Quasi and pseudo orthogonal rational functions for quadrature on the positive real line*

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We consider a positive measure on $[0, \infty)$ and a sequence of nested spaces $\mathcal{L}_0 \subset \mathcal{L}_1 \subset \mathcal{L}_1 \cdots$ or rational functions with prescribed poles in $[-\infty, 0]$. Let $\varphi_k \in \mathcal{L}_k$ be the associated sequence of orthogonal rational functions. The zeros of φ_n can be used as the nodes of a rational Gauss quadrature formula that is exact for all functions in $\mathcal{L}_n \cdot \mathcal{L}_{n-1}$, a space of dimension $2n$. Quasi- and pseudo-orthogonal functions are functions in $\mathcal{L}_n$ that are orthogonal to some subspace of $\mathcal{L}_{n-1}$. Both of them are generated from φ_n and φ_{n-1} and depend on a real parameter τ. Their zeros can be used as the nodes of a rational Gauss-Radau quadrature formula where one node is fixed in advance and the others are chosen to maximize the subspace of $\mathcal{L}_n \cdot \mathcal{L}_{n-1}$ where the quadrature is exact. The parameter τ is used to fix a node at a pre-assigned point. The space where the quadratures are exact have dimension $2n-1$ in both cases but it is in $\mathcal{L}_{n-1} \cdot \mathcal{L}_{n-1}$ in the quasi-orthogonal case and it is in $\mathcal{L}_n \cdot \mathcal{L}_{n-2}$ in the pseudo-orthogonal case.

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