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Generalized Space-Time Engineered Modulation (GSTEM) Metamaterials

A global and extended perspective.

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This article presents a global and extended perspective of electrodynamic metamaterials formed by space and time engineered modulations, which we name *generalized space-time engineered modulation (GSTEM) metamaterials*, or *GSTEMs*. This perspective describes metamaterials from a unified *spacetime viewpoint* and introduces *accelerated metamaterials* as an extra type of dynamic metamaterials. First, it positions GSTEMs in the even broader context of electrodynamic systems that include (nonmodulated) moving sources in vacuum and moving bodies, explains the difference between the moving-matter nature of the latter and the moving-perturbation nature of GSTEMs, and enumerates the different types of GSTEMs considered, namely space EMs (SEMs), time EMs (TEMs), uniform space-time EMs (USTEMs), and accelerated space-time EMs (ASTEMs). Next, it establishes the physics of the related interfaces, which includes direct-spacetime scattering and inverse-spacetime transition transformations. Then, it exposes the physics of the GSTEMs formed by stacking these interfaces and homogenizing the resulting crystals; this includes an original explanation of light deflection by USTEMs as being a *spacetime weighted averaging phenomenon* and the demonstration of ASTEM light curving and black hole light attraction. Finally, it discusses some future prospects. Useful complementary information and animations are provided in the downloadable supplementary materials available at http://doi.org/10.1109/MAP.2022.3216773.

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INTRODUCTION

Metamaterials are artificial structures consisting of supramolecular but subwavelength particles that are engineered to provide medium properties beyond ($\mu\epsilon\tau\acute{\alpha}$) those available in conventional materials [1]–[3]. Following rudimentary ancient nanocomposites, medieval stained glasses, and 20th century artificial dielectrics, they have experienced spectacular developments in the past two decades, where they have diversified and expanded to a point that they represent nowadays a powerful new paradigm in science and technology. This evolution has been largely facilitated by the advent of *metasurfaces* [4], [5], which may be seen as the 2D counterpart of voluminal metamaterials, with the benefits of easier fabrication, lower loss, and greater flexibility as well as drastic functional extensions of frequency- or polarization-selective surfaces, reflect- or transmitarrays and spatial light modulators.

Most of the metamaterials and metasurfaces investigated until recently have been *static*, i.e., modulated only in space; we shall therefore refer to them as *SEMs*. A major advance in the field has been realized by making metamaterials *dynamic*, either by replacing the space modulation by a time modulation or by adding a time modulation to the space modulation. This introduction of the *dimension of time* as a new structural medium parameter has resulted in the metamaterial classes of TEM metamaterials, or TEMs [6]–[18], and STEM metamaterials, or STEMs [19]–[40]. [Note: The terminology "time-modulated metamaterials" and "space-time modulated metamaterials" applies to metamaterials that already have a spatial modulation *before* being temporally modulated, but *not* to—equally relevant!—metamaterials whose dynamic structure is really *formed* (and *engineered*) by a time or space-time modulation. Hence our introduction of the *general* terms TEMs and STEMs, and related terminology in Table 1.] Specifically, TEMs and STEMs are metamaterials that are formed by the variation (modulation) of a medium parameter in time and in both space and time, respectively, induced by an external drive. In the case of electromagnetic metamaterials, on which this article focuses, the modulated parameter may be the refractive index, the permittivity, the permeability, or any of the bianisotropic and higher order spatial-dispersion constitutive parameters and combination thereof, while the modulation drive may be acoustic (e.g., surface/bulk acoustic waves in a piezoelectric crystal), electronic (e.g., electric voltage variations in varactor chips), optical (e.g., laser pulses in semiconductor slabs), etc. [41], [42]. TEMs and STEMs may thus be seen as *medium—generally 3 + 1D, or 4D—extensions* of electronic and optical active lumped element and circuit systems, such as parametric amplifiers [43], [44] and acoustoelectric/optic modulators [45], [46].

This article presents a global and extended perspective of dynamic metamaterials. The *global aspect* consists of describing all metamaterials, including SEMs and TEMs, in terms of *space-time*—or *spacetime*—modulations, with various degrees of complexity and in connection with the physics of moving bodies, while the extended aspect concerns the generalization of STEMs with *uniform* (constant in both space and time) modulation velocity, i.e., USTEM metamaterials, or USTEMs, to STEMs with accelerated modulation, i.e., ASTEM metamaterials, or ASTEMs. [Note: The two spellings (with and without a hyphen) of the word spacetime are found in the literature on dynamic systems, whether for the noun or for the adjective. The one-word spelling is the universal standard when referring to the mathematical model that describes the merged nature of the space and time dimensions into a four-dimensional manifold in relativity physics (e.g., curved spacetime), while the spelling with a hyphen is preferable in reference to modulated structures, where the spatial and temporal features of the modulation are distinct and may exist independently of each other (e.g., space-time modulated metasurface). The present article follows this convention.] We shall refer to these diverse possible types of metamaterials as *GSTEMs*, where it is noted that ASTEMs may feature different orders (derivatives) of acceleration and, hence, subdivide in further classes. Table 1 summarizes the terminology.

TABLE 1. TERMINOLOGY AND ACRONYMS.

EM	Engineered Modulation (metamaterial)
SEM	Space EM
TEM	Time EM
STEM	Space-Time EM
USTEM	Uniform (-velocity) Space-Time EM
ASTEM	Accelerated Space-Time EM
GSTEM	Generalized Space-Time EM

RELATED ELECTRODYNAMIC SYSTEMS

Electromagnetic GSTEMs are not the only *electrodynamic systems*. They represent only the category of *moving-perturbation* (or moving-modulation) electrodynamic systems. Two other fundamental types of electrodynamic systems should be considered here, *vacuum moving-source* systems and *moving-matter* (or moving-body) systems [47], because GSTEMs support physical effects that are inherited from them, although, as we shall see, in distinct embodiments. We shall next describe and compare the three categories, with the help of the illustrations provided in Figure 1.

Vacuum moving-source systems, illustrated in Figure 1(a), are systems involving objects (e.g., star or car) that emit or reflect light while moving in *vacuum* relatively to the observer (e.g., Earth or road), with *vacuum* being defined as a portion of space that is essentially devoid of matter. [Note: We use here the term "light", as commonly done in the optics community, to designate electromagnetic waves and photons of any frequency or wavelength, for brevity, but a broader spectrum, including radio and terahertz waves, is implicitly assumed.] The earliest reported related effect is the *Bradley aberration* [Figure 1(a), top panel], whereby a terrestrial observer sees a star in a direction that is tilted toward the direction of the motion of Earth in its orbit around the sun [48]. Another effect, which commonly manifests itself in daily life with sound sources, is the *Doppler shift* [Figure 1(a), bottom panel], whereby an observer of a moving source sees the frequency of the wave emitted or reflected by that source as depending on its velocity, with larger and lower frequency for approaching and receding motion, respectively [49]. Vacuum moving-source systems are the simplest electrodynamic systems since they are restricted to light propagation without light-matter interaction.

Moving-matter/body systems, illustrated in Figure 1(b), are systems involving *matter* (e.g., water or dielectric) that moves relatively to the observer (e.g., a laboratory experimenter) and that supports the propagation of light emitted from the reference frame of the observer, with matter motion defined as a collective translation or/and rotation of atoms and molecules over distances that are much larger than the molecular scale; these systems involve thus typically moving solids, fluids or gases. A related effect is the *Fresnel-Fizeau drag* [Figure 1(b), top panel], whereby the speed of light is reduced or increased for downstream or upstream propagation in a moving fluid [50], [51]. Another effect that is of major importance in electrodynamics is *Röntgen magnetoelectric coupling* [Figure 1(b), bottom panel], whereby the motion (here, the rotation) of a solid submitted to an electric field induces a magnetic field in the frame of a rest observer due to the creation of surface polarization currents [52], [53]. Moving-matter/body systems are more complex than vacuum moving-source systems because of the addition of their matter drag and magnetoelectric-coupling effects on top of the aberration and Doppler shift effects occurring in vacuum moving-source systems.

Finally, *moving-perturbation/modulation systems*, illustrated in Figure 1(c), are systems involving a *perturbation* (e.g., an acoustic wave in a piezoelectric crystal) that moves relatively to the observer (e.g., a frame of an optical or microwave device) and that scatters light emitted from the reference frame of the observer, with perturbation motion defined as a traveling-wave (or standing-wave) *modulation* of some electromagnetic medium parameter, *without any net transfer of matter*, i.e., with motion restricted to oscillations of bound charges over submolecular distances (dielectric or magnetic polarization). [Note: Thermodynamics provides an insightful analogy to distinguish moving perturbation and moving matter in associating the former with heat conduction and the latter with heat convection.] A common example of such a system is the *acousto-optic modulator (AOM)* [Figure 1(c), top panel], whereby a periodic propagating perturbation ("spacetime modulation grating"), induced by variations of the molecular density of the medium from an electric signal (piezoelectricity), *deflects the diffraction* orders of the incident light in the direction of the perturbation via Bragg-Brillouin scattering [45], [46]. *GSTEMs* [Figure 1(c), bottom panel], particularly USTEMs and ASTEMs, belong to this category of electrodynamic systems, where they generalize AOM-type systems to multidimensional (2 + 1D = 3D and 3 + 1 D = 4D), multivelocity (uniform or nonuniform) [54], homogenized [38] and "new-physics" [36], [37] electrodynamic systems.

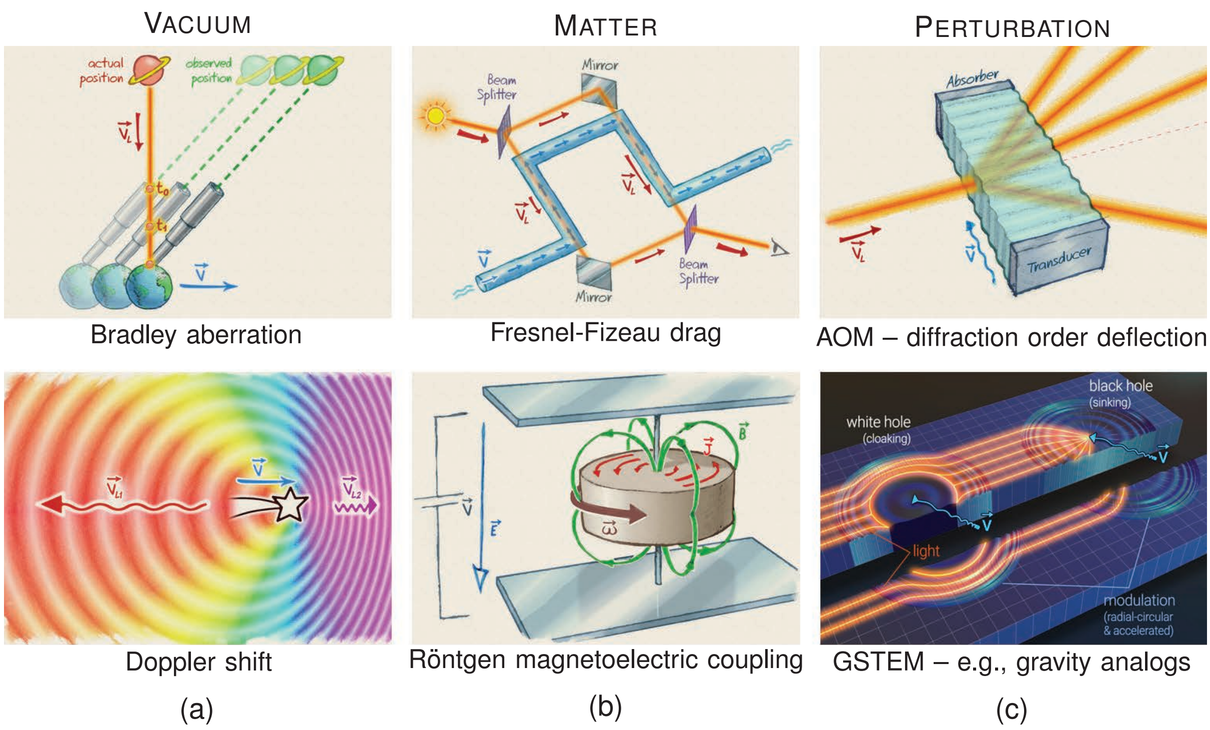

FIGURE 1. Different types of electrodynamic systems and related physical effects. (a) Moving sources in vacuum. (b) Moving matter or bodies. (c) Moving perturbation or modulation. AOM: acousto-optic modulator.

MOVING PERTURBATION VERSUS MOVING MATTER

Figure 2 compares the electrodynamic structures of the moving-matter/body systems [Figure 1(b)] on the one hand and the moving-perturbation/modulation systems [Figure 1(c)], which include GSTEMs, on the other hand. Figure 2(a) shows a moving-matter system (e.g., a sliding curling stone), where the atoms and molecules (matter) move together with the body, along with the comoving frame, K', at a velocity v with respect to the (fixed) laboratory frame, K. Figure 2(b) shows a moving-perturbation system (e.g., a dielectric slab excited by a laser-pump pulse or piezoelectric slab excited by a voltage source). In this case, the atoms and molecules oscillate about their bound position within the (solid) body, under the polarizing effect of the drive excitation, but do not experience any net motion in K. Only the related two perturbation interfaces move, inducing a STEM *pulse modulation* of the form $n(z,t) = n_0 + \Delta n \cdot \Pi[(z - v_m t)/D_m]$, where n_0 is the average refractive index, Δn is the modulation depth, $\Pi(\cdot)$ is the pulse function, D_m is the spatial extent of the pulse and $v_m = v$ is the velocity of the corresponding perturbation. Note that the atoms and molecules, while stationary in K, move with respect to K, in the opposite direction, with velocity $v'_{atom} = -v$.

Figure 2(c) shows a continuous (periodic) version of the pulse structure in Figure 2(b) (e.g., using a periodically pulsed laser-pump or electroacoustic drive), with the STEM medium function $n(z,t) = n_0 + \Delta n \cdot \mathrm{sgn}[(\cos(k_m z - \omega_m t) + 1]/2$, where $\mathrm{sgn}(\cdot)$ is the sign function, and k_m and ω_m, with $v_m = \omega_m/k_m$, are the spatial and temporal frequencies, respectively, of the modulation. Finally, Figure 2(d) shows a variant of Figure 2(c), where the drive excites the system from the top of the structure (e.g., an oblique laser pump illumination or electroacoustic source), under an angle θ with respect to the propagation axis, which provides *superluminal* modulation with velocity $v_m = v_{mz} = c/\sin\theta$ [36], reducing to instantaneous (pure time) modulation for $\theta = 0$.

Moving perturbation/modulation systems [Figure 2(b)–(d)], and particularly GSTEMs, are more promising than their moving-matter/body counterparts [Figure 2(a)] toward real-life applications because 1) they do not require cumbersome moving parts; 2) they easily attain relativistic velocities and accelerations; and 3) they possess richer functionality potential, resulting both from their dimensional extension of previous modulated systems and from their capability to mimic and transcend cosmological systems (e.g., equivalent horizons and black holes; superluminality and negative mass equivalent) [55], [56]. These are the reasons why GSTEMs are so attractive at this point of research in the field of metamaterials. We shall hereafter restrict our attention to GSTEMs and refer to the moving-matter/body dynamic systems only for the purpose of structural or property comparison.

PERSPECTIVE AND GENERALIZATION

Figure 3 depicts the proposed *global and extended perspective* of GSTEMs. The central part of the figure lists the related metamaterials—SEMs, TEMs, USTEMs, and ASTEMs (Table 1)—in the order of increasing dynamics generality from the bottom up. The periphery of the figure shows the spacetime (or Minkowski) diagrams [57] corresponding to the four main types of GSTEMs considered in this article,

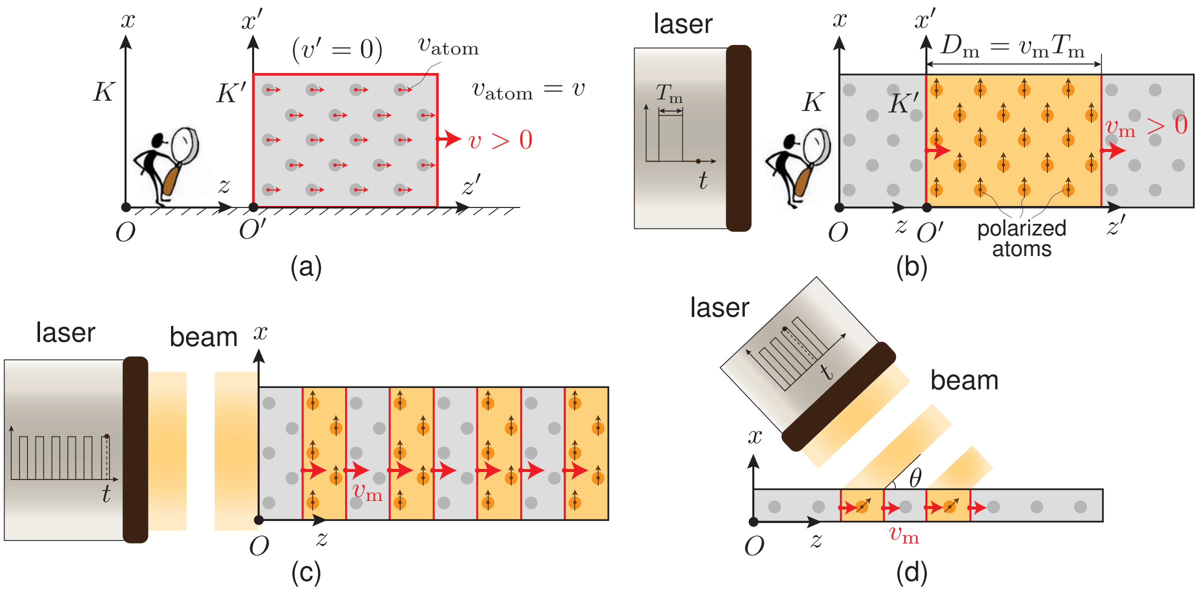

FIGURE 2. Comparison of the electrodynamic structures of the media involved in Figure 1(b) and (c). (a) Moving (matter) body. (b) Moving (perturbation) modulation. (c) Periodic version of (b). (d) Oblique excitation version of (c), where $v_m = v_{mz} = c/\sin\theta$, with even greater diversity attainable with a SEM or STEM mask at the output of the laser. Although the figures consider the specific example of laser-driven modulations, the principle holds for any type of modulation (optical, electronic, acoustic, mechanical, chemical, etc.) [41], [42].

with suggestive artistic illustrations (supplementary material section A). Such a global perspective offers multiple benefits, including 1) an elegant *classification*, based on the natural concept of spacetime structuration; 2) a powerful *unification*, suggesting insightful comparisons and cross-fertilization concepts (e.g., time duals of space systems [15], [58], [59] or space-time extensions of pure space/time systems [35], [60]); and 3) a *connection* to the physics of special relativity [61], [62], [63] and general relativity [62], [64], [65] (supplementary material section B.1), which rather involves sources [Figure 1(a)] or bodies [Figure 1(b)] moving in vacuum.

The concept of a *continuous medium* [66] is an idealization. In reality, all materials are formed by a more or less (crystal or amorphous) periodic collection of particles—atoms and molecules in the case of conventional materials and resonant scatterers in the case of metamaterials—which subtend the macroscopic response of the medium in terms of dipolar/multipolar responses at the microscopic scale. All materials can therefore be represented as periodically alternating regions of vacuum and particles, as suggested by the alternating gray-golden bands in the spacetime diagrams of Figure 3. This is true even when the medium exhibits a gradual variation, such as a GSTEM with the common sinusoidal modulation $n(z,t) = n_0 + \Delta n \cos(k_\mathrm{m} z - \omega_\mathrm{m} t)$, with $\lambda_\mathrm{m} = 2\pi/k_\mathrm{m}$ and $T_\mathrm{m} = 2\pi/\omega_\mathrm{m}$, which may be considered as a gradual version of the structure in Figure 3 and which is common in practice. In the *metamaterial—subwavelength and subperiod—regime*, corresponding to the twofold condition $\lambda_\mathrm{m} \ll \lambda$ and $T_\mathrm{m} \ll T$, where λ and T are the smallest wavelength and period of the wave, respectively, the wave propagating in the medium is myopic to such fine detail of the sinusoidal modulation; it probes only the index extrema, with blurred transitions between them, and the modulation can therefore be safely approximated by the discrete *bilayer function* $n(z,t) = n_0 + \Delta n \cdot \mathrm{sgn}[(\cos(k_\mathrm{m} z - \omega_\mathrm{m} t) + 1]/2$ ["Moving Perturbation Versus Moving Matter" section and Figure 2(c)]. [Note: Although this article focuses on the metamaterial (i.e., homogeneous) regime, the concepts of GSTEMs naturally extend to the Bragg regime, where the GSTEM structures, better called then GSTEM *crystals*, exhibit interesting oblique bandgap configurations and physical properties [19], [30], [35].] Thus, the spacetime diagrams of GSTEMs can *generally* be represented by the periodic bilayer spacetime patterns shown in Figure 3.

SEMs [Figure 3(a)] are GSTEMs whose parameters vary only in space. They represent the particular static limit case of GSTEMS with zero modulation velocity, $v_\mathrm{m} = 0$, and include conventional metamaterials [1]–[3] as well as photonic crystals in the long-wavelength regime [67]. The TEMs in Figure 3(b) are GSTEMs whose features do not vary in space but in time, as with some parametric amplifiers [43] and solid-state time crystals [68], [69]. They represent the particular *instantaneous* limit

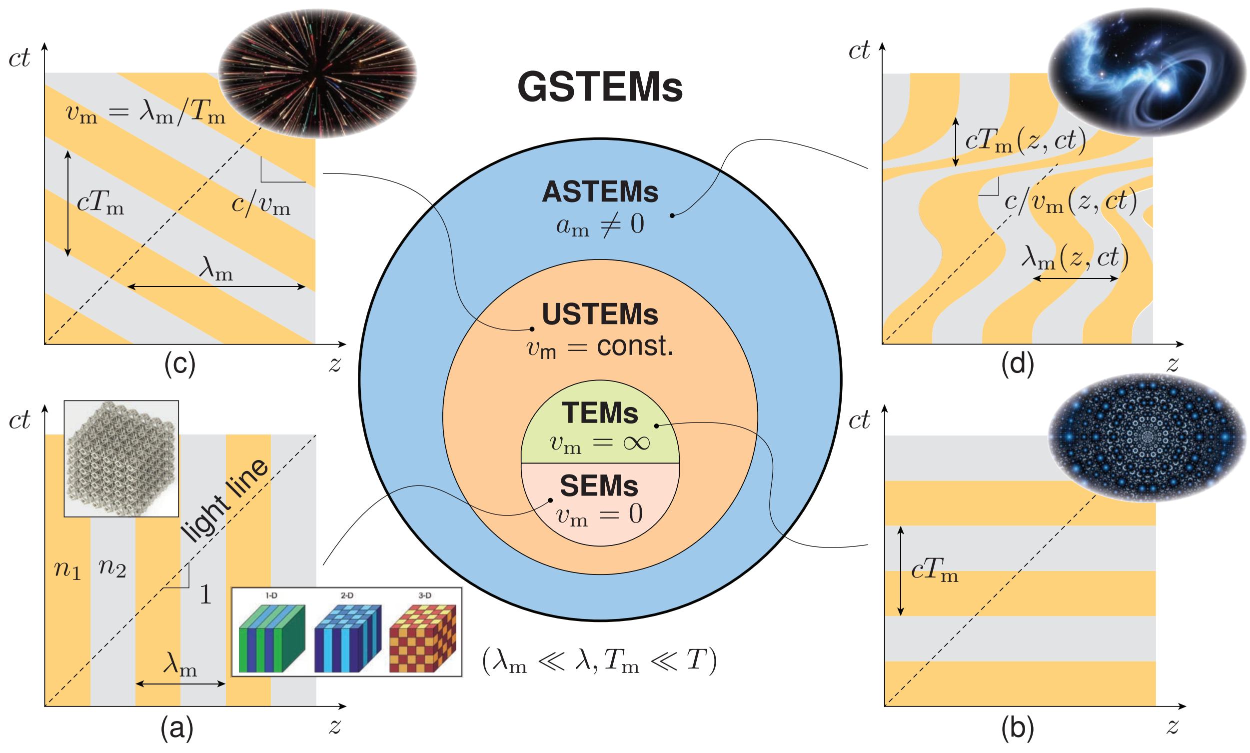

FIGURE 3. GSTEMs. (a) SEMs. (b) TEMs. (c) USTEMs. (d) ASTEMs. The subscript "m" refers to "modulation", n_1 and n_2 are the refractive indices of the constituent media, which are assumed to be isotropic and nondispersive, and z represents the spacetime hyperspace, which may include up to three spatial dimensions (x, y, z). Const.: constant.

case of GSTEMS with infinite modulation velocity, $v_{\mathrm{m}} = \infty$ [6]–[18]. We have next actual STEMs, with modulation occurring both in space and in time. USTEMs [Figure 3(c)] are characterized by a constant modulation velocity, $v_{\mathrm{m}} = \text{const.}$, as typical acousto-optical modulators [top panel of Figure 1(c)] [19]–[40], while ASTEMs [Figure 3(d)], introduced only recently [54], are characterized by an accelerated modulation, $a_{\mathrm{m}} \neq 0$, which may be constant in the moving frame (constant proper acceleration), or have nonzero temporal derivatives—jerk ($\partial a_{\mathrm{m}}/\partial t \neq 0$), snap ($\partial^2 a_{\mathrm{m}}/\partial t^2 \neq 0$), crackle ($\partial^3 a_{\mathrm{m}}/\partial t^3 \neq 0$), pop ($\partial^4 a_{\mathrm{m}}/\partial t^4 \neq 0$), etc. [70].

INTERFACE PHYSICS

As discussed in the "Perspective and Generalization" section and illustrated by the spacetime diagrams in Figure 3, GSTEMs can be modeled by alternating isotropic medium layers. The *interfaces* delimiting these layers are therefore the main discontinuities or nonuniformities of the structure and represent hence the entities that underpin the light-matter interaction of the metamaterial. For this reason, this section is dedicated to GSTEM interfaces, while the "Metamaterial Physics" section will reveal how the related principles extend to complete GSTEM media.

Let us start with the simplest cases of SEM and TEM interfaces. The electrodynamics of these interfaces is described in Figure 4, with Figure 4(a) and (b) representing the SEM and TEM cases, respectively [37]. When a wave hits a simple interface, or *SEM interface*, it splits into a reflected wave and a transmitted wave, which propagate in opposite directions over time, with well-known scattering (Fresnel) coefficients γ and τ, as shown in the left panel of Figure 4(a). These coefficients are found by enforcing the conservation of the tangential **E** and **H** fields at the (spatial) interface discontinuity, which is required to avoid making the **B** and **D** fields singular (at the interface) through the spatial derivative ($\nabla\times$) in Maxwell equations [37]. On the other hand, the transmitted wavelength is compressed (or the wavenumber increases) if the second medium is denser, as shown in the right panel of Figure 4(a). Note that this transformation does not involve any change of temporal frequency ($\Delta\omega = 0$, energy conservation) since the discontinuity is purely spatial.

The problem of a simple instantaneous interface, or *TEM interface*, is the perfect dual of that of the SEM interface. Now, the incident wave splits into a later backward wave and a later forward wave, which also propagate in opposite directions, but in the same (later) medium and with different scattering coefficients [6], ζ and ξ, as shown in the left panel of Figure 4(b). These coefficients are found by enforcing the conservation of **B** and **D** at the (temporal) interface discontinuity, which is required to avoid making **E** and **H** singular (at the interface) through the temporal derivative ($\partial/\partial t$) in Maxwell equations [37]. The two scattered waves are red-shifted (or their frequency decreases) if the second medium is denser, as shown in the right panel of Figure 4(b). It is thus the temporal frequency that changes in this transformation, while the spatial frequency remains unchanged ($\Delta k_z = 0$, momentum conservation) since the discontinuity is purely temporal. This space-to-time duality has several applications, including the *inverse prism*, a device that, instead of decomposing colors into angles as the Newton prism, maps angles into colors [58].

STEM interfaces may be considered as spacetime extensions of their SEM and TEM counterparts. The electrodynamics of USTEM interfaces is described in Figure 5, with Figure 5(a) and (b) representing the *subluminal* regime [$v_{\mathrm{m}z} < c/\max(n_1, n_2)$, e.g., as in Figure 2(c)] and the *superluminal* regime [$v_{\mathrm{m}z} > c/\min(n_1, n_2)$, e.g., as in Figure 2(d)], respectively [37]. [Note: We avoid here the *interluminal regime*, corresponding to $c/\max(n_1, n_2) < v_{\mathrm{m}z} < c/\min(n_1, n_2)$, which involves complex physics that has not been fully investigated so far [20], [71], [72].] Now, the modulation occurs simultaneously in space and time, as in all the electrodynamics systems represented in Figure 1. Scattering is space like—with reflected and transmitted waves—for the subluminal case and time-like—with later backward and later forward waves—in the superluminal case [37]. The corresponding scattering coefficients are found by enforcing the continuity of $(\mathbf{E}', \mathbf{H}')$ in the subluminal comoving frame (K', where $\Delta\omega' = 0$) and of $(\mathbf{D}', \mathbf{B}')$ in the superluminal simultaneity frame (K', where $\Delta k'_z = 0$) [60], which reveals, upon inverse-Lorentz transformation [63], [73], [74] (supplementary material section B.2), the conservation of the quantities $E_x - v_{\mathrm{m}} B_y$ and $B_y - v_{\mathrm{m}} E_x/c^2$ for the plane wave with components (E_x, B_y, k_z) and of $E_y + v_{\mathrm{m}} B_x$ and $B_x + v_{\mathrm{m}} E_y/c^2$ for $(E_y, -B_x, k_z)$ [37], [74]–[76]. On the other hand, the spectral transitions, whose reflective and transmissive parts are manifestations of the *Doppler effect* and of an *index contrast effect*, respectively, are oblique since the

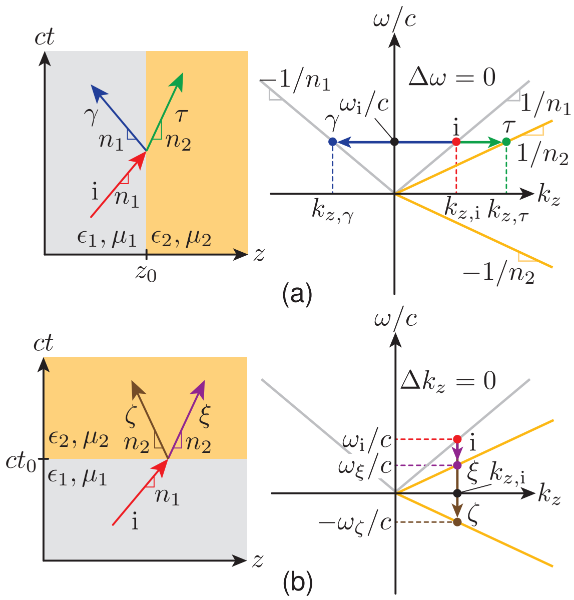

FIGURE 4. Electrodynamics of the simplest GSTEM interfaces, represented in terms of direct (left) and inverse (right) spacetime diagrams, for the case of normal incidence. (a) SEM interface [Figure 3(a)]. (b) TEM interface [Figure 3(b)].

discontinuity is both spatial and temporal, leading to simultaneous spatial and temporal frequency transformations [77]. In the case of *oblique incidence*, the dispersion curves, between which the oblique transitions occur, are altered by the change of the incident momentum on the interface, and the scattered angles are deflected toward/against the direction of motion in the sub/superluminal cases [60] (supplementary material section C).

Finally, an ASTEM interface may be seen as a generalization of a USTEM interface, where both the direct-spacetime interfaces and the normal-incidence dispersion lines change from straight to curved, as illustrated in Figure 3(d) for the case of an ASTEM metamaterial with a complex acceleration profile, including direction reversal, and, hence, jerk $(\partial a_{\mathrm{m}}/\partial t \neq 0)$. While an accelerated system is *locally* uniform, so that special relativity and Lorentz transformation apply *locally*, it globally requires a much more complex treatment that belongs to the realm of general relativity and, hence, differential geometry [65], [78], [79]. In this case, the (linear) Lorentz transformations must be replaced by nonlinear transformations corresponding to the type of acceleration at hand [56], [65], the simplest being the K'-constant and rectilinear (or proper) acceleration, which is associated with Rindler transformations (supplementary material section B.3). Physically, the USTEM space-time transformation (diffraction-Doppler) effect is promoted to a *space-time chirping effect* [80]–[83], with still little explored physics and application potential. A first application is the recently reported ASTEM waveform generator, whose properly designed acceleration trajectory allows virtually arbitrary pulse shaping [54].

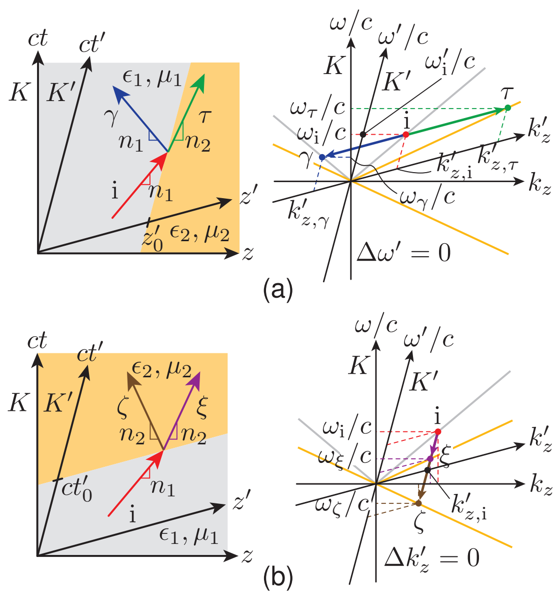

FIGURE 5. Electrodynamics of USTEM interfaces [Figure 3(c)], represented in terms of direct (left) and inverse (right) spacetime diagrams, for the case of normal incidence. (a) Subluminal (space-like) regime. (b) Superluminal (time-like) regime.

METAMATERIAL PHYSICS

Stacking the different GSTEM bilayer unit cells introduced in the "Perspective and Generalization" section, which involve the interfaces analyzed in the "Interface Physics" section, leads to the formation of the corresponding GSTEM structures in Figure 3 [Figure 4(a) → Figure 3(a), Figure 4(b) → Figure 3(b), Figure 5 → Figure 3(c), curved version of Figure 5 → Figure 3(d)]. Before homogenization, these structures are generally periodic spacetime structures (or, possibly, only locally quasi-periodic with a period gradient for the case of ASTEMs) or *GSTEM crystals*, and they involve the same direct (scattering) and inverse (transition) spacetime transformations as their interface counterparts. However, these crystals represent *spacetime-distributed structures*, as opposed to spacetime-localized discontinuities, and support therefore in addition multiple spacetime scattering and multiple spacetime transitions, leading to specific spacetime crystal properties [35].

The GSTEM crystal can be *spatially* 1D, 2D, or 3D, as shown in the bottom right inset of Figure 3(a). If the wave of interest propagates in a space of a dimension that is larger than the spatial dimension of the crystal [specifically, oblique incidence in a 1D crystal and off-plane incidence in a 2D crystal], the crystal is seen by the wave as anisotropic, with different tangential field components for different (e.g., p and s) wave polarizations. Since this represents the most general and most common configuration in early GSTEMs, let us consider henceforth this case of an *anisotropic crystal*, noting in passing that anisotropy, with an anisotropic medium being defined as a medium that exhibits different properties in different directions, is a purely spatial concept since time is monodimensional.

In moving-matter dynamic systems [e.g., Figure 2(a)], the problems are ideally solved in the comoving frame (K'), where everything is stationary, and hence, the boundary conditions and the constitutive relations are both trivial. The situation is more complicated in moving-perturbation systems [e.g., Figure 2(b)] because they involve motion in *both* frames [e.g., moving interfaces in K and (backward) moving atoms and molecules in K']! Let us still choose the K' frame on the grounds that moving matter with stationary boundaries is a known problem [84], whereas moving boundaries is a more difficult problem. To enter the metamaterial regime, we homogenize the GSTEM by prescribing subwavelength $[\max(\lambda_{\mathrm{m}}) \ll \min(\lambda)]$ and subperiod $[\max(T_{\mathrm{m}}) \ll \min(T)]$ operation. This operation leads to GSTEM-metamaterial anisotropic constitutive parameters with the same tensorial structure as that of their crystal parent [38], [85]. However, another effect is present in K': the contradirectional motion of polarized atoms and molecules [e.g., focus on the moving K' frame with K-stationary atoms and molecules in Figure 2(b)] ($v'_{\mathrm{atom}} = -v$); the problem is thus a *moving-matter* problem in K', with the usual *Fresnel-Fizeau* drag and related *motion bianisotropy* [i.e., Röntgen magnetoelectric coupling, bottom panel of Figure 1(b)] (supplementary material sections D.1 and E.1), due to the magnetic part of the Lorentz force, $\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$] [74], [75] that is associated with the magnetic part of the Lorentz force (supplementary material section

D.2). GSTEMs are thus generally *bianisotropic* in K' [75], [76], [86], [87] (supplementary material section E.1).

The fundamental properties of a GSTEM metamaterial, in the frame of interest (K), may be inferred from these preliminary considerations with the help of the spectral graphs provided in Figure 6, with Figure 6(a) representing the problem from the K' viewpoint and Figure 6(b) representing the problem from the K viewpoint. Since the Fresnel-Fizeau drag occurs in the $-z'(=-z)$ direction in K', as suggested in the panel at the extreme right bottom of the figure, the wave velocity, $v'_{\mathrm{gz}} = v'_{\mathrm{pz}} = c/n'$ (nondispersive medium assumption), is increased in the $-k'_z$ (or $-z'$) direction and decreased in the $+k'_z$ direction, i.e., $1/n^{-\prime} > 1/n^{+\prime}$, so that the spectral cone is tilted toward the positive k'_z direction [top panel of Figure 6(a)]. As a result, the isofrequency curve at $\omega' = \omega'_0$ is a right-shifted ellipse [bottom panel of Figure 6(a)], located between the $n'_1(\mathbf{k}')$ and $n'_2(\mathbf{k}')$, also right-shifted ellipses, which is a manifestation of the expected bianisotropy (off-centered ellipse → bianisotropy, centered ellipse → anisotropy, centered circle → isotropy) (supplementary material section E.2) and associated *nonreciprocity* [88]. The counterclockwise rotation, by the angle φ', of the group velocity, $\mathbf{v}'_{\mathrm{g}}(\mathbf{k}') = \nabla_{\mathbf{k}'}\omega'(\mathbf{k}')$, with respect to the phase velocity, $\mathbf{v}'_{\mathrm{p}} = (\omega'/k')\hat{\mathbf{k}}'$, corresponds to the addition of a $-z'$ directed component to the velocity and, thus, clearly shows the backward effect of the Fresnel-Fizeau drag.

We finally need to perform the required (Lorentz, Rindler, etc.) inverse transformation from K' to K to complete the resolution of the electrodynamics problem at hand. Obviously, the elimination of the motion of the atoms and molecules in the transition from K' to K eliminates motion-related bianisotropy since $v_{\mathrm{atom}} = 0$. In contrast, the exact nature (bianisotropic, anisotropic, biisotropic, homoisotropic) of the K solution is difficult to predict and can be precisely found only by performing the exact K'-to-K inverse transformation (inverse-*Lorentz* transformation, $\mathcal{L}^{-1}$, in the USTEM case of Figure 6) (supplementary material section E.3). Figure 6(b) shows a typical result. Note that the $n'_1(\mathbf{k}')$ and $n'_2(\mathbf{k}')$ isofrequency ellipses have transformed to simple centered circles, corresponding to the expected curves for the assumed isotropic constituent media with (scalar) refractive indices n_1 and n_2.

Interestingly, the USTEM curve is still a right-shifted ellipse, with a deflection of the group velocity by an angle of φ, still in the counterclockwise direction with respect to the phase velocity; this deflection is necessarily an effect of the moving-modulation *interfaces* since matter motion does not exist anymore. We have here $\varphi < \varphi'$, indicating that the deflection due to the modulation (K frame) is smaller than that due to the matter (K frame), but the effect is greater and can reach $\varphi > \varphi'$ if the constitutive media have less-than-one indices ($n_1, n_2 < 1$) (plasma-type media). Most importantly, the observed *light deflection* is contradirectional to the modulation ($+z$) and, hence, *opposite to the direction of the drag for a moving body*, consistently with the finding in [89].

This GSTEM deflection effect is quite distinct from the Fresnel-Fizeau drag. Indeed, it does not involve any motion of matter that would "push" or "pull"—i.e., *drag*—light. It is rather an effect of *spacetime weighted averaging*, as first suggested in [35]. This effect is illustrated Figure 7. In the SEM problem, represented in Figure 7(a), light spends on average the same amount of time in medium 1 and in medium 2 in the forward and backward directions so that the corresponding effective metamaterial indices are equal, i.e., $n^+ = n^-$. In the K'-USTEM problem, represented in Figure 7(b), light is subjected to the conventional Fresnel-Fizeau drag and propagates therefore faster in the backward (downstream) direction than in the forward (upstream) direction so that $n^{+\prime} > n^{-\prime}$. In the K-USTEM problem, represented in Figure 7(c), the spacetime slopes are the same as in the SEM problem since no matter motion occurs, but the medium is spacetime-wise oblique, here tilting in the forward direction, and the following—rather subtle—effect occurs due to this tilting. Consider, for simplicity, that $n_2 \gg n_1$, as in Figure 7 [where the vertical axes have been denormalized to ensure a restricted graphical aspect ratio without introducing (unphysical) superluminal light curves]. Then, light spends much more time (see dashed lines) in medium 2, the slower medium, in the forward direction, where it propagates almost parallel to the medium trajectory, than in the backward direction, where the propagation is relatively more perpendicular to the medium trajectory, while the

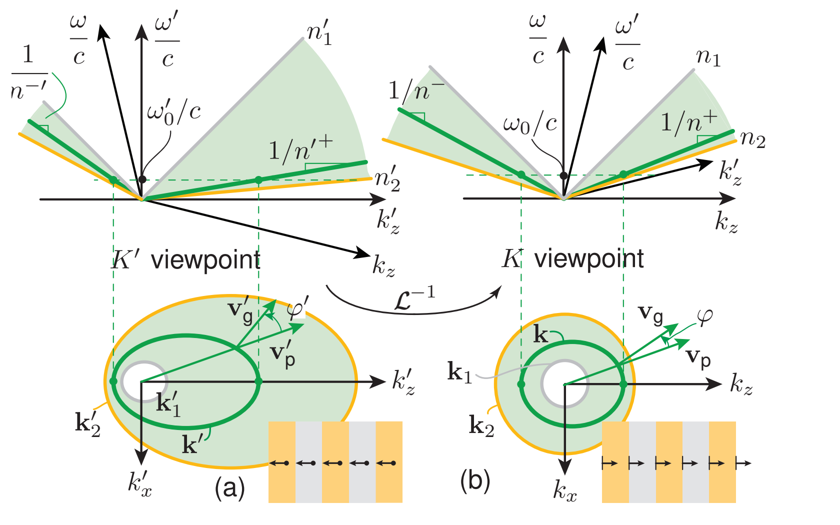

FIGURE 6. Spectral analysis for a 1 + 1D-(z; t) USTEM metamaterial under oblique [(k_z, k_x)] incidence with modulation traveling in the $+z$ direction, including the $(\omega, k_z)^{(\prime)}$ -cuts at $k_x^{(\prime)} = 0$ of the $(\omega, k_z, k_x)^{(\prime)}$ cones (top panels) and $(\omega_0, k_z, k_x)^{(\prime)}$ isofrequency projections (bottom panels), compared with the cases of the bulk constituent media, with refractive indices $n'_1(\mathbf{k}')$ and $n'_2(\mathbf{k}')$ or n_1 and n_2. (a) K' viewpoint, with Fizeau-Fresnel drag, due to the motion of matter, in the $-z$ direction. (b) K viewpoint, with "inverse drag," due to the motion of the interfaces.

ratio of the forward to backward traveled distances increases in a much smaller ratio, as clearly apparent in the figure. As a result, $n^+ > n^-$, and, hence, $v^+ < v^-$, consistently with the observation in Figure 6. This is really, as announced, a spacetime weighted averaging phenomenon; an exact mathematical formula for this phenomenon is provided in [35] (supplementary material section E.4), and a related animation is provided in the file GSTEM_Space-Time_Weighted_Averaging_phenomenon.mp4 of the supplementary material.

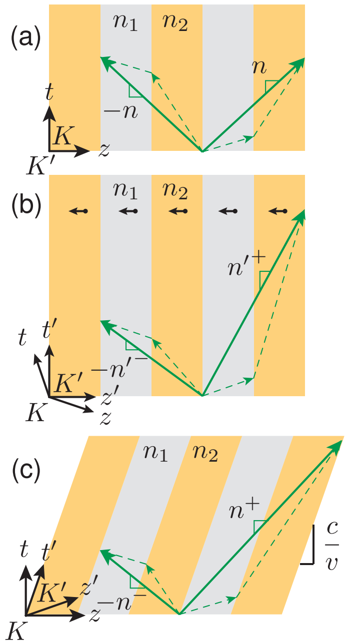

FIGURE 7. GSTEM weighted averaging deflection. (a) SEM. (b) USTEM, K' viewpoint. (c) USTEM, K viewpoint.

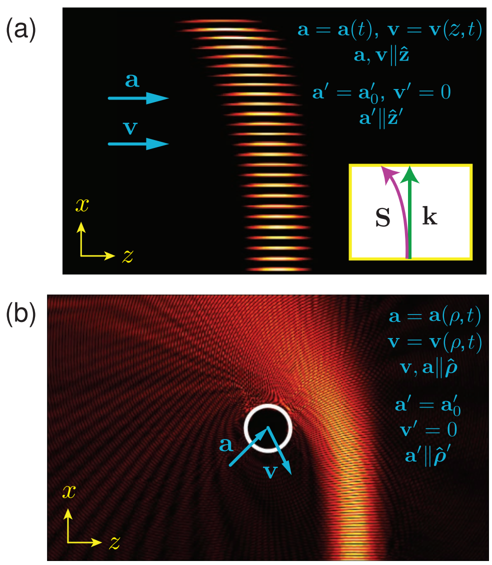

FIGURE 8. Examples of ASTEM metamaterials and related transformations of a light beam (injected from the bottom). (a) Rectilinear (Rindler metric [91], [92]) ASTEM, inducing curved light deflection. (b) Black hole (Schwarzschild metric [56], [93]) ASTEM, attracting and partly absorbing light.

As ASTEM interfaces are the curved-spacetime generalization of USTEM interfaces ("Interface Physics" section), ASTEM metamaterials are the curved-spacetime generalization of USTEM metamaterials. Therefore, the principles exposed in conjunction with the USTEM graphs in Figures 6 and 7 largely extend to ASTEMs metamaterials, although their rigorous treatment requires a quantum jump from the *theory of special relativity* [61], [63], routinely applied to USTEMs, to the *theory of general relativity* [56], [64]. Figure 8 presents two illustrative examples of ASTEM metamaterials [85]. Figure 8(a) shows a rectilinear ASTEM metamaterial, which exhibits the modulation-contradirectional group velocity deflection $[\mathbf{v}_\mathrm{g}(\mathbf{r}) \| \mathbf{S} = \mathbf{E} \times \mathbf{H}]$ and straight phase velocity $[\mathbf{v}_\mathrm{p}(\mathbf{r}) \| \mathbf{k}]$ propagation predicted in Figure 6; the beam curvature can be here qualitatively inferred from piecewise (straight) USTEM deflection. Figure 8(b) shows a Schwarzschild ASTEM black hole, which attracts and absorbs light like a cosmic black hole, an effect that is unattainable in simple graded-index lenses [90].

FUTURE PROSPECTS

Given their very fundamental nature and virtually unlimited diversity, GSTEMs have a formidable *potential for scientific and technological innovation*. The *scientific prospects* include

1) the study of the properties of higher dimensional ($2+1\mathrm{D} = 3\mathrm{D}$ and $3+1\mathrm{D} = 4\mathrm{D}$) GSTEM (unbounded) structures.
2) the analysis of the scattering and diffraction at the interfaces and wedges of *spacetime-truncated* [35] *GSTEMs*.
3) the exploration of new *GSTEM geometries* (e.g., Rindler, Schwarzschild, Kerr, jerk, snap, crackle, etc.) in both the homogeneous and Bragg regimes.
4) the extension of *gravity analogs*, currently restricted to interface horizons [94], such as white hole or Bose-Einstein condensate analogs.
5) the elaboration of new electrodynamic computational tools for spacetime nonuniformities, using, for instance, foliation decomposition [95], and for modulation-related multiphysics,
6) the investigation of novel GSTEM physics (e.g., superluminal and interluminal scattering; spacetime reversal; generalized spacetime metrics; spacetime quantum; and subcycle phenomena).

The *technological prospects* include, on the one hand, the development of efficient modulation platforms and techniques (e.g., acoustic, electronic, and optical) for the experimental implementation of the new GSTEM phenomena, and on the other hand, the identification and demonstration of novel related applications. Many potential USTEM-related applications have been identified in [37]; we expect that these applications will generalize to ASTEM-type systems, with extra opportunities offered by various spacetime curvatures and generalized spacetime "chirping".

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REFERENCES

[1] C. Caloz and T. Itoh, *Electromagnetic Metamaterials: Transmission Line Theory and Microwave Applications*. Hoboken, NJ, USA: Wiley, 2005.

[2] N. Engheta and R. W. Ziolkowski, Eds. *Metamaterials: Physics and Engineering Explorations*. Hoboken, NJ, USA: Wiley, 2006.

[3] F. Capolino, Ed. *Metamaterials Handbook*, vol. 2. Boca Raton, FL, USA: CRC Press, 2009.

[4] S. I. Bozhevolnyi, P. Genevet, and F. Ding, Eds. *Metasurfaces: Physics and Applications*. Basel, Switzerland: MDPI, 2018.

[5] K. Achouri and C. Caloz, *Electromagnetic Metasurfaces: Theory and Applications*. Hoboken, NJ, USA: Wiley, 2021.

[6] F. R. Morgenthaler, "Velocity modulation of electromagnetic waves," *IRE Trans. Microw. Theory Techn.*, vol. 6, no. 2, pp. 167–172, Apr. 1958, doi: 10.1109/TMTT.1958.1124533.

[7] D. Holberg and K. Kunz, "Parametric properties of fields in a slab of time-varying permittivity," *IEEE Trans. Antennas Propag.*, vol. 14, no. 2, pp. 183–194, Mar. 1966, doi: 10.1109/TAP.1966.1138637.

[8] J. A. Richards, *Analysis of Periodically Time-Varying Systems*. Berlin, Germany: Springer-Verlag, 1983.

[9] D. K. Kalluri, *Electromagnetic of Time Varying Complex Media: Frequency and Polarization Transformer*. Boca Raton, FL, USA: CRC Press, 2018.

[10] A. Shlivinski and Y. Hadad, "Beyond the Bode-Fano bound: Wideband impedance matching for short pulses using temporal switching of transmission-line parameters," *Phys. Rev. Lett.*, vol. 121, no. 20, Nov. 2018, Art. no. 204301, doi: 10.1103/PhysRevLett.121.204301.

[11] M. Mirmoosa, G. Ptitcyn, V. Asadchy, and S. Tretyakov, "Time-varying reactive elements for extreme accumulation of electromagnetic energy," *Phys. Rev. Appl.*, vol. 11, no. 1, Jan. 2019, Art. no. 014024, doi: 10.1103/PhysRevApplied.11.014024.

[12] G. Ptitcyn, M. S. Mirmoosa, and S. A. Tretyakov, "Time-modulated meta-atoms," *Phys. Rev. Res.*, vol. 1, no. 2, Sep. 2019, Art. no. 023014, doi: 10.1103/PhysRevResearch.1.023014.

[13] D. Ramaccia, D. L. Sounas, A. V. Marini, A. Toscano, and F. Bilotti, "Electromagnetic isolation induced by time-varying metasurfaces: Nonreciprocal Bragg grating," *IEEE Antennas Wireless Propag. Lett.*, vol. 19, no. 11, pp. 1886–1890, Nov. 2020, doi: 10.1109/LAWP.2020.2996275.

[14] Y. Mazor and A. Alù, "One-way hyperbolic metasurfaces based on synthetic motion," *IEEE Trans. Antennas Propag.*, vol. 68, no. 3, pp. 1739–1747, Mar. 2020, doi: 10.1109/TAP.2019.2949134.

[15] V. Pacheco-Peña and N. Engheta, "Antireflection temporal coatings," *Optica*, vol. 7, no. 4, pp. 323–331, Apr. 2020, doi: 10.1364/OPTICA.381175.

[16] Y. Mazor, M. Cotrufo, and A. Alù, "Unitary excitation transfer between coupled cavities using temporal switching," *Phys. Rev. Lett.*, vol. 127, no. 1, Jul. 2021, Art. no. 013902, doi: 10.1103/PhysRevLett.127.013902.

[17] D. Ramaccia, A. Alù, A. Toscano, and F. Bilotti, "Temporal multilayer structures for designing higher-order transfer functions using time-varying metamaterials," *Appl. Phys. Lett.*, vol. 118, no. 10, Mar. 2021, Art. no. 101901, doi: 10.1063/5.0042567.

[18] E. Galiffi et al., "Photonics of time-varying media," *Adv. Photon.*, vol. 4, no. 1, Jan. 2022, Art. no. 014002, doi: 10.1117/1.AP.4.1.014002.

[19] E. S. Cassedy and A. A. Oliner, "Dispersion relations in time-space periodic media: Part I–stable interactions," *Proc. IEEE*, vol. 51, no. 10, pp. 1342–1359, Oct. 1963, doi: 10.1109/PROC.1963.2566.

[20] E. S. Cassedy, "Dispersion relations in time-space periodic media: Part II–unstable interactions," *Proc. IEEE*, vol. 55, no. 7, pp. 1154–1168, Jul. 1967, doi: 10.1109/PROC.1967.5775.

[21] J. T. Mendonça, *Theory of Photon Acceleration*. Bristol, U.K.: Institute of Physics Publishing, 2007.

[22] A. B. Shvartsburg, "Optics of nonstationary media," *Phys. Uspekhi*, vol. 48, no. 8, pp. 797–823, Aug. 2005, doi: 10.1070/PU2005v048n08ABEH002119.

[23] K. A. Lurie, *An Introduction to the Mathematical Theory of Dynamic Materials*. Cham, Switzerland: Springer-Verlag, 2007.

[24] F. Biancalana, A. Amann, A. V. Uskov, and E. O'Reilly, "Dynamics of light propagation in spatiotemporal dielectric structures," *Phys. Rev. E*, vol. 75, no. 4, Apr. 2007, Art. no. 046607, doi: 10.1103/PhysRevE.75.046607.

[25] Z. Yu and S. Fan, "Optical isolation based on nonreciprocal phase shift induced by interband photonic transitions," *Appl. Phys. Lett.*, vol. 94, no. 17, p. 171,116, Apr. 2009, doi: 10.1063/1.3127531.

[26] A. Bahabad, M. M. Murnane, and H. C. Kapteyn, "Quasi-phase-matching of momentum and energy in nonlinear optical processes," *Nat. Photonics*, vol. 4, pp. 570–575, Aug. 2010, doi: 10.1038/nphoton.2010.122.

[27] A. M. Shaltout, A. Kildishev, and V. M. Shalaev, "Time-varying metasurfaces and Lorentz non-reciprocity," *Opt. Mater. Exp.*, vol. 5, no. 11, pp. 2459–2467, Nov. 2015, doi: 10.1364/OME.5.002459.

[28] N. A. Estep, D. L. Sounas, J. Soric, and A. Alù, "Magnetic-free non-reciprocity and isolation based on parametrically modulated coupled-resonator loops," *Nat. Phys.*, vol. 10, no. 12, pp. 923–927, Dec. 2014, doi: 10.1038/nphys3134.

[29] Y. Hadad, D. L. Sounas, and A. Alù, "Space-time gradient metasurfaces," *Phys. Rev. B*, vol. 92, no. 10, Sep. 2015, Art. no. 100304, doi: 10.1103/PhysRevB.92.100304.

[30] N. Chamanara, S. Taravati, Z.-L. Deck-Léger, and C. Caloz, "Optical isolation based on space-time engineered asymmetric photonic band gaps," *Phys. Rev. B*, vol. 96, no. 15, Oct. 2017, Art. no. 155409, doi: 10.1103/PhysRevB.96.155409.

[31] J. Vila, R. K. Pal, M. Ruzzene, and G. Trainiti, "A Bloch-based procedure for dispersion analysis of lattices with periodic time-varying properties," *J. Sound Vib.*, vol. 406, pp. 363–377, Oct. 2017, doi: 10.1016/j.jsv.2017.06.011.

[32] S. Taravati, "Giant linear nonreciprocity, zero reflection, and zero band gap in equilibrated space-time varying media," *Phys. Rev. A*, vol. 9, no. 6, Jun. 2018, Art. no. 064012, doi: 10.1103/PhysRevApplied.9.064012.

[33] S. Taravati, "Aperiodic space-time modulation for pure frequency mixing," *Phys. Rev. B*, vol. 97, no. 11, Jun. 2018, Art. no. 115131, doi: 10.1103/PhysRevB.97.115131.

[34] N. Chamanara, Z.-L. Deck-Léger, C. Caloz, and D. Kalluri, "Unusual electromagnetic modes in space-time modulated dispersion-engineered media," *Phys. Rev. A*, vol. 97, no. 6, Oct. 2018, Art. no. 063829, doi: 10.1103/PhysRevA.97.063829.

[35] Z.-L. Deck-Léger, N. Chamanara, M. Skorobogatiy, M. G. Silveirinha, and C. Caloz, "Uniform-velocity spacetime crystals," *Adv. Photon.*, vol. 1, no. 5, Oct. 2019, Art. no. 056002, doi: 10.1117/1.AP.1.5.056002.

[36] C. Caloz and Z.-L. Deck-Léger, "Spacetime metamaterials, part I: General concepts," *IEEE Trans. Antennas Propag.*, vol. 68, no. 3, pp. 1569–1582, Mar. 2019, doi: 10.1109/TAP.2019.2944225.

[37] C. Caloz and Z.-L. Deck-Léger, "Spacetime metamaterials, part II: Theory and applications," *IEEE Trans. Antennas Propag.*, vol. 68, no. 3, pp. 1583–1598, Mar. 2020, doi: 10.1109/TAP.2019.2944216.

[38] P. A. Huidobro, M. G. Silveirinha, E. Galiffi, and J. B. Pendry, "Homogenization theory of space-time metamaterials," *Phys. Rev. Appl.*, vol. 16, no. 1, Jul. 2021, Art. no. 014044, doi: 10.1103/PhysRevApplied.16.014044.

[39] X. Wang, V. S. Asadchy, S. Fan, and S. Tretyakov, "Space-time metasurfaces for power combining of waves," *ACS Photon.*, vol. 8, no. 10, pp. 3034–3041, Sep. 2021, doi: 10.1021/acsphotonics.1c00981.

[40] G. Milton and O. Mattei, "Field patterns: A new mathematical object," *Proc. Roy. Soc. A Math. Phys. Eng. Sci.*, vol. 473, no. 2198, Feb. 2017, Art. no. 20160819, doi: 10.1098/rspa.2016.0819.

[41] M. A. Gaafar, T. Baba, M. Eich, and A. Y. Petrov, "Front-induced transitions," *Nat. Photonics*, vol. 13, no. 11, pp. 737–748, Sep. 2019, doi: 10.1038/s41566-019-0511-6.

[42] A. M. Shaltout, V. M. Shalaev, and M. L. Brongersma, "Spatiotemporal light control with active metasurfaces," *Science*, vol. 364, no. 6441, pp. 1–10, May 2019, doi: 10.1126/science.aat3100.

[43] P. K. Tien, "Parametric amplification and frequency mixing in propagating circuits," *J. Appl. Phys.*, vol. 29, no. 9, pp. 1347–1357, Sep. 1958, doi: 10.1063/1.1723440.

[44] A. Cullen, "Theory of the travelling-wave parametric amplifier," *Proc. Inst. Elect. Eng. B, Electron. Commun. Eng.*, vol. 107, no. 32, pp. 101–107, Apr. 1960, doi: 10.1049/pi-b-2.1960.0085.

[45] W. Rhodes, "Acousto-optic signal processing: Convolution and correlation," *Proc. IEEE*, vol. 69, no. 1, pp. 65–79, Jan. 1981, doi: 10.1109/PROC.1981.11921.
[46] B. E. A. Saleh and M. C. Teich, *Fundamentals of Photonics*, vol. 2. Hoboken, NJ, USA: Wiley, 2019, ch. 20.
[47] J. D. Jackson, *Classical Electrodynamics*, 3rd ed. Hoboken, NJ, USA: Wiley, 1998.
[48] J. Bradley, "IV. A letter from the Reverend Mr. James Bradley Savilian Professor of Astronomy at Oxford, and F. R. S. to Dr. Edmond Halley Astronom. Reg. &c. giving an account of a new discovered motion of the fix'd stars," *Philos. Trans. Roy. Soc.*, vol. 35, no. 406, pp. 637–661, Dec. 1728, doi: 10.1098/rstl.1727.0064.
[49] C. Doppler, *Ueber Das Farbige Licht Der Doppelsterne Und Einiger Anderer Gestirne Des Himmels: Versuch Einer Das Bradley'sche Aberrations-Theorem Als Integrirenden Theil in Sich Schliessenden Allgemeineren Theorie*. London, U.K.: Forgotten Books, 1903 (from a treatise written in 1842).
[50] A. Fresnel, "Lettre d'Augustin Fresnel à François Arago sur l'influence du mouvement terrestre dans quelques phénomènes d'optiques," *Ann. Chem. Phys.*, vol. 9, pp. 57–66, Sep. 1818.
[51] H. Fizeau, "Sur les hypothèses relatives à l'éther lumineux, et sur une expérience qui parait démontrer que le mouvement des corps change la vitesse avec laquelle la lumière se propage dans leur intérieur," *Comptes Rendus Hebdomadaires Séances Acad. Sci.*, vol. 33, pp. 349–355, 1851.
[52] W. C. Röntgen, "Ueber die durch Bewegung eines im homogenen electrischen Felde befindlichen Dielectricums hervorgerufene electrodynamische Kraft," *Ann. Phys.*, vol. 271, no. 10, pp. 264–270, 1888, doi: 10.1002/andp.18882711003.
[53] A. Sommerfeld, *Electrodynamics: Lectures on Theoretical Physics*, vol. 3. New York, NY, USA: Academic, 1952.
[54] A. Bahrami and C. Caloz, "Cloaking using spacetime curvature induced by perturbation," in *Proc. 15th Int. Congr. Artif. Mater. Novel Wave Phenomena (Metamaterials)*, 2021, pp. 29–31, doi: 10.1109/Metamaterials52332.2021.9577130.
[55] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*. New York, NY, USA: Wiley, 1972.
[56] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation*. Princeton, NJ, USA: Princeton Univ. Press, 2017.
[57] H. Minkowski, "Raum und Zeit," *Phys. Z.*, vol. 10, pp. 104–111, 1909.
[58] A. Akbarzadeh, N. Chamanara, and C. Caloz, "Inverse prism based on temporal discontinuity and spatial dispersion," *Opt. Lett.*, vol. 43, no. 14, pp. 3297–3300, Jul. 2018, doi: 10.1364/OL.43.003297.
[59] V. Pacheco-Peña and N. Engheta, "Temporal equivalent of the Brewster angle," *Phys. Rev. B*, vol. 104, no. 21, Dec. 2021, Art. no. 214308, doi: 10.1103/PhysRevB.104.214308.
[60] Z.-L. Deck-Léger, A. Akbarzadeh, and C. Caloz, "Wave deflection and shifted refocusing in a medium modulated by a superluminal rectangular pulse," *Phys. Rev. B*, vol. 97, no. 10, Mar. 2018, Art. no. 104305, doi: 10.1103/PhysRevB.97.104305.
[61] A. Einstein, "Zur elektrodynamik bewegter Körper," *Ann. Phys.*, vol. 322, no. 10, pp. 891–921, Jun. 1905, doi: 10.1002/andp.19053221004.
[62] W. Pauli, *Theory of Relativity*. London, U.K.: Pergamon, 1958.
[63] A. P. French, *Special Relativity*. New York, NY, USA: Norton, 1968.
[64] A. Einstein, *Zur Allgemeinen Relativitätstheorie. Akademie der Wissenschaften*. New York, NY, USA: Wiley, 1915.
[65] S. M. Carroll, *Spacetime and Geometry*. Cambridge, U.K.: Cambridge Univ. Press, 2019.
[66] L. D. Landau, J. Bell, M. Kearsley, L. Pitaevskii, E. Lifshitz, and J. Sykes, *Electrodynamics of Continuous Media*, vol. 8. Amsterdam, The Netherlands: Elsevier, 2013.
[67] J. D. Joannopoulos, S. G. Johnson, J. N. Winn, and R. D. Meade, *Photonic Crystals: Molding the Flow of Light*, 2nd ed. Princeton, NJ, USA: Princeton Univ. Press, 2008.
[68] A. Shapere and F. Wilczek, "Classical time crystals," *Phys. Rev. Lett.*, vol. 109, no. 16, Oct. 2012, Art. no. 160402, doi: 10.1103/PhysRevLett.109.160402.
[69] F. Wilczek, "Quantum time crystals," *Phys. Rev. Lett.*, vol. 109, no. 16, Oct. 2012, Art. no. 160401, doi: 10.1103/PhysRevLett.109.160401.
[70] C. M. Harris and A. G. Piersol, *Harris' Shock and Vibration Handbook*, vol. 5. New York, NY, USA: McGraw-Hill, 2002.
[71] L. A. Ostrovskiĭ, "Some 'moving boundaries paradoxes' in electrodynamics," *Sov. Phys. Uspekhi*, vol. 18, no. 6, pp. 452–458, Jun. 1975, doi: 10.1070/PU1975v018n06ABEH001967.
[72] Z.-L. Deck-Léger and C. Caloz, "Scattering at interluminal interface," in *Proc. IEEE Int. Symp. Antennas Propag. USNC-URSI Radio Sci. Meeting*, Atlanta, GA, USA, Jul. 2019, pp. 367–368, doi: 10.1109/APUSNCURSINRSM.2019.8889224.
[73] H. A. Lorentz, "Electromagnetic phenomena in a system moving with any velocity smaller than that of light," in *Proc. Roy. Netherlands Acad. Arts Sci. (KNAW)*, 1904, vol. 6, pp. 809–831.
[74] J. V. Bladel, *Relativity and Engineering*. Berlin, Germany: Springer-Verlag, 1984.
[75] J. A. Kong, "Theorems of bianisotropic media," *Proc. IEEE*, vol. 60, no. 9, pp. 1036–1046, Sep. 1972, doi: 10.1109/PROC.1972.8851.
[76] J. A. Kong, *Electromagnetic Wave Theory*. Cambridge, MA, USA: EMW Publishing, 2008.
[77] N. Chamanara, Y. Vahabzadeh, and C. Caloz, "Simultaneous control of the spatial and temporal spectra of light with space-time varying metasurfaces," *IEEE Trans. Antennas Propag.*, vol. 67, no. 4, pp. 2430–2441, Apr. 2019, doi: 10.1109/TAP.2019.2891706.
[78] R. B. Sher and R. J. Daverman, *Handbook of Geometric Topology*. New York, NY, USA: Elsevier, 2001.
[79] L. W. Tu, *An Introduction to Manifolds*, 2nd ed. New York, NY, USA: Springer-Verlag, 2011.
[80] J. Anderson and J. Ryon, "Electromagnetic radiation in accelerated systems," *Phys. Rev.*, vol. 181, no. 5, May 1969, Art. no. 1765, doi: 10.1103/PhysRev.181.1765.
[81] K. Tanaka, "Relativistic study of electromagnetic waves in the accelerated dielectric medium," *J. Appl. Phys.*, vol. 49, no. 8, pp. 4311–4319, Aug. 1978, doi: 10.1063/1.325483.
[82] K. Tanaka, "Reflection and transmission of electromagnetic waves by a linearly accelerated dielectric slab," *Phys. Rev. A*, vol. 25, no. 1, pp. 385–390, Jan. 1982, doi: 10.1103/PhysRevA.25.385.
[83] U. Leonhardt and P. Piwnicki, "Optics of nonuniformly moving media," *Phys. Rev. A*, vol. 60, no. 6, pp. 4301–4312, Dec. 1999, doi: 10.1103/PhysRevA.60.4301.
[84] Z.-L. Deck-Léger, X. Zheng, and C. Caloz, "Electromagnetic wave scattering from a moving medium with stationary interface across the interluminal regime," *Photonics*, vol. 8, no. 6, pp. 1–15, Jun. 2021, doi: 10.3390/photonics8060202.
[85] A. Bahrami, Z.-L. Deck-Léger, and C. Caloz, "Electrodynamics of metamaterials formed by accelerated modulation," Jul. 2022, *arXiv:2208.11506*.
[86] C. Caloz and A. Sihvola, "Electromagnetic chirality, part I: The microscopic perspective," *IEEE Antennas Propag. Mag.*, vol. 62, no. 1, pp. 58–71, Feb. 2020, doi: 10.1109/MAP.2019.2955698.
[87] C. Caloz and A. Sihvola, "Electromagnetic chirality, part II: The macroscopic perspective," *IEEE Antennas Propag. Mag.*, vol. 62, no. 2, pp. 82–98, Apr. 2020, doi: 10.1109/MAP.2020.2969265.
[88] C. Caloz, A. Alù, S. Tretyakov, D. Sounas, K. Achouri, and Z.-L. Deck-Léger, "Electromagnetic nonreciprocity," *Phys. Rev. Appl.*, vol. 10, no. 4, Oct. 2018, Art. no. 047001, doi: 10.1103/PhysRevApplied.10.047001.
[89] P. A. Huidobro, E. Galiffi, S. Guenneau, R. V. Craster, and J. B. Pendry, "Fresnel drag in space–time-modulated metamaterials," *Proc. Nat. Acad. Sci. USA*, vol. 116, no. 50, pp. 24,943–24,948, Dec. 2019, doi: 10.1073/pnas.1915027116.
[90] U. Leonhardt and T. Philbin, *Geometry and Light: The Science of Invisibility*. New York, NY, USA: Dover, 2010.
[91] C. Møller, *The Theory of Relativity*. Oxford, U.K.: Clarendon, 1952.
[92] W. Rindler, "Hyperbolic motion in curved space time," *Phys. Rev.*, vol. 119, no. 6, pp. 2082–2089, Sep. 1960, doi: 10.1103/PhysRev.119.2082.
[93] K. Schwarzschild, "Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie," *Sitzungsberichte Koeniglich Preussischen Akademie Wissenschaften Berlin*, vol. 18, pp. 189–196, Jan. 1916.
[94] D. Faccio, F. Belgiorno, S. Cacciatori, V. Gorini, S. Liberati, and U. Moschella, Eds. *Analogue Gravity Phenomenology: Analogue Spacetimes and Horizons, from Theory to Experiment*. Berlin, Germany: Springer-Verlag, 2013.
[95] E. Gourgoulhon, *3+1 Formalism in General Relativity: Bases of Numerical Relativity*, vol. 846. Berlin, Germany: Springer-Verlag, 2012.
[96] J. A. Kong, "Time-harmonic fields in source-free bianisotropic media," *J. Appl. Phys.*, vol. 39, no. 12, pp. 5792–5796, Nov. 1968, doi: 10.1063/1.1656050.
[97] J. Van Bladel, "Electromagnetic fields in the presence of rotating bodies," *Proc. IEEE*, vol. 64, no. 3, pp. 301–318, Mar. 1976, doi: 10.1109/PROC.1976.10111.