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An optimization-based algorithm for simultaneous shaping of poles and zeros for non-collocated vibration suppression ⋆

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Abstract: This article presents a control design method for simultaneous shaping of the poles and zeros of linear time-invariant systems, motivated by the application of non-collocated vibration suppression to flexible multi-body systems. An entire suppression of vibrations at a target mass for a given excitation frequencies can be recast into the problem of assigning zeros of the transfer function from the excitation force to the target mass' position. The design requirement of achieving sufficient damping in the closed loop system combined with suppressing vibrations at the target, leads to the minimization of the spectral abscissa function of the closed loop system as a function of the controller parameters, subject to zero location constraints. These constraints exhibit polynomial dependence on the controller parameters. We present two approaches to solve the optimization problem which are both based on constraint elimination followed by application of an algorithm for non-smooth unconstrained optimization. The design approach is applicable to delay-free models as well as time-delay models of retarded and neutral type. Simulations results illustrate its applicability to a spring-mass-damper system.

Keywords: vibration suppression, zero placement, time-delay systems, spectral optimization

1. INTRODUCTION

Vibration suppression remains an interesting topic of study in a number of engineering applications. The detrimental effects of vibration are well known and are therefore considered a phenomenon which must be removed or reduced. Some examples of undesired vibrations include machine tool vibrations which affect the surface finish of the machined workpiece, fatigue failure of mechanical instruments or adverse effects of base vibrations in precision instruments, etc. Extensive studies have been conducted on both passive and active absorbers for the purpose of vibration suppression, see Preumont (2018). The advantages of passive absorbers have been known for a long time and are extensively used in practice (e.g. Rana and Soong, 1998, Den Hartog, 1985). While passive absorbers are effective in suppressing vibrations, these devices cannot achieve complete vibration suppression as there always exists some residual vibration resulting from the non-ideal nature of the absorber. In what follows, we shall focus on active vibration suppression only. Some of the existing studies include: Hosek and Olgac (2002), Vyhlídal et al. (2019), Pilbauer et al. (2016), Rivaz and Rohling (2007). Numerous other examples of active methods can also be found in Preumont (2018).

Most of the existing investigations on the topic of vibration suppression address the basic collocated vibration absorption task, where the absorber is attached directly to the target mass whose vibrations are to be suppressed. However, it may be impossible to mount an absorber or sometimes even a sensor directly on the target object. For example, it is impossible to place an absorber at the tip of a machining tool or at the end-point of the micromanipulator of a surgery robot for practical reasons. In such circumstances, it becomes necessary to apply an alternative approach which involves placing the absorber at a location different from the target location, which, in turn performs the desired task of suppression. This is termed as 'non-collocated vibration suppression'. Studies on non-collocated vibration suppression have been so far limited. Existing work in a similar area include: Wie et al. (1993), Balas and Doyle (1994), Yang and Mote (1992). The cited texts typically use the term *'non-collocated'* to indicate separately located sensor-actuator pairs. In the context of the current work, however, the term 'non-collocated'

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vibration suppression shall be restricted to the situation wherein the vibration absorption device is located at a position remote from the target mass at which the vibrations are to be suppressed.

Recently, in Olgac and Jenkins (2021), a benchmark problem was presented for the active tuning of an absorber to silence a point on the structure which is remote from the absorber. Prior to this, in Jenkins and Olgac (2019), the authors' team devised a method for actively suppressing a discrete vibration frequency by identifying a *'resonant substructure'* which is connected to the target and tuning the absorber such that the eigenvalues of the substructure lie on the imaginary axis at the desired frequencies of suppression. Due to the direct eigenvalue assignment of the resonant substructure, the substructure behaves as an ideal absorber and thereby completely suppresses the desired frequency at the target object. However, as per Theorem 1 in Jenkins and Olgac (2019), such resonance-based absorption is not always possible to achieve. Specifically, it was demonstrated in Silm et al. (2021) that this method of eigenvalue assignment of the resonant substructure is rendered infeasible in situations wherein it is not possible to identify such a resonant substructure. Fig. 1 shows a typical example of such a scenario. In these cases, vibration suppression can be achieved by considering the force balance at the target mass, and directly assigning the transmission zeros from the external excitation force to the position of the target mass. This approach remains consistent with the findings in Miu (1991) and Buhr (1997).

Shifting a zero in the complex plane however impacts the dynamics of the overall setup due to which the stability of the closed-loop system must be accounted for. Therefore, in addition to the absorber, a second controller was added in Silm et al. (2021) to improve the closed-loop response. The addition of a second control-loop is, however not ideal since, in the case of non-collocated suppression the design of the absorber and the second controller are interdependent and therefore these must be designed together. This is unlike the collocated setting wherein the absorber and the controller can be designed separately. Therefore, a more appropriate approach would be to use a single controller which could possibly attain multiple objectives, e.g. in terms of suppressing a desired vibration frequency as well as any higher level control task to be performed. As stated earlier, suppressing a certain frequency of vibration can be interpreted as placing the transmission zeros of the target system, while optimizing the controller parameters to achieve any higher level objective relate to shaping the closed-loop poles. This leads to a design problem for a single controller which can be interpreted as a constrained optimization problem of simultaneous shaping of the poles and zeros of the system. The article presents a solution to the above problem using constraint elimination, following which, the remaining controller parameters are optimized to achieve the desired control objective.

The next sections are arranged as follows: Section 2 describes the problem in detail and presents the motivation towards the specific case. From this, Section 3 lays out the detailed procedure for controller parameterization following which, in Section 4, the same is implemented and simulated on a laboratory setup model. Finally, Section 5 provides a conclusion to the article.

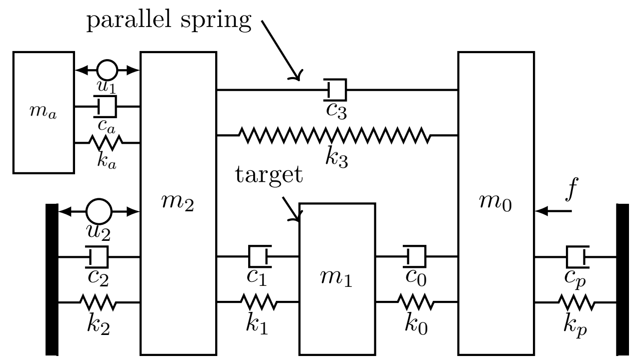

Fig. 1. Mass-spring-damper system excited by an external vibration force f.

2. PROBLEM STATEMENT

Fig. 1 shows an example setup of the task. In the spring-mass-damper system shown an external excitation force f, typically a harmonic disturbance, acts on the mass m_0. The force propagates through the system and excites the remaining masses causing them to oscillate at the same frequency. The desired control objective in this example is to completely suppress vibrations at the target mass m_1 while simultaneously achieving a higher level control task, which we will specify in what follows as having a sufficient stability margin of the closed-loop system. For instance, the equivalent of mass m_1 could represent a tool tip or the tip of a surgery robot or a similar device, on which it is not possible or feasible to attach an absorber. To correctly suppress the target frequency at this point, the control effort must be applied remotely using the control inputs u_1 and u_2.

2.1 State-Space Model

To begin with, consider the state-space model of a system given by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bf(t) + B_1u(t) \\ y(t) &= Cx(t) \\ y_1(t) &= C_1x(t), \end{aligned} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$ describes the dynamics, $B \in \mathbb{R}^{n\times 1}$ is the input matrix describing where a vibration force $f(t)$ acts and $C \in \mathbb{R}^{1\times n}$ specifies the position of the target mass to be suppressed. The matrices $B_1 \in \mathbb{R}^{n\times p}$ and $C_1 \in \mathbb{R}^{q\times n}$ are determined by the location of the actuators and sensors of the static output-feedback controller

$$u(t) = K^T y_1(t-\tau) \tag{2}$$

where $K \in \mathbb{R}^{q\times p}$ is the controller gain matrix to be designed. In order to generalise the problem, we assume a non-negligible measurement lag, which is represented by the time delay $\tau > 0$. Note that for the setup shown in Fig. 1, the control input u is given by $u = [u_1 \; u_2]$.

Substituting the controller input (2) in (1) yields the closed-loop system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bf(t) + B_1K^TC_1x(t-\tau) \\ y(t) &= Cx(t). \end{aligned} \tag{3}$$

2.2 Transmission Zeros

Given the system (3) excited by a disturbance $f(t) = Fe^{\lambda t}$, $F \neq 0$, the state-response is $x(t) = Xe^{\lambda t}, X \in \mathbb{C}$. For the excited system, the transmission zeros are characterized by any $\lambda \in \mathbb{C}$ for which $f(t)$ results in a zero steady-state output, i.e. $y(t) \equiv 0$. By substituting $y(t) = 0$ in (3), the imposed conditions can be reformulated as

$$\begin{bmatrix} \lambda I - A - B_1 K^T C_1 e^{-\lambda\tau} & -B \\ C & 0 \end{bmatrix} \begin{bmatrix} X \\ F \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (4)$$

Assuming the system described by the state-space model in equation (3) is both controllable and observable with f as the input and y as the output, the transmission zeros are obtained from the non-trivial solutions of (4) and are thus represented by

$$\det\left(\begin{bmatrix} \lambda I - A - B_1 K^T C_1 e^{-\lambda\tau} & -B \\ C & 0 \end{bmatrix}\right) = 0. \quad (5)$$

For a given frequency of vibration to vanish from the output, a corresponding pair of transmission zeros must be located on the imaginary axis. Suppressing m frequencies $\omega_1, \dots, \omega_m$, thereby leads to assigning $2m$ zeros at locations $\pm j\omega_1, \dots, \pm j\omega_m$ respectively, each of which must satisfy equation (5).

2.3 Formulation as an Optimization Problem

It is important to note that in placing zeros the dynamics of the overall system are changed as a result, which could lead to instability of the overall setup. Due to this and in order to achieve the desired higher-level objectives, further analysis of the closed-loop poles must be carried out. The characteristic matrix of the closed-loop system is

$$M(\lambda, K) = \lambda I - A - B_1 K^T C_1 e^{-\lambda\tau}, \quad (6)$$

where λ represents the characteristic roots of the closed-loop system (3).

For a time-delay system expressed by equation (3) there exist in general infinitely many characteristic roots. The condition for ensuring overall stability of the system is that all characteristic roots are located in the open left half-plane (Michiels and Niculescu, 2007). Due to the infinite dimensionality of system (3) and the finite degrees of freedom of the controller in K, stabilizing the system and maximizing the decay rate of solutions leads to an optimization problem of minimization of the spectral abscissa function $\alpha(K)$ subject to constraints on the location of transmission zeros. This can be restated as follows:

$$\min_{K \in \mathbb{R}^{q\times p}} \alpha(K) := \sup(\Re(\lambda) : \det(M(\lambda, K)) = 0) \quad (7a)$$

$$\text{s.t.} \quad h_i(K) = 0, \quad i = 1, \dots, m \quad (7b)$$

where

$$h_i(K) = \det\left(\begin{bmatrix} j\omega_i I - A - B_1 K^T C_1 e^{-j\omega_i\tau} & -B \\ C & 0 \end{bmatrix}\right).$$

In formulating the problem as above, we have transformed our physical problem of non-collocated vibration suppression into a mathematical optimization problem. Problem (7) is challenging since the objective function is in general a non-convex function of K and typically not smooth in the minima (Vanbiervliet et al., 2008), while the constraints are multi-variable polynomial functions in K. In this work, we present a procedure to convert the constrained optimization problem into an unconstrained one by elimination. Elimination is especially advantageous in our setting, where it is important that the zero-location constraints are satisfied without the need to have fully converged to the minimizers (typically this corresponds to multiple defective rightmost characteristic roots, which are highly sensitive). To perform the constraint elimination, we exploit the property that, in general, the constraints are affine in a *subset* of the controller parameters, while for the single input case they are affine functions of K. The procedure is detailed in the next section.

3. CONTROLLER DESIGN

In what follows, we distinguish between two cases, the multi-input case which is universally applicable and the single-input case.

3.1 Multi-Input Case

Consider a controller for a system with p inputs and q outputs, the controller gain K is of dimension $q \times p$. The general idea behind constraint elimination is to select elements (dependent variables) of the matrix K and express these elements in terms of the remaining elements (independent variables) of K.

Controller Parameterization: The intuition behind our dependent parameter selection is to transform the non-linear set of constraints given in (7b) into a set of equations which are affine in the dependent variables, which can subsequently be eliminated. This is achieved by selecting the dependent parameters (equal to the number of placed zeros), from either a *single row* or a *single column* of the gain matrix K. The procedure for assigning $2m$ zeros using a controller with p inputs and q outputs, where $2m < q$ is therefore as follows:

(1) Designate $2m$ elements of the controller gain matrix K, taken from a single row using $2m$ column entries as dependent variables. To simplify the notations in what follows we will designate the first $2m$ elements from the first row, using the first $2m$ entries of the gain matrix as $g^T = [g_1, \dots, g_{2m}]$.
(2) Express the dependent variables as a function of the independent ones to satisfy the constraints.
(3) With the $2m$ parameters from the first row fixed for positioning the zeros, the remaining $pq - 2m$ parameters can be optimized for the control task at hand. These independent parameters are represented by the independent parameter matrix K_L, where

$$K_L = \begin{bmatrix} 0 & \cdots & 0 & k_{1,2m+1} & \cdots & k_{1,p} \\ k_{2,1} & \cdots & k_{2,2m} & & & k_{2,p} \\ \vdots & & & & \ddots & \vdots \\ k_{q,1} & \cdots & k_{q,2m} & & & k_{q,p} \end{bmatrix}. \quad (8)$$

The matrix of the feedback part of the state-space equation (3) can be written as:

$$B_1 K C_1 = B_{1,1} g(K_L)^T C_{1,1:2m} + B_1 K_L^T C_1 \quad (9)$$

where $B_{1,1} \in \mathbb{R}^{n\times 1}$ denotes the first column of the input matrix B_1 and $C_{1,1:2m} \in \mathbb{R}^{2m\times n}$ denotes the first $2m$ rows of the output matrix C_1.

Constraint Elimination: Positioning the transmission zeros at the desired locations can be achieved by substituting $\omega_1, \ldots, \omega_m$ in the equation $h_i(K) = 0$ from (7), leading to $2m$ equations in $g_1, \ldots, g_{2m}$, by which, these dependent parameters are eliminated. Using the relation from (9) and by applying the Weinstein–Aronszajn identity, i.e. $\det(I - UV) = \det(I - VU)$ to equation (7b), for a single frequency ω_i we obtain that

$$\det(I - R(\omega_i, K_L)^{-1} \begin{bmatrix} B_{1,1} \\ 0 \end{bmatrix} g^T \,[C_{1,1:2m}\ \ 0]\, e^{-j\omega_i\tau}) = 0$$
$$\iff 1 - g^T\,[C_{1,1:2m}\ \ 0]\, R(\omega_i, K_L)^{-1} \begin{bmatrix} B_{1,1} \\ 0 \end{bmatrix} e^{-j\omega_i\tau} = 0 \tag{10}$$

provided that R is invertible, where

$$R(\omega_i, K_L) = j\omega_i \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} A & B \\ -C & 0 \end{bmatrix} - \begin{bmatrix} B_1 K_L^T C_1 e^{-j\omega_i\tau} & 0 \\ 0 & 0 \end{bmatrix}.$$

Since the entries of K are real-valued, assigning a single pair of imaginary zeros $\pm j\omega$ leads to two equations:

$$\begin{aligned} g^T \Re\{z(\omega_i, K_L)\} &= \Re\{e^{j\omega_i\tau}\} \\ g^T \Im\{z(\omega_i, K_L)\} &= \Im\{e^{j\omega_i\tau}\} \end{aligned} \tag{11}$$

where

$$z(\omega_i, K_L) = [C_{1,1:2m}\ \ 0]\, R(\omega_i, K_L)^{-1} \begin{bmatrix} B_{1,1} \\ 0 \end{bmatrix}.$$

By means of the above equations, we have effectively transformed the polynomial constraints in K, to a set of linear equations in g. For m frequencies to be suppressed, we can concatenate (11) to obtain a system of linear equations of the form:

$$P(K_L)g = Q, \tag{12}$$

where $P \in \mathbb{R}^{2m\times 2m}$ and $Q \in \mathbb{R}^{2m\times 1}$.

For a square and invertible matrix P,

$$g(K_L) = P(K_L)^{-1}Q. \tag{13}$$

Optimization: The characteristic matrix of the time-delay system characterized by (3) is given as:

$$\tilde{M}_1(\lambda, K_L) = \lambda I - A - B_1 K_L^T C_1 e^{-\lambda\tau} - B_{1,1} g(K_L)^T C_{1,1:2m} e^{-\lambda\tau} \tag{14}$$

using the equations (8) and (9), where λ represents the characteristic roots of the closed-loop system. As laid out in Section 2.3, for ensuring overall stability the independent parameters K_L can be optimized to minimize the spectral abscissa function

$$\min_{K_L \in \mathbb{R}^{q\times p}} \tilde{\alpha_1}(K_L) := \sup(\mathbb{R}(\lambda) : \det(\tilde{M}_1(\lambda, K_L)) = 0). \tag{15}$$

The above function is a non-smooth, non-convex function. To solve this problem, we use the MATLAB program HANSO (Burke et al., 2005) for optimization. The method uses BFGS method with weak Wolfe line search and gradient sampling. It relies on computing the rightmost eigenvalues along with the derivatives of the objective function with respect to the controller parameters, wherever the derivatives exist (Michiels et al., 2010). The rightmost eigenvalues are computed using the package *Software for analysis and control of time-delay system*, from Michiels (2011). An entry to the gradient of the above equation, whenever it exists, can be computed using:

$$\frac{d\lambda}{dk_{ij}} = -\frac{w^* \frac{\partial \tilde{M}_1}{\partial k_{ij}} v}{w^* \frac{\partial \tilde{M}_1}{\partial \lambda} v}, \tag{16}$$

where w and v are the left and right eigenvectors of the characteristic matrix $\tilde{M}_1$ respectively and k_{ij} denotes an entry from the free parameter matrix K_L. From equation (13), it is clear that g itself is a function of the independent parameters K_L and thus, the derivative of this term in the equation (14) is obtained by using the chain rule.

3.2 Single-Input Case

For a single input controller, the original constraints (7b) are already affine in the controller parameters K. The adopted approach is similar to the one presented in Michiels et al. (2010), wherein algebraic techniques for direct assignment of right-most poles were devised. In this article, the approach is extended to the case of direct assignment of the transmission zeros for suppressing a target vibration frequency.

Constraint Elimination: For a single-input controller we have $K \in \mathbb{R}^q$ with $q > 2m$. For each frequency to be suppressed, the constraints $h_i(\omega) = 0$ from equation (7) can be reduced to the following:

$$\det(I - R(\omega_i)^{-1} \begin{bmatrix} B_1 \\ 0 \end{bmatrix} K^T\,[C_1\ \ 0]\, e^{-j\omega_i\tau}) = 0$$
$$\iff 1 - K^T\,[C_1\ \ 0]\, R(\omega_i)^{-1} \begin{bmatrix} B_1 \\ 0 \end{bmatrix} e^{-j\omega_i\tau} = 0 \tag{17}$$

by applying the Weinstein-Aronszajn identity (provided R is invertible) where $R(\omega_i) = j\omega_i \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$.

Assignment of a pair of zeros thus leads to two equations in K:

$$\begin{aligned} \Re\{K^T z(\omega_i)\} &= \Re\{e^{j\omega_i\tau}\} \\ \Im\{K^T z(\omega_i)\} &= \Im\{e^{j\omega_i\tau}\} \end{aligned} \tag{18}$$

where

$$z(\omega_i) = [C_1\ \ 0]\, R(\omega_i)^{-1} \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \quad i = 1, \ldots, m.$$

Therefore, for m pairs of imaginary zeros placed at locations $\pm j\omega_1, ..., \pm j\omega_m$, we obtain $2m$ linear equations in K which can be expressed in the form

$$PK = Q \tag{19}$$

where $P \in \mathbb{R}^{2m\times q}$ and $Q \in \mathbb{R}^{2m\times 1}$.

The above equation (19) is an underdetermined system of equations in K and generically all solutions to (19) can be expressed as

$$K = K_0 + GL \tag{20}$$

where $K_0 \in \mathbb{R}^{q\times 1}$, $G \in \mathbb{R}^{q\times q-2m}$, $L \in \mathbb{R}^{q-2m\times 1}$. Here, K_0 and G are fixed, while L represents the free parameters.

Optimization: The optimization follows a similar procedure as with the multi-input case. Using (20) the constraints are automatically satisfied. We then minimize the spectral abscissa

$$\tilde{\alpha}(L) := \sup(\mathbb{R}(\lambda) : \det(\tilde{M}_2(\lambda, L)) = 0)$$

where

$$\tilde{M}_2(\lambda, L) = \lambda I - A - B_1 K_0^T C_1 e^{-\lambda\tau} - B_1 L^T G^T C_1 e^{-\lambda\tau}.$$

4. NUMERICAL EXPERIMENTS

To validate the above procedure, we apply the algorithm to a simulated setup shown in Fig. 1. The masses, spring constants and damping coefficients of the elements in the setup are shown in Table 1. Here, the target mass to be suppressed is the mass m_1 and the target frequency of suppression is 19 rad/s. The state variable x and the system state, input and output matrices A (given on the bottom of this page), B, C are:

$$x = [x_0\ \dot{x}_0\ x_1\ \dot{x}_1\ x_2\ \dot{x}_2\ x_a\ \dot{x}_a]^T$$

$$B = \left[0\ \tfrac{1}{m_0}\ 0\ 0\ 0\ 0\ 0\ 0\right]^T,\ C = [0\ 0\ 1\ 0\ 0\ 0\ 0\ 0]^T.$$

4.1 Controller Design with Multiple Inputs

The control input and output matrices for the multi-input controller are given by:

$$B_1 = \begin{bmatrix} 0\ 0\ 0\ 0\ 0 & -\frac{1}{m_2} & 0 & \frac{1}{m_a} \\ 0\ 0\ 0\ 0\ 0 & \frac{1}{m_2} & 0 & 0 \end{bmatrix}^T,\ C_1 = \begin{bmatrix} 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{bmatrix}.$$

Using the above data, we parameterize the controller gain K using the procedure described in Section 3.1. Fig. 2 shows the pole-zero plot of the overall system after optimization with dependent parameters corresponding to input u_1 and outputs $x_a, \dot{x}_a$. We see that the zeros are indeed placed correctly and the rightmost eigenvalues lie in the left half-plane. Fig. 4 shows the simulated output of the displacement of target mass m_1 upon excitation by external force $f(t)$. The controller is turned on at $t = 10s$ at which point, we see that the vibration is completely suppressed at the output.

4.2 Controller Design with Single-Input

For the single-input case, the input control force is applied on the mass m_2 only. The input matrix B_1 is:

$$B_1 = \left[0\ 0\ 0\ 0\ 0\ \tfrac{1}{m_2}\ 0\ 0\right]^T$$

while the output matrix C_1 is the same as in the multi-input case. The controller gain K is parameterized using the procedure outlined in Section 3.2. The results of the optimization can be seen from the pole-zero plot in Fig. 3.

The plots show that the zeros are indeed placed correctly meaning that the vibrations of frequency 19 rad/s are correctly suppressed. The advantage of using a single input controller is that with this controller both objectives viz. suppressing a given frequency as well as any other higher level control objective are simultaneously fulfilled using a single input. Additionally, unlike the multi-input case, dependent parameters for placing the zeros do not need to be preselected as the constraints are already affine in all variables. A drawback of using the single-input controller is that it has a smaller number of degrees of freedom. For this reason, a smaller spectral abscissa could be reached with the controller steering two different inputs. On comparing the spectral abscissa function of the two, it is evident that the multi-input case yields far better results due to the greater number of free parameters available for optimization.

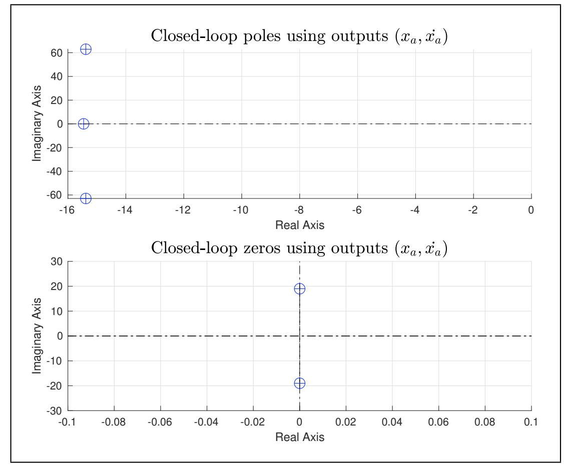

Fig. 2. Pole-Zero plots for the multi-input case. The plot shows the optimized poles and zeros, based on the selection of the dependent parameters

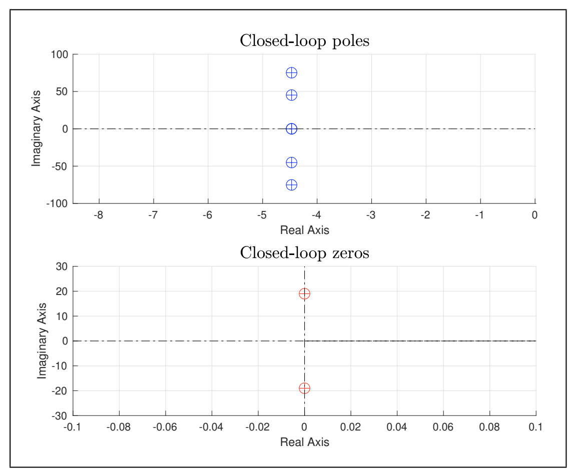

Fig. 3. Pole-Zero plot for the single-input case

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{k_1+k_3+k_p}{m_0} & -\frac{c_0+c_3+c_p}{m_0} & \frac{k_0}{m_0} & \frac{c_0}{m_0} & \frac{k_3}{m_0} & \frac{c_3}{m_0} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{k_0}{m_1} & \frac{c_0}{m_1} & -\frac{k_0+k_1}{m_1} & -\frac{c_0+c_1}{m_1} & \frac{k_1}{m_1} & \frac{c_1}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{k_3}{m_2} & \frac{c_3}{m_2} & \frac{k_1}{m_2} & \frac{c_1}{m_2} & -\frac{k_1+k_2+k_3+k_a}{m_2} & -\frac{c_1+c_2+c_3+c_a}{m_2} & \frac{k_a}{m_2} & \frac{c_a}{m_2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & \frac{k_a}{m_a} & \frac{c_a}{m_a} & -\frac{k_a}{m_a} & -\frac{c_a}{m_a} \end{bmatrix}$$

Table 1. Parameter values

Mass in kg		Stiffness in Nm^{-1}		Damping in Nsm^{-1}	
m_a	0.5	k_a	400	c_a	1.9
m_0	1	k_0	410	c_0	2.1
m_1	0.5	k_1	1450	c_1	4.9
m_2	1.15	k_2	380	c_2	2.2
		k_3	405	c_3	2
		k_p	1500	c_p	5

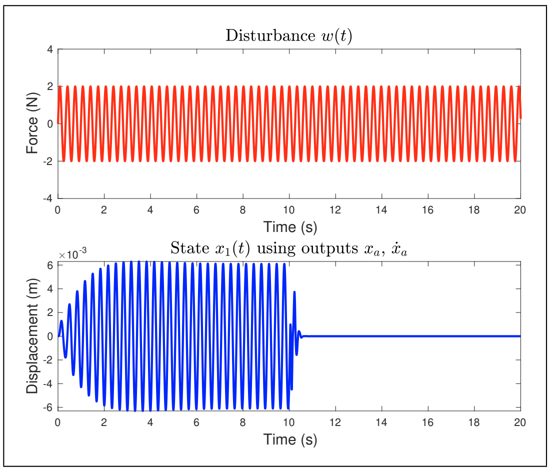

Fig. 4. Displacement of target mass m_1 subject to external excitation force f acting on mass m_0. The multi-input controller with dependent parameters corresponding to input u_1 and outputs $x_a, \dot{x}_a$ is turned on at $t = 10$s.

5. CONCLUSION

A procedure for controller parameterization applied to non-collocated vibration suppression is presented in this article. The method involves solving a constrained optimization problem by constraint elimination. Future work in this topic is the study of the optimal selection of dependent and independent parameters for the multi-input setup. Experimental verification of the above concept is also planned as one of the next steps as part of the research work.

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