Future preschool teachers' mathematical questions during shared book reading.

Abstract

Recent studies demonstrated that the adult-preschooler interaction during shared book reading (SBR) contributes to its effectiveness (Mol et al., 2008). The level of abstraction, or complexity, of the mathematical questions adults formulate during SBR serves as an indicator of the interaction quality. We aimed to investigate the chance of spontaneously formulating a mathematical question and the level of abstraction of the mathematical questions future preschool teachers propose to formulate during SBR, and their association with teachers' professional knowledge and beliefs, and type of picture book. Participants were 111 future preschool teachers. We investigated their chance of formulating a mathematical question and the level of abstraction of their mathematical questions using a video-based instrument, and distinguished between two types of picture books, namely mathematical and non-mathematical picture books. We additionally assessed their (1) mathematical content knowledge, (2) mathematical pedagogical content knowledge, and (3) beliefs about mathematics in general and about the teaching and learning of mathematics, with three online questionnaires. Data were analyzed using multilevel analyses. Results revealed that mathematical picture books increase the likelihood of formulating a mathematical question and provoked more abstract mathematical questions compared to non-mathematical picture books. There were no significant associations between teachers' professional knowledge and beliefs and the dependent variables. Our findings point to the importance of adequately selecting picture books to stimulate mathematical preschoolers' development via SBR, and also call for further investigations on the learning-supportive picture book characteristics and teacher characteristics.

Keywords: Early mathematics · Picture books · Shared book reading· Teachers' mathematical questions

1 Introduction

Given the importance of early mathematical skills for later (mathematical) success (Duncan et al., 2007), it is crucial to provide young children with various opportunities that stimulate their mathematical development via mathematically-rich materials and activities in natural, everyday situations such as play or shared book reading (e.g., Gasteiger & Benz, 2018; van Oers, 2010). Shared book reading (SBR) refers to an educational activity in which the teacher reads aloud to a group of children and which often includes teacher-child interactions about the picture book outside of the actual reading (van Kleeck & Vander Woude, 2003; Walsh & Hodge, 2018). Picture books are defined as books that are typically developed for young children and that contain a story which is conveyed through visual enriching pictures and narrative or descriptive text (Arizpe & Styles, 2003; Marston & Mulligan, 2012). Studies in the domain of early literacy consistently revealed that SBR is effective for young children's acquisition of oral language, vocabulary, and print knowledge (Lonigan et al., 2013; Mol et al., 2008; Zucker et al., 2013). Recently studies point to the potential of this activity for the acquisition of early mathematical skills as well (Purpura et al., 2017; van den Heuvel-Panhuizen & Elia, 2011). In line with studies in the domain of early literacy, researchers analyzed the characteristics of picture books (e.g., Ward et al., 2017) and the interaction behavior of the adult during SBR in the domain of mathematics (e.g., Hojnoski et al., 2014). However, empirical studies on the complex interplay between teachers' professional knowledge and beliefs, the characteristics of picture books, and the interaction quality during SBR within preschool mathematics education are currently missing. The present study aims at disentangling this complex interplay.

1.1 Shared book reading

Studies in the domain of early literacy demonstrated that the quantity of SBR activities (Barnes & Puccioni 2016; Mol et al., 2009) and the quality of the adult-preschooler interaction during this activity, often operationalized as the level of abstraction or complexity of children's and adults' utterances during SBR, contribute to early literacy development (Blewitt et al., 2009; van Kleeck et al., 1997; Zucker et al., 2010). Contrasting the domain of early literacy, research interest into this topic in the domain of mathematics is scarce and mainly focused on the number and type of mathematical utterances during SBR (Anderson et al., 2004; 2005; Hendrix et al., 2019). These studies revealed a large variety in mathematical utterances across adults and children. Moreover, the number of mathematical utterances was associated with the type of picture book (Hendrix et al., 2019; Hojnoski et al., 2014). Mathematical picture books, i.e., books written to stimulate preschoolers' mathematical development, were shown to result in more mathematical utterances than non-mathematical picture books, i.e., books without an explicit mathematical goal. To the best of our knowledge, only one study focused on the level of abstraction of the mathematical questions adults propose to formulate during SBR (Uscianowski et al., 2020). These researchers distinguished between four abstraction levels of questions (cf. Blank et al., 1978). Level 1 questions required children to respond to salient perceptual information, as inviting children to name, notice or locate objects or characters that are visually represented on the page (e.g. "What number do you see?"). Level 2 questions required children to focus on specific aspects and integrate separate components of salient perceptual information, as asking them to describe a scene, recall information, or name characteristics and functions of objects (e.g. "How many animals do you see?"). Level 3 questions required children to give an answer that is not saliently visible on the page but involved reordering and/or restructuring information from the page or previous pages to respond, as defining words, summarizing parts of the story, and making judgements (e.g. "How many animals do you have?"). Level 4 questions required children to predict what would happen next in the story or to solve problems (e.g. "If one of the animals runs away, how many animals will there be?"). Parents were shown to mainly propose questions at Level 2, with more abstract questions for the domain of number than shapes (Uscianowski et al., 2020). The level of abstraction was not associated with parents' characteristics (e.g., years of education, math anxiety, math beliefs and confidence). The child's numerical ability, as estimated by the parent, was significantly associated with the level of abstraction of questions parents formulated within the area of number.

1.2 Teacher Competence

Previous research demonstrated that teachers' professional knowledge contributes to instructional quality in the domain of mathematics (e.g., Author et al., 2020; Baumert et al., 2010; Hill et al., 2008). According to Blömeke et al. (2015), teacher competence can be framed as a continuum in which teachers' dispositions, and more specifically their professional knowledge and beliefs, affect their situation-specific skills, i.e., their skills to perceive and interpret a situation and decide how to act in that situation, which in their turn impact teachers' performance, i.e., their observable behavior.

The first component of Blömeke et al.'s framework, dispositions, refers to teachers' professional knowledge and beliefs. For teachers' professional knowledge, a distinction is made between domain-specific knowledge, and domain-general knowledge (Shulman, 1986). Regarding teaching mathematics, teachers' domain-specific knowledge involves their mathematical content knowledge (MCK) and mathematical pedagogical content knowledge (MPCK). MCK refers to the knowledge of the various mathematical domains (e.g., geometry, numbers) that ensures that teachers have sufficient background to teach mathematics (Ball et al., 2008). Teachers must know the mathematical content they are responsible for teaching not only from a more advanced perspective but beyond the level they are assigned to teach (National Mathematics Advisory Panel, 2008). MPCK includes knowledge of mathematics for the purpose of teaching, and more specifically: knowledge of mathematical tasks as instructional tools, and knowledge of students' thinking and assessment of understanding (e.g., misconceptions) (Ginsburg & Ertle, 2008). Teachers' domain-general knowledge involves their general pedagogical knowledge (GPK) which covers knowledge about teaching and learning in general, not related to a specific subject (e.g., classroom management) (König et al., 2016). Teachers' professional beliefs refer to their beliefs concerning learning and teaching in general as well as in a specific content domain (e.g., early mathematics) (Benz, 2012). The second component of the competence framework, situation-specific skills, involve teachers' ability to perceive and interpret a specific situation in the classroom and then make a decision about how to act in that situation, also referred to as the PID (perception, interpretation, decision) model (Blömeke et al., 2015; Kaiser et al., 2017). So, these situation-specific skills are related to one specific classroom situation, while teachers' professional knowledge includes more generalized cognition. Teachers' dispositions (component 1) are assumed to impact their situation-specific skills (component 2), which, at their turn, affect their actual behavior (component 3). However, empirical studies on the interplay between these different components in the domain of early mathematics are scarce.

To the best of our knowledge, only one empirical study explicitly focused on the association between preschool teachers' dispositions and their situation-specific skills in the domain of early mathematics (Dunekacke et al., 2016). Dunekacke et al. (2016) studied future preschool teachers' dispositions focusing on their beliefs concerning mathematics in general, their MCK, and their MPCK. Teachers' situation-specific skills were operationalized as perception (i.e., perceiving) and planning (i.e., decision-making) skills and measured using video-based tests. In line with the competence framework of Blömeke et al. (2015), these researchers found an association between, on the one hand, teachers' MPCK and application-oriented beliefs, and, on the other hand, their perception skills. Teachers' MPCK was also indirectly, via their perception skills, associated with their planning skills. Contrary to what could be expected on the basis of the competence framework of Blömeke et al. (2015), no association was observed between teachers' MCK and situation-specific skills. The authors explain this unexpected finding by referring to the role of MCK as a precondition for MPCK (Ball, 1988), as they found that teachers' MCK impacted their MPCK.

1.3 Current study

The current study addressed two gaps in our current knowledge about SBR and preschool teachers' competences in the domain of mathematics. First, empirical evidence on the interaction quality during SBR and its association with teacher characteristics is currently missing. Second, although the framework of Blömeke et al. (2015) would be useful to examine this association, empirical studies on the relationship between the different components of this model (i.e., dispositions, situation-specific skills, and performance) are scarce. To address these gaps, the current study aimed to investigate the association between teachers' dispositions and their situation-specific skills, more particularly the mathematical questions they propose to formulate during SBR. Since research point to the impact of picture book characteristics during SBR, we also investigated the contribution of the type of picture book on teachers' situation-specific skills (i.e., their proposed mathematical questioning behavior), as well as the interaction between dispositions and type of picture book on situation-specific skills.

Regarding teachers' situation-specific skills, we focused on their decision-making skills and more specifically, on the chance of proposing to formulate a mathematical question and the level of abstraction of the mathematical questions teachers propose to formulate during SBR. Teachers' dispositions were operationalized as their domain-specific professional knowledge, i.e., MCK and MPCK, and beliefs concerning the nature and the teaching and learning of mathematics (Blömeke et al., 2015; Dunekacke et al., 2016). Following previous studies in the domain of mathematics, we defined picture books as mathematical, i.e., books written to stimulate preschoolers' mathematical development, versus non-mathematical, i.e., books without an explicit mathematical goal (Hendrix et al., 2019; Hojnoski et al., 2014).

Given both the importance and the development of teacher competencies during teacher training (Kaiser et al., 2017), we invited future preschool teachers to participate. We aimed to answer the following six research questions for the questions these future preschool teachers propose to formulate during SBR:

- Is the chance of formulating a related mathematical question associated with their MCK, MPCK, and/or professional beliefs (i.e., dispositions) (RQ1)?
- Is the chance of formulating a related mathematical question associated with the type of picture book (i.e., mathematical versus non-mathematical picture book) (RQ2)?
- Are there interaction effects between their MCK, MPCK, and professional beliefs on one hand, and the type of picture book on the other hand on the chance of formulating a related mathematical question (RQ 3)?
- Is the level of abstraction of the mathematical questions associated with their MCK, MPCK, and/or professional beliefs (i.e., dispositions) (RQ4)?
- Is the level of abstraction of the mathematical questions associated with the type of picture book (i.e., mathematical versus non-mathematical picture book) (RQ5)?
- Are there interaction effects between their MCK, MPCK, and professional beliefs on one hand, and type of picture book on the other hand on the level of abstraction of the mathematical questions (RQ6)?

Based on previous research, we hypothesized that teachers' dispositions are positively associated with their situation-specific skills, and thus both the chance of formulating a mathematical question and the level of abstraction of their mathematical questions (Blömeke et al., 2015; Dunekacke et al., 2016;). Regarding the type of picture book, we expected teachers are more likely to formulate a mathematical question when receiving a mathematical picture book compared to a non-mathematical picture book (Hendrix et al., 2019; Hojnoski et al., 2014). Since no previous studies focused on this impact of the type of picture book on the level of abstraction of teachers' mathematical talk during SBR, or on the interaction between teachers' dispositions and the type of picture book, we did not formulate any hypotheses for these research questions.

2. Method

2.1 Participants

The original sample consisted of 111 students (9 male, 100 female and 2 unknown gender), coming from one [blinded for review] teacher training institute. All students were in their first (n = 63, M_{age} = 19.87, SD = 2.22) or second year (n = 48, M_{age} = 20.38, SD = 1.47) of a non-university preschool teacher training. The first-year students had not yet received any theoretical course on MPCK, while the second-year students had received already two theoretical courses on MPCK. No courses related to MCK were provided by the teaching training institute. More information about the context of the study can be found as Supplementary Material.

Instruments

We assessed future preschool teachers' dispositions using three online questionnaires, and investigated their situation-specific skills using a video-based instrument. More detailed information about the instruments can be found as Supplementary Material.

Mathematical Content Knowledge

MCK was measured using a slightly adapted (in view of its online administration) version the standardized mathematical achievement test for elementary school students Mid-Grade 6 (LVS-VCLB, 2017). It consisted of 52 multiple choice and open questions in the domains of number and arithmetic, measurement, and geometry. Fifty questions were scored dichotomously, the maximum scores for the other two questions were respectively 3 and 4 (maximum score = 57; internal consistency current sample α = .92).

Mathematical Pedagogical Content Knowledge

We administered the MPCK test of Author and colleagues (2021). Participants received five scenarios of mathematically-rich preschool situations (two in the domain of number, two in the domain of measurement, one in the domain of geometry), each accompanied with five to eight multiple choice questions on preschoolers' mathematical competences in the situation, and two multiple choice questions on potentially effective teaching strategies. All items were scored dichotomously (maximum score = 46; internal reliability current sample α = .76).

Professional Beliefs

We used the beliefs instrument of Benz (2012), with 27 statements about beliefs concerning (1) mathematics in general and (2) teaching and learning mathematics in preschool. Participants had to indicate whether they agreed with the statement on a 4-point Likert scale (1 = completely disagree; 4 = completely agree). Fifteen statements referred to beliefs about mathematics in general and 12 to beliefs concerning the teaching and learning of mathematics. For mathematics in general three possible beliefs were questioned (five statements per belief): scheme and formalism, process, and application. The statements concerning the teaching and learning of mathematics in preschool (six statements per belief) focused on the beliefs instruction and construction. We computed the mean score per belief. The internal consistency of three beliefs was insufficient ($\alpha_{\text{process}} = .55$; $\alpha_{\text{instruction}} = .54$; $\alpha_{\text{construction}} = .57$), while for the other two beliefs it was sufficient ($\alpha_{\text{scheme and formalism}} = .68$; $\alpha_{\text{application}} = .79$). Taking into account the insufficient internal consistencies, we decided (1) to exclude the subscales concerning beliefs of the learning and teaching of mathematics (i.e., instruction and construction), and (2) to make a dichotomy for the beliefs concerning mathematics in general based on the study of Thiel (2010). For all further analyses, we included two types of beliefs which both had a sufficient internal consistency: (1) scheme and formalism ($\alpha = .68$), and (2) process and application ($\alpha = .79$).

Situation-specific Skills (decision-making skills)

We administered participants' situation-specific skills, more specifically their decision-making skills, using a researcher-developed video-based instrument. This instrument consisted of two conditions: spontaneous and forced. In each condition, participants were offered 10 items, i.e., 10 videos that involved a typical SBR situation in preschool. All participants completed first the spontaneous condition and then the forced condition. In each condition, the first five videos involved non-mathematical picture books, and the last five mathematical picture books.

In the spontaneous condition, participants firstly received the cover and a description of the picture book's content. Participants were informed about the number of children attending the SBR activity, their age and the fact that they all were normally-developing children who heard the picture book for the first time. Next, a yes/no question was posted, asking participants to indicate whether they knew the picture book. The video of the SBR activity started immediately thereafter. After approximately two minutes the video stopped and participants were asked the following question: "Formulate one question that relates to the page read and that enhances the development of the preschoolers”. Participants were shown the last page read and had to formulate a question within 90 seconds. This process was repeated for each picture book. In the forced condition the same 10 video fragments were shown, but the question asked after the video stopped focused on mathematics: “Formulate one question that relates to the page read and that enhances the mathematical development of the preschoolers.”.

We scored participants’ familiarity with the picture book dichotomously (not familiar = 0), resulting in a familiarity score from 0 to 10. The questions in the spontaneous condition were scored in terms of their mathematical focus. The main aim of this condition was to measure whether participants spontaneously perceived mathematical possibilities and decided to formulate a mathematical question. The questions in the forced condition were scored in terms of their mathematical focus and level of abstraction.

With respect to the mathematical focus, we scored each item in the spontaneous and forced condition with 1 or 0. A score of 1 indicated that the question was a related mathematical question, i.e., focused on mathematical content and related to the page. A score of 0 indicated that the participant did not formulate a question or that the question did not focus on mathematical content (non-mathematical question) and/or did not relate to the page (non-related question). To be scored as a mathematical question, both the question and the most plausible answer(s) to be expected from the preschooler had to focus on - and, importantly: uniquely focus on - mathematics, as defined in Table 1.

Table 1 Mathematical Focus

Category	Definition
Numbers and operations	*Numbers and counting*: recitation of numbers in sequence (verbal counting, one-to-one counting correspondence). Determining the number in a collection (how many?). Ordinal numbers and “one” when used in reference to an object. *Comparing and ordering:* comparing and ordering using number words. Use of and references to numbers such as first, next, etc. The evaluation of the amount/ number in a set or sets using general quantity words (e.g., more) general number words (e.g., a few). *Operations*: Changing the number in a collection by adding or taking away. Use of appropriate concepts such as adding, dividing, etc.
Geometry	*Shape*: Basic (e.g., circle) and complex (e.g., hexagon) shape names and attributes of shapes. *Locations, directions, and coordinates/spatial relations*: Concepts such as over, under, above, below, in, out, on top of, underneath, next to, in front of, behind, between. Taking different points of view and understanding the symbols on pictograms concerning directions. *Patterns*: Continuation of a pattern.

Measurement	*Attributes of object:* Discussion or comparison of characteristics of objects that are related to a mathematical dimension. General words that refer to distance, such as near or far. Putting objects in a group according to some mathematical attribute—that is, an attribute that can be expressed quantitatively (e.g., inches, feet, pounds, time). Referring to the act of measuring.

Note. Coding scheme based on Hojnoski et al. (2014) and the [blinded for review] developmental objectives.

With respect to the level of abstraction, each related mathematical question in the forced condition received a score from 1 to 4, corresponding to its level of abstraction (cf. Blank et al., 1978). If participants asked more than one related mathematical question in the forced condition, all related mathematical questions were scored and the highest score was preserved.

All questions were independently scored by the first author and a trained student researcher, resulting in almost perfect agreement for related mathematical questions in the spontaneous condition ($\kappa = .97$, $p < .001$) and for the level of abstraction of these questions in the forced condition ($\kappa = .97$, $p < .001$), and perfect agreement for the relatedness and mathematical focus of the questions in the forced condition ($\kappa = 1$, $p < .001$).

2.2 Procedure

All instruments were offered online and in the same order: situation-specific instrument, MPCK test, beliefs questionnaire, MCK test. The mathematical focus of the study was not communicated to prevent bias. The tests were administered collectively in groups ranging from 11 to 30 participants at the start of the second semester (i.e., the second week of February) in their teacher training institute.

2.3 Analyses

Because participants were tested multiple times, the data demonstrated a nested structure: measurements (level 1) nested within participants (level 2). We controlled for this non-independent nature of our data by using multilevel logistic regression analyses (see Supplementary Material).

3 Results

3.1 Descriptive results

Table 2 presents the means and standard deviations of the disposition variables and the chances of formulating a related mathematical question in the spontaneous condition. Results showed that, on average, participants answered half of the MCK and slightly more than half of the MPCK items correctly. Participants showed a moderate agreement for the scheme and formalism belief and a moderate to high agreement for the process and application belief. In general, the chance that participants spontaneously formulated a related mathematical question was around 30%. This means that participants generally formulated a related mathematical question on three of the 10 pages in the spontaneous condition. As shown in Table 3, more than half of the related mathematical questions in the forced condition were scored as questions at Level 2, which means that the answer is perceptually available but requires children to focus on specific aspects of objects and events and/or integrate separate components (e.g., "How many apples do you see?"). Results further revealed that participants were familiar with hardly any of the picture books. On average, they knew less than one of the 10 picture books ($M = 0.38$, $SD = 0.83$).

We conducted a one-way ANOVA with year of education as independent variable to check whether there were differences between first- and second-year students for the measured variables. There were no statistically significant differences between first-year versus second-year-students' MCK ($F(1,108) = 2.34$, $p = .13$), MPCK ($F(1,108) = .95$, $p = .33$), and scheme and formalism belief ($F(1,108) = 1.37$, $p = .24$). There was a statistically significant difference in first- versus second-year students' process and application belief ($F(1,108) = 11.71$, $p = .001$): Second-year students scored significantly higher on this belief than first-year students. We therefore controlled for year of training in all further analyses. We observed no statistically significant differences between first- and second-year students related to the familiarity with the picture books ($F(1,109) = 1.82$, $p = .18$). Given students' overall unfamiliarity with the picture books, we did not include this variable in our further analyses.

Table 2 Descriptive Statistics of Disposition Variables and Related Mathematical Questions

Variables (max score)	M (SD)	
	1st year	2nd year
MCK (57)	26.78 (10.26)	23.65 (11.18)
MPCK (46)	27.47 (5.70)	28.58 (6.24)
Scheme and formalism belief (4)	2.84 (0.49)	2.94 (0.41)
Process and application belief (4)	2.93 (0.38)	3.17 (0.35)
Related mathematical question (spontaneous condition) (1)	.31 (.46)	.32 (.47)
Non-mathematical picture books (1)	.05 (.22)	.05 (.22)
Mathematical picture books (1)	.56 (.50)	.60 (.49)

Table 3 Level of Abstraction of Related Mathematical Questions in the Forced Condition

Level	Proportion	
	1st year	2nd year
Level 1	.03	.03
Level 2	.74	.69
Level 3	.10	.08
Level 4	.13	.20

Finally, we examined bivariate correlations among the measured variables using Spearman Rho. As displayed in Table 4, we observed significant positive correlations between participants' MCK and MPCK. Participants' process and application beliefs were also positively correlated with both the number and the level of abstraction of the related mathematical questions in respectively the spontaneous and the forced condition.

Table 4 Correlation Matrix of the Measured Variables

	1	2	3	4	5	6
1. MCK	-					
2. MPCK	.47**	-				
3. Scheme and formalism belief	-.01	-.10	-			
4. Process and application belief	.04	.19	.12	-		
5. Sum related mathematical questions (spontaneous condition)	.08	.13	.05	.22*	-	
6. Level of abstraction of related mathematical questions (forced condition)	.07	.12	.08	.20*	.09	-

**. Correlation is significant at the 0.01 level (2-tailed); *. Correlation is significant at the 0.05 level (2-tailed).

3.2 The chance of formulating a related mathematical question

We first estimated an empty two-level model for the chance of formulating a related mathematical question in the spontaneous condition (Model 1). This model Equation (4) is shown below:

$$\text{Logit}\ (\pi_{ij}) = \eta_{ij} = \gamma_{00} + U_{0j} \quad \text{with } U_{0j} \sim \text{N}(0, \tau_{00}) \text{ and } Y_{ij} \sim \text{Bernoulli}\ (\pi_{ij}) \tag{4}$$

in which η_{ij} is the log-odds that participants formulate a related mathematical question, and γ_{00} is the fixed intercept, whereas U_{0j} represents the random effect associated with participant j. The model showed an intercept of -0.78 (SE = .06, p < .001) (Table 5). Transforming this expected logit to a probability, shows that the overall chance of formulating a related mathematical question was 31.43%. The ICC was almost 0 (τ_{00} = 2.84E-17), which means that this chance hardly varies over participants.

In Model 2, we included the following predictors: MCK, MPCK, scheme and formalism belief, process and application belief, and year of training to answer the first research question, which resulted in the following Equation (5):

$$\text{Logit}(\pi_{ij}) = \eta_{ij} = \gamma_{00} + \gamma_1.\text{MCK} + \gamma_2.\text{MPCK} + \gamma_3.\text{scheme and formalism} + \gamma_4.\text{process and application} + \gamma_{year\ of\ training} + U_{0j} \tag{5}$$

As could be expected based on the ICC value of the empty model, we found no significant effects of these predictors on the chance of formulating a related mathematical question (Table 5). The R square McFadden indicated that only 0.23% of the variance could be explained by these variables (R^2_{MF} = .0023).

The second research question focused on the association between the chance of formulating a related mathematical question and type of picture book. Therefore, we included a dummy indicator for non-mathematical picture books in Model 3. All effects of the predictors from Model 2 remained non-significant, but the effect of the type of picture book was statistically significant (γ = -3.26, t = -15.28, p < .001). To calculate the chance for second-year students of formulating a related mathematical question when receiving a mathematical picture book and all other predictors equal to zero, we filled in the different predictor scores in the model and transformed this expected logit (η_{ij} = 0.33) to a probability, which resulted in a chance of 58.18%. The estimated chance for second-year students of formulating a related mathematical question when receiving a non-mathematical picture book and with all other predictors equal to zero was 5.07%. By including type of picture book, the proportion of the variance explained increases to .2952.

In Model 4, we included all possible interactions between participants' dispositions and picture book characteristics. Participants' dispositions, and year of education remained non-significant and the type of picture book remained significant, while none of the interaction effects was significant. By adding the interaction terms, the proportion of variance explained increased only to a small extent (R^2_{MF} = .3018). Based on the AIC and BIC model fit indices, we can conclude that Model 3 is the best fitting model (see Table 5).

Table 5 Multi-level Analyses Predicting the Chance of Posing a Related Mathematical Question in the Spontaneous Condition

	Model 1 (Empty model)	Model 2 (Dispositions)	Model 3 (Picture book characteristics)	Model 4 (Interactions)
Fixed Effects				
Intercept	-0.78**	-0.77**	0.33*	-1.02
Year of training (first year)		- 0.01	- 0.02	-0.10
MCK		0.03	0.04	0.41
MPCK		0.06	0.09	0.12
Scheme and formalism belief		0.01	0.02	0.02
Process and application belief		0.08	0.12	0.05
Non-mathematical picture book			-3.26**	-3.66**
Year of training*type of picture book				0.51
MCK* type of picture book				-0.42
MPCK* type of picture book				-0.18
Scheme and formalism * type of picture book				-0.04
Process and application* type of picture book				0.40
Model Fit				
-2LL	1369.92	1366.72	965.55	956.51
AIC	1371.92	1378.80	979.55	980.51
BIC	1374.62	1394.92	998.45	1012.92

*. Correlation is significant at the 0.05 level; **. Correlation is significant at the 0.01 level

3.3 The level of abstraction of related mathematical questions

We first estimated an empty two-level model for the chance of formulating a related mathematical question at each level of abstraction in the forced condition (Equation 6; called Model 1 in Table 6).

$$\text{Logit}\ [P(Y_{ij} \leq k)] = \alpha_k + U_{0j} \ \text{ with } U_{0j} \sim N(0, \tau_{00}) \text{ and } Y_{ij} \sim \text{Bernoulli}\ (\pi_{ij}) \qquad (6)$$

in which the intercept α_k is the log-odds of falling into or below category *k* (i.e., Level 1, 2, or 3) when all other predictors equal 0, and U_{0j} represents the random effect associated with participant *j*. We transformed the expected cumulative logits of the intercept coefficients for each level and determined the probability of formulating a related mathematical question at Level 1, 2, 3, and 4. For example, the expected logit of -3.52 for a Level 1 question corresponds with a probability of .0287. For Level 2, the expected logit (1.12) corresponds with a probability of .7540, which means that the overall chance of formulating a related mathematical question at Level 2 or below is 75.40%. To calculate the chance for a Level 2 question, we subtract the probability on a Level 1 question (0.0287) from the probability on a questions at Level 2 or below (.7540), which results is an probability of 0.7253. Following this approach, the overall chances of formulating a related mathematical question at Level 1, 2, 3, and 4 were respectively 2.87%, 72.53%, 9.14%, and 15.46%. The ICC indicated that 3.24% of the differences in the level of abstraction of related mathematical questions is due to differences between participants.

In Model 2, we included MCK, MPCK, scheme and formalism, process and application, and year of education as predictors. None of these five variables was a significant predictor for the chance of formulating a related mathematical question at Level 1, 2, 3, or 4 of abstraction. A positive γ coefficient indicates that when the predictor variable (e.g., MPCK) increases, the log-odds of falling into or below a category increases, in other words, that lower levels for the response variable Y (i.e., level of abstraction) become more likely. The R square McFadden was .0044, which indicated that only 0.44% of the variance could be explained by participants' dispositions.

In Model 3 we again included a dummy indicator for non-mathematical picture books and found that all teacher predictors remained non-significant. Type of picture was a statistically significant predictor ($\gamma = 0.90$, $t = 6.11$, $p < .001$). To calculate the chances for second-year students of formulating a related mathematical question at Level 1, 2, 3, and 4 when receiving a non-mathematical picture book and all other predictors equal to zero, we transformed the expected logits and obtained the following probabilities: 3.63%, 77.83%, 7.54%, and 11.01% for each level respectively. When receiving a mathematical picture book and all other predictors equal to zero, the chance for the second-year students to formulate a Level 1 question was 1.51%, 62.60% for a Level 2 question, 12.57% for Level 3, and 23.33% for a Level 4 question. The R square McFadden demonstrated that an additional 2.34% of the variance was explained by type of picture book (R^2_{MF} = .0278).

In Model 4, the possible interaction effects between type of picture book and participants' dispositions were added to the model. The disposition predictors remained non-significant, type of picture book remained significant predictor and none of the interaction effects was significant. This model showed a McFadden's R square of .0329, which indicated that only 0.51% of the variance could be uniquely explained by the interactions. Based on the

AIC and BIC model fit indices, we conclude that Model 3 is the best fitting model for our data (see Table 6).

Table 6 Multi-level Analyses Predicting the Chance of Posing a Related Mathematical Question at Level 1, 2, 3, or 4 of Abstraction in the Forced Condition

	Model 1 (Empty model)	Model 2 (Dispositions)	Model 3 (Picture book characteristics)	Model 4 (Interactions)
Fixed Effects				
Intercept Level 1	-3.52**	-3.64**	-4.18**	-4.27**
Intercept Level 2	1.12**	0.99**	0.58**	0.52**
Intercept Level 3	1.70**	1.57**	1.19**	1.13**
Year of training (first year)		0.21	0.21	0.35
MCK		-0.11	-0.11	-0.09
MPCK		0.003	0.01	-0.10
Scheme and formalism belief		0.07	0.07	-0.03
Process and application belief		-0.12	-0.13	-0.18
Non-mathematical picture book			0.90**	1.07**
Year of training * type of picture book				-0.30
MCK* type of picture book				-0.05
MPCK* type of picture book				0.25
Scheme and formalism belief * type of picture book				0.24
Process and application belief * type of picture book				0.13
Model Fit				
-2LL	1583.95	1576.94	1539.95	1531.86
AIC	1591.95	1595.14	1560.20	1561.86
BIC	1602.75	1619.25	1586.96	1602.37

**. significant at the 0.01 level

4 Discussion

Previous studies have revealed the effectiveness of SBR for early literacy and mathematical development (Mol et al., 2008; van den Heuvel-Panhuizen & Elia, 2011), and pointed to the importance of the interaction quality for its effectiveness. Empirical studies on this interaction quality in the domain of mathematics and its association with characteristics of the teacher are currently missing. A useful framework to investigate this complex interplay is the competence framework of Blömeke et al. (2015), distinguishing between (1) teachers' dispositions, (2) situation-specific skills, and (3) instructional behavior. However, empirical evidence on the relationship between these components in the context of preschool education is scarce. The present study addressed these gaps by focusing on the relationship between preschool teachers' dispositions, and more particularly their MCK, MPCK, and professional beliefs, and their situation-specific skills, more particularly the chance of proposing to formulate a related mathematical question and the level of abstraction of mathematical questions. As research has revealed that picture book characteristics might also impact SBR, we also investigated whether the type of picture book (i.e., mathematical versus non-mathematical) impact teachers' chance of proposing to formulate a mathematical question and the level of abstraction of teachers' proposed mathematical questions. Furthermore, the interaction between teachers' dispositions and type of picture book on their situation-specific skills was investigated.

Our findings revealed first that teachers' dispositions were not associated with their situation-specific skills. By contrast, the type of picture book was positively associated with teachers' situation-specific skills: Future preschool teachers had a higher chance to propose to formulate a mathematical question and also to propose a more abstract mathematical question when receiving a mathematical compared to a non-mathematical picture book. Furthermore, no significant interaction effects between teachers' dispositions and the type of picture book on teacher's situation-specific skills were found. In what follows, we first discuss the results regarding the association between teachers' dispositions and their situation-specific skills. Second, we elaborate on the contribution of the type of picture book. Lastly, the limitations and implications of the current study are discussed.

4.1 Teachers' dispositions

As stated above, we did not find evidence for the associations between teachers' dispositions and their situation-specific skills. Neither the chance of formulating a mathematical question nor the level of abstraction of mathematical questions were associated with preschool teachers' dispositions. This finding is inconsistent with the previous study of Dunekacke et al. (2016) in which the association between future preschool teachers' dispositions and their situation-specific skills was also examined in the domain of mathematics.

A first possible explanation for this finding refers to participants' generally low MPCK scores which might have made it impossible for them to see the mathematical content and formulate (abstract) mathematical questions related to that content. Future research should include (future) preschool teachers who had more opportunities to learn the core concepts related to MPCK due theoretical courses and practical experience, as for example future preschool teachers in their last year of training or in-service preschool teachers.

Second, our focus on only teachers' decision-making skills might explain the findings. As suggested in the PID-model, teachers' decision-making skills are impacted by their perceiving and interpreting skills. Dunekacke et al. (2016) found that teachers' MPCK was positively associated with their perception skills, but only indirectly associated with their decision-making skills. Since we only included teachers' decision-making skills, we were not able to investigate the indirect effect(s) of teachers' dispositions on these skills. Future research should also include teachers' perceiving and interpreting skills to analyze the direct and indirect effects between teachers' dispositions and their PID skills separately.

Third, it might be that teachers' intentionally formulated mathematical questions at a low level of abstraction to enhance preschoolers' mathematical development by providing sufficient scaffolding and motivating input. The high proportion of Level 2 questions can also be positively interpreted: Such questions allow preschool teachers to gain information about the actual competence level of their preschoolers and, on the basis of this information, provide sufficient scaffolding within their zone of proximal development to optimize learning (cf. Vygotsky, 1978). Taking into account preschoolers' responses to their Level 2 questions, participants might have formulated questions at higher, or lower, levels of abstraction on the next pages of the picture book. Likewise, we cannot exclude that participants formulated Level 2 questions to make the preschoolers feel confident and successful, and as such motivate them to participate to the SBR activity (van Kleeck, 2003). Future studies are needed to evaluate these hypothetical explanations and will help to understand whether questions at lower levels of abstraction point to weaknesses in teachers' competence to formulate higher level questions or are rather indications of teachers' intentions and competencies to enhance preschoolers' mathematical development by providing sufficient scaffolding and motivating input.

4.2 The type of picture book

First, we observed that in general 30% of the questions participants spontaneously proposed to formulate during SBR were mathematical, but with large differences between mathematical and non-mathematical picture books. The chance of spontaneously formulating a mathematical question when receiving a non-mathematical picture book was generally low. Participants rather formulated questions related to literacy or language, or the socialemotional behavior of the characters in the non-mathematical picture books. This might be due to the fact that we used picture books for the present study, which are mainly used to enhance children's language competencies (Lonigan & Whitehurst, 1998; Mol et al., 2008). Participants might therefore not have been stimulated to spontaneously search for and perceive mathematical possibilities in the situation or, even when perceived, to make the decision to formulate a mathematical question. When shown a mathematical picture book, the chance of spontaneously formulating a mathematical question was about ten times higher than with non-mathematical picture books, resulting in mathematical questions on about half of the trials.

Second, results revealed that future preschool teachers had a higher chance to propose to formulate a mathematical question and a more abstract mathematical questions when receiving a mathematical compared to a non-mathematical picture book. The findings with regard to the chance of formulating a mathematical question are in line with previous studies on the amount of mathematical talk in general (Hendrix et al., 2019; Hojnoski et al., 2014) and might be explained on the basis of Price and colleagues' (2009) observation that the parent-child interaction during SBR is limited to the content of the picture book (i.e., text and illustrations), in our case, the mathematical content of the picture books. With regard to the level of abstraction, picture books without explicit mathematical content might have motivated the participants to first draw preschoolers' attention to the "hidden" mathematical content via mathematical questions at lower abstraction levels. By contrast, as the mathematical content was clearly present in the mathematical picture books, such lower level questions were not necessary to reveal the mathematical content of this type of picture books. The explicit mathematical content in mathematical picture books thus might have enabled participants to formulate questions that invite preschoolers to look further than the clearly presented mathematical content, i.e., more abstract questions, than the implicit mathematical content of the non-mathematical picture books.

4.3 Limitations and future research

As a first limitation, we only focused on future preschool teachers' decision-making skills, and did not take into account their perceiving and interpreting skills. For futures research, an assessment of the entire PID model, i.e., participants' perceiving, interpreting, and decision-making skills, would be helpful to get insight in participants' intentions and to be able to analyze the (in)direct effects of teachers' dispositions on these skills separately.

A second limitation concerns our strict scoring system to score the mathematical focus of a question. To be scored as a related mathematical question, the question as well as the most plausible expected answer(s) had to uniquely focus on the content included in our coding scheme. This might have resulted in an underestimation of the likelihood of formulating a mathematical question, as participants might have proposed questions that they thought would invite the preschooler to give a mathematical answer, without considering plausible non-mathematical answers as well. It is important that future studies also include the (expected) answer to get a better view on the focus of the question and the answer as intended by the participant.

Third, it is important to notice that we measured preschool teachers' situation-specific skills and not their actual behavior. The competence framework of Blömeke and colleagues (2015) assumes that teachers' situation-specific skills influence their actual teaching behavior. However, to the best of our knowledge, this relationship was not yet empirically validated in the preschool context. Through the video-based instrument participants were introduced into an authentic SBR situation in which they had to make a decision about the next teacher move. We cannot ascertain that these future preschool teachers would formulate the same questions in a real SBR situation. Future research focusing on the association between preschool teachers' situation-specific skills and their actual behavior is needed to deepen our theoretical and empirical insights into this topic.

Lastly, the relative small sample size restricted the study's power. Further research is needed to investigate preschool teachers' situation-specific skills in larger samples, increasing the power of the study as well as the inter-individual variation in MCK and MPCK.

4.4 Educational implications

On the basis of our findings, two major educational implications can be formulated. First, the present study indicates that the chance that future preschool teachers formulate a mathematical question during SBR is relatively low, and that, in case of mathematical questions, the level of abstraction is also relatively low. Therefore, it is important that teacher training institutes provide ample opportunities to acquire this competency. Given the relatively low MCK and MPCK scores in our sample, a second implication is that teacher training institutes should provide sufficient learning opportunities to acquire this important knowledge.

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