A survey on the transit network design and frequency setting problem

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Abstract

Appropriate public transport systems are crucial in modern cities. Given the high costs that they represent and the impact they have on people's lives, effective tools are required to support their design. With this in mind, the Transit Network Design problem (TNDP) and the Transit Network Design and Frequency Setting problem (TNDFSP) have been extensively studied in the domain of Operations Research. However, due to the complexity of these problems, multiple simplifications are typically made when modelling and designing solution algorithms. Therefore, still no optimization techniques are available to address these problems in practice. Moreover, different studies address different versions of the problem, with varying assumptions and constraints, complicating the comparison of results or solution approaches.

This paper presents an extensive survey of studies addressing the TNDP and the TNDFSP. It discusses the different assumptions, constraints, objectives, solutions approaches and testing instances that have been considered in the literature. Furthermore, a detailed analysis is done regarding the case studies considered for the TNDFSP. Moreover, the variants of the passenger assignment subproblem that have been applied within the TNDP and the TNDFSP are discussed. The analysis shows that extensive research has been done regarding these problems. However, it also identified the significant gap that still exists between theory and practice, even in the studies addressing case studies.

Keywords

Public transport optimization; Transit network design; Frequency setting; Line planning; Passenger assignment

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1 Introduction

Urban public transport systems are crucial in modern cities since they allow commuters to move in an efficient, sustainable, and affordable manner. These systems generate multiple benefits, such as reducing traffic congestion (and the many externalities associated to it), reducing the dependency on private transportation and allowing a better use of space, among others. However, it is essential that public transport systems are well designed and managed in order to achieve these objectives. Moreover, public transportation represents a high cost and typically depends on public subsidies. Therefore, it is important to have tools and methods that allow the design and operation of efficient networks, maximizing the service quality provided to the passengers and minimizing the costs of its operation.

Given the practical interest and the challenges that correspond to the design and operation of public transport systems, there are several well-known problems in the domain of Operations Research related to it. These problems can be classified, according to the time horizon of the decisions considered, in strategic, tactical, and operational problems. Ceder and Wilson (1986) proposed a commonly adopted classification, which is illustrated in Figure 1. At the strategic level, the two most relevant problems are the Transit Network Design Problem (TNDP) and the Frequency Setting Problem. The TNDP, also called Line Planning Problem, consists of designing the set of lines that will conform the transit network. Typically, the underlaying infrastructure network and the demand matrix between the stops are given. Therefore, the objective is to determine the sequence of stops visited by each line, considering the travel times between stops and the existing demand. There is not a unique definition of the problem, so several objectives, constraints and assumptions are typically considered. However, due to the complexity of the problem, commonly only simple assumptions and constraints are included.

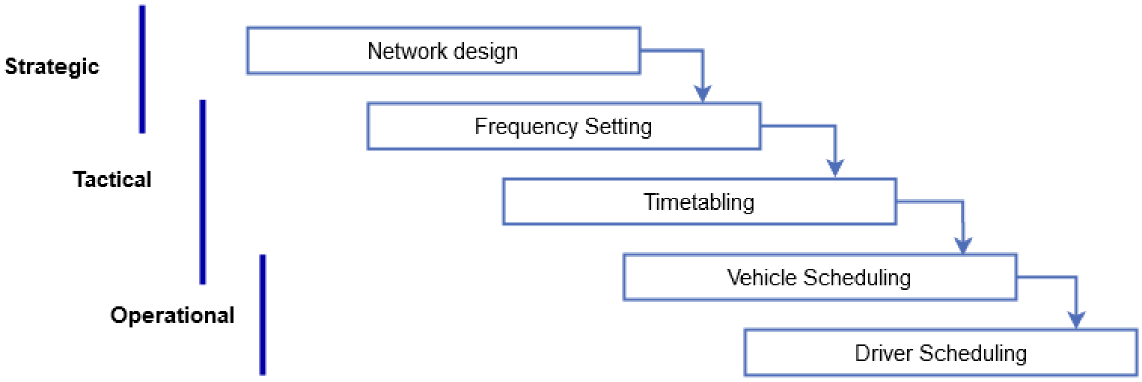

Figure 1: Sub-problems in the design of public transport systems.

The Frequency Setting Problem consist of, as its name indicates, setting the service frequency for each of the lines conforming a given transit network. The frequencies have important implications for the expected passenger waiting times, and therefore also for the service quality, as well as for the number of vehicles required to operate the system. Consequently, frequency setting is required for a proper evaluation of a candidate transit network. Despite this relationship between these two strategic problems, and given their complexity, they have been traditionally solved in sequence, first designing the transit network and then solving the frequency setting problem. However, in the last decade it has become common to integrate them, in what is commonly called the Transit Network Design and Frequency Setting Problem (TNDFSP) (López-Ramos 2014).

The TNDFSP has been proven to be a very complex problem that requires many assumptions before it can be properly addressed in research. This results in different studies making different assumptions, leading to different problems for which the results cannot be directly compared. Moreover, due to these assumptions there is a gap between the problems addressed in theory and the extremely complex problems found in real life. For example, the problem is very often modelled with a single objective, despite the many conflicting objectives that can be identified in practice. Moreover, a homogeneous fleet is usually considered, with only one type of vehicle, while in fact public transport networks are commonly operated by vehicles of different sizes and characteristics. Additionally, the behaviour of passengers is almost always greatly simplified. For example, it is typically assumed that passengers do not consider crowding issues in their route choice, and that they can always board the first vehicle to arrive at the stop.

A crucial assumption with important implications in many implementations of the TNDP or TNDFSP is related to the Passenger Assignment (sub)Problem (PAP). The PAP models the expected decisions of the passengers regarding which lines they will take in a given network to reach their destination. It is a complex problem that has been studied independently, although it is also part of larger problems. Multiple models and solution approaches have been proposed for the PAP, with different levels of detail and complexity (Liu et al. 2010b). The PAP is an unavoidable step when evaluating the expected passenger travel time or passengers loads of a certain transit network, so it is often included within the TNDP and TNDFSP. Although mostly only simple models are adopted for the integration in the TNDP and TNDFSP, this remains the most time consuming part in most solution approaches (Farahani et al. 2013) .

This paper presents an exhaustive review of recent studies related to the TNDP and the TNDFSP, focussing on the different assumptions and problem definitions used in different studies. This analysis allows to identify the existing gap between theory and practice and to identify future research paths. Due to this focus, this review is complementary to previous surveys considering the development of the TNDP and TNDFSP (Guihaire and Hao 2008; Kepaptsoglou and Karlaftis 2009; Farahani et al. 2013; López-Ramos 2014; Ibarra-Rojas et al. 2015; Iliopoulou et al. 2019a). Moreover, many more recent contributions are included. Compared to the most recent survey paper, our focus is completely different. Iliopoulou et al. (2019a) concentrate on the metaheuristics that have been proposed to solve the problem and they present a comparative analysis of the results obtained by different studies for a small benchmark instance. On the contrary, this survey paper focusses on the different variants of the problem that have been addressed. Additionally, it discusses case studies related to the TNDFSP, discussing the characteristics of some larger and more realistic instances.

Although the definitions of the TNDFSP may be applicable to different modes of public transportation, the studies in literature with this denomination deal mostly with urban bus networks. When considering other modes of transportation, such as train or metro, aspects related to infrastructure limitations have a greater relevance, so the problem in these cases tend to have additional specific constraints. Therefore, studies addressing network design of these transportation modes are more specific. Moreover, although some studies address the problem with multimodal networks, in these cases the problem consists of optimizing the network of one mode while the others are assumed as given. This survey focuses on studies addressing the design of bus or generic transit networks. Within the design of bus networks, some studies address the specific problem of designing feeder bus networks or bus rapid transit lines. Since the modelling of these problems differ significantly from the TNDFSP, these studies are not considered in this survey.

The papers included in this survey were selected using scientific search engines and looking at the references of the selected studies, focusing on papers published after 2009. The search was performed in Scopus with the following search terms: “transit network design problem”, “urban public transport network design problem” and “bus line planning problem”. The relevant papers were selected according to the scope given in the previous paragraphs. Therefore, studies addressing the design bus feeder networks, bus rapid transit corridors or single transit lines were discarded. Moreover, studies addressing only the frequency setting problem are not included. According to these criteria, 30 studies addressing the TNDP and 65 studies addressing the TNDFSP were selected.

The rest of this survey is structured as follows. Sections 2 and 3 discuss the literature related to the TNDP and the TNDFSP, respectively, presenting the different objectives, assumptions and solution approaches that have been considered. Section 4 discusses in more detail the studies describing case studies for the TNDFSP. Section 5 presents different approaches to solve the Passenger Assignment Problem (PAP) within the TNDP or TNDFSP. Finally, in Section 6 the main conclusions of this review are discussed together with possible future research lines.

2 Transit Network Design Problem

The basic version of the TNDP consists of designing a transit network conformed by a set of lines served in both directions. Each line is defined as a sequence of stops. The inputs of this problem are the infrastructure network and the demand matrix, which is typically assumed to be static during the studied period. The underlying infrastructure network is represented by a graph where the nodes represent the stops, and the edges are characterized by the distance or travel time between stops. The basic constraints in the problem are that the lines can visit each node at most once, and a lower and upper bound that limit the number of nodes that a line can contain. In this version of the problem, it is typically assumed that passengers take the shortest path within the transit network, including transfer times, but without considering waiting times or capacity issues. The most commonly used objectives in literature correspond to the user’s and operator’s point of view, which can be used as objectives separately or together. The user’s point of view is mostly represented by the minimization of the average travel time of the passengers, corresponding to the in-vehicle travel time and a fixed penalization per transfer. The operator’s cost is usually represented by the combined length of the lines. This problem corresponds to the most basic version of the TNDP and is frequently used to compare different solution approaches. Even then, this problem has been shown to be extremely difficult, since even special cases of the subproblems that conform it are NP-Hard (Magnanti and Wong 1984; Schöbel 2012). Of course, there are studies that approach variants of this problem, but in those cases different studies are hard to compare. Table 1 presents a summary of the papers since 2009 that address the TNDP, indicating in each case the objectives (travel time, network length or other), the passenger assignment technique (shortest path or other), additional elements that are added to the problem, and the solution approach.

Table 1: Papers about the TNDP.

Authors	Objective[1]			Passenger assignment[2]		Additional elements	Solution approach[3]	
	ATT	NL	Other	SP	Other		Exact	MH
(Fan et al. 2009)	x	x		x				H
(Fan and Mumford 2010)	x			x				H
(Miandoabchi et al. 2012)	x	x			x	Fixed terminals, alternative mode		EA/SA
(Mumford 2013)	x	x		x				EA
(Nikolić and Teodorović 2013)	x			x				NI
(John et al. 2014)	x	x		x				EA
(Kechagiopoulos and Beligiannis 2014)	x			x				NI
(Nayeem et al. 2014)	x			x				EA
(Yao et al. 2014)	x			x		Travel time uncertainty and reliability		H
(Cadarso and Marín 2016)	x		x		x	Risk aversion, alternative mode	x	
(Wu and Wang 2016)	x			x				NI
(Cadarso et al. 2017)	x		x		x	Risk aversion, alternative mode	x	
(Gutiérrez-Jarpa et al. 2017)	x		x	x		Predefined topology, alternative mode	x	
(Feng et al. 2018)	x	x		x				EA/SA
(Krylatov and Shirokolobova 2018)			x	x				EA
(Ahmed et al. 2019a)	x	x		x		Set of terminals		H
(Ahmed et al. 2019b)	x	x		x				H
(Feng et al. 2019)	x			x				EA
(Heyken Soares et al. 2019)	x	x		x		Fixed terminals		EA
(Iliopoulou and Kepaptsoglou 2019)	x		x	x		Location electric charging stations		H
(Islam et al. 2019)	x			x				H
(Suman and Bolia 2019)			x		x	Fixed terminals	x	
(Fan et al. 2020)	x			x				NI
(Liu et al. 2020c)	x	x		x		Location electric charging stations		NI/EA
(Wu et al. 2020b)	x		x	x		Exclusive lanes assignment	x	
(Yang and Jiang 2020)	x	x		x				EA
(Yoon and Chow 2020)			x		x	Stochastic demand		H
(De-Los-Santos et al. 2021)	x			x		Walking alternative, circular lines	x	
(Heyken Soares 2021)	x	x		x		Zone based demand		EA
(Heyken Soares et al. 2021)	x	x	x		x	Zone based demand, Alternative mode		H

[1] ATT = Average passenger travel time; NL = Network length.
[2] SP = Shortest path.
[3] MH = Metaheuristic; EA = Evolutionary algorithm; SA = Simulated annealing; NI = Other nature inspired metaheuristics; H = other heuristic/metaheuristics.

2.1 Objective function

Most studies consider as objective function a combination between the user's and operator's point of view. The user's point is mostly modelled by the total travel time of all users, corresponding to the in-vehicle travel time and a fixed penalization per transfer (usually 5 minutes). Typically, the demand is assigned to specific stops, and frequencies or timetables are not part of the problem. Therefore, access or waiting times are not included in the TNDP. In some cases, an additional penalization is included for unsatisfied demand or if more than two transfers are required (Mumford 2013; John et al. 2014; Heyken Soares 2021). However, in most cases, the whole demand must be satisfied with at most two transfers. A few other representations of the user's point of view include measures on the network coverage (Cadarso and Marín 2016; Cadarso et al. 2017), the directness of the paths available for the users (Suman and Bolia 2019) or the captured demand (Yoon and Chow 2020).

On the contrary, the operator's point of view is mostly modelled by the sum of the length of all the lines, or a function directly related to it. Some studies consider additional cost elements related to infrastructure, such as the cost of assigning segregated lanes (Wu et al. 2020b) or the construction of charging stations for electric buses (Iliopoulou and Kepaptsoglou 2019; Liu et al. 2020c). Instead of just minimizing the costs, a few studies maximize the profits including the revenue, which depends on how much demand is captured by the network under design (Cadarso and Marín 2016; Cadarso et al. 2017). When more than one objective is considered, many studies combine the objectives using a weight function. A few use a bi-objective approach where both objectives are minimized at the same time, generating a set of non-dominated solutions (Miandoabchi et al. 2012; Mumford 2013; John et al. 2014; Yang and Jiang 2020).

2.2 Passenger Assignment

Most studies addressing the TNDP use a shortest path approach to model the PAP. This approach assumes that all passengers take the shortest path available, considering only the in-vehicle travel time and the penalizations for transfers. This makes sense in the basic version of the TNDP, where there is no further information available, such as expected waiting times, access times or level of crowding in the vehicles. However, in studies that consider additional aspects in the TNDP, different passenger assignment models can be applied. For example, in (Yao et al. 2014; Cadarso and Marín 2016; Cadarso et al. 2017) a variant of the TNDP with stochastic travel times is proposed. In that case, although still the shortest path is used, the generalized cost of an alternative path includes a measure of the uncertainty effect, thus affecting the passenger's decisions. In (Yoon and Chow 2020), a multinomial logit-model is used for the route choice. In (Heyken Soares et al. 2021), an agent-based simulation software is used to solve the PAP, so that stochastic arrival processes affect the paths taken by the users. Although in most studies that deal with multimodal networks the different modes are evaluated independently, in (Miandoabchi et al. 2012) the interaction between buses and cars are modelled by considering congestion in the streets, thus affecting also the route choice of the users.

2.3 More realistic variants

Table 1 shows that in recent years there is a growing trend to address variants of the basic TNDP, by including additional elements in the model to make it more realistic. For example, although the basic version of the TNDP does not take into account the characteristics of the vehicles, recent studies consider the operation of electric buses. This assumption may impose stricter constraints related to the lines that they can operate or decisions related to the location of charging stations (Iliopoulou and Kepaptsoglou 2019; Liu et al. 2020c). Moreover, in the basic TNDP it is assumed that lines can start and end at any node, but some studies impose that lines can only use a subset of nodes as terminals, which can be related to locations were buses can turn around or stay on hold (Ahmed et al. 2019b). Additionally, although typically lines are assumed to be bidirectional, following the same route in both directions, the problem can be extended to consider a more flexible line structure (De-Los-Santos et al. 2021). Other variants allow to simplify the problem, and potentially address larger instances. For example, most studies assume that the number of lines is fixed beforehand, and in some cases it is assumed that the terminal nodes of each line are predetermined, significantly reducing the number of possible options (Miandoabchi et al. 2012; Heyken Soares et al. 2019; Suman and Bolia 2019). In a more innovative approach, in a first step the overall topology of the network is defined (between star, triangle or cartwheel shape), so that the lines can only follow a set of predetermined corridors (Gutiérrez-Jarpa et al. 2017).

A crucial assumption usually made in the TNDP is that demand can only be satisfied by the transit network under design and that there are no interactions with other modes of transportation. However, some studies consider the existence of alternative transportation modes, which can be another transit mode, private transportation, or walking. In the simplest versions, it is assumed that passengers will simply prefer the mode that offers the shortest travel time (Cadarso and Marín 2016; Cadarso et al. 2017; De-Los-Santos et al. 2021), while other studies consider more complex modal-choice models, such as binary logit models (Miandoabchi et al. 2012; Heyken Soares et al. 2021). However, in the latter cases, the characteristics of a logit model are not fully exploited, since key elements to calculate how attractive a transit mode is are not properly estimated, such as the expected waiting or access time. In (Heyken Soares et al. 2021) a zone-based demand approach is used, in which demand is assigned to a separate set of nodes instead of directly to the stops. Therefore, it models the option of passengers walking to alternative stops, so in this case there is a better estimation on the access time.

2.4 Solution approaches

Table 1 shows that most studies use metaheuristics. Just a few studies have proposed mathematical formulations and solve these with exact methods. However, this can be done only in very simplified versions of the problem or in very small instances (Cadarso and Marín 2016; Cadarso et al. 2017; Gutiérrez-Jarpa et al. 2017; Suman and Bolia 2019; Wu et al. 2020b; De-Los-Santos et al. 2021). Metaheuristics have shown to be efficient to find good solutions for the TNDP, and their flexibility allows to tackle the different variants of the problem. In general, metaheuristic implementations solve the TNDP by iteratively proposing and improving alternative line plans, using different frameworks and operators to generate those networks. The evaluation of each network involves solving the PAP, so the evaluation step is typically the most time demanding step of the metaheuristics. The most commonly used metaheuristic frameworks to solve the TNDP are different variations of evolutionary algorithms. Many studies propose genetic algorithms with search operators specifically designed for the TNDP (Nayeem et al. 2014, p. 201; Krylatov and Shirokolobova 2018; Feng et al. 2019; Heyken Soares et al. 2019; Heyken Soares 2021). Additionally, bi-objective implementations such as the NSGA-II have been used (Mumford 2013; John et al. 2014), as well as hybrid algorithms, combining evolutionary techniques with simulated annealing (Feng et al. 2018) or swarm optimization techniques (Liu et al. 2020c). Other metaheuristics that have been proposed include bee colony optimization (Nikolić and Teodorović 2013) , tabu search (Yao et al. 2014), swarm optimization (Kechagiopoulos and Beligiannis 2014), wolf pack search (Wu and Wang 2016), beam search (Islam et al. 2019), and hyper heuristics (Ahmed et al. 2019b; Heyken Soares et al. 2021).

2.5 Benchmark instances

Most papers addressing the TNDP use a few benchmark instances to compare the results. The most used benchmark was proposed by Mandl (1979). This instance consists of a network with only 15 nodes (Figure 2) and a symmetric OD matrix with 142 non-zero OD pairs. In most cases, solutions are compared using different numbers of lines for this network, typically ranging between 4 and 12. Despite the small size and the numerous papers using this network, optimal solutions for different instances on this network have only been obtained recently (Vermeir et al. 2021). In 2013, a set of four networks was proposed, that span between 30 and 127 nodes (Figure 3), each of them with a specific number of lines (Mumford 2013). It should be noted that the appropriate number of lines for a TNDP instance depends not only on the infrastructure network and the OD matrix. Indeed, it is also highly sensitive to the optimization objectives considered and other aspects, such as the intended demand coverage or service level of the network under design. Some of these aspects can be considered implicitly during the generation of the infrastructure network and OD matrix, but in other cases these could be explicitly included in the problem definition. Considering these aspects, some studies argue that in some cases the number of bus lines considered in Mumford's network is highly unrealistic (Yang and Jiang 2020).

The use of these five networks allows to compare algorithms designed to solve the basic version of the TNDP. However, it must be noted that there are still some differences in parameters used in different studies, such as the maximum length of the lines (Mumford 2013; Nikolić and Teodorović 2013; Nayeem et al. 2014; Ahmed et al. 2019b; Islam et al. 2019).

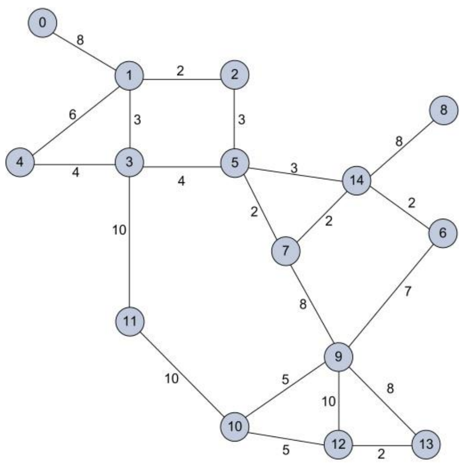

Figure 2: Mandl's network (Mandl 1979).

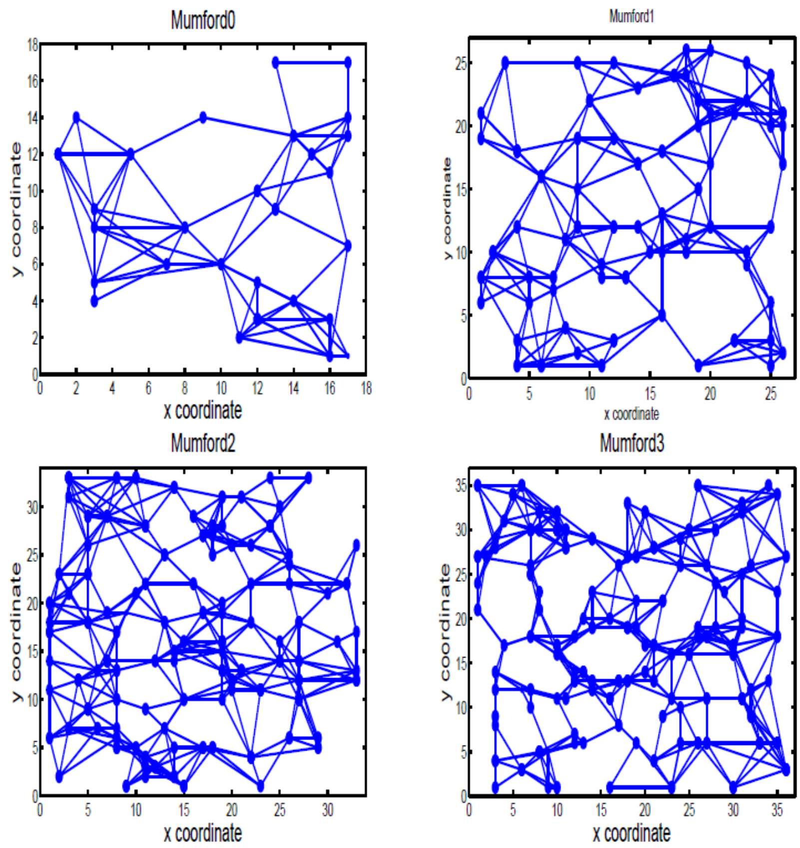

Figure 3: Mumford's networks (Mumford 2013).

2.6 Conclusion

The TNDP is a well-known problem that has been extensively studied in literature. Multiple variants of the problem have been addressed, which in many cases complicate the comparison between studies. Many of those variants aim to extend the problem to make it more realistic, although in general the TNDP is still a very simplified version of the real-life network design problem. However, despite the simplifications and assumptions done in theory, the TNDP is a very difficult problem which is mostly addressed with heuristic and metaheuristic methods. The simplifications made in theory complicate the practical application of the proposed solution methods. In particular, by not considering the service frequencies of the lines, it is hard to estimate the expected operator's costs and the real behaviour of the passengers within the proposed transit network.

3 Transit Network Design and Frequency Setting Problem

In the last decade, it has been more common to address the TNDFSP, thus including the frequency setting, rather than the network design alone. The assignment of frequencies is fundamental to properly evaluate a candidate transit network. Indeed, the frequencies have a big impact on the two most common objectives of the TNDFSP: the user's and operator's costs. The service frequencies determine the expected waiting times, having a great influence on the passenger's satisfaction levels. Moreover, the frequencies affect the route choice of the users.

It is possible that passengers take routes with a longer in-vehicle travel time if the frequencies are higher, resulting in lower waiting times. The frequencies also have a great impact in the operator's costs, since they determine the number of buses that will be required to operate a line. The fleet size is a better estimation of the operator's cost compared to the length of the network, since it reflects cost associated to the number of vehicles and drivers. It is important to note that the impact of the frequencies from the user's and operator's point of view are in conflict. Higher frequencies reduce the waiting time, therefore reducing the user's cost, but at the same time they increase the operator's costs. Therefore, it is relevant and interesting to address the TNDFSP as an integrated problem.

The studies addressing the TNDFSP can be classified in two main approaches: continuum approximation models and discrete models. The former one simplifies the problem by assuming that an idealized network can be characterized by a few continuous variables, such as the spacing between lines, distance between stops, and the frequencies. This allows to easily solve the problem to optimality, but determining the validity of such results in practice is not straightforward. On the other hand, discrete models make use of discrete variables, in particular the decision if specific stops are included in specific lines. This allows to represent complex networks in heterogenous scenarios, but with higher computational costs. The latter approach has been more popular in literature.

3.1 Continuum approximation models

A summary of recent papers using continuum approximation models to solve the TNDFSP is presented in Table 2, indicating in each case the objectives (user's cost, operator's cost or other) and additional elements that are added to the problem. The concept was introduced by Holroyd (1967), which modelled a transit network with lines forming a square grid over a large area with uniformly distributed demand. In this case, the decision variables are the spacing between lines and the service frequencies, assuming that the stops are located at the intersections and that all the lines have the same frequency. Under this approach, the average travel time and the operator's costs can be derived with a continuous function, and therefore an analytical solution can be obtained in order to minimize those objectives. This type of model has been extended to consider different network topologies, such as circular networks (Chen et al. 2015; Medina-Tapia et al. 2020) or networks where the lines form a grid in the centre but branch as they move away from the centre (Daganzo 2010; Estrada et al. 2011). Additionally, hybrid networks have been studied, which consist of overlapping express and local services (Fan et al. 2018; Chen et al. 2018; Li et al. 2020). Moreover, other demand patterns have been considered, such as differentiating between the downtown and the periphery of a city (Luo and Nie 2020). Additionally, in (Wu et al. 2020a; Li et al. 2020) decisions related to a shared-bikes network are incorporated, which are used to feed the transit system. Most studies consider the minimization of the travel time and the operator's cost, although alternative objectives have also been considered, such as the minimization of polluting emissions (Griswold et al. 2013; Amirgholy et al. 2017). Despite the multiple extensions made to the continuum approximation models, they still represent heavily simplified versions of the TNDFSP, that assumed idealized geometrical networks and demand patterns.

Table 2: Papers about the TNDFSP using continuum approximation models.

Authors	Objective[1]			Additional elements
	UC	OC	Other	
(Daganzo 2010)	x	x		Grid network with branches
(Estrada et al. 2011)	x	x		Grid network with branches
(Griswold et al. 2013)	x		x	Emissions minimization
(Badia et al. 2014)	x	x		Radial network
(Chen et al. 2015)	x	x	x	Radial network and variable headways
(Amirgholy et al. 2017)	x	x	x	Multimodal
(Chen et al. 2018)	x	x		Local networks
(Fan et al. 2018)	x	x		Local and express lines
(Li et al. 2020)	x	x		Local and express lines, fed by shared bikes
(Luo and Nie 2020)	x	x		Spatial heterogeneity
(Wu et al. 2020a)	x	x		Fed by shared bikes
(Nocera et al. 2020)	x	x		
(Medina-Tapia et al. 2020)	x	x		Radial network

[1] UC = Users' cost; OC = Operator´s cost.

3.2 Discrete models

A summary of recent papers using discrete models to solve the TNDFSP is presented in Table 3, indicating in each case the objectives (user's cost, operator's cost or other), the passenger assignment technique (frequency share, system optimal, shortest path or other), additional elements that are added to the problem, and the solution approach (exact method or metaheuristic). These models represent an extension of the studies about the TNDP discussed in Section 2, therefore sharing their characteristics and complexity.

Table 3: Papers about the TNDFSP using discrete models.

Authors	Objective[1]			Passenger assignment[2]				Additional elements	Solution approach[3]	
	UC	OC	Other	FS	SO	SP	Other		Exact	MH
(Barabino 2009)	x	x			x			Zone-based demand		H
(Beltran et al. 2009)			x	x				Multimodal, heterogeneous fleet		EA
(Mauttone and Urquhart 2009)	x	x		x				Discrete frequencies		H
(Wan and Lo 2009)	x	x					x	Multimodal		H
(Liu et al. 2010a)	x	x					x	-		EA/SA
(Wang and Lin 2010)	x	x				x				EA
(Bagloee and Ceder 2011)	x						x	Stop selection		EA/NI
(Blum and Mathew 2011)	x	x		x						H
(Cipriani et al. 2012)	x	x		x				Multimodal		EA
(Yu et al. 2012)			x			x		Crowding, lines hierarchy		NI
(Afandizadeh et al. 2013)	x	x					x	Depot assignment		EA
(Fan et al. 2013)	x	x				x		Zone-based demand, elastic demand		EA
(Yan et al. 2013)		x					x	Stochastic travel times		H
(Nikolić and Teodorović 2014)	x	x		x						NI
(Szeto and Jiang 2014)			x		x			Zone-based demand, terminal nodes, crowding		NI
(Zhang et al. 2014)			x					Multimodal, zone-based demand	x	
(Arbex and da Cunha 2015)	x	x		x						EA
(Cancela et al. 2015)	x	x			x			Discrete frequencies	x	

Table 3 (cont): Papers about the TNDFSP using discrete models.

Authors	Objective[1]			Passenger assignment[2]				Additional elements	Solution approach[3]	
	UC	OC	Other	FS	SO	SP	Other		Exact	MH
(Pternea et al. 2015)			x				x	Zone-based demand, heterogeneous fleet		EA
(Zhao et al. 2015)	x			x						EA
(Canca et al. 2017)			x			x		Multimodal		H
(Goerigk and Schmidt 2017)	x	x				x		Discrete frequencies		EA
(Buba and Lee 2018)	x			x						EA
(Owais and Osman 2018)	x	x		x				Crowding		EA
(Camporeale et al. 2019)	x	x			x			Equity constraint		EA
(Duran et al. 2019)	x		x	x				CO_2 emissions, multimodal		EA
(Iliopoulou et al. 2019b)	x			x				Location electric charging stations		NI
(Jha et al. 2019)	x	x		x				No bus capacity		EA/ NI
(Kim et al. 2019)			x	x				Multimodal		H
(Liang et al. 2019)	x	x			x			Multimodal, stochastic demand and time	x	
(Mahdavi Moghaddam et al. 2019)	x			x						EA
(Nayeem et al. 2019)			x	x						EA
(Nnene et al. 2019)	x	x					x			EA
(Almasi et al. 2020)	x	x		x				Multimodal		EA
(Capali and Ceylan 2020)	x	x		x						NI
(Chai and Liang 2020)	x	x		x				Multimodal, crowding		EA
(Duran-Micco et al. 2020)	x		x	x				CO_2 emissions, heterogenous fleet		EA
(Liang et al. 2020)	x	x		x						EA
(Liu et al. 2020b)		x		x				Location electric charging stations		NI
(Liu et al. 2020d)	x	x		x				Location electric charging stations		NI
(Manser et al. 2020)		x					x	Multimodal, heterogeneous fleet		EA
(Sachan and Mathew 2020)	x	x		x				Multimodal		EA
(Oliker and Bekhor 2020a)	x			x				Zone-based demand		H
(Ranjbari et al. 2020)	x	x				x			x	
(Sadeghi et al. 2020)			x	x				Heterogenous fleet		NI
(Wang et al. 2020)	x	x					x	Lines hierarchy		H
(Bourbonnais et al. 2021)	x	x				x		Zone-based demand, terminal nodes		EA
(Zhou et al. 2021)	x	x			x				x	

[1] *UC = Users' cost; OC = Operator´s cost.*
[2] *FS = Frequency share; SO = System optimal; SP = Shortest path.*
[3] *MH = Metaheuristics; EA = Evolutionary algorithm; SA = Simulated annealing; NI = Other nature-inspired metaheuristics; H = other heuristic/metaheuristics.*

3.2.1 Objective function

As in the TNDP, the most common objectives in the TNDFSP correspond to the user's and operator's point of view. In this case, the user's point of view is represented by the travel time, including in-vehicle time, waiting time and penalizations for transfers. In some papers, the components of the travel time are weighted, to represent that passengers perceive the waiting times to last longer compared to the in-vehicle travel times (Arbex and da Cunha 2015; Jha et al. 2019; Liu et al. 2020d; Oliker and Bekhor 2020a). The operator's point of view is usually represented just by the fleet size. A few studies incorporate decisions regarding the construction of infrastructure, such as charging stations, so they incorporate those elements in the operator cost (Iliopoulou et al. 2019b; Liu et al. 2020d, b). Bas in the TNDP, both points of view are relevant for the design of a transit network, but they are often in conflict. Some studies consider one point of view in the objective and define one or more constraints for the other point of view. In earlier studies it was common to minimize a simple proxy of the operator's costs as the main objective under the only constraint of serving all demand (Kepaptsoglou and Karlaftis 2009). However, now usually the user's point of view is taken as the most important one, so the operator's costs are subject to a constraint on the number of buses (Szeto and Jiang 2014; Zhao et al. 2015; Buba and Lee 2018). Many studies actually consider both objectives in the optimization. In general, both objectives are weighted in one function that represents the social welfare looking for a compromise between them. The social welfare may include additional elements such as the social cost of emissions or unsatisfied demand (Beltran et al. 2009; Zhang et al. 2014; Pternea et al. 2015; Sadeghi et al. 2020). Instead of weighting the objectives into one function, a few papers consider bi-objective techniques which generate a set of non-dominated solutions to evaluate the trade-off between both objectives (Mauttone and Urquhart 2009; Blum and Mathew 2011; Nikolić and Teodorović 2014; Arbex and da Cunha 2015; Goerigk and Schmidt 2017; Nayeem et al. 2019; Chai and Liang 2020; Duran-Micco et al. 2020). This seems to be the best option in order to make a correct decision considering both points of view, but in general it is much more difficult to solve. An alternative objective is the explicit minimization of the CO_2 emissions, together with the passenger travel time, considering the operator's cost as a constraint (Duran et al. 2019; Duran-Micco et al. 2020). Other objectives that have been used include equity indicators (Kim et al. 2019) and the demand density captured by the transit system (Yu et al. 2012; Fan et al. 2013). Apart of the aforementioned objectives, other metrics have been proposed to be used as objective, such as the average number of transfers, demand satisfaction, seating capacity and empty seats, but these have not been implemented in other studies (Ul Abedin Zain et al. 2018).

3.2.2 Passenger assignment

The PAP associated to the TNDFSP is usually more complex compared to the TNDP. In the PAP it is assumed that individual passengers try to minimize a cost function that can take different forms. As discussed in Section 2.2, the shortest path approach (SP in Table 3) assumes that all passengers take the shortest path regarding the in-vehicle travel time, typically adding also penalties for transfers. However, when frequencies are considered, the division of passenger flows among parallel paths becomes relevant, which is tackled by the frequency-share rules. In this section, and considering the methods commonly used in the TNDFPS, we define the frequency-share method (FS in Table 3) including the assumption that all passengers can take the first bus to arrive to the stop. However, when capacity issues are included in the model, additional assumptions should be made. An important distinction is made between user equilibrium and system optimal approaches. In user equilibrium, it is assumed that passengers act selfishly, trying to minimize their own time or cost. On the contrary, in the system optimal approach (SO in Table 3) it is assumed that the overall cost of all passengers is minimized. Further discussion on the different approaches to model the PAP is given in Section 5.

As shown in Table 3, within the TNDFSP the most common approach to model the PAP has been the frequency-share method, in the form proposed in (Baaj and Mahmassani 1990), which assumes that the route choice is influenced by the frequencies of the lines but disregards the possible effects of crowding. This method is discussed in more detail in Section 5. Some variations of the frequency-share method have been proposed as well. For example, it is combined with an incremental algorithm, in which the demand is divided in portions and assigned iteratively, so that crowding effects might be considered (Owais and Osman 2018; Chai and Liang 2020). Also, the frequency-share method is combined with mode-choice models when dealing with multimodal networks (Beltran et al. 2009; Almasi et al. 2020). Besides the frequency-share method, some studies use alternative methods to solve the PAP. More simple methods include using only the shortest path, without considering the frequencies in the route choice (Wang and Lin 2010; Yu et al. 2012; Canca et al. 2017; Goerigk and Schmidt 2017; Ranjbari et al. 2020). Some studies have used the system optimal approach. The system optimal approach could be represented by complex non-linear models that give a reasonable representation of real life. However, in the context of the TNDFSP, typically linear models are used, which can be solved faster, but provide less realistic representations (Barabino 2009; Szeto and Jiang 2014; Cancela et al. 2015; Camporeale et al. 2019; Liang et al. 2019; Zhou et al. 2021). Other studies find a solution to the PAP by embedding different heuristic methods (Liu et al. 2010a; Yan et al. 2013; Pternea et al. 2015; Wang et al. 2020) or agent-based simulation approaches (Nnene et al. 2019; Manser et al. 2020). Note that even with those modifications, all these methods correspond to very simple versions of the PAP.

3.2.3 More realistic variants

Several extensions have been made to the basic TNDFSP. Some of these are similar to the ones made in the TNDP, such as the consideration of multimodal networks (Wan and Lo 2009; Beltran et al. 2009; Canca et al. 2017; Duran et al. 2019; Kim et al. 2019; Sachan and Mathew 2020; Almasi et al. 2020), stochastic travel times (Yan et al. 2013; Liang et al. 2019) or the location of electric charging stations (Iliopoulou et al. 2019b; Liu et al. 2020d, b). Additional extensions have been proposed related to the frequencies. The frequencies do not only determine the waiting times and the number of buses required, but they also determine the capacity of the lines, so in the TNDFSP the capacity of each bus becomes relevant. It is usually assumed that all buses have the same size, although some studies have addressed the problem with heterogeneous fleet, either related to the capacity of the buses (Manser et al. 2020), to the technology of the vehicles (Beltran et al. 2009; Pternea et al. 2015; Sadeghi et al. 2020), or both (Duran-Micco et al. 2020). In other studies, the capacity of the line is used to consider crowding issues and its impact on passengers route choice (Yu et al. 2012; Szeto and Jiang 2014; Owais and Osman 2018; Chai and Liang 2020).

3.2.4 Solution approaches

The solution approaches used for the TNDFPS are similar to the ones proposed for the TNDP. The most repeated metaheuristic frameworks are evolutionary algorithms, in particular genetic algorithms, although other variants have been proposed as well, such as multi-objective implementations (Duran et al. 2019; Nayeem et al. 2019; Chai and Liang 2020), co-evolutionary algorithms (Manser et al. 2020; Liang et al. 2020) or memetic algorithms (Zhao et al. 2015; Duran-Micco et al. 2020). Other nature inspired metaheuristics include swarm-based algorithms (Iliopoulou et al. 2019b; Liu et al. 2020d, b), ant colony algorithm (Yu et al. 2012), bee colony algorithms (Szeto and Jiang 2014; Nikolić and Teodorović 2014), water drops algorithm (Capali and Ceylan 2020) and cuckoo algorithm (Sadeghi et al. 2020). Additionally, local-search based metaheuristics have been proposed, such as GRASP (Mauttone and Urquhart 2009) and large neighbourhood search (Canca et al. 2017). Moreover, other heuristic procedures have been proposed considering the specific characteristics of the problem (Barabino 2009; Wan and Lo 2009; Blum and Mathew 2011; Yan et al. 2013; Kim et al. 2019; Oliker and Bekhor 2020a). Unfortunately, since most studies use different constraints, objectives, or instances, it is not possible to make a direct comparison of the results obtained. Therefore, it is not possible to assess comparative advantages or disadvantages of the different solution approaches.

Most studies have tested the algorithms for the TNDFSP using Mandl's network. However, as mentioned before, this instance is too small to provide meaningful conclusions. Additionally, the instances proposed in (Mumford 2013) are not appropriate for the TNDFSP without modifications, because the demand is for an entire day and it is not explained if the demand corresponds only to public transport demand or to all modes of transportation. As a result, if these values are used, the resulting frequencies of the bus lines tend to be extremely high. Several studies use alternative instances, in many cases generated using real data. However, unfortunately, the complete data is rarely made publicly available, so these are rarely used by other authors. A deeper discussion on these instances is given in Section 4.

3.2.5 Conclusion

Multiple discrete models for the TNDFSP have been addressed with a variety of solution approaches. The TNDFSP is more realistic than the TNDP, but still there are many gaps between theory and practice that complicate the implementation of optimization tools for solving the network design problem in practice. Indeed, many studies include additional assumptions or constraints to make the problem more realistic, but they usually simplify the problem in other aspects. The next section discusses studies that approach the TNDFSP from a more practice-oriented point of view, although even then the gap remains large.

4 Case studies for the Transit Network Design and Frequency Setting Problem

Despite the extensive literature about the TNDFSP, it is important to note that these optimization techniques are hardly used in the design of real transit networks. In practice, the line planning or network design process is still mostly an iterative manual process, that relies on the experience of the transit planners and their knowledge of the areas under study. There are some optimization techniques that are used during this design process. For example, a simulation software may be used to evaluate the performance of proposed line plans. However, they are not available as true optimization tools telling how to improve a certain network, much less to generate a new line plan from scratch. Although the experience of transit planners is of course a valuable asset, the implementation of appropriate optimization algorithms would allow to speed-up the design process and would lead to better solutions.

The fact that algorithms are not used in practice yet, is a consequence of the many assumptions and simplifications that are done in theory. These simplifications lead to solutions that many times are unreliable, unpractical, or even unfeasible in practice. Moreover, the application of such techniques is limited by the large amounts of data that is required, for example, to estimate the demand or to calibrate the models that represent the passenger's behaviour. However, besides the aforementioned limitations, there are also new tools and scenarios which make the application of these algorithms more attractive than before. For example, current management policies incentivise the system-wide optimization of the transit systems by the operators. Moreover, new information and communication technologies help to smoothen the transition after modifications in a transit network, for both passengers and operators. Therefore, the implementation of changes in a transit network are not as disruptive as they used to be. Finally, advances in data collection and processing, such as GPS data or the use of smartcards, allows a better estimation of key parameters to solve the TNDFSP, such as the travel times or the demand. Considering these elements, there is an increasing interest in studying the potential of applying optimization techniques to solve the TNDFSP in practice, which is reflected by the number of case studies on this topic.

The case studies of the TNDFSP are characterized by the use of real data to generate realistic instances or to validate the results. Moreover, some of these papers compare the networks generated by the algorithm with the actual transit network operating in the respective city. This section discusses in more detail the papers describing a case study related to the TNDFSP and presents the main characteristics of the instances that they propose. These studies are interesting because they allow to estimate the actual potential of applying optimization techniques to solve the TNDFSP in practice. Nevertheless, it should be noted that none of these papers describe an actual implementation in practice of the results. They consist of proposing a solution approach to address a variant of the TNDFSP and testing it on an instance that is generated using real data. Moreover, in many cases the instances are generated only partially using real data. For example, it is common that the infrastructure network is generated based on the real street network, but the demand is randomly generated.

Table 4 displays the main characteristics of the instances that have been used in case studies for the TNDFSP. Since typically these papers do not include details about the proposed instances or solutions, the information publicly available is limited and incomplete. However, this information allows to evaluate the size and complexity of the instances. For a comparison, also the values of the common benchmark instances are given (Mandl and Mumford's networks). Other instances have been used in the TNDP without frequencies, but in this section we focus in the TNDFSP, since this is the more complex problem that is closer to practice. For each instance, this data is summarized: the reference of the paper in which the instance is used for the first time; the name of the city that is represented; the number of nodes that conform the infrastructure network; the number of nodes of the OD matrix; the number of non-zero elements in the OD matrix; and the number of lines in the transit network. The number of nodes is the main indicator of the size of the instances. However, the size of the OD matrix is also a relevant factor that may influence the performance of the algorithm. In most cases, all the nodes in the infrastructure network are also the origin and destination nodes of the OD matrix.

However, in some papers the OD matrix uses only a limited set of nodes, which can be a subset of the nodes of the infrastructure network or a different set of nodes. Therefore, in some papers, the number of nodes that conform the OD matrix is given, in others the number of non-zero OD pairs is given, or there is no information about the demand. The number of lines refers to the representation of the real transit network, which also indicates the complexity of the transit network under design. This value is typically only given in papers that compare the results with the current network.

Table 4: Instances of the TNDFSP used in case studies.

Authors	Represented city	Nodes	Demand-nodes	Non-zero OD-pairs	Lines
(Cipriani et al. 2006)	Rome, Italy	49	21	-	32
(Borndörfer et al. 2007)	Potsdam, Germany	410	-	4685	31
(Enrique Fernández L. et al. 2008)	Santiago, Chile	-	450	-	147
(Barabino 2009)	Sardinia, Italy	855	22	51	20
(Mauttone and Urquhart 2009)	Rivera, Uruguay	84	-	378	-
(Bagloee and Ceder 2011)	Winnipeg, Canada	903	153	5394	106
(Bagloee and Ceder 2011)	Chicago, US	648	-	-	54
(Blum and Mathew 2011)	Delhi, India	1332	-	-	-
(Cipriani et al. 2012)	Rome, Italy	>1300	450	-	214
(Yu et al. 2012)	Dalian, China	1007	-	-	89
(Pternea et al. 2015)	Heraklion, Greece	50	-	-	-
(Camporeale et al. 2019)	Molfetta, Italy	210	28	-	5
(Liang et al. 2019)	Beijing, China	52	-	318	-
(Nnene et al. 2019)	Cape Town, South Africa	472	-	-	46
(Almasi et al. 2020)	Daejeon, South Korea	2675	76	-	70
(Chai and Liang 2020)	Baotou, China	50	-	-	10
(Liu et al. 2020b)	Beijing, China	344	-	-	39
(Ranjbari et al. 2020)	Phoenix, US	51	-	638	-
(Wang et al. 2020)	Zhaoyuan, China	22	-	462	12
(Bourbonnais et al. 2021)	Sherbrooke, Canada	1503	-	6096	46
(Bourbonnais et al. 2021)	Saguenay, Canada	1639	-	8171	63
(Bourbonnais et al. 2021)	Trois-Rivières, Canada	1169	-	9469	22
(Zhou et al. 2021)	Hong Kong, China	44	-	-	12
(Mandl 1979)	-	15	14	172	4-12
(Mumford 2013)	-	30	30	870	12
(Mumford 2013)	-	70	70	4830	15
(Mumford 2013)	-	110	110	11990	56
(Mumford 2013)	-	127	127	16002	60

Table 4 shows the wide range of sizes of the proposed instances, between 22 and 2675 nodes. However, it is important to note that in the instances with infrastructure networks above 1000 nodes, the OD matrix is substantially smaller or there is no information about it. In most cases, the size of the instances is small or medium, due to the complexity of the TNDFSP. Considering the studies that indicate the number of non-zero OD pairs, it is possible to note that in general the proportion of non-zero OD pairs is small (below 25%), in contrast to the commonly used benchmark instances, where almost all the possible OD pairs have some positive demand. This is related to the fact that the estimation of an OD matrix may include an implicit policy criterion regarding demand coverage. For example, by using an OD matrix with a large proportion of non-zero OD pairs and with high demand levels, it is implicitly defined that the transit system should provide a high coverage and will capture a high proportion of the total demand. This shows the relevance of proposing new benchmark instances that provide a better representation of a real-life problem and that can be used by different authors to test their algorithms. The estimation of an OD matrix is a complex problem by itself, but it has a relevant impact on the resulting transit networks and in the performance of the algorithms.

Cipriani et al. (2006) proposed a small instance based on a neighbourhood of Rome, Italy. They addressed an extended version of the TNDFSP that considers a multimodal network, with elastic demand between the transit network and cars. They proposed a set of heuristic procedures to generate a pool of possible lines. Then, the set of lines that conform the transit network is selected using a genetic algorithm. The algorithm is applied to the proposed instance and the solution is compared to the real transit network. The same instance and a similar algorithm are used in (Beltran et al. 2009). The algorithm is extended to incorporate the assignment of a limited set of low-emission buses within the transit network under design. In this case, the objective function includes externalities related to the pollution generated by the buses. In these two cases an extended version of the TNDFSP is addressed. However, the small size of the instance, which has 49 nodes, is not large enough to properly represent a real city.

A few papers have applied linear programming techniques to address case studies of the TNDFSP, although that requires simplifications in the problem. A common assumption in this regard is to ignore the waiting times in the passengers' route-choice modelling, which simplifies the passenger assignment sub-problem. Borndörfer et al. (2007) modelled the problem as a multi-commodity flow model and proposed a column generation approach with heuristics to obtain integer solutions. They used a relatively large network based on the city of Potsdam, Germany, composed by 410 nodes. However, the problem is simplified in the passenger assignment step, since they do not consider the waiting times or transfers. Also in (Liang et al. 2019) a column generation technique is proposed, within a two-step algorithm. First, the column generation technique is used to generate the bus lines and the possible passenger paths. Then, the passenger flows and the frequencies are optimized solving a stochastic linear programming model, which includes uncertainties in the bus travel times. This study addresses a multimodal problem, considering an existing metro line. However, in this case also the passenger assignment model is simplified by ignoring the passenger waiting times. Moreover, they use a small infrastructure network used to represent Beijing, China, with only 52 nodes and 318 OD pairs. Similarly, Ranjbari et al. (2020) use a linear integer model to solve a simplified version of the TNDFSP. In this case, they design an intercity transit system, in which all lines and passenger paths travel between two urban areas. Therefore, the authors consider that all passengers travel without transfers. The method is tested on an instance based on the metropolitan area of Phoenix, US, which has 51 nodes and 638 OD pairs. A MILP was proposed in (Zhou et al. 2021) and was applied to design a transit network in a small instance based on Honk Kong, China. The network in this case has 44 nodes and the transit network is composed of 12 lines.

Enrique Fernández et al. (2008) studied the problem of redesigning the bus network in Santiago, Chile, considering the existing metro network. The transportation model of the city consists of a street network with 2500 nodes and 8000 links, and 450 demand zones. However, it is not mentioned exactly how this transportation network is applied to design the transit network. A heuristic procedure is used to generate a transit network consisting of 147 lines, of which 7 are metro services, 49 are trunk services and 91 are feeder services. This network is compared to the current situation, which has 377 lines. A detailed comparison shows that the proposed network offers lower passenger travel times and a substantially smaller fleet, although it imposes a higher number of transfers to the passengers. In (Zhao and Zeng 2008), a metaheuristic algorithm is applied to optimize the transit networks in Miami, US, consisting of 83 lines. In this case, the transportation network has 4500 stops, 2804 street nodes, 4300 street links and around 120000 OD pairs, although it is not mentioned how this transportation network is adapted to solve the TNDFSP. The comparison of the results with the current network shows a reduction in the expected travel times and the number of transfers, maintaining the same fleet size.

Barabino (2009) generated an instance with 855 nodes based on a town in Sardinia, Italy. Despite the large number of nodes in the infrastructure network, the instance considers only 22 demand zones and 51 OD pairs. He proposed a two-step heuristic to solve the TNDFSP, which first designs the line plan and then sets the frequencies. A comparison with the existing transit network in the area shows that the proposed solutions allow a reduction in the passenger travel times using a smaller transit network. However, in the same paper it is discussed that the limited demand data could affect the comparison.

Mauttone and Urquhart (2009) proposed a new instance with 84 nodes based on the small city of Rivera, Uruguay. The authors use a bi-objective GRASP algorithm that generates an approximated Pareto front considering the passenger travel time and the required fleet size. Given the characteristics of the city, the authors assume that all the demand can be satisfied without transfers, therefore simplifying the problem. This may be a feasible assumption for a small city with low public transport demand, but it is not the case in medium or large cities. The same instance is used in (Owais and Osman 2018), where a genetic algorithms is used to solve the TNDFSP, with the same objectives as in the previous case. However, in this case the problem is extended to consider the effect of crowding in the passenger route-choice modelling. Despite using the same instances, the solutions found in these two studies are not compared, and no comparison is done with the current transit network either.

Bagloee and Ceder (2011) proposed a three step algorithm and tested it using two large instances. Firstly, an instanced based on Winnipeg, Canada, conformed by 903 nodes and 153 demand zones. Additionally, a case study is performed using data of Chicago, US. In this case, the proposed algorithm is used to optimize the train network, considering an infrastructure network with 648 stops. In a first step, the proposed algorithm selects a subset of candidate stops using a clustering procedure. Then, a heuristic procedure is used to generate potential lines. Finally, a genetic algorithm is used to select the lines included in the transit network. In the instances studied, the generated solutions are compared with the current transit networks, obtaining solutions with lower travel time or fleet size.

Some studies have addressed the TNDFSP considering instances with large infrastructure networks, with more than 1000 nodes. However, in these cases, the size of the OD matrix is considerably smaller than the infrastructure network, which reduces the complexity of the problem. In (Blum and Mathew 2011), a representation of Delhi, India, with 1332 nodes is used. However, this paper does not mention the size of the OD matrix. An agent optimization technique that minimizes the passenger travel time and the fleet size is proposed. A network of similar size is used in (Cipriani et al. 2012), where the problem is solved using a network representing the city of Rome, Italy. They proposed a genetic algorithm and compare the results with the real transit network in the city, reporting significant improvements in the waiting times and operating costs. Moreover, the generated solutions have around 100 lines, compared to the 214 lines that conform the real network. In (Yu et al. 2012) an instance with 1007 nodes is used, generated based on the city of Dalian, China, although no information is given about the demand distribution. In this case, an ant colony optimization algorithm is proposed to solve the problem considering a sequential approach, in which, sequentially, the skeleton, main and feeder lines are designed. An even larger instance is used in (Almasi et al. 2020). In this case, the city of Daejeon, South Korea, is represented by a street network with 2675 nodes, although the demand matrix has only 74 nodes. This study also considers an extended version of the TNDFSP, in which the demand is assumed to be elastic. The demand is split between the different transportation modes (bus, train, and cars) with a logit model. A genetic algorithm is proposed, and the results are compared with the real transit network. The results show that the proposed solution captures a larger demand share with a lower operational cost. In (Bourbonnais et al. 2021) three instances are used, which are based on medium-sized cities in Canada. Detailed data from the street networks and demand surveys are used to generate realistic instances. However, the paper does not detail the size of the instances after a pre-processing step, in which relevant nodes and OD pairs are selected. The large size of the street network implies a high degree of complexity. For this reason, a relatively small pool of feasible lines is generated in advance. Then, a genetic algorithm is used to select the lines that will conform the transit network. A comparison with the current transit networks shows that the generated solutions reduce the operational costs by a 15%.

An instance based on Heraklion, Greece, was used in (Pternea et al. 2015). The infrastructure network in this case consists of 50 nodes. The authors propose a genetic algorithm that solves the TNDFSP and incorporates the decision of allocating a limited set of electric buses within the transit network, considering an additional length constraint in the lines that can be operated by electric buses. In (Camporeale et al. 2019) an instance with a larger infrastructure network is considered, with 210 nodes, representing the city of Molfetta, Italy. However, it is a small instance because the OD matrix has only 28 nodes, and the transit network consists of only 5 lines. In this case, a novel objective is considered, which includes comprehensive equity measures regarding the service offered for the different passenger groups. A genetic algorithm is proposed to solve the problem and the results are compared to the current transit network. The results show that the proposed methodology generates transit networks that represent a lower cost, satisfy more demand and achieve a higher level of equity in the service. A larger instance based on the city of Cape Town, South Africa, is used in (Nnene et al. 2019). In this case, the evaluation of the solutions is done using an agent-based simulation procedure, which gives a more precise estimation of the passengers' behaviour. However, they do not indicate the size of the demand input. In (Chai and Liang 2020) a network with 50 nodes is proposed, based on the city of Baotou, China. The problem considers the TNDFSP with elastic demand and a penalization for crowding inside the buses. A bi-objective genetic algorithm is used, which minimizes the total travel time and the fleet size. A set of non-dominated solutions with 10 lines is generated, but in this case the results are not compared to the current network.

In (Liu et al. 2020d, b) additional costs and constraints related to the operation of the electric buses and the allocation of charging stations are considered. An artificial fish swarm algorithm is proposed, which is tested on a large instance with 344 nodes, based on the city of Beijing, China. The results are compared to the current transit network, showing an improvement in the passenger travel time and the operational costs. Wang et al. (2020) use a very small instance based on Zhaoyuan, China, which has only 22 nodes. In this case, the TNDFSP is addressed considering a hierarchical structure of the transit network, distinguishing between skeleton, arterial and feeder lines. A hybrid approach is used, in which the problem of designing the skeleton and feeder lines is reduced to a minimum cost and maximum flow (MCMF) problem, while the arterial lines are constructed using a simulated annealing algorithm. The results show a significant reduction of the user's costs compared to the current network.

As discussed above, there are several papers that apply optimization techniques to solve more realistic instances of slightly different versions of the TNDFSP, and some of them compare the obtained results with the current situation in the respective city. However, many of the papers discussed above represent the cities by small networks that do not fully represent the real topology and demographics of the city. For practical purposes, the infrastructure networks of the instances should be considerable larger. It is relevant to test the algorithms in real size instances, since it has been shown before that metaheuristics that take a few seconds in solving an instance of the TNDP with 15 nodes may require hours to solve an instance with 100 nodes (Mumford 2013; John et al. 2014; Ahmed et al. 2019b; Islam et al. 2019). In addition, several works greatly simplify the models, for example, in the passenger assignment sub-problem. Obviously, these studies have their merits and lead to interesting results and insights in the considered cases, which will be helpful to solve the actual practical problem. However, more research is required to narrow the gap between theory and practice.

5 Passenger Assignment Problem

The passenger assignment problem (PAP), also called transit assignment problem, models the behaviour of passengers within a transit network. The problem consists of determining which lines and paths passengers will use to reach their respective destinations, given a transit network and an OD matrix. As mentioned before, the PAP is a subproblem of the TNDP and the TNDFSP, necessary to evaluate a solution of these problems. The two approaches most used within the TNDP and TNDFSP were presented in the previous sections (the shortest path and the frequency share methods), which correspond to some of the simplest implementations of the PAP. However, the PAP is a very complex problem by itself with extensive literature dedicated to it, and there are many models and solution approaches that tackle more detailed representations of the problem. In fact, some solution approaches may take several hours to solve one instance of the PAP, while the iterative algorithms implemented to solve the TNDFSP usually need to solve the PAP hundreds or thousands of times in a few hours at most. In this regard, it is important to note that the most detailed models are typically used in tactical or operational planning, while the TNDP and TNDFSP correspond to strategic problems. Therefore, high amounts of detail in the PAP are not required or realistic in the context of the TNDP or TNDFSP.

However, the different versions of the PAP provide insights on how the models within strategic problems can be enhanced while maintaining the required simplicity.

There are many possible classifications on the different versions of the PAP. Firstly, PAP studies can be categorized between static and dynamic transit assignment (Liu et al. 2010b). They differ in whether they consider time dimensions in the passengers' choices. In dynamic transit assignment, it is usually considered that some parameters vary over time during the planning horizon, such as the demand, the travel times, or the line schedules. The level of detail on the variability ranges from broad peak/off-peak intervals to real time dynamics. Dynamic transit assignment usually employs schedule-based formulations, where user´s decisions and travel times are a function of the timetable information and the passenger arrival patterns. Some studies have incorporated dynamic transit assignment approaches in the TNDFSP, through the incorporation of agent-based simulation procedures in the evaluation step of a metaheuristic, requiring additional assumptions regarding the timetable of the transit network under design (Nnene et al. 2019; Manser et al. 2020). On the contrary, static transit assignment usually employs frequency-based formulations, where lines are assumed to operate with a constant frequency, without a specific timetable. Obviously, the dynamic transit assignment is more realistic, but it is also much more complex and difficult to solve. Additionally, it requires a large amount of data that in practice is rarely available. As a result of this, static transit assignment is mostly used in the TNDFSP and, therefore, the rest of this section will focus on that type of approach.

Within static transit assignment models, further classifications can be made. A relevant distinction is made between system optimal and user equilibrium models (Farahani et al. 2013). In the PAP, it is assumed that some disutility function is minimized, which normally represents the passenger travel time, but it can be extended to include fares or other cost components. In this regard, it is commonly assumed that passengers decide based on perfect information. In the system optimal passenger assignment, the overall disutility considering all the passengers is minimized, while in the user equilibrium passenger assignment it is assumed that passengers act in a non-cooperative manner, trying to minimize their own disutility even if it increases the overall disutility. The user equilibrium approach follows the Wardrop's (1952) principle, initially proposed for the traffic assignment problem, which states that an assignment represents an equilibrium when no passenger has an incentive to change its route choice. In presence of capacity issues, the user equilibrium approach is a more realistic representation of passengers' behaviour in real life, so it is mostly used in the most detailed versions of the PAP. However, it must be noted that under the many assumptions made in the TNDP and TNDFSP, the solutions generated with both approaches tend to be the same. A particular assumption in this regard is that crowding issues tend to be ignored, even when vehicle capacities are considered. Therefore, in these cases, the disutility of a passenger is not affected by the choices of other passengers. Of course, this is not true in more detailed models of the PAP.

The PAP version that is commonly used in the TNDP corresponds to a static user equilibrium approach that does not consider crowding issues. This corresponds to an all-or-nothing shortest path method, as was described in Section 2. It assumes that passengers take the shortest path considering only the in-vehicle travel time and a fixed penalization for each transfer. As a result, all the passengers of one OD pair take the same route. A common implementation to solve this problem uses an extended network, sometimes called the Change & Go network (Schöbel and

Scholl 2006), trajectory graph (Cancela et al. 2015) or train service network (Liu et al. 2020a). The extended network consists of two type of nodes. The original stops (representing the origin and destinations of the passengers) and an additional node for each line in each stop. The set of edges include edges between the nodes of the same line (representing on-board travels) and edges between nodes of the same station (representing waiting, transfers and alighting at destination). An example of an extended network is displayed in Figure 4. Then, the PAP can be solved by searching the shortest path in the extended network for each OD pair. An issue that is not addressed in the TNDP is when multiple lines are equally attractive for the passengers, denoted as common lines. Since in the TNDP the capacity of the lines is not considered, this issue can be ignored and the passengers can be assigned all to one arbitrary line, but that is not the case in the TNDFSP.

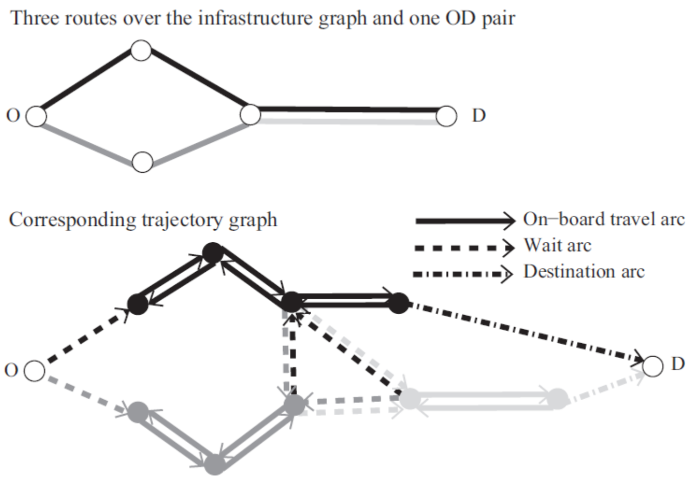

Figure 4: Transit network and extended network (trajectory graph) (Cancela et al. 2015).

In the TNDFSP, even if crowding issues are not considered, it is at least required that the frequency assigned to each line is high enough so the capacity of the line can transport all the assigned passengers. Therefore, when common lines exist, the passenger flows must be distributed between those lines, as it happens in real life. This is addressed with the notions of strategies and hyperpaths (Nguyen and Pallottino 1988; Spiess and Florian 1989), which describe the multiple attractive lines and paths that a passenger can take to reach its destination, possibly at multiple decision points. This method assumes that the passengers select a set of attractive lines for the origin stop and every possible transfer stop, to then take the first bus arriving from the set of attractive lines at any stop. Therefore, the distribution of passengers of an OD pair between those attractive lines is proportional to the relative frequency of each line. This assumes regular headways and uniform passenger arrival rates, which are both common assumptions in the frequency-based passenger assignment (Liu et al. 2010b). These are the fundaments of the frequency-share method, commonly used to solve the PAP in the TNDFSP. In the TNDFSP, this approach is usually combined with the frequency setting step, using an iterative method proposed by Baaj and Mahmassani (1990). This process starts by assigning the flows of passengers according to an initial set of frequencies, to then update the frequencies of every line to the minimum frequency so that the capacity of the line is just enough to transport all the passengers assigned to that line. Then, the passenger flows are updated according to the new frequencies and the frequencies are updated again according to the new flows. This process is repeated until the frequencies converge. Although this method does not ensure convergence, in practice it converges in most cases.

More complex PAP models involve the consideration of multimodal networks, in which users can choose between different transportation modes. The multimodality can be captured by different approaches (Farahani et al. 2013). In the simplest versions there are no interactions between the flows of passengers using different modes. The interactions that are considered in more complex approaches can be congestion in shared streets or the possibility that passengers make transfers between modes along the journey. In the few studies that address multimodal networks in the TNDFSP, the decisions normally refer to only one mode and no interaction between modes is assumed. In these cases, normally in a first step the demand is split between modes and then the PAP of each mode is solved independently. The demand split usually depends on the travel time of each mode and can be performed using an all-or-nothing approach (Canca et al. 2017) or a logit model (Beltran et al. 2009; Duran et al. 2019; Kim et al. 2019; Chai and Liang 2020; Almasi et al. 2020).

An issue that is usually ignored in the TNDFSP is the impact of crowding on the passenger's decision. Crowding has multiple effects on user's experience, such as extended travel times, failing to board at stops, reduced reliability of the service, and the discomfort experienced inside the vehicles. Ignoring these effects may lead to an overestimation of demand in a route or transit mode (Tirachini et al. 2013). Multiple approaches have been proposed to deal with crowding in the PAP, but given their added complexity they have been rarely applied in the TNDFSP (Fu et al. 2012). Crowding is commonly incorporated in the PAP by assuming penalizations that increase together with the passenger flows. Those penalizations can be related to the in-vehicle travel time. In this case, if in a section of a line there is crowding, passengers will perceive a longer travel time on that section (Sun and Szeto 2018). In these cases, the penalization is related to the discomfort generated by crowding. An alternative is to penalize the waiting times, reflecting the failure-to-board effect. This is done using the notion of effective frequencies (de Cea and Fernández 1993), which assumes that in presence of crowding, passengers perceive a frequency that is lower than the nominal frequency of the line. In these two approaches, since the travel times or the effective frequencies become endogenous variables, the PAP becomes more difficult. Moreover, in general, these approaches do not ensure that the capacity constraints are satisfied (Cominetti and Correa 2001). There are approaches that ensure the satisfaction of the capacity constraints, but they require to solve difficult mathematical models (Codina and Rosell 2017) or rely on heuristics that do not generate user equilibrium solutions (Cheung and Shalaby 2017; Oliker and Bekhor 2020b). Within the TNDFSP, crowding has been included using the concept of effective frequencies and implementing incremental algorithms. The demand of all the OD pairs is divided in portions that are assigned iteratively. After each iteration, the expected waiting time at each stop is updated considering the passengers flows. This means that the effective frequencies are updated and that the passenger choices might change in the next iterations (Owais and Osman 2018; Chai and Liang 2020).

Other aspects that can be included in the PAP are the effects of uncertainties in demand and travel times, the preference of passengers to travel seated, or the impact of weather conditions, to mention a few. However, such aspects require additional input data and make the models even more complex, so they have not been included in the TNDFSP. As discussed in this section, from the many models that have been proposed to address the PAP, only the simplest ones have been incorporated within the TNDFSP. Therefore, in order to model and solve more realistic versions of the TNDFSP, it is also important to develop efficient algorithms to solve more realistic versions of the PAP.

6 Conclusions

This paper presents an exhaustive review of the literature related to the Transit Network Design Problem (TNDP) and the Transit Network Design and Frequency Setting Problem (TNDFSP), with a focus on papers from the last decade, and on the many different problem definitions and assumptions. Due to the complexity of the problem and the large size of the real instances in practice, there is still a gap between the problems addressed in theory and the extremely complex problem found in practice. However, despite those difficulties, there have been several factors that have facilitated the research in the last decade. For example, better computer capacity allows to solve quicker complex models and algorithms. Moreover, GPS tools and information systems facilitate the implementation of the changes in a transit network, by diminishing the negative effects that such changes can generate for passengers and operators. Similarly, new data collection and processing techniques, facilitate the generation of more accurate instances. This is the case for smart cards and mobile record technologies, which complement the traditional use of costly household surveys for the estimation of passengers' demand. These aspects have generated an increasing interest in the study of the TNDP and TNDFSP, illustrated by the extensive literature on the topic, particularly in the last years.

Many assumptions and simplifications are made when modelling the TNDP and TNDFSP in theory, such as imposing constraints in the network topology, ignoring the interaction with other modes of transportation, assuming a homogeneous fleet, etc. Still, several studies have addressed extended versions of the TNDP and TNDFSP, including additional assumptions or constraints to make it more realistic in one of these aspects. For this reason, most studies address the problem with different assumptions, so the results and solution approaches cannot be directly compared. This survey paper discusses the main characteristics and differences of the different variants of the problem addressed in different studies. Moreover, different approaches to model and solve the passenger assignment (sub)problem (PAP) are discussed. The PAP is a very complex problem in itself, since it models the behaviour of the passengers within the transit network. Therefore, it is typically a computationally very expensive part within the solution algorithms for the TNDFSP. For this reason, only simple implementations of the PAP are included in the TNDFSP, failing to represent key issues such as the effect of crowding in the passenger's route choice. In order to get a more realistic version of the TNDFSP, efficient algorithms to solve more realistic versions of the PAP are required as well.

This survey paper also analyses different instances that have been used to test the solution approaches proposed for the TNDFSP. Many papers consider only Mandl´s network, which is a very small benchmark instance. There are no larger instances widely adopted in the literature. Instead, many papers use their own instances. Although many of these instances are generated considering real data, they typically consider several simplifications and are relatively small. This shows that there is still an important gap between theory and practice. More research is needed to generate models and algorithms that can better represent the true complexity of the problem and that can solve realistic instances of large size in reasonable computing times.

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