Data-driven state-space identification of nonlinear feedback systems

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1 Background

Many nonlinear systems can be represented as linear time-invariant (LTI) systems with a static nonlinear function in the feedback path (see Fig. 1). As for system identification, modelling of the nonlinear function is particularly challenging since this typically requires prior system information, expert knowledge and/or engineering judgement. Another issue in identification of nonlinear systems is that the optimisation algorithm may converge to a local minimum of the typically non-convex cost function.

2 Problem statement

Consider the discrete-time state-space representation of a nonlinear feedback system

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + w_k, \\ y_k &= Cx_k + Du_k, \end{aligned} \tag{1a}$$

where A, B, C and D are the linear state, input, output, and direct feedthrough matrices, respectively. Moreover, x_k is the latent state vector and u_k and y_k are the measured inputs and outputs, respectively, at discrete time instant k. The nonlinear function is represented by an additional multivariate input w_k, which is modelled as a feedforward neural network with one hidden layer:

$$\begin{aligned} w_k &= W_w \sigma(W_z z_k + b_z) + b_w, \\ z_k &= Ex_k + Fu_k + Gy_k, \end{aligned} \tag{1b}$$

where W_z and W_w are the inner and outer weights of the neural net, respectively, and b_z and b_w their associated biases. Any suitable nonlinear activation function $\sigma(.)$ can be chosen. The neural net input is z_k, which is comprised of a linear combination of the states, inputs, and outputs, through coefficient matrices E, F and G, respectively. The aim of this work is to first infer the nonlinear input w_k and the latent state x_k in the time domain, such that their functional mapping in the form of a neural net can be learnt afterwards.

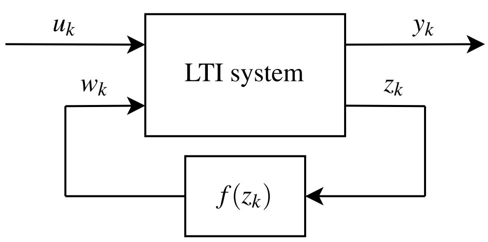

Figure 1: Block-diagram of a nonlinear feedback system.

3 Method

Based on the measured input-output data only, the identification procedure consists of four steps:

1. Initialise A, B, C and D, through the best linear approximation (BLA) [1]. This facilitates the ability to gather crucial data about the system, such as the order of its dynamic behaviour.
2. Use the BLA and the input-output data to find w_k in the time domain by solving a convex optimisation problem similar to unconstrained model predictive control [2]. Here, the reference that we track is the original output data y_k, while the original input data u_k is treated as a known disturbance. This step automatically yields an estimate of the latent state x_k.
3. Define E, F and G; decide on the activation function and the number of neurons; possibly perform some dimensionality reduction on z_k and w_k; and train the feedforward neural net (1b).
4. Perform final optimisation on all model parameters to further reduce the simulation error.

Steps 1-3 thus serve as an initialisation of the final optimisation step, and are meant to mitigate the risk of getting stuck in a local minimum. The considered method also requires almost no prior system knowledge, therefore overcoming one of the main challenges in nonlinear system identification.

4 Results

The effectiveness of the proposed method is evaluated on a number of nonlinear benchmark data sets (from www.nonlinearbenchmark.org), including the Silverbox system and the Bouc-Wen hysteric system.

References

[1] R. Pintelon and J. Schoukens, *System identification: a frequency domain approach*. John Wiley & Sons, 2012.

[2] E. F. Camacho and C. B. Alba, *Model predictive control*. Springer science & business media, 2013.