Transit network design considering link capacities

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Abstract

Appropriate public transport systems are crucial in modern cities. Given the high costs that they represent and the impact they have on people's lives, effective tools are required to support their design. The Transit Network Design and Frequency Setting problem (TNDFSP) has been extensively studied in operations research. This problem consists of designing a set of public transport lines and a service frequency to each line. The main contribution of this work is to consider, for some links of the underlying network, a maximum 'combined' frequency among all the lines using that link and addressing the crowding issues that can result from that. These additional constraints intend to limit, for instance, the number of buses circulating in certain areas, in line with current urban design policies. A bi-objective memetic algorithm is proposed to solve the problem. The algorithm generates a set of non-dominated solutions that comply with the link-capacity constraints in one hour of computing time. Additionally, alternative solutions are generated by designing the line plan without considering the link-capacity constraints and adapting the frequencies afterwards, to compare the two approaches. The algorithm is tested on an instance representing the bus network in the city of Utrecht, The Netherlands. The algorithm that takes into account the link-capacity constraints during the optimization process, generates better results. Moreover, the algorithm generates transit networks with less lines traversing the city centre, allowing higher individual frequencies for those lines. The algorithm could become an important tool for policy makers and transit operators, allowing the design of efficient transit systems that adjust better to contemporary urban requirements.

Keywords

Transit network design; Bus line planning; Frequency setting; Crowding; Memetic algorithm

1 Introduction

The design of public transport networks is a very complex problem with important practical implications. Public transport provides an affordable transportation option and contributes to reducing car dependency. However, it also represents a high cost, and if not well designed, it can generate negative externalities such as increased traffic congestion. Therefore, it is relevant to work with tools that support the design of transit networks. This implies decisions about which stops will be served by each public transport line, in which order and at which frequency. In the field of operation research, this problem is commonly addressed as the Transit Network Design and Frequency Setting Problem (TNDFSP) and has received increasing attention in the last decade. However, despite the extensive research, optimization techniques are rarely used in practice. Indeed, the design of transit networks typically relies on the experience of transit planners, who perform this task manually based on their experience and knowledge of the area. In some cases, modelling techniques are used as support tools and for specific tasks, but not as proper optimization tools for the whole design of the network. However, the use of optimization algorithms has the potential of improving the design process and lead to more efficient transit networks.

One of the reasons why optimization techniques are not used in practice to solve the TNDFSP is that many assumptions and simplifications are made to represent the problem, due to its complexity. Therefore, the results obtained by such techniques are not easily translated into practice. Among the many simplifications made in theory, a common one is related to the capacity of the streets. It is commonly assumed that the capacity of the links in the infrastructure network is unlimited. In many instances it is indeed reasonable to ignore traffic congestion, since public transport vehicles tend to circulate on segregate lanes. Moreover, even if streets with shared traffic are considered, the congestion levels can be estimated in advance and the travel times can be set accordingly. However, there are other reasons to consider strict capacities on the streets. Currently, policy makers tend to recognize the benefits of having limited traffic flows, in order to generate more liveable and pedestrian friendly neighbourhoods. However, even in these conditions, it might be desirable to allow a limited number of buses circulating through these streets. In this regard, it is interesting to have transit network design tools that take into account such capacity constraints in the street network.

In the optimization models for the TNDFSP, further simplifications are made when modelling the behaviour of passengers. Typically, the effects of crowding are ignored, assuming that passengers can always take the first bus that arrives to the stop. However, in reality passenger crowding may increase the expected waiting times and may affect passengers route choice, particularly during peak hours. Therefore, it is relevant to consider it during the transit network design stage.

The main contribution of this paper is the development of an algorithm that allows to study the impact of strict capacity constraints in some links of the infrastructure and passenger crowding on optimizing the TNDFSP. Although there are models that consider these aspects in related problems, such as in the frequency setting problem, the approaches used in those cases are not directly applicable to the TNDFSP, due to its higher complexity. Therefore, in this paper, we propose an approach that include these aspects in the TNDFSP efficiently. Capacity constraints and crowding issues are considered together since the capacity constraints on the links impose a strict upper limit on the frequencies of the lines. These upper bounds for the frequencies limit the capacity of the lines, making the crowding issues, commonly ignored in the TNDFSP, more relevant. The proposed algorithm is tested on a large realistic instance, showing its potential to be applied in practice. Therefore, such an algorithm could be used by policy makers and transit planners as a tool to support the design of real transit networks, allowing the design of efficient transit systems that adjust better to modern urban requirements.

The rest of the paper is structured as follows. Section 2 presents a brief literature review about the TNDFSP. Section 3 gives a detailed description of the problem addressed in this study. Section 4 describes the solution algorithm proposed to solve the problem. Then, Section 5 presents and discusses the experimental results. Finally, in Section 6 the main conclusions of this study are presented.

2 Literature review

The transit network design is a very complex problem, with a great number of decisions, constraints, conflicting objectives, and uncertainties. Due to this complexity, there are no optimization techniques that can be applied in practice to solve the whole problem at once. However, models and algorithms are used by practitioners to support the design or solve subproblems. For example, clustering techniques are applied to group stops or neighbourhoods, to simplify the design process in later stages. Similarly, optimization algorithms are used to design a simplified layout, indicating the main corridors that will serve as a basis to design the whole network. Moreover, other techniques, such as simulation, can be used to support the design. Although simulation models cannot generate an optimal solution, they allow to evaluate different scenarios and consider complex interactions within the system. In the case of the design of transit networks, commercial simulation software are commonly used to evaluate proposed line plans. These models consider detailed operations and passengers' behaviour, allowing to observe the consequences of emerging behaviours resulting from these interactions. This allows to check the feasibility of the proposed networks regarding constraints that are difficult to evaluate in previous design stages without modelling techniques, such as passenger crowding or resilience of the system. However, simulation experiments are time consuming for large instances, so typically only a few alternatives can be evaluated. Moreover, this simulation software is typically used only to evaluate a proposed network, and cannot generate better networks. Therefore, the design process could be improved by using optimization algorithms that tackle the problem as a whole, which is modelled in the field of operations research as the TNDFSP.

The study of the TNDFSP has attracted increasing attention in the last decades, as is reported in several comprehensive reviews (Durán-Micco and Vansteenwegen, 2021; Guihaire and Hao, 2008; Ibarra-Rojas et al., 2015; Iliopoulou et al., 2019; López-Ramos, 2014). The TNDFSP has proven to be a very complex problem, so mostly heuristic and metaheuristic algorithms have been proposed to solve it. Examples of recently proposed algorithms include various evolutionary algorithms (Bourbonnais et al., 2021; Camporeale et al., 2019; Duran-Micco et al., 2020; Manser et al., 2020; Owais and Osman, 2018), hyper-heuristics (Ahmed et al., 2019), swarm-based metaheuristics (Liu et al., 2020), among others. Some exact methods have been proposed as well, although these are applied to solve simplified version of the problem or only very small instances (Suman and Bolia, 2019; Vermeir et al., 2021; Wu et al., 2020).

Traditionally, the Transit Network Design Problem (TNDP) and the Frequency Setting Problem were studied independently. In simple words, the TNDP consists of defining the set of lines to conform a transit network, given an underlying infrastructure network and a demand matrix. Because no frequencies are considered, the expected waiting times or the required fleet size are not considered either. Since it is a complex problem in itself, still many studies address it as an independent problem (Ahmed et al., 2019; Islam et al., 2019; Suman and Bolia, 2019). However, in the last decade it has become more common to address the TNDFSP as an integrated problem (López-Ramos, 2014). The consideration of the frequencies allows a better estimation of the average passenger travel time, the passengers' route choice, the required fleet size, among other relevant metrics. In addition, different studies have extended the TNDFSP in several aspects, for example, by incorporating constraints related to electric buses (Liu et al., 2020; Pternea et al., 2015), considering multimodal networks and elastic demand (Almasi et al., 2020; Amirgholy et al., 2017; Manser et al., 2020), looking for the explicit minimization of the CO_2 emissions (Duran-Micco et al., 2020), incorporating equity measures (Camporeale et al., 2019), among others. Despite these efforts, optimization techniques are still not widely used in practice, in part due to the many assumptions done in the modelling, the small size of the commonly used benchmark instances, or the long computing times required to solve real test cases.

An important element within the TNDFSP is the modelling of the behaviour of the passengers, regarding the paths they are expected to follow in the transit network under design. This forms the Passenger Assignment Problem (PAP), which is a complex problem in itself, and therefore has been extensively studied as an independent problem (Liu et al., 2010). Several models and algorithms have been proposed to solve the PAP. Some of these approaches are highly complex and computationally expensive, intended to obtain detailed solutions for operational or tactical proposes. Such models can be applied in simpler or more specific problems, such as the frequency setting problem. However, in the context of the TNDFSP, such level of detail is not required. Moreover, the iterative solution approaches typically used in the TNDFSP require to solve the PAP multiple times, so fast methods are necessary. Therefore, mostly simple approaches to model and solve the PAP are used within the TNDFSP. The most commonly used approaches rely on the concepts of hyperpaths and frequency-sharing rules (Baaj and Mahmassani, 1990; Spiess and Florian, 1989). These methods assume that passengers pre-select a set of attractive lines and board the first bus to arrive from any of those lines. Therefore, the proportion of passengers captured by each line is related to the relative frequency values. A common assumption in this regard is that all passengers are able to board the first bus that arrives, therefore disregarding the possible effects of passenger crowding.

Crowding may have multiple effects on the behaviour of passengers, since it can generate longer travel times, reduced reliability of the service and discomfort, among others. Therefore, ignoring crowding effects when modelling the PAP may lead to an overestimation of demand in certain lines or in the overall system (Tirachini et al., 2013). Although there are different models for the PAP that consider crowding effects, they are typically more complex and time consuming. Therefore, they have been applied to solve the frequency setting problem, but they are rarely applied to the TNDFSP (Fu et al., 2012). There are multiple approaches when modelling crowding effects, but they typically consist of adding some penalization to crowded lines, to reflect the fact that passengers tend to avoid these paths when possible. A common approach is to consider these penalizations as extended waiting times, considering the failing-to-board effect, when passengers cannot board the first vehicle that arrives to the stop. This can be modelled using the concept of effective frequencies, which assumes that when trying to board a crowded line, passengers perceive a lower frequency compared to the nominal one (de Cea and Fernández, 1993). This approach has some limitations, such as not ensuring that the line capacity constraints are fully satisfied (Cominetti and Correa, 2001). However, the concept of effective frequencies is intuitive and easier to apply than alternative methods that do ensure the satisfaction of capacity constraints. In general, these alternative methods require longer computing times or do not ensure user-equilibrium solutions (Codina and Rosell, 2017). In the context of the TNDFSP, the concept of effective frequencies has been applied, but the models are simplified in other aspects and are tested in very small instances, so further research is needed (Chai and Liang, 2020; Owais and Osman, 2018). This paper addresses this gap by using effective frequencies within an efficient solution algorithm that can solve large instances of the TNDFSP.

To conclude, the TNDFSP is a complex problem that has been extensively studied, but there is still a relevant gap that complicates the application of optimization techniques in practice. This paper attempts to narrow this gap, by considering an extended version of the TNDFSP, including additional capacity constraints in some links and considering the effects of crowding in the PAP. Additionally, the performance of the proposed algorithm is tested in a large realistic instance, in order to test the real potential of applying the approach in practice.

3 Problem description

This section describes the problem addressed in this paper. The main inputs of the problem are the infrastructure network and the demand matrix. The infrastructure network is represented by a graph $G = (V, E)$, where the set of nodes V respresents the stops and the set of edges E represents a simplified version of the street network. Each edge has an associated in-vehicle travel time (expressed in minutes). Moreover, a subset of edges $E^c \subseteq E$ has a maximum capacity b_e, expressed in buses/h. The demand is given in an origin-destination (OD) matrix, indicating the number of passengers traveling per hour between the stops in V.

3.1 TNDFSP formulation

The aim of the TNDFSP is to define a set of lines L, where each line $l \in L$ is defined as a sequence of nodes. The number of lines is not fixed, but a lower (N_{min}) and an upper (N_{max}) bound are considered. Additionally, a frequency f_l is assigned to each line, expressed in buses/h. The frequencies can only take certain values, given in the set F. This constraint allows to design periodic timetables in later design stages. In this study, it is assumed that each line can visit a node only one time, except in the case of circular lines, where a single loop is allowed. It is also assumed that the lines are served in both directions with the same frequency. The maximum length of a line, measured as the travel time in one direction, is set to l_{max}.

Two optimization objectives are considered in this paper: the average passenger travel time and the required fleet size. The average passenger travel time is a representation of the users' cost and includes the in-vehicle travel time, the estimated waiting time (considering the crowding effect) and a penalization for transfers. Additionally, a penalization for each passenger that is not satisfied is considered. On the other hand, the fleet size is a representation of the operator's cost and is calculated as the sum of the number of buses required to serve each line. For each line, the number of required buses is calculated by Equation 1.

$$n_l = \lceil r_l * f_l \rceil \quad [1]$$

Where:

- n_l = number of buses required to operate line l;
- r_l = roundtrip duration of line l (h);
- f_l = service frequency of line l (buses/h).

In summary, the problem addressed in this paper can be represented with the following mathematical formulation, where the decisions variables are: the set of lines L; the frequencies $\boldsymbol{f}$ assigned to each line; and the flows of passengers $\boldsymbol{v}$ in each link of each line.

$$Min\ Z_1 = T(\boldsymbol{v}, \boldsymbol{f}) \quad [2]$$

$$Min\ Z_2 = \sum_{l \in L} n_l \quad [3]$$

$$S.t.$$

$$N_{min} \leq |L| \leq N_{max} \quad [4]$$

$$\sum_{l \in L_e} f_l \leq b_e \quad , \forall e \in E_c \quad [5]$$

$$\boldsymbol{v} = \Phi(L, \boldsymbol{f}) \quad [6]$$

$$L \subseteq L^* \quad [7]$$

$$\boldsymbol{f} \in F \quad [8]$$

$$\boldsymbol{v} \geq 0 \quad [9]$$

As mentioned before, two objectives are considered in this problem. Equation 2 corresponds to the minimization of the total travel time, which depends on the passenger flows $\boldsymbol{v}$ within the transit network and the frequencies assigned to the lines. More details on how it is calculated are given in Section 3.2. Equation 3 corresponds to the second objective, which is the minimalization of the total bus fleet required to operate the transit network. Equation 4 constraints the number of lines that can conform the transit network. Equations 5 indicate that for the edges with a maximum capacity, the sum of the frequencies of the lines using that edge must be lower than the given capacity. Equation 6 indicates that the flows of passengers $\boldsymbol{v}$ within the transit network depends on the structure of the transit network and the selected frequencies. The relation between these elements corresponds to the PAP, and the main assumptions used in this paper are discussed in Section 3.2. Equation 7 indicates that a set of lines L must be defined, where L^* contains all the feasible lines according to the constraints described before, such as the maximum length of the lines and the sequences of stops that are allowed. Equation 8 indicates that for each line in L a frequency from the set F must be assigned. Finally, Equation 9 indicates that the passenger flows must take positive values.

3.2 Passenger assignment sub-problem

The PAP is a subproblem that must be addressed to solve the TNDFSP, in particular to evaluate a candidate transit network. In this paper, the PAP is modelled using an adapted version of the frequency share rules. This method assumes that passengers select a set of attractive lines before accessing their origin stop (or transfer stop). The set of attractive lines is composed by the line that correspond to the shortest path (considering in-vehicle travel time and penalizations for transfers), plus the lines corresponding to the paths whose travel time is at most 10% longer than the shortest path. This criterion is based on the algorithm proposed by Baaj and Mahmassani (1990). This method is a simple and intuitive representation of the behaviour of the passengers, that avoids assigning passengers to paths that are considerably longer than the shortest alternative. Of course, alternative methods to define the attractive lines could be used instead (de Cea and Fernández, 1993; Spiess and Florian, 1989), since this does not affect how the algorithm works. Then, it is assumed that passengers take the first bus arriving to the stop corresponding to any of those attractive lines. Therefore, for a given OD pair, the proportion of passengers boarding a given line at a given stop depends on the relative frequencies of the lines. Using the frequency-share rule, the demand split at one stop is done in the following way: from the passengers of the OD pair k that need to board a bus in stop i, the share that will take line l, is given by Equation 10 (Baaj and Mahmassani, 1990):

$$s_{k,l,i} = \frac{f_l}{\sum_{l \epsilon L_{k,i}} f_l} \qquad \textbf{[10]}$$

Where:

- $s_{k,l,i}$ = share of passengers from OD pair k in stop i that board line l;
- f_l = service frequency of line l (buses/h);
- $L_{k,i}$ = set of attractive lines for passenger of OD pair k in stop i.

Typically, nominal frequencies are used in the frequency-share rule, assuming that the passengers can always board the first bus to arrive. However, since in this study the effect of crowding is considered, that assumption is not valid anymore. For this reason, the concept of effective frequency is used to model the effect of crowding in the transit network (de Cea and Fernández, 1993). In this case, it is assumed that if a line is crowded, passengers may not be able to board the first bus to arrive, hence their waiting time may be longer, and therefore they will perceive a lower frequency compared to the nominal frequency. This assumption affects the calculation of the waiting times, but also may alter the route-choice of passengers.

The effective frequency is a function of the nominal frequency of the line and the ratio between the average number of boarding passengers and available seats in the vehicles operated on the line at a given stop. Therefore, the effective frequency depends on the line, the direction, and the stop. The available capacity in the bus at the stop is calculated after the passengers with that stop as destination alight, but before the new passengers board the bus. To calculate the effective frequency, first the effective waiting time is calculated as indicated in Equation 11.

$$t_{l,d,i}^{w*} = \frac{1}{2}\frac{60}{f_l}\left(\frac{b_{l,d,t}}{s_{l,d,t}}\right)^{\beta} \quad \textbf{[11]}$$

Where:

- $t_{l,d,i}^{w*}$ = effective waiting time of line l in direction d at stop i (min);
- f_l = nominal frequency of line l (buses/h);
- $b_{l,d,t}$ = passengers boarding line l in direction d at stop i (pass/h);
- $s_{l,d,t}$ = available capacity of line l in direction d at stop i (pass/h);
- β = coefficient.

If there is no available capacity, then the effective waiting time is set equal to an arbitrary maximum effective waiting time. After that calculation, the effective waiting time is bounded within the nominal waiting time (half of the headway) and the maximum effective waiting time. Then, the effective frequency can be calculated with Equation 12.

$$f_{l,d,i}^{*} = \frac{1}{2}\frac{60}{t_{l,d,i}^{w*}} \quad \textbf{[12]}$$

Where:

- $f_{l,d,i}^{*}$ = effective frequency of line l, direction d and stop i (bus/h);
- $t_{l,d,i}^{w*}$ = effective waiting time in line l, direction d and stop i (min).

Finally, the demand split considering the effects of crowding can be calculated by Equation 10, but using the corresponding effective frequencies instead of the nominal frequencies.

4 Solution algorithm

A bi-objective memetic algorithm (MA) is proposed to solve the studied problem. This algorithm was proposed in (Duran-Micco et al., 2020) to solve the TNDFSP without considering capacitated edges or crowding, showing the good performance of the algorithm compared to the state-of-the-art. Therefore, in this paper only a general description of the algorithm is given, focusing on the modifications made to address the extended version of the problem. For a more detailed description of the basic algorithm, readers are referred to (Duran-Micco et al., 2020).

Memetic algorithms, first proposed by Moscato (1989), basically consist of an evolutionary algorithm with an embedded local search operator. The local search operator differentiates memetic algorithms from other well-known evolutionary algorithms, such as genetic algorithms. This operator allows to benefit from the advantage of evolutionary algorithms, such as the ability to explore large areas of the solution space, but with a faster convergence to better solutions. This is particularly attractive for problems like the TNDFSP, in which the evaluation of each solution is computationally expensive. In this implementation, each solution corresponds to a candidate transit network, represented by a list of lines, with each line consisting of a sequence of adjacent nodes. Additionally, each line has associated a frequency. The basic flowchart of the algorithm is shown in Figure 1.

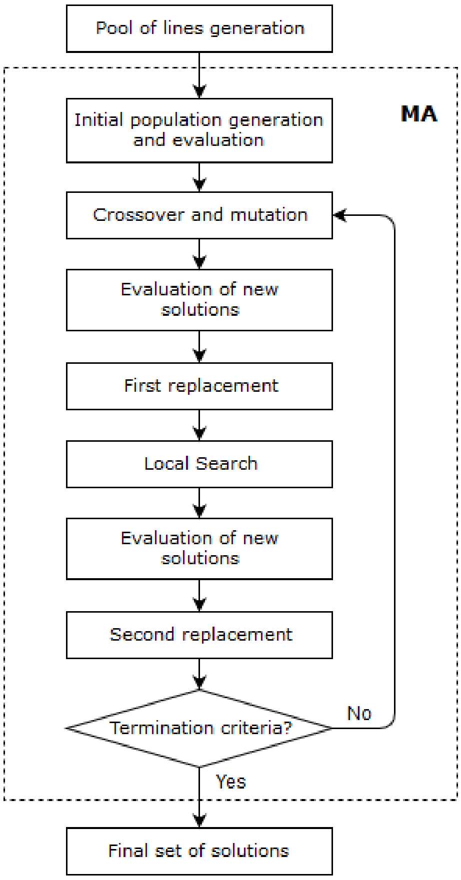

Figure 1: Flowchart of the bi-objective memetic algorithm (Duran-Micco et al., 2020).

4.1 Initial population

Before the execution of the algorithm, a pool of feasible lines is generated. A feasible line is defined as a sequence of adjacent stops, that either contain no cycles or conforms a single closed loop (circular line). In both cases, the length of the line must be smaller than l_{max}. In general, the most attractive lines in the TNDFSP are the lines that satisfy a lot of demand as direct as possible. Therefore, the initial feasible lines are obtained by generating k-shortest paths between the OD pairs with greater demand. After preliminary experiments, it was decided to consider the 5 shortest paths between the OD pairs that concentrate 50% of the total demand. Only feasible lines are included in the pool, by discarding the lines that are longer than the maximum length. It should be noted that although at the beginning of the algorithm this limited set of feasible lines is considered, the search operators used in the MA can modify them, generating new lines not included at this point. The pool of feasible lines is used to generate the initial population. For each solution in the population, first the number of lines is randomly decided, considering the lower and upper bounds. Then, the lines are selected one at the time from the pool of feasible lines. Each time, lines that cover a larger amount of unsatisfied demand in the current network have a greater chance of being selected.

4.2 Search operators

Once the initial population is generated, this population is improved through an iterative process using three operators: cross-over, mutation and local search. In the cross-over, two randomly selected parent solutions are combined to generate a new offspring solution. The offspring takes half of the lines of each parent. The lines are selected one at the time, alternating the parent, in each case maximizing the additional satisfied demand. Then, the new solutions are randomly modified by the mutation operator. In this case, the modification may be small, by randomly adding or removing a single node to a line, or large, by randomly replacing an entire line by a new line from the pool of feasible lines. It must be noted that when adding a node to a line the length constraint must be verified. If after the mutation this constraint is violated, then the modification is discarded and the original network (before the mutation) is maintained. Contrary to many implementations of TNDFSP, in this paper it is not required that the transit network covers all the nodes. However, in general better solutions are obtained when a large proportion of the total demand is satisfied. Therefore, a repair operator is executed after the mutation operator, in which the transit network may be extended to connect unsatisfied OD pairs. For each unconnected OD pair, it is randomly decided if it is connected or not. The connection is performed by extending the line which generates a feasible path for the OD pair at the minimum additional length, always considering the maximum length of the lines. After these operators, the solution could be unfeasible due to the maximum capacity of the edges. If for any edge the capacity constraint is violated even if the minimum frequency is assigned to every line, then the solution is discarded. The new solutions that are not discarded are evaluated using the procedure described in Section 4.3. After the evaluation, a first replacement is done, to generate a new population, with the same size as the original one, by selecting the best solutions among parents and offspring. Since two objectives are considered, the selection is done considering measures of quality and diversity, so a balanced approximation of the Pareto front can be obtained. For this purpose, the *dominance depth* and the *crowding distance* indicators are used, which are part of the well-known NSGA-II algorithm (Talbi, 2009). First, the solutions are ranked using the *dominance depth*, which assigns solutions to different levels according to the solutions that dominate or are dominated by them. Then, solutions in the same level are ranked using the *crowding distance*, which is a measure of the distance between a solution and its neighbours in the solution space.

The population obtained after the first replacement is subject to the local search operator. This operator attempts to improve some of the new solutions, which are randomly selected. To speed-up the process, only a simple operator is used to generate new solutions, consisting of adding one node at the time to the extreme of a line. At each step of this process, it is ensured that the line length constraint is not violated. Moreover, since the calculation of the objective functions is computationally expensive, an alternative objective is used. During the local search, the proportion of demand satisfied without transfers is maximized. The final solutions of the local search operator are fully evaluated using the procedure described in Section 4.3. Then, a second replacement step is executed, considering the same quality and diversity measures as in the first replacement. The second replacement generates the final population of the iteration. The iterative procedure is repeated until a maximum computing time is reached.

4.3 Evaluation procedure

To evaluate a given transit network and calculate the value of the objective functions, it is necessary to solve the PAP. Therefore, the evaluation of a solution of the TNDFSP is computationally expensive. The evaluation procedure used in this paper is based on the frequency share method first proposed in (Baaj and Mahmassani, 1990). In the original method, applied to the TNDFSP without capacitated edges or crowding, the frequencies are set together with the PAP by means of an iterative heuristic procedure. Starting with some initial frequencies, the PAP is solved using the frequency share rules described in Section 3.2. Once the passenger flows are defined, the frequencies are updated so that the capacity of the lines is just enough to transport all the assigned passengers, as indicated in Equation 13.

$$f_l = \frac{v_{max}^l}{q} \qquad \textbf{[13]}$$

Where:

f_l = frequency of line l (buses/hour);

v_{max}^l = maximum flow of passengers on line l (pass/hour);

q = capacity of one bus (pass/bus).

Since in this case a discrete set of frequency values is considered, after the calculation with Equation 13, the lowest value from the discrete set that is larger than the calculated value is selected. After updating the frequencies, the passenger flows are re-calculated using these new frequency values. This iterative process is repeated until the frequencies converge. Although this method does not ensure convergence, in practice it typically converges after a few iterations.

This method is modified in this paper, in order to address the problem with capacitated edges and crowding. In this case, a distinction is made between nominal and effective frequencies. The nominal frequencies correspond to the actual number of buses running, while the effective frequencies correspond to the apparent frequency perceived by passengers, which is lower than the nominal one in case of crowding (Equation 11 and 12). The capacitated edges affect the calculation of the nominal frequencies. On the other hand, the effective frequencies are used to calculate the passenger flows and to determine the convergence of the algorithm. These aspects are discussed in the following paragraphs.

The capacitated edges refer to additional constraints that limits the number of buses per hour that can run on certain edges. In each iteration of the evaluation procedure, the nominal frequencies are first calculated with the normal method (Equation 13). Then, once the nominal frequency of every line is updated, the capacity constraints of the edges are checked. If on an edge the capacity is surpassed, then the nominal frequency of every line using that edge is reduced in the same proportion, so that the capacity constraint is satisfied. After that, since discrete frequencies are considered in this research, the frequencies are assigned to the greatest value from the set of feasible frequencies that is lower than or equal to the calculated value. Finally, after reducing the value of the nominal frequencies, it is checked if it is feasible to increase the nominal frequency of one or more lines without violating the constraint, prioritizing the lines that transport more passengers.

Once the nominal frequencies are updated and adjusted, the effective frequencies are calculated for each stop in each line, as indicated in Equation 12. The effective frequencies are calculated using the passenger flows and the nominal frequencies obtained in the respective iteration. The new effective frequencies are compared to the ones obtained in the previous iteration, to check for the convergence of the procedure. If they do not converge, then these effective frequencies will be used in the next iteration to re-calculate the flows of passengers and repeat the process. As described in Section 3.2, the use of the effective frequencies to calculate the demand split implies that passengers take into account crowding during the route choice process, potentially avoiding crowded lines if possible.

It is important to note that the convergence of the effective frequencies is more difficult to achieve, compared to the nominal frequencies. In Section 5.1 this issue is discussed and it is shown that, indeed, there is a considerable proportion of solutions where the convergence is not reached due to iterative oscillations. However, the same results show that the impact of such iterations on the average passenger travel time is negligible. Therefore, it seems reasonable that, in the case that the effective frequencies do not converge, the passenger assignment calculated in the last iteration (before the maximum number of iterations I_{max} is reached) can be used to compute the objective functions for that network.

5 Results

Since this problem has not been considered before, it is not possible to compare the results obtained to results from the state of the art. However, we can assume that our algorithm will perform well in general, based on previous results of this algorithm presented in (Duran-Micco et al., 2020). In that study, the algorithm is used to solve simpler versions of the TNDP and TNDFSP, for which there are benchmark instances and previous results from the state of the art available to make a comparison. The results presented in that paper show that the algorithm can find good solutions, in some cases improving the previous best results. Moreover, the algorithm generates the solutions after short computing times. For example, the algorithm was able to find good solutions in less than 1 hour for an instance for which previous studies had reported computing times longer than 10 hours. In this paper, we will evaluate if the algorithm is successful in addressing the extended variant of the TNDFSP described in Section 3, with acceptable computation times, when applied to a large realistic instance. Furthermore, we will focus on evaluating the impact of considering capacitated edges and crowding. The instance used in this study is a representation of the city of Utrecht, The Netherlands. It is represented by an infrastructure network consisting of 271 nodes and 470 undirected edges, displayed in Figure 2. To test the impact of considering capacitated edges, a perimeter is defined around the historic centre, in which strict capacity constraints are imposed. These constraints affect 16 edges, in which a maximum capacity of 12 buses/hour in each direction is allowed. The OD matrix has 16823 non-zero OD pairs, corresponding to 22.9% of the total number of OD pairs. A detailed description on how this instance is generated is given in (Durán-Micco et al., 2022).

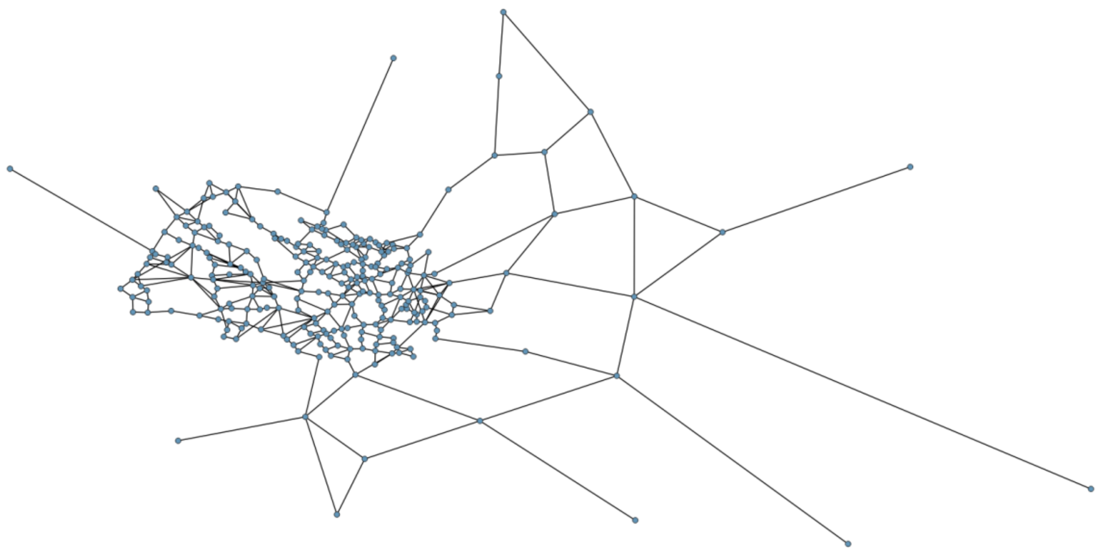

Figure 2: Utrecht network.

Table 1 shows the values of several algorithm and problem related parameters that are used in the experiments presented in this section. The table show the most relevant algorithm parameters, such as the population size (s_p), which determine how many solutions are maintained in the population during the execution of the algorithm. Also, the probability that the mutation operator is applied to a new solution (p_{MUT}) is presented, as well as the probability that such mutation corresponds to a small modification (p^*_{MUT}), as described in Section 4.2. Similarly, the table shows the probability that the local search operator is applied to a new solution (p_{LS}). Finally, it shows the maximum number of iterations used in the evaluation procedure of a solution (I_{max}). The selected parameter values may have a big impact on the performance of a metaheuristic, so the parameter setting is an important step in the implementation of such algorithms. Therefore, the algorithm parameter values used in the following experiments were set using the results of preliminary experiments, in which different values were tested and the ones delivering the best results were selected. These preliminary experiments also showed that after 1 hour of computing time the solutions generated by the algorithm stabilize for the instances used in this paper.

Table 1 also shows some key problem parameters, which are parameters that define a specific instance of the problem, as described in Section 3. For example, it includes the penalization on the total travel time for each passenger that must do a transfer (t_{tr}) or whose demand is not satisfied (t_{ud}). It also includes the minimum frequency that can be assigned to a line (f_{min}), the maximum length of a line measured in travel time (l_{max}), the minimum (N_{min}) and maximum (N_{max}) number of bus lines allowed in the network, the bus capacity of the buses operating the network (q), and the coefficient value used to calculate the effective frequencies (β). These parameters were set considering the standard values used in literature and information gathered from the real transit system in Utrecht (Durán-Micco et al., 2022).

Table 1: Algorithm and problem parameters' values.

Parameter	Symbol	Unit	Value
Algorithm parameters:			
Population size	s_p	-	30
Mutation probability	p_{MUT}	-	0.05
Small mutation probability	p^*_{MUT}	-	0.5
Local Search probability	p_{LS}	-	0.75
Max. iterations in frequency setting	I_{max}	-	25
Problem parameters:			
Transfer penalization	t_{tr}	min	5
Unsatisfied demand penalization	t_{ud}	min	200
Min. frequency	f_{min}	bus/h	2
Max. line length	l_{max}	min	90
Minimum number of lines	N_{min}	-	20
Maximum number of lines	N_{max}	-	60
Bus capacity	q	pass/bus	60
Effective frequency coefficient	β	-	4

All the experiments presented in this paper were performed on an Intel® Core™ i7-3770 CPU 3.40 GHz machine with 16.00 GB RAM. The MA was coded in C++ and compiled with Visual Studio 2019. For each scenario, four executions of the MA are run in parallel during 1 hour. The comparison of the results obtained by the different executions for each of the experiments described below, show that the non-dominated solution sets found are similar in quality and shape, showing the consistency of the results generated by the algorithm. The non-dominated solution sets presented in the following subsections are obtained by combining the final populations of these four executions. Detailed data about the instance and relevant results are available here: https://www.mech.kuleuven.be/en/cib/lp/mainpage#section-22.

The presentation of the results is divided in three subsections. Firstly, the crowding effects are analysed without considering the link-capacity constraints, to check if the algorithm is successful in generating solutions with less crowding in the vehicles. Secondly, the two elements are considered together, to determine if the algorithm can generate solutions that comply with the capacity constraints while mitigating the negative impacts on service quality. Thirdly, a discussion is given on how the results compare to real networks and about limitations that remain to apply the algorithm in practice.

5.1 Crowding effect

In the basic TNDFSP, the PAP is solved without considering the capacity of the lines, and then the frequencies are updated according to the passenger flows. Therefore, the resulting frequencies could be set too high or, if a strict limit is assigned to them, the algorithm could assign more passengers to a line than its capacity. The evaluation procedure proposed in Section 4.3 addresses this issue using the concept of effective frequencies, which penalize the waiting time on crowded lines, making them less attractive for passengers. Therefore, the passenger assignment is modified when there is crowding. To visualize the impact of considering crowding in the MA, Figure 3 compares three sets of non-dominated solutions: solutions found without considering crowding (S_{NC}), solutions found considering crowding (S_C), and the result of re-evaluating the solutions found without considering crowding, now considering the crowding effects (S_{RE}). The re-evaluation implies that the lines are maintained, but the frequency setting and passenger routes can change.

Figure 3 shows the respective sets of non-dominated solutions considering the two objectives described in Section 3.1: the average passenger travel time (ATT) and the required fleet size. Additionally, Table 2 shows some additional information related to crowding levels in the same sets of non-dominated solutions. For each set, the proportion of solutions without crowded lines is displayed. Moreover, the crowding level of each solution is quantified using a crowding-indicator, which computes the distance travelled by passengers in lines where the number of passengers is above capacity. This is calculated by multiplying, for each link in each line, the travel time by the number of passengers above the line capacity. Then, these values are added obtaining a single value for the entire network.

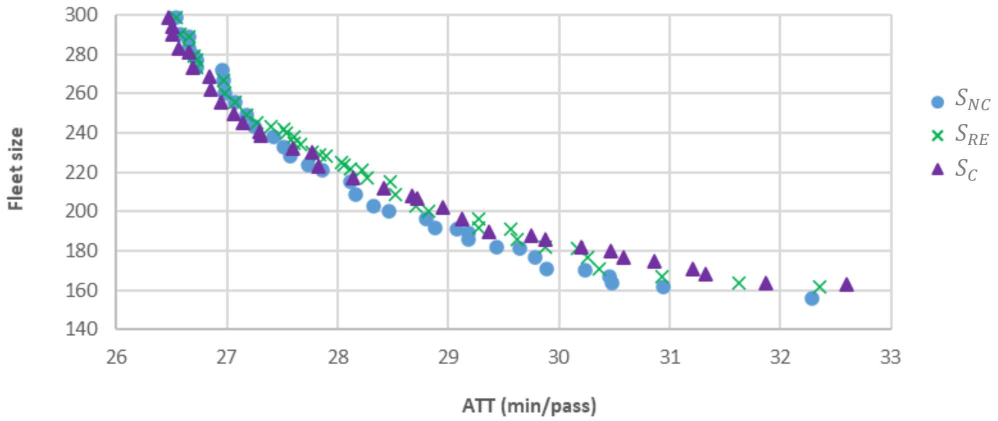

Figure 3: Comparison of objective-function values of non-dominated solutions with and without considering crowding effect.

Table 2: Comparison of crowding levels of non-dominated solutions with and without considering crowding effect.

	S_{NC}	S_{RE}	S_C
Solutions without crowding	24%	44%	56%
Average crowding-indicator [min*(pass/h)]	4533	3504	2570
Median crowding-indicator [min*(pass/h)]	4685	4286	0
Max. crowding-indicator [min*(pass/h)]	12467	12467	14670

It is expected that the solutions ignoring the crowding effects look better, since the crowding effect implies longer waiting times. It should be noted, however, that these solutions are not realistic. As shown in Figure 3, in this example this difference is very small and is noticeable only in the solutions with lower fleet size, since these solutions tend to have lines with lower capacity. Furthermore, the difference after re-evaluating the solutions is also small. This is due to the characteristics of the instance, since the demand levels are not particularly high. Moreover, in this experiment, in which there are no capacitated edges, crowding is present only when the calculated frequency of a line is greater than the maximum frequency of 20 bus/h. In this instance, such high frequencies normally happen in only one line, which connects a specific OD pair with particularly high demand. A more constrained scenario is considered in the next section, by considering the link-capacity constraints, which indirectly limit the frequency of the lines.

It is interesting to note that in this case, although the difference in the objective function values is not significant, there is a noticeable impact on the number of solutions in which there is crowding. In this case, a solution with crowding is a solution in which at least one line has more passengers assigned than its capacity. The capacity of a line is given by the multiplication of the frequency by the capacity of each bus. Indeed, as shown in Table 2, among the solutions in S_{NC}, only 24% has no crowding. However, in S_C 56% of the solutions have no crowding. This is also reflected in the values of the crowding-indicator. Despite the maximum crowding level being higher in S_C, corresponding to a solution with low fleet size, the average and median values are significantly lower compared to S_{NC} and S_{RE}. This shows that when taking crowding effects explicitly into account during the optimization, the algorithms generate solutions that avoid part of that crowding, through the generation of alternative paths for the crowded links and the resulting re-routing of passengers. Remember that, as described in Section 4.3, the proposed method does not intend to completely avoid crowding, but instead to represent the fact that passengers could prefer alternative paths if these exist. Also, by penalizing the waiting times, the algorithm should generate solutions in which crowding can be reduced. In the case of the example discussed above, the algorithm indeed generated more solutions in which crowding can be reduced, and in many cases completely avoided. However, when the operator's cost is minimized, solutions with crowding will still be present, unless it is strictly avoided.

As discussed in Section 4.3, a potential issue of the proposed method to address the problem with crowding is related to the convergence of the evaluation procedure. In the evaluation procedure of the basic TNDFSP, the nominal frequencies must converge, while in the extended TNDFSP, the effective frequencies must converge. This is expected to be more difficult since the effective frequencies may be different for every node and direction within a line. Indeed, the results in the basic TNDFSP show that almost all the solutions converge. However, that is not the case when addressing the extended TNDFSP, as is demonstrated with some additional experiments. For these experiments, firstly a set of 50 solutions is generated for the Utrecht instances using the MA without considering crowding. Then, a random solution is selected and the frequency of 20% of the lines (randomly selected) is fixed to a lower value than the one previously set by the MA, so that the capacity is reduced and crowding is induced. Then, the solution with these fixed frequencies is re-evaluated, solving the PAP now considering crowding effects, to test the convergence of the effective frequencies. This process was repeated 500 times, selecting a random solution each time. It was found that in almost 30% of the cases the effective frequencies did not converge. This happens because, even in small examples, the iterative process of updating passenger flows and effective frequencies can lead to oscillations. However, it can be observed that, in general, the impact of these oscillations in the ATT is very small, with variations between iterations of less than 0.2% on average. Therefore, although in some cases the solution does not lead to an equilibrium state, the selection of the last passenger assignment solution after the maximum number of iterations can be considered a good estimation for the purposes of this study.

5.2 Link-capacity constraints

To test the effect of considering capacitated edges and the performance of the algorithm to solve this problem, a capacity is assigned to a subset of edges in the historic centre of Utrecht. This area is located just next to the central railway station, which is an important transit hub and attracts a lot of demand. Therefore, if no constraint is imposed, many bus lines with high frequencies are assigned to this area by the algorithm. The objective of this experiment is to determine if the algorithm can find solutions that comply with these additional constraints and how the resulting bus line plans are affected in this particular instance.

Figure 4 shows the comparison between the non-dominated solution sets generated by the MA with and without considering the link-capacity constraints. The solutions that do consider the capacity on the edges are slightly worse. This is expected since these solutions require some lines to take detours and/or have reduced frequencies. However, the difference is relatively small, with a maximum difference in ATT of around 2.5% (between solutions with equivalent fleet size). This shows that the algorithm can also find good solutions for the extended version of the problem. It is important to note that when the non-dominated solutions generated without the capacity constraints are re-evaluated to comply with the maximum frequency, all of them turn to be infeasible. This happens because in those solutions, even if the frequency of all the lines traversing through the restricted area is set to the minimum allowed frequency (f_{min}) , still the capacity constraints are violated. This clearly demonstrates that, in this case, it is important to consider such constraints already during the line plan design stage, since this allows to design efficient transit networks that comply with modern urban policies that limit the number of vehicles allowed in certain areas.

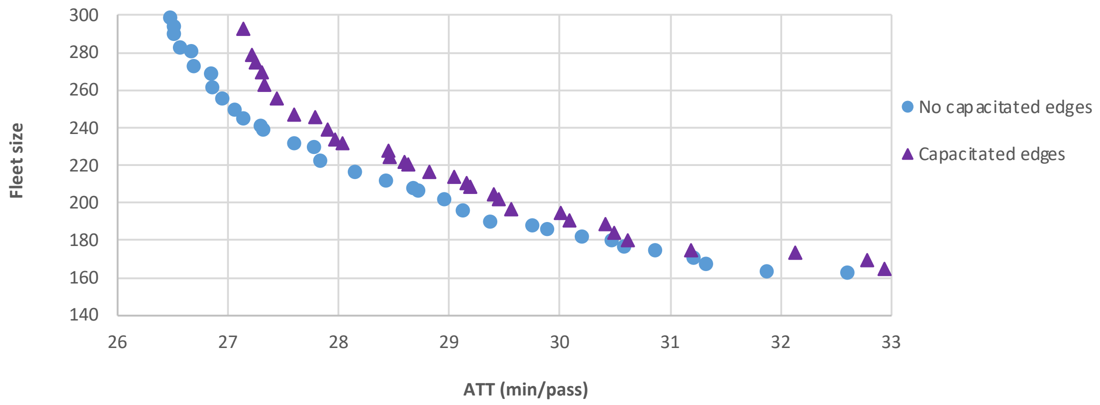

Figure 4: Comparison of non-dominated solutions with and without considering capacitated edges.

To have a closer look on how the resulting line plan is affected by considering the capacity constraints or not, one solution is selected from each set of non-dominated solutions in Figure 4. In each case, a solution with a fleet size of approximately 200 buses is selected, which is approximately the fleet size in the current bus network in Utrecht. Figure 5 (Network 1) and Figure 6 (Network 2) display a visualization of each proposed line plan in the centre of Utrecht. In the figures, each line is drawn in a different colour. Moreover, the area containing the 16 capacitated edges is marked by the hatched polygon. In both cases, the high concentration of lines right to the west of the delimited area corresponds to the railway central station. It is possible to see in Figure 5 that, in the line plan generated without considering the capacitated edges, there are ten lines running through the east-west corridor that traverses right through the middle of the delimited area. On the contrary, in the line plan displayed in Figure 6, generated considering the capacity constraints, only five lines run through the same corridor. By analysing the frequencies of those lines, it can be seen that in Network 1 the combined frequency of the 10 lines is 30 bus/h, while in Network 2 the combined frequency of those 5 lines is exactly 12 bus/h, which is the maximum capacity. Moreover, it can be seen that the rest of buses are mostly re-routed through the east-west corridor located at the south of the delimited area. Indeed, at the busiest section of this corridor, in Network 2 there are 2 additional lines (increasing from 6 to 8 lines), and the number of buses increases by 47% (from 34 to 50 bus/h). A further comparison between these two networks shows that both networks have almost the same number of lines in total, with 32 and 33 lines, respectively. Nevertheless, the total length of the network generated without the capacity constraints is approximately 13% larger, and this network has 8% more demand satisfied without transfers. However, since the Network 1 does not consider crowding effects, and considering a maximum frequency of 20 bus/hour for each line, it results in a passenger assignment with a crowding level of 4200 [min*(pass/h)], using the indicator described in Section 5.1. On the contrary, the crowding is completely avoided in Network 2, generated considering the capacitated edges.

These results provide an insight in the importance of considering both the crowding effects and the capacity of the edges, already during the network design stage of a transit network. Indeed, if the capacity constraints are ignored during the design stage, the solution could become unfeasible when these constraints are later addressed, as was the case in this experiment. Moreover, even if the solutions turn out to be feasible, probably their quality will be severely diminished by a forced reduction of the frequencies in some lines. Of course, the relevance of the link-capacity constraints depends on the particular instance that is being addressed.

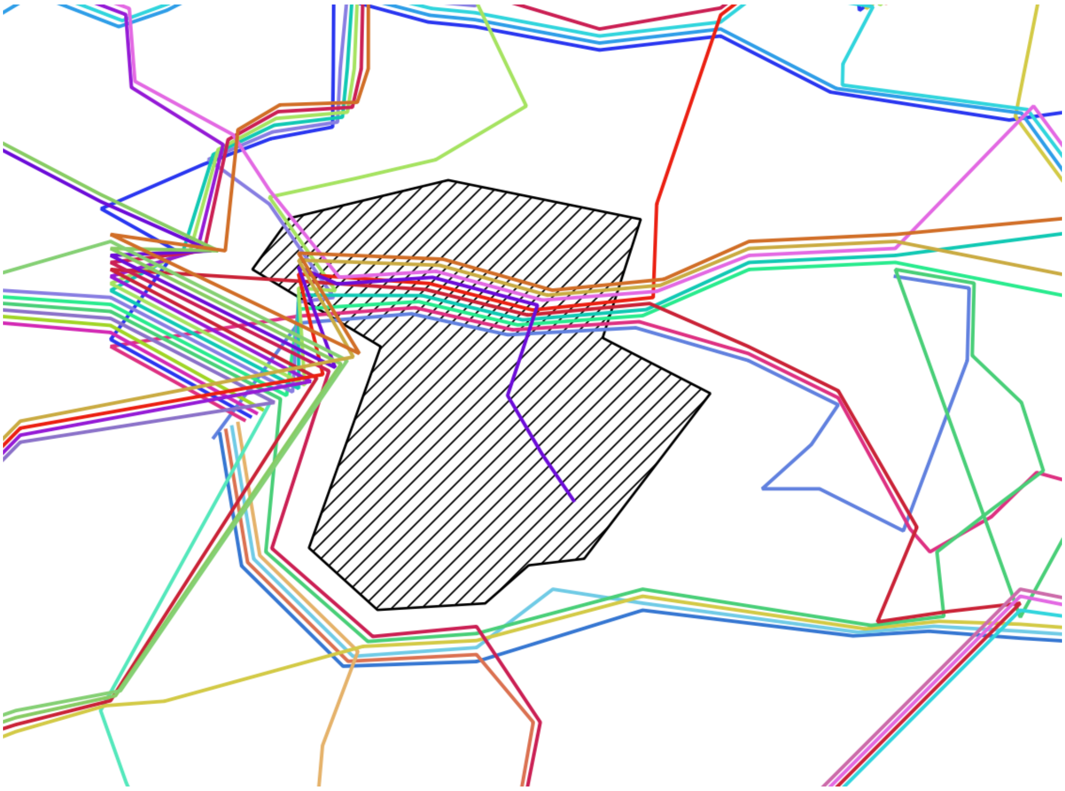

Figure 5: Visualization of a proposed line plan in centre of Utrecht network, without considering capacitated edges.

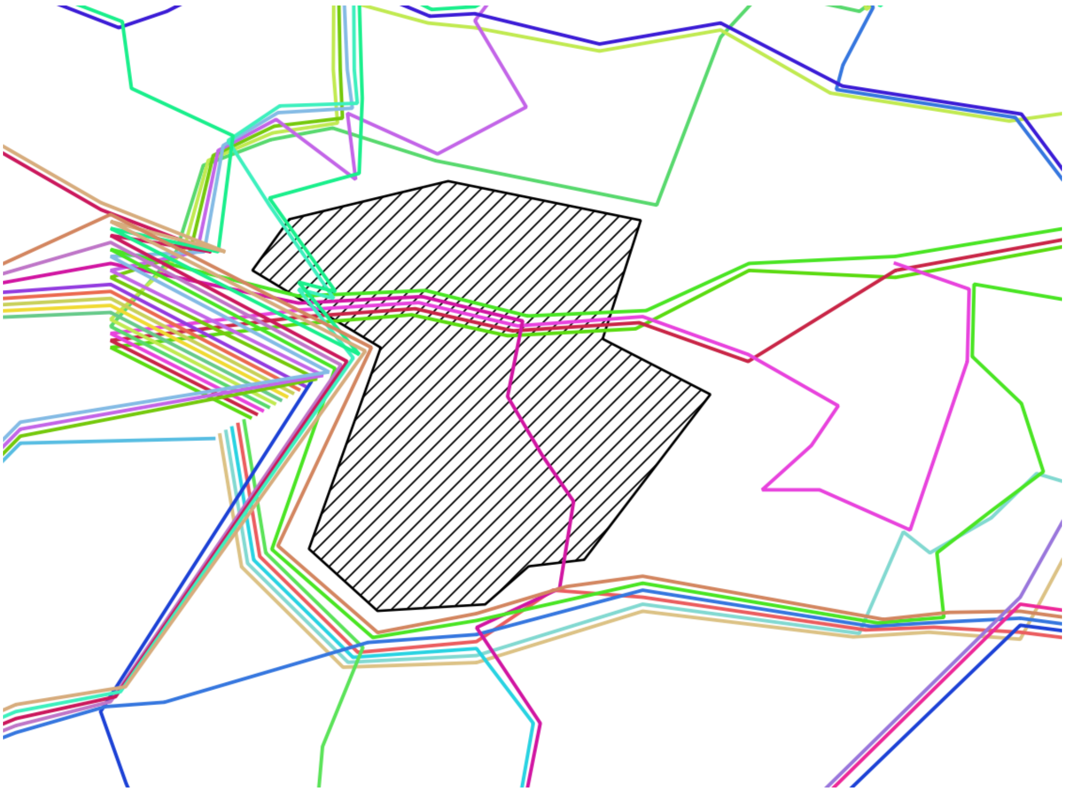

Figure 6: Visualization of a proposed line plan in centre of Utrecht network, considering capacitated edges.

5.3 Application to practice

The previous results show that the algorithm can address large instances and that it is relevant to consider crowding and link capacities during the design stage. In this section we compare the generated solutions to the real transit network in Utrecht and we discuss which limitations still hinder the application of such techniques in practice.

The current bus network operating in Utrecht was modelled and evaluated using the algorithm described in Section 4.3. The results show that the algorithm can generate solutions that improve both the passenger travel times and the fleet size, for the different versions of the problem. For example, there are solutions that keep the same fleet size as the in the current network, but with an average passenger travel time that is 6% lower. Similarly, the algorithm finds solutions with the same passenger travel time but with a fleet size 19% smaller, compared to the current network. The results also show that the algorithm consistently generates solutions with less lines than the current network. Indeed, the current network has 52 lines while the algorithm generates solutions with 25-35 lines. A lower number of lines allows higher frequencies and therefore reducing the waiting times with the same fleet size. More details about these comparisons between the algorithm results and the real bus network in Utrecht are provided in (Durán-Micco et al., 2022).

Despite the good results and the improvements made to the model compared to previous ones, there are still some aspects that limit the direct application of the algorithm in practice. A relevant issue is how the demand is estimated and used by the model. Our algorithm assumes a one-period demand, so it does not consider the variations in demand along the day or between days. Moreover, it considers an inelastic demand, which does not vary with the service level. These simplifications are done to make the model tractable, but also because such data is rarely available. Other limitations include not considering explicitly other modes of transportation, although existing alternative transit modes, such as train or tram, could be incorporated with minor adjustments to the algorithm. Furthermore, this type of techniques struggles to consider cultural or political aspects that may become relevant during the design of real transit networks, and that can force decisions that are not necessarily optimal from an engineering point of view. However, despite these persisting limitations, the proposed algorithm and similar ones can be used during the design process as a support tool or as a starting point. Therefore, the design of more realistic and efficient algorithms remains a relevant challenge.

6 Conclusions

This paper addresses an extended version of the TNDFSP, which includes a strict capacity in the number of buses that can circulate on certain links of the infrastructure network and considers the effects of crowding in passengers' route choice. These assumptions lead to a more realistic representation of reality, and are therefore useful for policy makers and transit planners to design better transit networks. A bi-objective memetic algorithm is proposed to solve the problem, which minimizes the passenger travel time and the required fleet size. The algorithm is tested on a large instance generated with real data, obtaining good results in 1 hour of computing time, which is a reasonable value for the TNDFSP. The experimental results show the convenience of considering this extended version of the problem. The results show that the modified algorithm is capable to deal with the crowding levels present in the instance, reflecting the fact that passengers try to avoid crowded lines, and to generate transit networks that mitigate these issues. Moreover, the crowding effects are more relevant when also considering capacity constraints in some edges. Indeed, in this case the algorithm could generate solutions that comply with these additional capacity constraints and avoid the crowding issues at the same time. Moreover, in this case, trying to handle the capacity constraints once the line plan is already designed, resulted in unfeasible solutions. This shows the importance of considering these constraints already when solving the TNDFSP.

Overall, the proposed algorithm successfully addresses the problem and could serve as a relevant design tool to support the implementation of transit networks in practice. However, there is still a relevant gap between the complex problem found in real life and the simplified versions that are addressed by optimization algorithms. Further research in the topic could focus on the design of efficient algorithms that address even more realistic versions of the problem, considering aspects such as elastic demand or the integration with other modes of transportation.

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