Mechanical property characterization of a 3D printing manufacturing system

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Abstract. Additive manufacturing is making it possible to increase the complexity of designed mechanical structures. However, the variability inherent to this manufacturing process can influence significantly the performance of structural elements, specially in phononic crystals and metamaterials since their working principles relies on the repetition of identical cells with a dedicated designed geometry. In this work, first, a design of experiments approach is applied to a determine a sampling strategy in order to characterize an additive manufacturing machine. Then, mechanical properties of the samples are inferred using material properties measured with an ultrasound transducer. The material density was measured using the weight of the samples, both dry and immersed in water, using the buoyancy force expression. It is known that the elastic modulus measured via ultrasound is biased. Therefore, the distributions inferred using ultrasound measurements were updated using experimental forced responses of sample rods and dynamic models via the Spectral Element Model. Updated values are used in statistical regression modeling to infer the stochastic field over print are of the 3D printer. The presented work is a first step in the longer term research goal: to show how to model the overall variability of a given additive manufacturing process, which is usually obtained in the statistical process control, and explain how to use it in the design of robust phononic crystal and metamaterial designs. The printing direction presented a statistically significant relationship with the elastic modulus and with the mass density, while only the printing direction presented a statistically significant relationship for the shear modulus.

Keywords: uncertainty quantification, statistical regression, statistical inference, Kernel smoother

1 Introduction

Geometrically complex designs, which includes metamaterials and phononic crystals, can be printed using additive manufacturing [14]. However, the variability of such manufacturing process can influence substantively the printed structures, specially the mechanical properties [16] are more impacted than what occurs typically in other manufacturing processes [15,**?**] and, thus, statistical process control can be a tool to assure that the manufacturing process will coincide with the design [18]. The inferred variability can be propagated through a deterministic model to obtain the stochastic result, and it can be used in a robust optimization approach as showed in [2].

The objective of this research is to show how statistical modeling can be applied to the data obtained from statistical process control to estimate stochastic fields that represents the variability of the mechanical properties of 3D printed parts. This estimation can be, for instance, combined with a robust optimization for designing phononic crystals and metamaterials that are robust against these types of variability.

2 Design of experiments

In the current research, we assumed variability in the mass density (ρ), Young's modulus (E), and shear modulus (G). The samples were defined as rectangles, and these variables were assumed as dependent variables, which could be tested if they are statistically related to the independent variables: thickness (T), printing position ($\boldsymbol{P} = \{P_x, P_y, P_z\}$), which is the position the parts are printed inside the 3D printer, printing direction ($\boldsymbol{D} = \{D_x, D_y, D_z\}$). In addition, samples were taking over the weeks to see if the machine settings would influence the mechanical properties. For this paper a 3D Printer of the type "Prusa MK3S" with a print volume of approximately 11 dm^3 and making use of the fused deposition modeling technique, is studied. In addition, by taking samples over the time, the assumption of the variability being the same over the time due to substantive changes in the used material and the setting were checked through statistical test.

Before the measurements, we assumed each observation for E follows a normal distribution with mean 2.1 and standard deviation 0.5, as found by [3] in a similar manufacturing process, and consequently, the mean follows a t-distribution. Assuming the α value was defined as 5%, and the type II error for 11 statistical degrees-of-freedom for a distribution X_A with mean $\bar{X}_A$ 1.5 distant from $\bar{X}$ is lower than 0.001%, and assuming that no more than 4 variables are going to be tested at the same time, the sample size of 15 is defined.

Each week, for 3 weeks, 15 samples were printed in one print, whose position were defined via dividing the batch into 216 (6 on x direction, 6 on y direction, and 6 on z direction) smaller boxes of 30x30x30 mm. Then, the printing position of each sample were defined via taking samples without replacement from a discrete uniform distribution $U(1, 6)$ for x, y, and z directions, defining, thus, the vector $\boldsymbol{P}$. Hence, rectangular parallelepipeds with dimensions of $1.5d_1$x1.5xd_1xd_1 mm, were defined, where the term $1.5d_1$ was sampled from the continuous uniform distribution $U(5, 25)$. Thus, the thickness can be defined as $T = d_1$. The variable time was also included in the analysis as week number (0, 1, 2).

For each sample, the values of E and G were observed using a Olympus 38DL Plus acquisition system with the M106 and V152 longitudinal shear wave contact ultrasound transducer of 2.25 MHz. The mass density was estimated using the highly precise Acculab Atilon scale where the samples weights were measured dry and submerged in water. Once the estimations of E, G, and ρ are done, their statistical relationships with the independent variables can be made using statistical regression modeling.

3 Statistical regression models

For each dependent random variable, the vector of values $\boldsymbol{y}$ can be defined and modeled using the matrix of observations of independent variables weighted by the parameter vector $\boldsymbol{\beta}$ as presented in Eq. (1) [10].

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}. \tag{1}$$

Assuming that each element on vector $\boldsymbol{\varepsilon}$ follows a normal distribution with zero mean and variance σ^2, and that all these distributions are independent from each other, i.e., $\boldsymbol{\varepsilon} \sim N(\boldsymbol{0}, \sigma^2 I_{N_{iv}})$, where $I_{N_{iv}}$ is the identity matrix. Then, the maximum likelihood estimator to estimate $\boldsymbol{\beta}$ and σ^2 are respectively given by [9]

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{y}, \tag{2a}$$

$$\hat{\sigma}^2 = \frac{(\boldsymbol{y} - \boldsymbol{X}\hat{\boldsymbol{\beta}})'(\boldsymbol{y} - \boldsymbol{X}\hat{\boldsymbol{\beta}})}{N}. \tag{2b}$$

After that, statistical test can be applied to test the hypothesis of β_j being statistically significant, i.e., $\beta_j \neq 0$ using a significance level of 5% [9].

The relationships between the dependent variables E, G, and ρ, with the independent variables T, $\boldsymbol{P}$, and $\boldsymbol{D}$ were also verified. Variables $\boldsymbol{P}$ and $\boldsymbol{D}$ were included as random variables ($\delta(P_c)$ and $\delta(D_c)$ with $c = \{x, y, z\}$) [11], where 0 indicates absence and 1 presence of these variables. All the combinations of the independent variables at the power of one, two and three were used in the analyses. The Akaike information criterion [7,8] was in the selection of the models that presented statistically significant parameters and presented valid assumptions.

The Breusch–Pagan test [4] was used to check if the variance is the same for all the terms in the vector $\boldsymbol{\varepsilon}$, the Shapiro–Wilk test [5] was used to verify the normality, and the Durbin–Watson [6] test was used to verify the assumption of independency on the residuals.

Using the proposed methodology, the stochastic regression models in Eq. (3) were obtained.

$$E_m \sim \mathrm{N}(2.9740 + 0.34445 \times (\delta(P_x) + \delta(P_y)), 0.4035), \tag{3a}$$

$$G_m \sim \mathrm{N}(1.1247 + 0.1200 \times (\delta(P_x) + \delta(P_y)), 0.2718), \tag{3b}$$

$$\rho_m \sim \mathrm{N}(1.1956 + 0.0036 \times (\delta(P_x) + \delta(P_y)), 0.0974). \tag{3c}$$

As is can be observed by the estimated equations, the parts printed in the z direction have significantly lower Young's modulus, shear modulus, and mass density than the parts printed in x and y directions.

3.1 Kernel smoother

Using the models in Eq. (3), discrete samples of E, G, and ρ in the three-dimensional space can be defined as the vectors of length n $\boldsymbol{E}_{3d}$, $\boldsymbol{G}_{3d}$, and $\boldsymbol{\rho}_{3d}$. When premultiplying one of those vectors by the Kernel matrix $\boldsymbol{K}$. The sampled vector can be made smoother, whose r-th row and s-th column element is given by

$$K_{rs} = \frac{f_K\left(\frac{d_{3d,r}-d_{3d,s}}{zeta}\right)}{\sum_{s=1}^{n} f_K\left(\frac{d_{3d,r}-d_{3d,s}}{\zeta}\right)} \tag{4}$$

where the function $f_K(d)$ can be an exponential function of the spatial distance (d) between $d_{3d,r}$ and $d_{3d,s}$ with correlation length ζ [12,13].

The 1,000 samples of the spatial E, G, and ρ sampled from Eq. (3) and smoothed using the Kernel smoother, whose elements are given by Eq. (4), are illustrated in Fig. 1.

Two samples of the smoothed $\boldsymbol{E}_{3d}$, $\boldsymbol{G}_{3d}$, and $\boldsymbol{\rho}_{3d}$ are illustrated in Fig. 2. For a large enough samples, after convergence of the Monte Carlo method, the three-dimensional spatial field of E, G, and ρ can be simulated. Figure 3 illustrates two samples of the smoothed field of the mechanical properties on a specific frame structure.

4 Final remarks

In the current research, we have used some data, simulating the data from a statistical process control from a manufacturing process and we have showed how to infer and estimate the variability via stochastic field. The estimated stochastic field can be used to check is the manufacturing process is under control and ir can be used in a robust optimization, via combining engineering design and statistical process control. In the current research, we found a significant lower Young's modulus, shear modulus, and mass density for the parts printed in z direction than the ones printed in x and y directions.

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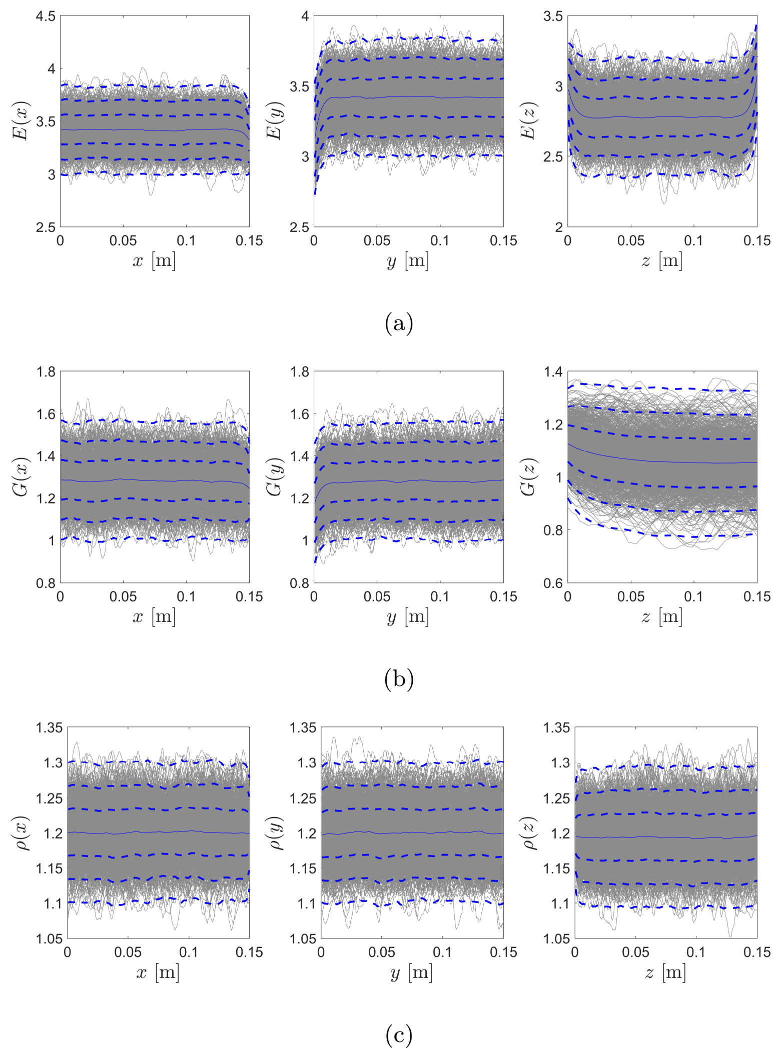
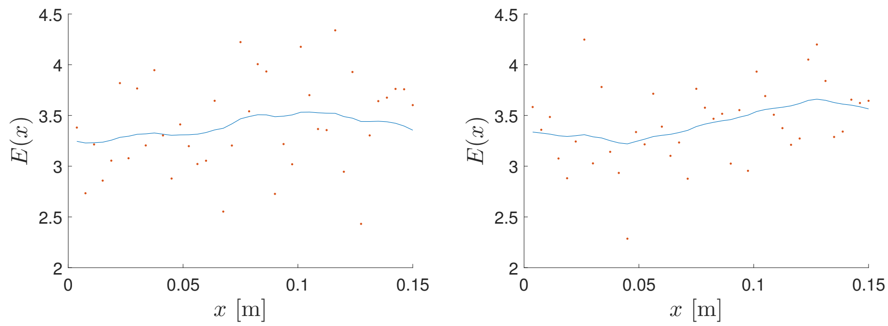
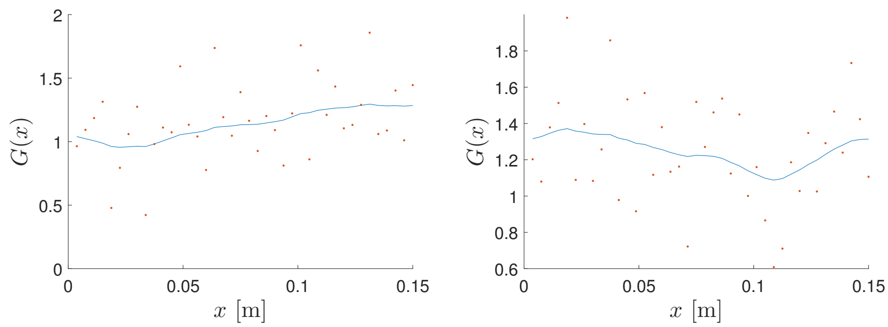


Fig. 1: Illustration of two simulated field of $\boldsymbol{E}_{3d}$ (a), $\boldsymbol{G}_{3d}$ (b), and $\boldsymbol{\rho}_{3d}$ (c) with 1,000 samples each. The straight line is the field mean, the dashed lines are the intervals that contains one, two, and three standard deviations.



(a)



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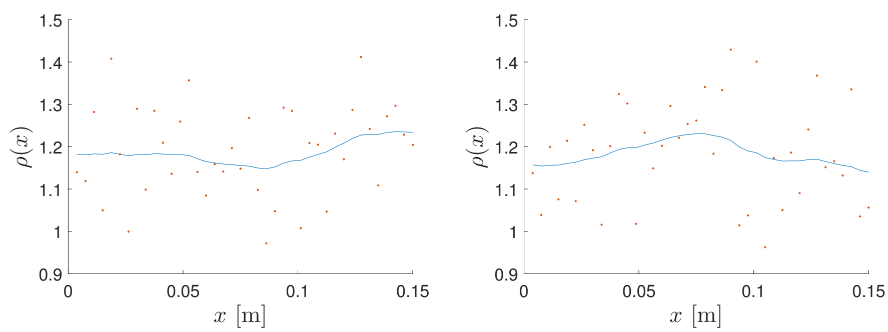
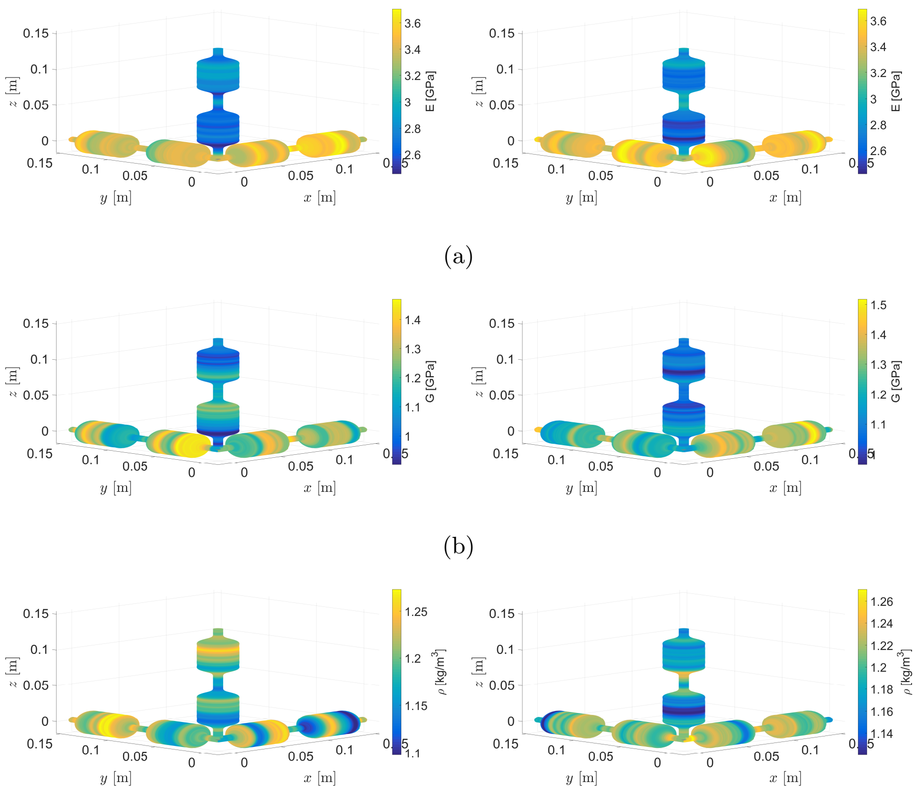


(c)

Fig. 2: Two raw (red dots) then smoothed (blue lines) samples of the stochastic fields of Young's modulus (a), shear modulus (b), and mass density (c).



(a)

(b)

(c)

Fig. 3: Illustration of two smoothed samples of $\boldsymbol{E}_{3d}$ (a), $\boldsymbol{G}_{3d}$ (b), and $\boldsymbol{\rho}_{3d}$ (c) with $n = 100$.