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Incorrect Ways of Thinking About the Size of Fractions

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Abstract

The literature has amply shown that primary and secondary school students have difficulties in understanding rational number size. Many of these difficulties are explained by the natural number bias or the use of other incorrect reasoning such as gap thinking. However, in many studies, these types of reasoning have been inferred from comparing students' accuracies in multiple-choice items. Evidence that supports that these incorrect ways of reasoning are indeed underlying is scarce. In the present work, we carried out interviews with 52 seventh grade students. The objective was to validate the existence of students' incorrect ways of thinking about fraction size that were previously inferred from patterns of correct and incorrect answers to multiple-choice items, by looking at students' verbalizations, and examine whether these ways of thinking are resistant to change. Students' verbalizations support the existence of the different incorrect ways of thinking inferred from previous studies in fraction size. Furthermore, the high levels of confidence in their incorrect reasoning and the fact that they were reluctant to change their answer when they were confronted with other reasoning suggest that these ways of thinking may be resistant to change.

Keywords Gap thinking · Natural number bias · Interviews · Rational number · Reverse bias

Introduction

The rational number concept is one of the major mathematical ideas that children have to master during their pre-secondary school years, since it is the basis for the more advanced mathematical understanding of calculus and algebra (Behr et al., 1983). However, rational numbers have been considered as one of the most difficult topics in elementary and middle school mathematics to teach and learn (Behr

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et al., 1984; Smith, 1995). One reason students have difficulties understanding in rational numbers is their inappropriate application of the properties of natural numbers (Moss, 2005; Ni & Zhou, 2005; Smith et al., 2005; Van Hoof et al., 2015). This phenomenon has been named *natural number bias* (NNB) (Van Dooren et al., 2015). As they begin to learn about rational numbers, especially fractions, students have the "temptation to deal with fractions in the same manner as with natural numbers" (Streefland, 1991, p. 70), leading them to numerous errors and misconceptions that can persist over the years (Vamvakoussi et al., 2012).

Research has hitherto repeatedly found evidence of the interference of natural numbers in students' thinking by comparing students' levels of accuracy in rational number tasks that were compatible with natural number knowledge and their levels of accuracy in tasks where this knowledge was incompatible (Nunes & Bryant, 2008; Stafylidou & Vosniadou, 2004). For instance, primary school students accurately determine that 0.67 is larger than 0.5 (the larger decimal number has the longer natural number after the decimal point), but they have difficulties in determining that 0.7 is larger than 0.35 (the larger number has the shorter natural number after the decimal point) (Moss, 2005).

Studies addressing the natural number bias phenomenon have considered three main domains in which rational numbers differ from natural numbers: operations with rational numbers, the density of the rational number line, and rational number size (Gómez & Dartnell, 2019; Obersteiner et al., 2020; Van Hoof et al., 2015). Furthermore, the different ways in which rational numbers can be represented (fractions and decimal numbers) have been considered to represent an additional difficulty in these domains (DeWolf & Vosniadou, 2011). The current study focuses specifically on the domain of rational number size. In the following section, we review the literature in this domain, presenting the main findings regarding natural number interference, as well as other possible reasons that could explain students' errors in understanding rational number size, particularly in fractions.

Theoretical and Empirical Background

Natural Number Bias Phenomenon in the Domain of Rational Number Size: Fractions

Determining fraction size implies a *holistic* understanding of a fraction, which involves access to its size as a single entity. In this way, it implies recognizing that the size depends on the multiplicative relationship between its numerator and denominator (Moss, 2005; Ni & Zhou, 2005; Smith et al., 2005). However, many students fail to recognize fractions as numbers that have a magnitude of their own and tend to interpret the symbol *a/b* as two independent natural numbers separated by a slash (Stafylidou & Vosniadou, 2004). Therefore, they misunderstand fractions considering them as a pair of numbers (Meert et al., 2010). A common misconception is that a fraction's numerical value increases when its numerator, denominator, or both increase (e.g. 5/9 is larger than 2/3 because 5 is larger than 2 and 9 larger than 3) (Behr et al., 1984; Pearn & Stephens, 2004).

Studies over the last decade have tested this possible interference by conducting studies focused on comparing performance on congruent and incongruent fraction comparison items. Congruent items are items compatible with natural number knowledge, so the largest fraction has the largest numerator and denominator (3/5 vs. 8/9), and incongruent items are items that are incompatible with natural number knowledge, i.e. the largest fraction has the smallest numerator and denominator (2/3 vs. 4/9). Results have shown that primary and secondary school students, and even undergraduates, accurately solve congruent items, but have more difficulties in solving incongruent ones (DeWolf & Vosniadou, 2011; González-Forte et al., 2020b; Meert et al., 2010).

Contradictory Results in Fraction Comparison Tasks: Students' Various Incorrect Ways of Thinking

In previous research, high accuracy in congruent items and low performance in incongruent ones has typically been interpreted as pointing towards an over-reliance on natural number knowledge. However, some studies have produced opposite results. Research has found that some primary school students are more accurate in solving incongruent items than congruent ones (Gómez & Dartnell, 2019; González-Forte et al., 2020a; Resnick et al., 2019; Rinne et al., 2017). Similar results have been obtained with undergraduates (Barraza et al., 2017; DeWolf & Vosniadou, 2015; Obersteiner et al., 2020). These results suggest the existence of other incorrect reasoning strategies which are not based on a straightforward application of natural number knowledge.

Resnick et al. (2019) considered that since the largest fraction has the smallest numerator and denominator, students might think that *smaller digits produce larger magnitudes*. A similar explanation is students' reasoning based on comparing the denominator of each fraction and considering that *the smallest the denominator, the largest the fraction* (Fazio et al., 2016; Stafylidou & Vosniadou, 2004). This reasoning, hereon called reverse bias, implies that smaller denominators indicate that pieces are larger; therefore, "2/3 is larger than 3/5 because 3 pieces are larger than 5 pieces" (Pearn & Stephens, 2004, p. 434). Thus, students who use this reasoning seem to recognize that larger numbers in the denominator can lead to smaller fractions, but they do not fully understand the relationship between numerator and denominator (Rinne et al., 2017). This reasoning has been attributed mostly to undergraduates and adults when they solve complex fraction comparison items (Barraza et al., 2017; DeWolf & Vosniadou, 2015; Gómez & Dartnell, 2015; Obersteiner et al., 2020), that is, fractions composed of two digits and without common numerators or denominators.

Furthermore, the use of gap thinking (Pearn & Stephens, 2004) when students solve fraction comparison items can also explain that some studies observe a higher accuracy in incongruent items than in congruent ones (Gómez et al., 2017; González-Forte et al., 2020b). Gap thinking is based on comparing the (absolute) difference between numerator and denominator in both fractions, interpreting this difference as the number of parts missing to complete the whole. Therefore, students consider that “a fraction is larger if the difference between the numerator and the denominator is smaller” (e.g. 2/3 is larger than 7/9 “because from 2 to 3 there is a gap of one and from 7 to 9 there is a gap of two”). In the same way, students can consider that both fractions are equal if the difference is equal (e.g. 2/3 and 3/4 are equivalent) (Clarke & Roche, 2009). This kind of incorrect thinking evidences a tendency to reason in additive rather than in multiplicative terms (Clarke & Roche, 2009; Moss, 2005). In this sense, Mitchell and Horne (2010) consider that this reasoning based on the gap is due to the “influence of part-whole counting and shading activities rather than activities framed in partitioning, unit forming and equivalence actions” (p. 417).

These previous studies with primary and secondary school students have suggested the existence of a reverse bias and of gap thinking, in addition to natural number bias thinking (Behr et al., 1984; Clarke & Roche, 2009; Mitchell & Horne, 2010; Pearn & Stephens, 2004; Smith, 1995). However, the studies were not explicitly focused on documenting these ways of thinking.

As far as we know, there is only one study that explicitly focused on these ways of thinking (Gómez & Dartnell, 2019) with 5th, 6th, and 7th grade students. In this study, profiles were identified from students’ performances in a test with congruent and incongruent fraction comparison items with common components and without common components. Results showed the existence of six distinct profiles. Besides the subgroup of students who achieved high accuracy in all types of items, results showed the existence of a subgroup of students who correctly answered congruent items but incorrectly the incongruent ones, and another subgroup who answered congruent items better compared to incongruent items in fractions with common components. This finding led them to conclude that students within both profile groups were biased by their natural number ordering knowledge. Furthermore, they identified a subgroup of students who answered incongruent items correctly and the congruent ones incorrectly. This finding led them to conclude that these students were reasoning in a way that is opposite to the natural number bias, i.e. what we called reverse bias. Finally, in addition to the subgroup of students who obtained low percentages of accuracy in all items, the results showed a subgroup of students who accurately answered all the items, except the congruent ones without common components. This profile raised questions about the possible use of gap thinking in this kind of items, i.e. where gap thinking leads to the incorrect answer. However, because the item set was not specifically designed to elicit gap thinking, their results may suggest the use of gap thinking by the students but could not prove unequivocally this.

The latter study takes a first step in explaining the previously obtained results where congruent items obtained lower accuracy rates compared to incongruent ones. However, its conclusions were based on indirect evidence, as inferences were made based on comparing accuracies in specific kinds of items. Studies focused on examining students’ verbalizations that support these inferences are needed (Gómez & Dartnell, 2019; González-Forte et al., 2020a; Obersteiner et al., 2020) in order to validate students’ ways of thinking when students determine rational number size. Nevertheless, as far as we know, no such studies exist.

Furthermore, it is interesting to validate not only students' ways of thinking through their verbalizations but also the level of confidence that students have in their ways of thinking. This will be clarified in the next section.

Assessing Students' Confidence

Written mathematics assessments generally focus on the procedural competence, but an important issue linked to this competence is the student's confidence in the answers that they give (Kyriacou, 2005). The confidence of response index (CRI) has its origin in the social sciences, where it is used particularly in surveys and where a respondent is requested to provide the degree of certainty he has in his own ability to select and utilize well-established knowledge or concepts to arrive at an answer (Hasan et al., 1999; Webb et al., 1994).

Regardless of whether the answer is correct or not, a low confidence indicates a guess which, in turn, implies a lack of knowledge. However, a high confidence and an incorrect answer points to a misplaced confidence in the subject matter knowledge, either misjudging his own ability or a sign of the existence of misconceptions (Engelbrecht et al., 2005; Hasan et al., 1999). While a lack of knowledge can be improved with instruction and subsequent learning, misconceptions — which tend to go along with higher levels of confidence — are believed to hamper the appropriate integration of new knowledge or skills (Hasan et al., 1999; Merenluoto & Lehtinen, 2002; Vamvakoussi & Vosniadou, 2004).

Previous studies have examined students' confidence by asking them to indicate how sure they were about the correctness of their answers (De Bock et al., 2002; Foster, 2016; Lundeberg et al., 1994). De Bock et al. (2002) carried out individual semi-standardized interviews aimed at analyzing the thinking process underlying secondary school students' improper linear reasoning and how this process is affected by their mathematical conceptions, beliefs, and habits. The student was asked to indicate how sure he was about the correctness of his answer, by choosing position on a five-point scale (from "certainly wrong" to "certainly correct"). When a student did not indicate "certainly correct," the interviewer asked why he was not absolutely sure. Then, the student was confronted with an alternative solution given by a group of fictitious peers. The student was asked which answer he preferred: the initial answer or the alternative that emerged in the peer group. After the student made his decision, the interviewer asked for a justification. If the student did not react spontaneously, the interviewer asked if the argumentation of the peer did (not) raise doubts about his initial answer. Results from De Bock et al.'s (2002) study showed that the great self-confidence observed in most students seemed to indicate that for them, the procedure employed was self-evident and any other answer would not even deserve consideration. Students' reasoning (even being incorrect) was perceived as correct without a need for any further justification, and students were reluctant to question the correctness of it when confronted with conflicting evidence.

Measuring students' level of confidence regarding the correctness of their own reasoning and asking for the reason of their (in)confidence, paying special attention to those who use incorrect reasoning, can allow us to know if they are overconfident with their way of thinking while being incorrect (thus pointing at the presence of incorrect knowledge, i.e. a misconception) or they are simply not sure about the correctness of their answers (thus pointing to a lack of knowledge). As it is mentioned previously, a lack of knowledge can be improved with instruction and subsequent learning; however, when students have certain misconceptions, these are believed to hamper the appropriate integration of new knowledge or skills. This incorrect prior knowledge can be resistant to change (as it is related to higher confidence levels) and may require a different kind of instruction.

Furthermore, presenting students with other alternative reasoning might additionally show if they are reluctant to question the correctness of their own way of reasoning and, thus, to what extent these ways of reasoning are stable and resistant to change. Previous research has pointed out that people are often unaware of incorrect answers and that they are usually overconfident regarding their knowledge (De Bock et al., 2002; Lichtenstein & Fishhoff, 1981).

The Present Study: Objective and Research Questions

This study is part of a larger study carried out on 1262 primary and secondary school students, which focused on students' various ways of thinking when solving fraction and decimal number comparison items. These students individually answered a multiple-choice (pencil) test with 31 fraction and decimal comparison items, where they had to circle the largest number; if they thought that both numbers were equally large, they had to circle both (González-Forte et al., 2020a).

Fraction items were carefully designed to examine responses based on the natural number bias, gap thinking, or reverse bias. In this way, there were congruent items (compatible with a reasoning based on natural number knowledge, such as 5/8 vs. 2/7), incongruent items (incompatible with a reasoning based on natural number knowledge, such as 2/3 vs. 3/7), items where a reasoning based on gap thinking leads to the correct answer (e.g. 3/7 vs. 7/9), and items where gap thinking leads to an incorrect answer (e.g. 5/9 vs. 1/3). The congruent and incongruent items also allowed us to examine the presence of reverse bias reasoning, since this reasoning allows to solve incongruent items correctly and congruent ones incorrectly.

Using a TwoStep Cluster Analysis, six student profiles were inferred (González-Forte et al., 2020a): students who answered all (or almost all) the items correctly (*All correct*); students who incorrectly solved all the incongruent items both in fractions and decimal numbers, and correctly solved the congruent ones (*Full NNB*); students who correctly solved the congruent fraction items and incorrectly solved the incongruent ones, but correctly solved the congruent and incongruent decimal items (*Fraction NNB*); students who had difficulties only in the items where gap thinking leads to the incorrect answer (*Gap thinker*); students who incorrectly solved the congruent fraction items and correctly solved the incongruent ones, but correctly solved both congruent and incongruent decimals items (*Reverse bias*); and students who solved the items without any identifiable pattern (*Remainder*). Based on these profiles, we can implicitly infer the reasoning used by students, but we do not obtain any direct evidence of their way of reasoning.

The objective of the present study was to validate the existence of the different incorrect ways of thinking about fraction size inferred from the profiles obtained in the previous study (using a multiple-choice test) (González-Forte et al., 2020a) and as suggested by previous studies in the field. This validation is done by conducting interviews with a specific selection of students who also participated in the previous study. These students were chosen based on the information of the previous study, in which the different profiles of learners as mentioned above were found. This allowed us to examine students' verbalizations in each student profile when confronted with similar items. This can also allow us to provide explanations for the previously conflicting results found with regard to the congruency effect. Furthermore, we were interested in examining if students' ways of thinking are resistant to change. To measure it, during the interview, (i) we examined students' level of confidence in their ways of thinking since high confidence levels are associated with the presence of certain misconceptions — resistant to change — and low confidence levels are associated with students' lack of knowledge — open to change — and (ii) we confronted students with other kinds of alternative reasoning that might allow us to know whether they are either prepared to change their answer or not, and thus whether they are reluctant to question the correctness of their reasoning (for a similar approach, see the study of De Bock et al., 2002).

The research questions formulated were:

- Can the incorrect ways of thinking about fraction size that were indirectly inferred from written responses in the previous study (profiles inferred from the multiple-choice test) be validated by means of the verbalizations of the students (interviews)?
- Are these ways of thinking resistant to change, as shown by their confidence levels and their preparedness to consider alternative reasoning?

Method

Participants

The participants of this study are a subsample 52 students out of the larger group of 1262 Spanish students who also participated in the previous study (González-Forte et al., 2020a). These students (36 were boys) in 7th grade (12- and 13-year-olds) belonged to the different profiles obtained: 15 students from the *All correct* profile, 14 students from the *Gap thinker* profile, 15 students from the *NNB* profile (*Fraction NNB* and *Full NNB* profiles), and 8 students from the *Reverse bias* profile. We chose 7th grade students since this was the grade that had sufficient numbers of participants in each profile in the study by González-Forte et al. (2020a). Schools belonged to different cities and students were from mixed socio-economic backgrounds. The instrument to collect that in the present study were interviews that were conducted 3 months after participating in the previous study where we inferred students' profiles (multiple-choice test).

Instrument and Procedure

The interviews consisted of two parts. In part 1, the 52 students were shown four of their own answers in the multiple-choice test and were asked to explain how they had found the largest fraction. If they considered that they had incorrectly chosen the largest fraction in the multiple-choice test, they could change their answer during the interview, and then justify their reasoning. Items used in the interview were 2/3 vs. 7/9; 4/7 vs. 1/3; 4/5 vs. 5/8; and 2/3 vs. 3/7. Items 2/3 vs. 7/9 and 4/7 vs. 1/3 are compatible with a reasoning based on natural number knowledge, and incompatible with a reasoning based on gap thinking or reverse bias. In contrast, items 4/5 vs. 5/8 and 2/3 vs. 3/7 are compatible with a reasoning based on gap thinking or reverse bias, and incompatible with a reasoning based on natural number knowledge. Table 1 summarizes the characteristics of the items used in part 1 of the interview (C indicates compatible with the reasoning and I incompatible).

Furthermore, in part 1, in order to know the students' level of confidence regarding their own reasoning, each student had to say how much confidence he/she had in his/her answer. Therefore, they had to pick an option from the following scale: (1) *I have serious doubts*, (2) *I have some doubts*, (3) *Almost sure*, and (4) *Absolutely sure*. If students did not choose "Absolutely sure," they had to explain their reasons.

In part 2, students had to read three fictional students' answers to two fraction comparison items with the same format as the test. Items were 1/3 vs. 5/8, which is compatible with a reasoning based on natural number knowledge, and incompatible with a reasoning based on gap thinking or reverse bias thinking; and the item 4/7 vs. 3/4, which is compatible with a reasoning based on gap thinking or reverse bias, and incompatible with a reasoning based on natural number knowledge. In each item, the three fictional students' answers were: a student's answer that used a reasoning based on natural number knowledge, a student's answer that used a reasoning based on gap thinking, and a student's answer that used a reasoning based on reverse bias (Table 2). Students had to indicate whether they agreed or disagreed with each of the students' answers and had to explain their reasoning.

The interviews were carried out individually and were videotaped, after prior consent of their parents. There was no time limitation and students were provided with pieces of paper in case they needed them to perform operations, annotations, or drawings. Generally, the duration of each interview was between 10 and 15 min.

Table 1 Items used in part 1 and their characteristics

Reasoning based on	2/3 vs. 7/9	4/7 vs. 1/3	4/5 vs. 5/8	2/3 vs. 3/7
Natural number bias	C	C	I	I
Gap thinking	I	I	C	C
Reverse bias	I	I	C	C

Table 2 Fictional students' answers as shown in part 2

Item	Answer
1/3 vs. 5/8	**Pere** (Gap thinking): *It's 1/3 because from 1 to 3 there is a difference of 2 and from 5 to 8 there is a difference of 3, so in 1/3 the difference between the numerator and the denominator is smaller*
	Marta (Natural number bias): *It's 5/8 because 5 is larger than 1 and 8 is larger than 3*
	Andrés (Reverse bias): *It's 1/3 because 3 is smaller than 8, and if the denominator is smaller the fraction is larger*
4/7 vs. 3/4	**Maria** (Gap thinking): *It's 3/4 because from 3 to 4 there is a difference of 1 and from 4 to 7 there is a difference of 3, so in 3/4 the difference between the numerator and the denominator is smaller*
	Roberto (Natural number bias): *It's 4/7 because 4 is larger than 3 and 7 is larger than 4*
	Alicia (Reverse bias): *It's 3/4 because 4 is smaller than 7, and if the denominator is smaller the fraction is larger*

Analysis

The analysis was carried out in two phases. First, students' verbalizations in both parts of the interview were analyzed to determine if they used a reasoning based on natural number ordering, gap thinking, or reverse bias thinking. This analysis allowed us to identify students' different ways of thinking when comparing fractions and, therefore, provide evidence to support the conclusions that were indirectly inferred from the information gathered by the multiple-choice test (profiles) and in previous studies in the field. Data from this analysis answer the first research question.

Second, to answer our second research question, whether these ways of thinking are resistant to change, we analyzed the confidence levels from students given in the part 1 of the interview. With the scores obtained in the confidence scale [1–4], the average scores were calculated for each profile (calculating percentages over 10). Furthermore, we analyzed how the students considered the alternative reasoning when they were confronted in the part 2 of the interview.

Results

In this section, we first discuss the different ways of thinking about fraction size identified exemplifying them with students' verbalizations. Second, we focus on the question whether these ways of thinking are resistant to change.

Different Ways of Thinking Identified

We present the results in this section considering student's thinking by profile (15 students belonged to *All correct* profile, 14 students to the *Gap thinker* profile, 15 students to the *NNB* profile (*Fraction NNB* and *Full NNB* profiles), and 8 students to the *Reverse bias* profile). During the interview, most students within each profile used the reasoning inferred from their answers in the preceding multiple-choice test. Therefore, our results validate the existence of the different incorrect ways of thinking about fraction size inferred from the profiles obtained in the previous study. Below, we detail each profile and give an example of a student's reasoning for the various items presented, as well as his or her reactions to the fictitious students' answers.

Thirteen out of the 15 students of the *All correct* profile used a correct reasoning in part 1 and did not choose any of the fictional students' answers presented in part 2. Tables 3 and 4 present examples of verbalizations given by one of these students (P154) in both parts. We can observe in both tables that this student did indeed follow a correct type of reasoning. In the first two items, the two fractions' exact values are compared (by creating fractions with the same denominator), while in the last two, half is used as a benchmark, which is a useful and correct strategy for these items. In response to the fictitious answers of other students, student P154 was able to put in words why each fictitious student showed incorrect reasoning.

Eleven out of the 15 students of the *NNB* profile used a reasoning based on the natural number ordering in the interview to determine the magnitude of fractions in part 1 and they chose Marta and Roberto's answers in part 2. They always considered the fraction with a larger numerator and denominator to be the largest fraction. Tables 5 and 6 exemplarily show the verbalizations given by one of these students (P027) in both parts. Both tables illustrate how this student indeed followed a reasoning based on natural number knowledge. In the four items in part 1, the student verbalized that the largest fraction was the fraction with the larger numerator and denominator which led him/her to the correct answer in the first and third items, but to the incorrect answer in the second and fourth items. In response to the fictitious answers of other students, this student understood that the other fictitious answers were incorrect, but could not see this for the answer based on natural number knowledge.

Nine out of the 14 students of the *Gap thinker* profile used a reasoning based on gap thinking in part 1 and chose Pere and Maria's answers in part 2. Thus, they always considered the fraction with the smallest difference between numerator and denominator as the largest fraction. Tables 7 and 8 exemplarily show the

Table 3 Answers of student P154 within the *All correct* profile in part 1

Item	Reasoning
2/3 vs. 7/9	*7/9 is larger because if we multiplied the numerator and denominator of 2/3 by 3, both fractions will have the same denominator: 6/9 and 7/9. Therefore, 7/9 is larger*
4/5 vs. 5/8	*4/5 is larger because if I multiply the numerator and denominator of 4/5 by 8, it will be 32/40, and the numerator and denominator of 5/8 by 5, it will be 25/40. Therefore, 4/5 is larger than 5/8*
4/7 vs. 1/3	*4/7 is larger because it is larger than a half, and 1/3 is less than a half*
2/3 vs. 3/7	*2/3 is larger because 3/7 is less than a half, and 2/3 is larger than a half*

Table 4 Answers of student P154 within the *All correct* profile in part 2

Item	Answer	Reasoning	
1/3 vs. 5/8	*None*	Pere	*I disagree with him. He used the difference between numerator and denominator but that is not correct. Because if we had 0/3 … even if the difference is equal to 5/8 … they will not be the same*
		Marta	*Marta's strategy is only correct for some fractions… such as this one. But for example, in 2/7, the numbers are larger than 1/3, but the fraction is smaller*
		Andrés	*I disagree. It is possible that a fraction has the largest denominator and be the largest fraction. Not only do you have to look at the denominator but also at the numerator*
4/7 vs. 3/4	*None*	Maria	*This demonstrates that Pere can get it wrong or get it right. In the previous answer, he got it wrong and now he got it right, using the same strategy. I can say that the strategy is totally incorrect*
		Roberto	*In this case, we can see Marta's error when she compares the numbers. It doesn't work now to use the same strategy as before*
		Alicia	*Alicia needs to take into account the numerator as well*

Table 5 Answers of student P027 within the *NNB* profile in part 1

Item	Reasoning
2/3 vs. 7/9	*The two numbers of the fraction always guide me, that is, I look for the largest numbers. In this case, 7 and 9 are larger than 2 and 3, so 7/9 is larger*
4/5 vs. 5/8	*5/8 is larger since 5 and 8 are larger than 4 and 5*
4/7 vs. 1/3	*4/7 is larger because 4 and 7 are larger than 1 and 3. I look at the two numbers, numerator and denominator, and I compare them*
2/3 vs. 3/7	*3/7 is larger because 3 and 7 are larger than 2 and 3*

verbalizations of one of these students (P014) in both parts. One can observe in both tables how this student indeed followed a reasoning based on gap thinking. In the four items of part 1, the student verbalized the difference between numerator and denominator, which led him/her to the correct answer in the second and fourth items, but to the incorrect answer in the first and third items. In response to the fictitious answers of other students, this student understood that the other fictitious answers were incorrect, but could not see this for the gap thinking answer.

Six out of the 8 students of the *Reverse bias* profile showed a reverse bias thinking in part 1 and chose Andrés and Alicia's answers in part 2. They always considered the fraction with the smallest denominator to be the largest fraction. Tables 9 and 10 exemplarily show the verbalizations given by one of these students (P131) in both parts. In both tables, we can observe how this student indeed followed a reasoning based on reverse bias. In the four items of part 1, the student could put in words that the larger fraction is the fraction with the smallest denominator, which led him/her to the correct answer in the second and the fourth items, but to the incorrect answer in the first and third items. In response to the fictitious answers of other students, this student understood that the other fictitious answers were incorrect but could not see this for the answer based on a reverse bias thinking.

Regarding students who did not use the reasoning that we inferred from the preceding multiple-choice test (profile) during the interview, 5 out of the 13 students in the first part of the interview still gave answers in the same way as the reasoning inferred from the profile, but in the second part, when they saw various other ways of reasoning, they changed their own way of reasoning. The remaining eight students answered both parts of the interview with a reasoning that was different from that inferred of their profile.

Are These Ways of Thinking Resistant to Change?

Table 11 shows the mean confidence percentages in each item and profile of students who reasoned in the interview in the same way as the reasoning inferred from the multiple-choice test (profile).

Students within the *All correct* profile were more confident about their reasoning than students in the other profiles. However, the percentages of *Gap thinker*, *NNB*, and *Reverse bias* profiles also showed a high level of student confidence (an

Table 6 Answers of student P027 within the *NNB* profile in part 2

Item	Answer		Reasoning
1/3 vs. 5/8	*Marta*	Marta	*I would say the same as Marta. 5/8 is larger because numbers are larger*
4/7 vs. 3/4	*Roberto*	Roberto	*Roberto thinks like me. For example, a fraction larger than 4/7 would be 5/8; and one larger than 5/8 would be 6/9. The larger the numerator and the denominator, the larger the fraction*

Student P027 did not say anything in the rest of the answers, he/she only indicated and justified that he/she agreed with Marta and Roberto's answers.

Table 7 Answers of student P014 within the *Gap thinker* profile in part 1

Item	Reasoning
2/3 vs. 7/9	*2/3 is larger because in 2/3 there is a difference of 1 and in 7/9 there is a difference of 2*
4/5 vs. 5/8	*4/5 is larger because in 4/5 there is a difference of 1 and in 5/8 there is a difference of 3. Therefore, the difference is smaller*
4/7 vs. 1/3	*1/3 is larger because in 1/3 there is a difference of 2, and in 4/7 there is a difference of 3*
2/3 vs. 3/7	*2/3 is larger because in 3/7 there is a difference of 4 and in 2/3 there is a difference of 1. So the difference is smaller*

Table 8 Answers of student P014 within the *Gap thinker* profile in part 2

Item	Answer		Reasoning
1/3 vs. 5/8	*Pere*	Pere	*This one is correct. The smaller the difference, the larger the fraction*
		Marta	*This would be incorrect*
		Andrés	*This would be incorrect*
4/7 vs. 3/4	*Maria*	Maria	*This one is correct to me. The fraction that has the smallest difference is the largest*
		Roberto	*This is incorrect to me*
		Alicia	*This is incorrect. It is possible for the largest fraction to have the largest denominator*

Table 9 Answers of student P131 within the *Reverse bias* profile in part 1

Item	Reasoning
2/3 vs. 7/9	*2/3 is larger than 7/9 because there are less pieces* (he/she observed the 3 in the denominator of 2/3) *but the pieces are larger*
4/5 vs. 5/8	*4/5 is larger. I know because I always look at the denominator, since there are fewer pieces, the pieces must be larger*
4/7 vs. 1/3	*1/3 is larger because there are larger pieces than in 4/7. I rely on the size of the parts. The fraction has to have the biggest piece, and not the biggest number of pieces*
2/3 vs. 3/7	*2/3 is larger because the pieces are larger. In 3/7, there are more pieces, but they are smaller*

approximate mean of 80%). Students' high level of confidence in their ways of thinking indicates that they believe the reasoning they used is totally correct, even in situations where it leads to an incorrect answer. Therefore, it points to a misplaced confidence in the subject matter knowledge and may be a sign of the existence of misconceptions (that are believed to be resistant to change).

Furthermore, the fact that in the part 2 of the interview we confronted students with other alternative reasoning allowed us to know whether students changed their answer, or whether they were reluctant to question the correctness of their reasoning, thus, showing a reasoning more resistant to change. In the previous section, it has been shown that 11 out of the 15 students of the *NNB* profile chose Marta and

Table 10 Answers of student P131 within the *Reverse bias* profile in part 2

Item	Answer		Reasoning
1/3 vs. 5/8	*Andrés*	Pere	*I think this answer is incorrect, I agree with Andrés' one*
		Marta	*It is not right to me*
		Andrés	*This is what I have done. I think this one is correct. The strategy that I choose is this one*
4/7 vs. 3/4	*Alicia*	Maria	*I am not sure whether this explanation is correct. I choose Alicia's answer*
		Roberto	*It is incorrect. I believe that if the denominator is smaller, the fraction must be larger, because the pieces are larger*
		Alicia	*I believe it is correct. This student has answered like me*

Table 11 Mean confidence percentages in each item and profile

	2/3 vs. 7/9	4/5 vs. 5/8	4/7 vs. 1/3	2/3 vs. 3/7
All correct	94.3	94.3	100	98.0
Gap thinker	86.0	83.3	80.5	83.3
NNB	79.6	77.3	81.8	77.3
Reverse bias	75.0	75.0	75.0	79.3

Roberto's answers based on the natural number ordering in part 2; 9 out of the 14 students of the *Gap thinker* profile chose Pere and Maria's answers based on gap thinking in part 2; and 6 out of the 8 students of the *Reverse bias* profile chose Andrés and Alicia's answers based on a reverse bias thinking in part 2. These results show that the majority of students did not change their incorrect way of thinking when they were confronted with other alternative reasoning; therefore for them, the way of thinking used was self-evident and any other answer would not deserve consideration. These students were reluctant to question the correctness of their reasoning showing resistance to change.

Discussion and Conclusions

Research on the understanding of fraction size has widely considered the natural number bias as a major source of learners' mistakes in fraction comparison tasks, not only in primary and secondary school students (González-Forte et al., 2020b; Meert et al., 2010) but also in undergraduates (DeWolf & Vosniadou, 2011). However, recent studies have also found evidence of other incorrect strategies, such as reverse bias or gap thinking (DeWolf & Vosniadou, 2015; Gómez & Dartnell, 2019; Obersteiner et al., 2020; Rinne et al., 2017). Nevertheless, these studies indirectly inferred these kinds of reasoning based on profiles of correct and incorrect answers to series of multiple-choice items. Until now, there were no studies that explicitly validated these students' reasoning by examining students' verbalizations by means of interviews. Next to this, as far as we know, this is the first study that investigated whether these ways of reasoning are resistant to change.

Students' verbalizations in both parts of the interview have confirmed the existence of different incorrect ways of thinking about fraction size supporting the results obtained from the multiple-choice test study (profiles inferred) (González-Forte et al., 2020a) and from other studies in the research field (Barraza et al., 2017; Gómez & Dartnell, 2019; Obersteiner et al., 2020; Resnick et al., 2019). Some students used a reasoning based on natural number ordering "a fraction is larger if numerator and denominator are larger" (Behr et al., 1984; Pearn & Stephens, 2004). Others used a reasoning based on gap thinking "a fraction is larger if the difference between the numerator and the denominator is smaller" (Pearn & Stephens, 2004; Stafylidou & Vosniadou, 2004). And still other students used a reverse bias thinking "a fraction is larger if the denominator is smaller" (Fazio et al., 2016; Rinne et al., 2017; Stafylidou & Vosniadou, 2004). In sum, some students used other incorrect ways of thinking different than natural number–based reasoning when comparing fractions, such as gap thinking and reverse bias thinking, which can explain the conflicting results obtained in previous studies with regard to the congruency effect: while many studies found higher accuracy rates in congruent items compared to incongruent items (DeWolf & Vosniadou, 2011; González-Forte et al., 2020b; Meert et al., 2010), other studies found high accuracies in incongruent items and low accuracies in congruent ones (DeWolf & Vosniadou, 2015; Gómez & Dartnell, 2019; Obersteiner et al., 2020; Rinne et al., 2017).

Our results also show that reverse bias thinking and gap thinking are present in secondary school when students solve simple fraction comparison items (proper fractions with a one-digit numerator and denominator), which had been previously attributed mostly to undergraduates and adults when solving complex fractions (DeWolf & Vosniadou, 2015; Gómez & Dartnell, 2015; Obersteiner et al., 2020).

Furthermore, our results provide information about the resistance to change of these ways of reasoning. First, mean confidence percentages of the three profiles that use an incorrect reasoning (*Gap thinker*, *NNB*, and *Reverse bias*) remained high — approximately 80%. These students had high confidence in their way of thinking, even in situations where it leads to an incorrect answer. Students' high levels of confidence in their incorrect reasoning can indicate a misconception, which are believed to hamper the appropriate integration of new knowledge or skills (Hasan et al., 1999; Merenluoto & Lehtinen, 2002; Vamvakoussi & Vosniadou, 2004). Furthermore, the fact that students are not aware that their thinking is incorrect can lead them to being overconfident about their knowledge (Lichtenstein & Fishhoff, 1981), and thus their way of thinking becoming consolidated and difficult to change during instruction (Fischbein, 1987).

Second, results of the second part of the interview showed that the majority of students did not change their answer when they saw other ways of reasoning. Thus, the incorrect ways of thinking are stable and resistant to change, rather than random errors due to guessing, for instance. As in De Bock et al.'s study (2002), students' incorrect reasoning was perceived as correct without a need for any further justification, students were overconfident in it, and were reluctant to question the correctness of it when confronted with conflicting alternative responses, as they perceived their own answer as logical, self-evident. In other words, students were reluctant to change their choice of reasoning, using the same kind of reasoning that they had used both in the test and in the first part of the interview.

Educational Implications and Further Research

Our results have educational implications. In the present study, the different incorrect ways of thinking that students use to determine fraction size were identified and validated. This information might help primary and secondary school teachers to identify students' different incorrect ways of thinking about the fraction size. In fact, identifying incorrect ways of thinking is not an easy task for teachers since, as it has been shown in previous studies, handbooks typically do not explicitly talk about incorrect strategies that students may use and can increase natural number–based reasoning (Van Dooren et al., 2019). In a natural number biased way of thinking, students think that numerator and denominator are two independent numbers that behave as natural numbers, without identifying the multiplicative relationship between both. In gap thinking, students identify an additive relationship between the numerator and the denominator. This incorrect way of thinking comes from additive instead of multiplicative thinking. In fact, previous research has shown students' difficulties in the transition from the additive to multiplicative thinking (Hart et al., 1981; Van Dooren et al., 2010). Finally, in the reverse bias thinking, students think that the larger fraction is the fraction with the smaller denominator since the size of the parts are bigger. This reasoning is valid for fraction comparison with the same numerator, but cannot be generalized to other kinds of fraction comparisons. During the instruction, teachers should emphasize the multiplicative relationship between the numerator and denominator and use fraction comparison items with different characteristics (compatible and incompatible with these ways of thinking) to avoid these incorrect ways of thinking.

Furthermore, our results revealed high levels of confidence in the use of these incorrect ways of thinking. Therefore, it is unlikely that overconfident students in their incorrect reasoning would overcome them spontaneously. It is thus important that teachers are aware of the existence of the profiles found, as well as the individual differences in the different ways learners of the same age think. Being aware of these different ways of thinking can help teachers introduce teaching examples that are incompatible with each incorrect way of thinking found. In other words, they should put students in situations in which a certain way of thinking is not always applicable, to avoid students from generalizing the use of this way of thinking. Therefore, our results provide useful information for developing the primary and secondary school curriculum. Furthermore, these findings are also useful for designing courses for initial and in-service teachers' training.

The present study brings about further lines of research. The existence of other incorrect ways of thinking that are not based on natural number knowledge raises questions about when and why these incorrect ways of thinking appear. It is widely assumed that the natural number bias is due to the interference of students' prior knowledge about natural numbers as they work with rational numbers. Thus, it mainly occurs at the beginning of the learning process. In addition, our results regarding the insecurity of students who reason this way (*NNB* profile) seem to show that it is a naïve way of thinking. However, gap thinking and reverse bias thinking seem to be acquired during middle school (González-Forte et al., 2020a). Future longitudinal studies could focus on the emergence and development of these types of thinking.

Further research on gap thinking is also necessary. The present study included fraction comparison items with a different gap between numerator and denominator. However, the inclusion of fraction comparison items with the same gap, where gap thinking leads to the incorrect answer ("both fractions are equal"), could help to find differences between students who use gap thinking only when the gap is different and students who use gap thinking in both cases. Furthermore, the gap thinking strategy is an "additive" way of looking at fractions instead of a multiplicative one, and therefore comes close to the correct way of thinking (since it is a relational way of reasoning); but at the same time, it reflects a typical and resistant error in multiplicative reasoning (Fernández et al., 2012; Hart et al., 1981; Van Dooren et al., 2010). Further research about the origin of this "additive" reasoning and its transition to multiplicative — and correct — reasoning could be valuable.

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Declarations

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References

Barraza, P., Avaria, R., & Leiva, I. (2017). The role of attentional networks in the access to the numerical magnitude of fractions in adults/El rol de las redes atencionales en el acceso a la magnitud numérica de fracciones en adultos. *Estudios De Psicología, 38*(2), 495–522. https://doi.org/10.1080/02109395.2017.1295575

Behr, M. J., Lesh, R., Post, T., & Silver, E. (1983). Rational number concepts. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 91–125). Academic Press.

Behr, M., Wachsmuth, I., Post, T., & Lesh, R. (1984). Order and equivalence of rational numbers: A clinical teaching experiment. *Journal for Research in Mathematics Education, 15*(5), 323–341.

Clarke, D. M., & Roche, A. (2009). Students' fraction comparison strategies as a window into robust understanding and possible pointers for instruction. *Educational Studies in Mathematics, 72*(1), 127–138. https://doi.org/10.1007/s10649-009-9198-9

De Bock, D., Van Dooren, W., Janssens, D., & Verschaffel, L. (2002). Improper use of linear reasoning: An in-depth study of the nature and the irresistibility of secondary school students' errors. *Educational Studies in Mathematics, 50*(3), 311–334. https://doi.org/10.1023/A:1021205413749

DeWolf, M., & Vosniadou, S. (2011). The whole number bias in fraction magnitude comparisons with adults. In L. Carlson, C. Hoelscher, & T. F. Shipley (Eds.), *Proceedings of the 33rd Annual Conference of the Cognitive Science Society* (pp. 1751–1756). Cognitive Science Society.

DeWolf, M., & Vosniadou, S. (2015). The representation of fraction magnitudes and the whole number bias reconsidered. *Learning and Instruction, 37*, 39–49. https://doi.org/10.1016/j.learninstruc.2014.07.002

Engelbrecht, J., Harding, A., & Potgieter, M. (2005). Undergraduate students' performance and confidence in procedural and conceptual mathematics. *International Journal of Mathematical Education in Science and Technology, 36*(7), 701–712. https://doi.org/10.1080/00207390500271107

Fazio, L. K., DeWolf, M., & Siegler, R. S. (2016). Strategy use and strategy choice in fraction magnitude comparison. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 42*(1), 1–16. https://doi.org/10.1037/xlm0000153

Fernández, C., Llinares, S., Van Dooren, W., De Bock, D., & Verschaffel, L. (2012). The development of students' use of additive and proportional methods along primary and secondary school. *European Journal of Psychology of Education, 27*(3), 421–438. https://doi.org/10.1007/s10212-011-0087-0

Fischbein, E. (1987). *Intuition in Science and Mathematics. An educational approach*. Reidel.

Foster, C. (2016). Confidence and competence with mathematical procedures. *Educational Studies in Mathematics, 91*(2), 271–288. https://doi.org/10.1007/s10649-015-9660-9

Gómez, D. M., & Dartnell, P. (2015). Is there a natural number bias when comparing fractions without common components? A meta-analysis. In K. Beswick, T. Muir, & J. Wells (Eds.), *Proceedings of the 39th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 1–8). PME.

Gómez, D., & Dartnell, P. (2019). Middle schoolers' biases and strategies in a fraction comparison task. *International Journal of Science and Mathematics Education, 17*(6), 1233–1250. https://doi.org/10.1007/s10763-018-9913-z

Gómez, D., Silva, E., & Dartnell, P. (2017). Mind the gap: Congruency and gap effects in engineering students' fraction comparison. In B. Kaur, W. K. Ho, T. L. Toh, & B. H. Choy (Eds.), *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 353–360). PME.

González-Forte, J. M., Fernández, C., Van Hoof, J., & Van Dooren, W. (2020a). Various ways to determine rational number size: an exploration across primary and secondary education. *European Journal of Psychology of Education, 35*(3), 549–565. https://doi.org/10.1007/s10212-019-00440-w

González-Forte, J. M., Fernández, C., & Van Dooren, W. (2020b). Is there a gap or congruency effect? A cross-sectional study in students' fraction comparison. Studia Psychologica, 62(2), 109–122. https://doi.org/10.31577/sp.2020.02.794

Hart, K. M., Brown, M. L., Kuchemann, D. E., Kerslake, D., Ruddock, G., & McCartney, M. (1981). *Children's understanding of Mathematics: 11–16*. John Murray.

Hasan, S., Bagayoko, D., & Kelley, E. L. (1999). Misconceptions and the certainty of response index (CRI). *Physics Education, 34*(5), 294–299.

Kyriacou, C. (2005). The impact of daily mathematics lessons in England on pupil confidence and competence in early mathematics: A systematic review. *British Journal of Educational Studies, 53*(2), 168–186. https://doi.org/10.1111/j.1467-8527.2005.00289.x

Lichtenstein, S., & Fischhoff, B. (1981). *The effects of gender and instructions on calibration* (Tech. Rep. PTR-1092–81–7). Decision Research.

Lundeberg, M. A., Fox, P. W., & Punćcohaŕ, J. (1994). Highly confident but wrong: Gender differences and similarities in confidence judgments. *Journal of Educational Psychology, 86*(1), 114–121. https://doi.org/10.1037/0022-0663.86.1.114

Meert, G., Grégoire, J., & Noël, M. P. (2010). Comparing the magnitude of two fractions with common components: Which representations are used by 10-and 12-year-olds? *Journal of Experimental Child Psychology, 107*(3), 244–259. https://doi.org/10.1016/j.jecp.2010.04.008

Merenluoto, K., & Lehtinen, E. (2002). Conceptual change in mathematics: Understanding the real numbers. In M. Limon & L. Mason (Eds.), *Reconsidering conceptual change: Issues in theory and practice* (pp. 233–258). Kluwer Academic Publishers.

Mitchell, A., & Horne, M. (2010). Gap thinking in fraction pair comparisons is not whole number thinking: Is this what early equivalence thinking sounds like? In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Vol. 2, pp. 414–421). MERGA.

Moss, J. (2005). Pipes, tubs, and beakers: New approaches to teaching the rational-number system. In M. S. Donovan & J. D. Bransford (Eds.), *How students learn: History, Math, and Science in the classroom* (pp. 309–349). National Academies Press.

Ni, Y., & Zhou, Y. D. (2005). Teaching and learning fraction and rational numbers: The origins and implications of whole number bias. *Educational Psychologist, 40*(1), 27–52. https://doi.org/10.1207/s15326985ep4001_3

Nunes, T., & Bryant, P. (2008). Rational numbers and intensive quantities: Challenges and insights to pupils' implicit knowledge. *Anales De Psicología/annals of Psychology, 24*(2), 262–270.

Obersteiner, A., Alibali, M. W., & Marupudi, V. (2020). Complex fraction comparisons and the natural number bias: The role of benchmarks. *Learning and Instruction*, *67*, 101307. https://doi.org/10.1016/j.learninstruc.2020.101307

Pearn, C., & Stephens, M. (2004). Why you have to probe to discover what year 8 students really think about fractions. In I. Putt, R. Faragher, & M. McLean (Eds.), *Mathematics education for the third millenium: Towards 2010 (Proceedings of the 27th Annual Conference of the Mathematics Education Research Group of Australasia)* (pp. 430–437). MERGA.

Resnick, I., Rinne, L., Barbieri, C., & Jordan, N. C. (2019). Children's reasoning about decimals and its relation to fraction learning and mathematics achievement. *Journal of Educational Psychology, 111*(4), 604–618. https://doi.org/10.1037/edu0000309

Rinne, L. F., Ye, A., & Jordan, N. C. (2017). Development of fraction comparison strategies: A latent transition analysis. *Developmental Psychology, 53*(4), 713–730. https://doi.org/10.1037/dev0000275

Smith, C. L., Solomon, G. E., & Carey, S. (2005). Never getting to zero: Elementary school students' understanding of the infinite divisibility of number and matter. *Cognitive Psychology, 51*(2), 101–140. https://doi.org/10.1016/j.cogpsych.2005.03.001

Smith, J. P. (1995). Competent reasoning with rational numbers. *Cognition and Instruction, 13*(1), 3–50. https://doi.org/10.1207/s1532690xci1301_1

Stafylidou, S., & Vosniadou, S. (2004). The development of students' understanding of the numerical value of fractions. *Learning and Instruction, 14*(5), 503–518. https://doi.org/10.1016/j.learninstruc.2004.06.015

Streefland, L. (1991). *Fractions in realistic mathematics education: A paradigm of developmental research*. Kluwer.

Vamvakoussi, X., & Vosniadou, S. (2004). Understanding the structure of the set of rational numbers: A conceptual change approach. *Learning and Instruction, 14*(5), 453–467. https://doi.org/10.1016/j.learninstruc.2004.06.013

Vamvakoussi, X., Van Dooren, W., & Verschaffel, L. (2012). Naturally biased? In search for reaction time evidence for a natural number bias in adults. *The Journal of Mathematical Behavior, 31*(3), 344–355.

Van Dooren, V., De Bock, D., & Verschaffel, L. (2010). From addition to multiplication… and back: The development of students' additive and multiplicative reasoning skills. *Cognition and Instruction, 28*(3), 360–381. https://doi.org/10.1080/07370008.2010.488306

Van Dooren, W., Lehtinen, E., & Verschaffel, L. (2015). Unraveling the gap between natural and rational numbers. *Learning and Instruction, 37*, 1–4. https://doi.org/10.1016/j.learninstruc.2015.01.001

Van Dooren, W., Christou, K., Depaepe, F., Inglis, M., Määtä, S., McMullen, J., Obersteiner, A., Heck Ribeiras, P., Van Hoof, J., Triandafyllou, M., Vamvakoussi, X., Verschaffel, L., Wittmann, G., Woollacott, B. (2019). *Tackling the natural number bias. A comparative textbook analysis*. Paper presented at the 18th Biennial Conference of the European Association for Research on Learning and Instruction (EARLI), Aachen, Germany.

Van Hoof, J., Vandewalle, J., Verschaffel, L., & Van Dooren, W. (2015). In search for the natural number bias in secondary school students' interpretation of the effect of arithmetical operations. *Learning and Instruction, 37*, 30–38. https://doi.org/10.1016/j.learninstruc.2014.03.004

Webb, J. M., Stock, W. A., & McCarthy, M. T. (1994). The effects of feedback timing on learning facts: The role of response confidence. *Contemporary Educational Psychology, 19*(3), 251–265. https://doi.org/10.1006/CEPS.1994.1020

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