} = [\tilde{A}_{n+1}^t\ 0^t\ B_{n+1}^t]^t$$
$$?\hat{T}_{n-1}\, Q_{n-1} = [\tilde{A}_{n-1}^t\ 0^t\ B_{n-1}^t]^t$$



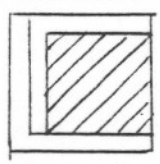
$$T_n = \lfloor T_{n+1} \rfloor^t$$
$$T_{n-1} = \lceil H_n \rfloor^t$$

$$a_{n+1} = -(q_n^{n+1} - q_{n-1}^n)$$
$$\Lambda_{n+1} = (q_{n-1}^{n+1} - q_{n-2}^n + q_{n-1}^n\, a_{n+1})^{-1}$$
$$\begin{bmatrix} b_k \\ b'_k \end{bmatrix} = \begin{bmatrix} t_{-M+k-1} \cdots t_{-M+1} \\ t_{-M+k-2} \cdots t_{-M} \end{bmatrix} \lfloor \lfloor\, Q_k,\ k=n,n+1$$
$$c_{n+1} = [\, t_{-M+n-1} \cdots t_{-M}\,] \lfloor\, Q_n$$

$$0 \swarrow\ x \leftarrow x$$

b

$$!\hat{H}_n\, Q_n = [\tilde{A}_n^t\ 0^t\ B_n^t]^t$$
$$\hat{H}_{n+1}\, Q_{n+1} = [\tilde{A}_{n+1}^t\ 0^t\ B_{n+1}^t]^t$$
$$?\hat{H}_{n-1}\, Q_{n-1} = [\tilde{A}_{n-1}^t\ 0^t\ B_{n-1}^t]^t$$



$$H_n = \lfloor H_{n+1} \rceil^t$$
$$H_{n-1} = \lceil H_n \rceil^t$$

$$a_{n+1} = -(q_1^{n+1} - q_1^n)$$
$$\Lambda_{n+1} = (q_2^{n+1} - q_2^n + q_1^n\, a_{n+1})^{-1}$$
$$\begin{bmatrix} b_k \\ b'_k \end{bmatrix} = \begin{bmatrix} h_{-M+1} \cdots h_{-M+k-1} \\ h_{-M} \cdots h_{-M+k-2} \end{bmatrix} \lceil \lceil\, Q_k,\ k=n,n+1$$
$$c_{n+1} = [\, h_{-M} \cdots h_{-M+n-1}\,] \lceil\, Q_n$$

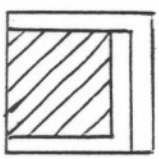
$$B_{n-1} = \left(\begin{bmatrix} 0 \\ B_{n+1} \end{bmatrix} - \begin{bmatrix} 0 \\ B_n \end{bmatrix} + B_n\, a_{n+1} \right) \Lambda_{n+1}$$

$$A_{n-1} = \begin{bmatrix} 0 \\ A_{n+1} \\ b_{n+1} \\ b'_{n+1} \end{bmatrix} - \begin{bmatrix} A_n \\ b_n \\ b'_n \end{bmatrix} + \begin{bmatrix} 0 \\ A_n \\ c_{n+1} \end{bmatrix} a_{n+1} \Lambda_{n+1}$$

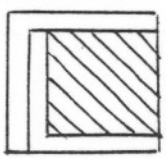
$$a_{n+1} = -(q_n^{n+1} - q_{n-1}^n)$$
$$\Lambda_{n+1} = (q_{n-1}^{n+1} - q_{n-2}^n + q_{n-1}^n\, a_{n+1})^{-1}$$
$$\begin{bmatrix} b_k \\ b'_k \end{bmatrix} = \begin{bmatrix} h_{N-k+1} \cdots h_{N-1} \\ h_{N-k+2} \cdots h_N \end{bmatrix} \lfloor \lfloor\, Q_k,\ k=n,n+1$$
$$c_{n+1} = [\, h_{N-n+1} \cdots h_N\,] \lfloor\, Q_n$$

$$0 \nwarrow\ x \leftarrow x$$

$$a_{n+1} = -(q_1^{n+1} - q_1^n)$$
$$\Lambda_{n+1} = (q_2^{n+1} - q_2^n + q_1^n\, a_{n+1})^{-1}$$
$$\begin{bmatrix} b_k \\ b'_k \end{bmatrix} = \begin{bmatrix} t_{N-1} \cdots t_{N-k+1} \\ t_N \cdots t_{N-k+2} \end{bmatrix} \lceil \lceil\, Q_k,\ k=n,n+1$$
$$c_{n+1} = [\, t_N \cdots t_{N-n+1}\,] \lceil\, Q_n$$



$$H_n = \lceil H_{n+1} \rfloor^t$$
$$H_{n-1} = \lfloor H_n \rfloor^t$$



$$T_n = \lceil T_{n+1} \rceil^t$$
$$T_{n-1} = \lfloor T_n \rceil^t$$

$$!\hat{H}_n\, Q_n = [\tilde{B}_n^t\ 0^t\ A_n^t]^t$$
$$\hat{H}_{n+1}\, Q_{n+1} = [\tilde{B}_{n+1}^t\ 0^t\ A_{n+1}^t]^t$$
$$?\hat{H}_{n-1}\, Q_{n-1} = [\tilde{B}_{n-1}^t\ 0^t\ A_{n-1}^t]^t$$

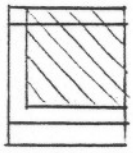
d

$$!\hat{T}_n\, Q_n = [\tilde{B}_n^t\ 0^t\ A_n^t]^t$$
$$\hat{T}_{n+1}\, Q_{n+1} = [\tilde{B}_{n+1}^t\ 0^t\ A_{n+1}^t]^t$$
$$?\hat{T}_{n-1}\, Q_{n-1} = [\tilde{B}_{n-1}^t\ 0^t\ A_{n-1}^t]^t$$

c

a

$$!\hat{S}_n\ P_n = [\tilde{C}_n^t\ 0^t\ D_n^t]^t$$
$$\hat{T}_n\ Q_n = [\tilde{A}_n^t\ 0^t\ B_n^t]^t$$
$$?\hat{T}_{n-1}\ Q_{n-1} = [\tilde{A}_{n-1}^t\ 0^t\ B_{n-1}^t]^t$$

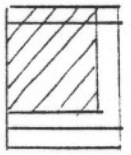


$$S_n = \downarrow T_n$$
$$T_{n-1} = \lrcorner\, T_{n+1}\, \urcorner^t$$

$$a_{n+1} = -(q_0^n)^{-1}\,[q_1^n\,(q_0^n)^{-1} - p_1^n\,(p_0^n)^{-1}]^{-1} q_0^n$$
$$b_{n+1} = p_0^n\,[q_1^n\,(q_0^n)^{-1} - p_1^n\,(p_0^n)^{-1}]^{-1} q_0^n$$
$$\begin{bmatrix} c_{n+1} & d_{n+1} \\ c'_{n+1} & d'_{n+1} \end{bmatrix} = \begin{bmatrix} t_{N-1} \cdots t_{N-n+1} \\ t_N \cdots t_{N-n+2} \end{bmatrix} [\urcorner\,\urcorner Q_n \mid \urcorner\,\urcorner P_n]$$
$$f_{n+1} = [\,t_N \cdots t_{N-n+1}\,]\,\urcorner\, Q_n$$

b

$$!\hat{K}_n\ P_n = [\tilde{C}_n^t\ 0^t\ D_n^t]^t$$
$$\hat{H}_n\ Q_n = [\tilde{A}_n^t\ 0^t\ B_n^t]^t$$
$$?\hat{H}_{n-1}\ Q_{n-1} = [\tilde{A}_{n-1}^t\ 0^t\ B_{n-1}^t]^t$$



$$K_n = \downarrow H_n$$
$$H_{n-1} = \lrcorner\, H_n\, \lrcorner^t$$

$$a_{n+1} = -(q_n^n)^{-1}\,[q_{n-1}^n (q_n^n)^{-1} - p_{n-1}^n (p_n^n)^{-1}]^{-1} q_n^n$$
$$b_{n+1} = (p_n^n)^{-1}\,[q_{n-1}^n (q_n^n)^{-1} - p_{n-1}^n (p_n^n)^{-1}]^{-1} q_n^n$$
$$\begin{bmatrix} c_{n+1} & d_{n+1} \\ c'_{n+1} & d'_{n+1} \end{bmatrix} = \begin{bmatrix} h_{N-n+1} \cdots h_{N-1} \\ h_{N-n+2} \cdots h_N \end{bmatrix} [\lrcorner\,\lrcorner Q_n \mid \lrcorner\,\lrcorner P_n]$$
$$f_{n+1} = [\,h_{N-n+1} \cdots h_N\,]\,\lrcorner\, Q_n$$

0 ↙ x ↑ x

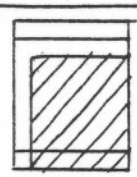
$$B_{n-1} = \begin{bmatrix} 0 \\ B_n \\ f_{n+1} \end{bmatrix} + \begin{bmatrix} B_n \\ c_{n+1} \\ c'_{n+1} \end{bmatrix} a_{n+1} + \begin{bmatrix} 0 \\ D_n \\ d_{n+1} \\ d'_{n+1} \end{bmatrix} b_{n+1}$$

$$A_{n-1} = A_n + \begin{bmatrix} 0 \\ \lrcorner\, A_n \end{bmatrix} a_{n+1} + \lrcorner\, C_n\, b_{n+1}$$

x ↓ x ↘ 0

d

$$!\hat{K}_n\ P_n = [\tilde{D}_n^t\ 0^t\ C_n^t]^t$$
$$\hat{H}_n\ Q_n = [\tilde{B}_n^t\ 0^t\ A_n^t]^t$$
$$?\hat{H}_{n-1}\ Q_{n-1} = [\tilde{B}_{n-1}^t\ 0^t\ A_{n-1}^t]^t$$

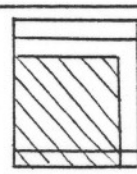


$$K_n = \uparrow H_n$$
$$H_{n-1} = \urcorner\, H_n\, \urcorner^t$$

$$a_{n+1} = -(q_0^n)^{-1}\,[q_1^n\,(q_0^n)^{-1} - p_1^n\,(p_0^n)^{-1}]^{-1} q_0^n$$
$$b_{n+1} = p_0^n\,[q_1^n\,(q_0^n)^{-1} - p_1^n\,(p_0^n)^{-1}]^{-1} q_0^n$$
$$\begin{bmatrix} c_{n+1} & d_{n+1} \\ c'_{n+1} & d'_{n+1} \end{bmatrix} = \begin{bmatrix} h_{-M+1} \cdots h_{-M+n-1} \\ h_{-M} \cdots h_{-M+n-2} \end{bmatrix} [\urcorner\,\urcorner Q_n \mid \urcorner\,\urcorner P_n]$$
$$f_{n+1} = [\,h_{-M} \cdots h_{-M+n-1}\,]\,\urcorner\, Q_n$$

c

$$!\hat{S}_n\ P_n = [\tilde{D}_n^t\ 0^t\ C_n^t]^t$$
$$\hat{T}_n\ Q_n = [\tilde{B}_n^t\ 0^t\ A_n^t]^t$$
$$?\hat{T}_{n-1}\ Q_{n-1} = [\tilde{B}_{n-1}^t\ 0^t\ A_{n-1}^t]^t$$

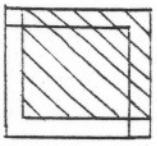


$$S_n = \uparrow T_n$$
$$T_{n-1} = \urcorner\, T_n\, \lrcorner^t$$

$$a_{n+1} = -(q_n^n)^{-1}\,[q_{n-1}^n (q_n^n)^{-1} - p_{n-1}^n (p_n^n)^{-1}]^{-1} q_n^n$$
$$b_{n+1} = (p_n^n)^{-1}\,[q_{n-1}^n (q_n^n)^{-1} - p_{n-1}^n (p_n^n)^{-1}]^{-1} q_n^n$$
$$\begin{bmatrix} c_{n+1} & d_{n+1} \\ c'_{n+1} & d'_{n+1} \end{bmatrix} = \begin{bmatrix} t_{-M+n-1} \cdots t_{-M+1} \\ t_{-M+n-2} \cdots t_{-M} \end{bmatrix} [\lrcorner\,\lrcorner Q_n \mid \lrcorner\,\lrcorner P_n]$$
$$f_{n+1} = [\,t_{-M+n-1} \cdots t_{-M}\,]\,\lrcorner\, Q_n$$

a

$$!\hat{S}_n\ P_n = [\tilde{C}_n^t\ 0^t\ D_n^t]^t$$
$$\hat{T}_{n+1}\ Q_{n+1} = [\tilde{A}_{n+1}^t\ 0^t\ B_{n+1}^t]^t$$
$$?\hat{T}_n\ Q_n = [\tilde{A}_n^t\ 0^t\ B_n^t]^t$$

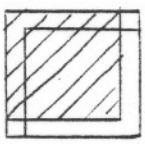


$$T_n = \lrcorner\, T_{n+1}\, \urcorner^t$$
$$S_n = \urcorner\, T_{n+1}\, \lrcorner^t$$

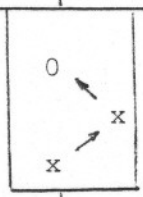
$$a_{n+1} = -(q_1^{n+1} - p_1^n (p_0^n)^{-1} q_0^{n+1})^{-1} q_0^{n+1}$$
$$b_{n+1} = -(p_0^n)^{-1}\ q_0^{n+1}\ a_{n+1}$$
$$\begin{bmatrix} c_{n+1} & d_{n+1} \\ c'_{n+1} & d'_{n+1} \end{bmatrix} = \begin{bmatrix} t_{N-1} \cdots t_{N-n} \\ t_N \cdots t_{N-n+1} \end{bmatrix} \left[\urcorner\, \urcorner\, Q_{n+1} \,\middle|\, \begin{matrix} \urcorner\, \urcorner\, P_n \\ 0 \end{matrix} \right]$$
$$f_{n+1} = [\, t_N \cdots t_{N-n}\,]\ \urcorner\, Q_{n+1}$$

b

$$!\hat{K}_n\ P_n = [\tilde{C}_n^t\ 0^t\ D_n^t]^t$$
$$\hat{H}_{n+1}\ Q_{n+1} = [\tilde{A}_{n+1}^t\ 0^t\ B_{n+1}^t]^t$$
$$?\hat{H}_n\ Q_n = [\tilde{A}_n^t\ 0^t\ B_n^t]^t$$



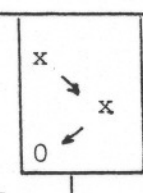
$$H_n = \lrcorner\, H_{n+1}\, \lrcorner^t$$
$$K_n = \urcorner\, H_{n+1}\, \urcorner^t$$



$$a_{n+1} = -(q_n^{n+1} - p_{n-1}^n (p_n^n)^{-1} q_{n+1}^{n+1})^{-1} q_{n+1}^{n+1}$$
$$b_{n+1} = -(p_n^n)^{-1}\ q_{n+1}^{n+1}\ a_{n+1}$$
$$\begin{bmatrix} c_{n+1} & d_{n+1} \\ c'_{n+1} & d'_{n+1} \end{bmatrix} = \begin{bmatrix} h_{N-n-1} \cdots h_{N-1} \\ h_{N-n} \cdots h_N \end{bmatrix} \left[\lrcorner\, \lrcorner\, Q_{n+1} \,\middle|\, \begin{matrix} 0 \\ \lrcorner\, \lrcorner\, P_n \end{matrix} \right]$$
$$f_{n+1} = [\, h_{N-n} \cdots h_N\,]\ \lrcorner\, Q_{n+1}$$

$$B_n = \begin{bmatrix} 0 \\ B_{n+1} \\ f_{n+1} \end{bmatrix} + \begin{bmatrix} B_{n+1} \\ c_{n+1} \\ c'_{n+1} \end{bmatrix} a_{n+1} + \begin{bmatrix} D_n \\ d_{n+1} \\ d'_{n+1} \end{bmatrix} b_{n+1}$$

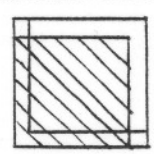
$$A_n = \lrcorner\, A_{n+1} + \begin{bmatrix} 0 \\ \lrcorner\, A_{n+1} \end{bmatrix} a_{n+1} + \lrcorner\, \lrcorner\, C_n\ b_{n+1}$$



$$a_{n+1} = -(q_1^{n+1} - p_1^n (p_0^n)^{-1} q_0^{n+1})^{-1} q_0^{n+1}$$
$$b_{n+1} = -(p_0^n)^{-1}\ q_0^{n+1}\ a_{n+1}$$
$$\begin{bmatrix} c_{n+1} & d_{n+1} \\ c'_{n+1} & d'_{n+1} \end{bmatrix} = \begin{bmatrix} h_{-M+1} \cdots h_{-M+n} \\ h_{-M} \cdots h_{-M+n-1} \end{bmatrix} \left[\urcorner\, \urcorner\, Q_{n+1} \,\middle|\, \begin{matrix} \urcorner\, \urcorner\, P_n \\ 0 \end{matrix} \right]$$
$$f_{n+1} = [\, h_{-M} \cdots h_{-M+n}\,]\ \urcorner\, Q_{n+1}$$

c

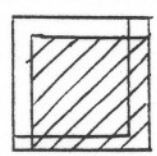
$$!\hat{S}_n\ P_n = [\tilde{D}_n^t\ 0^t\ C_n^t]^t$$
$$\hat{T}_{n+1}\ Q_{n+1} = [\tilde{B}_{n+1}^t\ 0^t\ A_{n+1}^t]^t$$
$$?\hat{T}_n\ Q_n = [\tilde{B}_n^t\ 0^t\ A_n^t]^t$$



$$T_n = \urcorner\, T_{n+1}\, \lrcorner^t$$
$$S_n = \lrcorner\, T_{n+1}\, \urcorner^t$$

$$a_{n+1} = -(q_n^{n+1} - p_{n-1}^n (p_n^n)^{-1} q_{n+1}^{n+1})^{-1} q_{n+1}^{n+1}$$
$$b_{n+1} = -(p_n^n)^{-1}\ q_{n+1}^{n+1}\ a_{n+1}$$
$$\begin{bmatrix} c_{n+1} & d_{n+1} \\ c'_{n+1} & d'_{n+1} \end{bmatrix} = \begin{bmatrix} t_{-M+n} \cdots t_{-M+1} \\ t_{-M+n-1} \cdots t_{-M} \end{bmatrix} \left[\lrcorner\, \lrcorner\, Q_{n+1} \,\middle|\, \begin{matrix} 0 \\ \lrcorner\, \lrcorner\, P_n \end{matrix} \right]$$
$$f_{n+1} = [\, t_{-M+n} \cdots t_{-M}]\ \lrcorner\, Q_{n+1}$$

$$!\hat{K}_n\ P_n = [\tilde{D}_n^t\ 0^t\ C_n^t]^t$$
$$\hat{H}_{n+1}\ Q_{n+1} = [\tilde{B}_{n+1}^t\ 0^t\ A_{n+1}^t]^t$$
$$?\hat{H}_n\ Q_n = [\tilde{B}_n^t\ 0^t\ A_n^t]^t$$



$$H_n = \urcorner\, H_{n+1}\, \urcorner^t$$
$$K_n = \lrcorner\, H_{n+1}\, \lrcorner^t$$

d