A model order reduction method for the dynamic simulation of bolt joints considering contact nonlinearity

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Abstract

Bolt joints are pervasively used in mechanical systems and can have an important effect on the dynamic behavior of the system and its performance. In order to predict the dynamic behavior of bolted structures the finite element method is commonly employed. However, the contact nonlinearity and large model size can make the simulation within a reasonable computational time challenging. In view of improving the computational efficiency of the finite element analysis for bolted structures in the time domain, in this work, model order reduction is applied to high-fidelity finite element models with contact nonlinearity and to the dynamic contact algorithm. The contact nonlinearity is modelled by the augmented Lagrange multiplier method, which is commonly implemented in commercial software. In order to achieve the model order reduction, the contact-related nonlinear terms are regarded as inputs for the system and an input matrix is introduced to represent the effect of the contact in the model order reduction. Next, the block second order Krylov subspace is calculated for the reduction basis. Since the number of DOFs at the contact surfaces influences the reduction performance, the interface reduction for the contact surfaces based on the joint interface modes is applied considering the contact force equilibrium. For the time domain simulation, the Newmark-Beta method is used for the time integration with the dual loop algorithm for contact convergence. The latter is adapted to include the reduced model into each time step, where the reduced variables are computed and the contact info is updated and reduced. This enjoys the benefit of only having to generate the reduction basis and reduced structural FE matrices once before the contact algorithm. The computational efficiency and accuracy of the proposed model order reduction method are investigated on a single bolt jointed structure.

1 Introduction

Bolt joints play an important role in mechanical systems, since they are commonly used to effectively assemble different components and materials. Since the bolt connection is guaranteed by the normal contact forces and therefore tangential friction forces, the change of contact condition and area at the bolt joint region will significantly influence the dynamic behavior and the performance of the entire system. Thus, predicting the dynamic behavior of bolt joints with a proper model is necessary.

To predict and analyze the dynamic behavior of a mechanical system, the finite element (FE) method is commonly employed due to its high accuracy and strong compatibility with different mechanisms by spatially discretizing analytical equations. For bolt joints, contact nonlinearity is an important mechanism, since the contact condition can change between gapping, sticking and slipping, which in turn varies the stiffness, produces additional energy dissipation, and changes the dynamic properties of the system. Additional interesting research related to the mechanisms happening at the bolt joints can be found in [1-3]. From the view of computational contact mechanics, methods such as the penalty method, the Lagrange multiplier method, and the augmented Lagrange multiplier method [4] are used to introduce frictionless/frictional contact into numerical models, and are commonly used in commercial software. The penalty method only adds the primal displacement variables into the formulation, but the selection of the penalty factor is restricted to avoid ill-conditioning and a tolerance of penetration has to be set. The Lagrange multiplier method works by introducing additional dual force variables, but can fulfill the contact constraints exactly. The augmented Lagrange multiplier method can be regarded as a combination of these two methods, that tries to take the merits of both methods, while a relatively more complex solution algorithm is needed. Besides considering different numerical methods to introduce contact into the bolt joints, there are also several strategies to model the bolt joints in a finite element model, such as no-bolt, spider bolt, coupled bolt, and solid bolt models [5]. As suggested by their names, these modelling strategies for bolt joints are achieved by different simplifications on the geometry, as well as the use and combination of different types of finite elements. Among these, the solid bolt model is the most realistic one and should be the most accurate for most cases. However, since there is no simplification on the geometry, the dimension of the FE model will be the largest compared to the others. Moreover, when a complex industrial application is under analysis, where the structure is complex and multiple bolt joints are used, the corresponding FE model may contain millions of degrees of freedom (DOFs), which may result in a too heavy and even unacceptable computational cost, especially with the additional contact nonlinearity in the model and its complex solution algorithm. Therefore, a more efficient model without loss of link to physics and accuracy is required.

Model order reduction (MOR) is a technique that transforms the high fidelity discretized model into another space that has a smaller dimension without additional simplifications of the underlying physics. It has been widely explored, and a lot of research has been conducted to apply MOR methods specifically on the bolt joint model with contact problem, such as component mode synthesis (CMS) [6], trial vector derivatives [7] and proper orthogonal decomposition [8]. Among existing research, CMS is a frequently-used method, including the Craig-Bampton method[9] and Rubin method[10]. This type of methods is originally used to reduce the substructures from a complete structure and keep the reduced model still feasible for assembly. Taking the Craig-Bampton method as an example, the constraint modes are introduced to maintain the interface part and the model reduction is achieved by the modal truncation of the internal part. Inspired by this idea, for bolt joint models with contact problem, the reduction can be achieved and the interface can be kept to apply the contact constraints [11]. However, the effectiveness of this method is highly dependent on the number of DOFs on the surfaces in contact, since this part is not reduced, and also the application of constraint modes could destroy the sparsity of the FE matrices which may result in more operations in matrix manipulations. Therefore, the interface reduction for the contact surfaces may be necessary although it is a localized nonlinearity. Originally in CMS, there is a solution for this issue, called characteristic constraint mode (CCM) [12], and this method has been employed for the bolt joint model with contact nonlinearity [13]. The main idea is to do a second modal computation on the interface part after the CMS is applied. Then, the interface part can be represented by a subset of modal information with much smaller dimension. To improve the accuracy of this method for the contact problem, an extension of the reduction basis is proposed, where the so-called joint interface mode (JIM) [14] is introduced by considering Newton's third law across the joint interface and concatenated with the reduction basis of CMS. Besides the improvement of CCM, another method to achieve interface reduction is proposed by further dividing the contact interface according to the contact conditions [15]. It is based on that only a small part of the contact surface is sliding, so that treating this part as the interface by introducing an internal penalty variable to compensate for the other two conditions results in a much smaller interface part. This strategy is based on the penalty method for contact constraints, and an adaptive procedure is needed during the computation of frequency domain dynamic responses. There is also another group of interface reduction methods that physically reduces the domain for the computation of contact [16], where two methods are proposed based on the virtual nodes and remeshing of the interface. Besides the types of methods based on CMS, the second order Krylov subspace method (SOK) [17] could also be an option, since it is more convenient to apply to damped systems [18]. To the author's best knowledge, it was first applied to bolt joint models in [19]. However, in that work, the bolt geometry and contact nonlinearity are not exactly considered and replaced by boundary conditions between fully-clamped and simply supported for the DOFs at the contact zone of bolt joints.

In this work, the block second order Krylov subspace method (BSOK) [20] is employed and it is shown that how it is applied to the bolt joint model with contact nonlinearity. The contact nonlinearity is modelled by the augmented Lagrange multiplier method. The interface reduction is also involved. The JIM is used to extend the CCM and combined with the BSOK method. With the reduced model, the time domain dynamics simulation is focused in this paper. It could effectively obtain the dynamic responses of the bolt joint model with the contact nonlinearity. The Newmark-Beta method is employed for the time integration. In each time step, the dual loop contact algorithm [4] is applied, and it is adapted to be compatible with the proposed reduced model. The rest of the paper is organized as follows: In section 2, the formulation of the contact nonlinearity is briefly reviewed, and the corresponding FE model is built up; In section 3, the BSOK is applied on the re-expressed system equation of motion with contact nonlinearity, and the combination of JIM and CCM is integrated into the BSOK to achieve interface reduction of the contact surfaces; In section 4, the dual loop contact algorithm is adapted to the reduction strategy and integrated into the Newmark-Beta method for the time domain dynamic computation; A proof-of-concept single bolt joint model is used to illustrate the performance of the proposed MOR method in section 5; A summary of the present work and remarks on future improvement are concluded in section 6.

2 Bolt modelling

2.1 Contact problem

The contact problem that happens at the bolt joint can be regarded as pairwise contacts between the bolt, washers, nut and connected components. Thus, we can analysis the contact problem at bolt joints as between two fixed deformable bodies. To fulfill the contact constraints, first the non-penetration condition at the contact surface should be guaranteed and is expressed as

$$g_N = (\boldsymbol{x}^2 - \boldsymbol{x}^1)\boldsymbol{n} + x_g \geq 0, \tag{1}$$

where g_N is the gap between two bodies in contact. $\boldsymbol{x}$ represents the coordinates of the deformable bodies in contact and the superscripts indicate different bodies, $\boldsymbol{n}$ is the normal direction of this contact pair, and x_g means the gap between the contact bodies at the initial configuration. Since all components of a bolt joint are connected to each other by applying preloads, this last value can be eliminated in the bolt joint model. With this non-penetration condition, the Signorini's condition, which is also known as Kuhn–Tucker–Karush condition, is given by

$$g_N F_N = 0, \tag{2}$$

where F_N is the contact force at the contact surface. This condition means that when there is a gap, the contact force is zero, and when the gap is zero, the contact forces must be negative such that it tends to push the two bodies away from each other for non-penetration. At the tangential direction in the contact, there is also a similar condition related to the friction and is expressed as

$$\boldsymbol{g}_T \boldsymbol{R} = 0. \tag{3}$$

In this equation, $\boldsymbol{g}_T$ is the relative movement at the tangential direction, and $\boldsymbol{R}$ represents the difference between the reaction force at the tangential direction and the friction force computed by the friction law, which should be less than or equal to zero. This condition means when contact is sticking, the relative tangential movement is zero, and when contact is slipping, $\boldsymbol{R}$ is equal to zero.

With the contact constraints, the strong form of a contact problem can be expressed as

$$T - \Pi + W_f = \Pi_c, \tag{4}$$

where T is the kinetic energy, Π is the internal elastic energy, and W_f is the work done by external loads, including body, face and concentrated forces. This equation is derived from the inequality stemming from the contact constraints, where Π_c is introduced as the contact contribution to make it an equality. More details of the formulation can be found in [4]. Then, with this equation, the weak form equation can be derived for discretization. There are several methods to formulate this contact contribution. In this work, the augmented Lagrange multiplier method is applied, which can be regarded as the combination of the penalty method and Lagrange multiplier method, and it is expressed as

$$\Pi_c = \int \left(\boldsymbol{\lambda g} + \frac{1}{2} \epsilon \boldsymbol{g g} \right) dA, \tag{5}$$

where $\boldsymbol{\lambda}$ and $\boldsymbol{\epsilon}$ represents the Lagrange multiplier and the penalty factor, respectively. Since the introduction of the penalty method, a small tolerance on the penetration is allowed. For the bolt joint in normal working conditions, the friction can be simplified in this model, although more detailed descriptions of the contact happening at a bolt joint exist in literature such as the micro-slip. Only the infinitesimal 'elastic slip' can occur due to the penalty factor at the tangential direction, while the plastic slip after the friction force reaches the maximum is ignored. However it will later be illustrated that the proposed MOR method is compatible with different contact models.

2.2 Finite element model

The modelling strategy used for the bolt joint is the solid bolt model, which means that all the components are modelled with 3D solid finite elements and the contact occurs in 2D surfaces. Five contact pairs exist between the bolt, two washers, nut and the connected components, as schematically shown in Figure 1. The nut is stiffly connected to the bolt, since the interaction through threads is neglected.

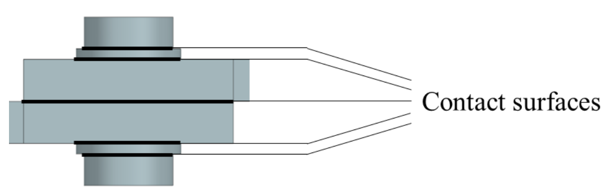

Figure 1: Contact pairs in the solid bolt model

Deriving the weak form from the strong formulation in equation (4) by variation, the equation can be discretized by the FE method. Under the assumption of elasticity and geometric linearity, node-to-node contact or isoparametric discretization of contact are capable to discretize the contact surfaces and describe the contact contribution [4]. In this work, the isoparametric discretization of the contact is employed, which can be directly derived from the surfaces of the existing structural finite elements in contact. The finite element model of the bolt joint structure is expressed as

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} + \boldsymbol{K}_c\boldsymbol{q} = \boldsymbol{F}_e - \boldsymbol{F}_c. \tag{6}$$

In this equation, $\boldsymbol{M}$, $\boldsymbol{K}$ and $\boldsymbol{F}_\mathrm{e}$ represents the mass matrix, structural stiffness matrix and external loads, respectively. The external loads $\boldsymbol{F}_\mathrm{e}$ include the external excitations to the entire structure and the preloads for the bolt joint. $\boldsymbol{C}$ represents the damping matrix, which is introduced as a structural damping. $\boldsymbol{K}_\mathrm{c}$ and $\boldsymbol{F}_\mathrm{c}$ are both the terms introduced by the augmented Lagrange multiplier method, representing contact stiffness matrix and contact forces, respectively. The contact stiffness matrix $\boldsymbol{K}_\mathrm{c}$ is derived from the penalty factor $\boldsymbol{\epsilon}$ in equation (5), which will change due to the active contact set during the vibration and deformation of the structure. The contact force $\boldsymbol{F}_\mathrm{c}$ is derived from the Lagrange multiplier $\boldsymbol{\lambda}$, which is unknown before the computation. Therefore, an iterative algorithm is needed to solve this model. For most bolt joints, since the components in each contact pair are close to each other and the potential contact areas are deterministic, the contact area detection for a general contact problem during the computation is not necessary. Thus, the contact area for judging the contact element activeness is fixed for each contact pair in the bolt joints.

For this FE model, the MOR method can be developed and applied.

3 Model order reduction of the bolt jointed structure

3.1 Model reduction by BSOK

Equation (6) describes the bolt joint model with contact nonlinearity. The finite element matrices in this equation can be in huge dimension, resulting in heavy and even unacceptable computational costs. Therefore, to make the computation more efficient, this model can be transformed to a reduced model with much smaller dimension by applying the MOR method. In this equation, the contact stiffness matrix $\boldsymbol{K}_\mathrm{c}$ will change with the deformation and therefore the contact condition at the contact interfaces. So the MOR method cannot be applied directly to this varying system. This contact stiffness is derived from the contact contribution Π_c in equation (5) and the contact is a localized nonlinearity which means that the contact stiffness is under low rank change [21] compared to the structural FE matrices. Therefore, the contact stiffness can be re-expressed together with the contact force term as

$$\boldsymbol{K}_c\boldsymbol{q} + \boldsymbol{F}_c = \boldsymbol{B}\boldsymbol{u}, \tag{7}$$

where $\boldsymbol{B} \in \boldsymbol{R}^{N\times b}$ is a Boolean matrix to indicate the location of contact. The number N represents the dimension of the entire model, and b represents the number of inputs of the system, namely the number of DOFs at the contact surfaces. Thus, each column of $\boldsymbol{B}$ indicates an individual contribution from the contact applied at the corresponding DOF. The vector $\boldsymbol{u}$ represents the distinct values of contact contribution at all related DOFs, and it is unknown before the computation. It should be noted that the contact forces at each pair of contact DOFs have the same amplitude and the opposite direction. Thus, one column can represent two DOFs of the contact contribution and therefore the column rank of this Boolean matrix can be reduced to half of the dimension of the DOFs at the contact surfaces. Using this new expression into the equation (6), it is rewritten as

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = \boldsymbol{B}_t\boldsymbol{u}_t. \tag{8}$$

This way, the low rank contact related terms are transferred as extra inputs into the system and the remaining system matrices are all constant. In equation (8), the external loads $\boldsymbol{F}_\mathrm{e}$ are also integrated into the combination of the Boolean matrix and extra inputs. There are two main types of forces present in $\boldsymbol{F}_\mathrm{e}$, namely the external excitations and the bolt preloads. For the external excitations, if the values are constant, no matter they are concentrated forces or the surface forces applied on an area, they can always be expressed by using only one column in the input matrix. When the periodic excitations are applied, it is presumed that usually only few distinct frequencies are involved for excitations. Thus, the external excitations can be represented within only a few columns in the Boolean input matrix. For the bolt preloads, no matter whether they are applied in the lumped way or the consistent way, there is always a pair of forces with the same amplitude and opposite directions. Thus, it is convenient to represent the contribution from the bolt preloads by only one column into the input matrix. Therefore, adding the external loads also as inputs has limited influence on the dimension of the input matrix $\boldsymbol{B}_t$. Because of this, it will be explained later that the reduction level is dominated by the dimension related to the contact contribution.

By rewriting the system matrices into the form of equation (8), the second order Krylov subspace method (SOK) can now be applied to directly reduce this model in the second order form. The damping terms related to the velocity can be of an arbitrary formulation, which is an advantage compared to the mode related reduction methods. Moreover, the Krylov subspace method only depends on the location of the inputs, namely the input matrix $\boldsymbol{B}_t$. Thus, the nonlinear effects from the contact contribution in the extra inputs can be avoided in the reduction procedure, and therefore this MOR method is compatible with various contact models. The equation is first transformed into the frequency domain by the Laplace transformation and expressed as

$$s^2\boldsymbol{MX} + s\boldsymbol{CX} + \boldsymbol{KX} = \boldsymbol{B}_t\boldsymbol{u}_t, \tag{9}$$

where s is the Laplace variable and $\boldsymbol{X}$ are the displacements in frequency domain. Due to the existence of the input matrix $\boldsymbol{B}_t$, this model becomes a multi-input system for which the classical SOK method cannot be applied. Therefore, an extension of the SOK method for multi-input systems, namely the block second order Krylov subspace method (BSOK), is employed to reduce the model in equation (9). By applying this method, an orthogonal reduction basis $\boldsymbol{V}_r \in \boldsymbol{R}^{N\times n}$ is generated and its column sequence $[\boldsymbol{r}_0, \boldsymbol{r}_1, \ldots, \boldsymbol{r}_{m-1}]$ spans the corresponding second order Krylov subspace. The number N and n represent the dimension of the full model and reduced model, respectively. It should be noted that $\boldsymbol{r}_{i,\, i=0,1,\ldots,m-1}$ is not a vector but a matrix, and the dimension of each of them should be equal to the number of inputs. This is just because, as previously mentioned, the model is extended to a multi-input system by transferring both the contact contributions and the external loads to extra inputs. It is also noteworthy that among all the inputs, the contact contribution takes the most part. Thus, the dimension of the reduction basis and therefore the dimension of the reduced model are dominated by the dimension related to the contact contribution. The expansion point s_0 is usually used in this method, which leads to the re-expression of the system equation (9) as

$$(s - s_0)^2\boldsymbol{MX} + (s - s_0)\widetilde{\boldsymbol{C}}\boldsymbol{X} + \widetilde{\boldsymbol{K}}\boldsymbol{X} = \boldsymbol{B}_t\boldsymbol{u}_t. \tag{10}$$

Due to the introduction of the expansion point, there are also some extra terms added into the equation for the compensation and integrated into the existing matrices. For the sake of brevity, the detailed formulation is omitted, and interested readers are referred to [17]. The block second order Krylov subspace sequence $[\boldsymbol{r}_0, \boldsymbol{r}_1, \ldots, \boldsymbol{r}_{m-1}]$ is then generated by an iterative computation process expressed as

$$\begin{cases} \boldsymbol{r}_1 = \boldsymbol{P}\boldsymbol{r}_0 \\ \boldsymbol{r}_l = \boldsymbol{P}\boldsymbol{r}_{l-1} + \boldsymbol{Q}\boldsymbol{r}_{l-2}, \ \ l \geq 2 \end{cases}, \tag{11}$$

based on $\boldsymbol{P} = -\widetilde{\boldsymbol{K}}^{-1}\widetilde{\boldsymbol{C}}$, $\boldsymbol{Q} = -\widetilde{\boldsymbol{K}}^{-1}\boldsymbol{M}$ and $\boldsymbol{r}_0 = -\widetilde{\boldsymbol{K}}^{-1}\boldsymbol{B}_t$. To generate an orthogonal reduction basis, the block second order Arnoldi algorithm [20] is employed and the iterative computation (11) is integrated into this algorithm. Applying this reduction basis $\boldsymbol{V}_r$ to the equation (10), expanding the reformulated terms and extra inputs to their original formulation, and then transforming it back to the time domain, the reduced model for the bolt joint structure with contact nonlinearity is expressed as

$$\boldsymbol{M}_r\ddot{\boldsymbol{q}}_r + \boldsymbol{C}_r\dot{\boldsymbol{q}}_r + \boldsymbol{K}_r\boldsymbol{q}_r + \boldsymbol{K}_{cr}\boldsymbol{q}_r = \boldsymbol{F}_{er} - \boldsymbol{F}_{cr}, \tag{12}$$

where

$$\boldsymbol{M}_r = \boldsymbol{V}_r^T\boldsymbol{M}\boldsymbol{V}_r,\ \boldsymbol{C}_r = \boldsymbol{V}_r^T\boldsymbol{C}\boldsymbol{V}_r,\ \boldsymbol{K}_r = \boldsymbol{V}_r^T\boldsymbol{K}\boldsymbol{V}_r,\ \boldsymbol{K}_{cr} = \boldsymbol{V}_r^T\boldsymbol{K}_c\boldsymbol{V}_r,\ F_{er} = \boldsymbol{V}_r^T\boldsymbol{F}_e,\ F_{cr} = \boldsymbol{V}_r^T\boldsymbol{F}_c. \tag{13}$$

3.2 Interface reduction of contact surfaces

From the computation procedure of the reduction basis introduced in the previous section, it is clear the reduction basis generated from the BSOK method is highly dependent on the number of inputs, namely the DOFs involved in the contact surfaces in our case. Thus, the dimension of the reduced model is also highly dependent on this factor. Although the contact nonlinearity is a localized phenomenon that happens at the bolt joint, the interface still has a strong impact on the model reduction in many cases, especially when the interface takes a considerable portion of DOFs. Therefore, the interface reduction of the contact surface is an effective method to further improve the reduction level of the model.

Since the BSOK method is directly dependent on the dimension of the contact surfaces, to combine with this method, the interface reduction method is preferred to be conducted in advance. Then, the BSOK method is applied on the interface reduced model so that further reduction level can be achieved. In this paper, the joint interface mode (JIM) is employed to augment the characteristic constraint mode (CCM) to reduce the contact surfaces, and the augmented reduction basis for the interface part is generated as

$$V_{aug} = [V_{CCM} \quad V_{JIM}]. \tag{14}$$

Both methods can be regarded as doing a second mode computation on the interface part after the condensation of the original model by constraint modes, and then the modal truncation achieves the reduction of the interface part. Different from CCM, the contact force equilibrium is enforced on the contact surfaces when JIM is computed. However, due to the existence of the damping term, the mode computation is not that straightforward. Thus, different from how they are originally calculated, this time, the second order Krylov subspace is used to replace the calculation of the modes of a second order system. Therefore, the extra inputs due to the contact contribution do not need to be considered and the input matrix will be a single vector of ones, while the scalar input is zero.

After being generated for the interface reduction of the contact surfaces, the interface reduction basis is combined with the reduction basis $\boldsymbol{V}_\mathrm{r}$, resulting in

$$\boldsymbol{V}_t = \begin{bmatrix} \boldsymbol{I} & \\ & \boldsymbol{V}_{IR} \end{bmatrix} \boldsymbol{V}_r, \tag{15}$$

where $\boldsymbol{V}_\mathrm{t}$ represents the final reduction basis for the model of the bolt joint with contact nonlinearity. $\boldsymbol{V}_\mathrm{IR}$ is only applied on the interface DOFs, so the internal DOFs remain unchanged by multiplying them with an identity matrix. It should be noted that before the reduction bases are calculated, the DOFs need to be re-ordered due to the division of interface and internal parts. To guarantee the correct mapping of DOFs between different matrices, extra transformation matrices are needed. For the sake of brevity, these matrices are omitted in the equation. By applying the final reduction basis, an effectively reduced model for the bolt joint model considering contact nonlinearity is generated in the same format as in equation (12).

4 Time integration with the reduced model

This section is to introduce how the reduced model is integrated into the time integration scheme to achieve the efficient computation of the structural dynamic response in the time domain.

To have a better stability with a relatively large time step, the implicit time integration scheme is considered. Since this type is usually more computationally complex than the explicit time integration, there is also a large need for a reduced model to improve the computational efficiency. Thus, the reduced model can also introduce added values from this perspective. The Newmark-Beta method is selected due to its unconditional stability. Although different time integration schemes will introduce different numerical damping during the computation, the purpose here is to effectively achieve the time integration with the reduced model and therefore the influence from the different time integration schemes will not be discussed. The general equations for the Newmark-Beta method are given as

$$\boldsymbol{K}_{eff}\boldsymbol{q}_{t+1} = \boldsymbol{F}_{eff}, \tag{16a}$$

$$\dot{\boldsymbol{q}}_{t+1} = \alpha_4(\boldsymbol{q}_{t+1} - \boldsymbol{q}_t) + \alpha_5\dot{\boldsymbol{q}}_t + \alpha_6\ddot{\boldsymbol{q}}_t, \tag{16b}$$

$$\ddot{\boldsymbol{q}}_{t+1} = \alpha_1(\boldsymbol{q}_{t+1} - \boldsymbol{q}_t) - \alpha_2\dot{\boldsymbol{q}}_t - \alpha_3\ddot{\boldsymbol{q}}_t, \tag{16c}$$

where $\alpha_{i,\ i=1\text{-}6}$ are the coefficients based on the two parameters of the Newmark-Beta method and the time step. For the reduced bolt joint model with contact nonlinearity,

$$\boldsymbol{K}_{eff} = \alpha_1\boldsymbol{M}_r + \alpha_4\boldsymbol{C}_r + \boldsymbol{K}_r + \boldsymbol{K}_{cr}, \tag{17a}$$

$$\boldsymbol{F}_{eff} = \boldsymbol{F}_{er,t+1} - \boldsymbol{F}_{cr,t+1} + (\alpha_1\boldsymbol{M}_r + \alpha_4\boldsymbol{C}_r)\boldsymbol{q}_{r,t} + (\alpha_2\boldsymbol{M}_r - \alpha_5\boldsymbol{C}_r)\dot{\boldsymbol{q}}_{r,t} + (\alpha_3\boldsymbol{M}_r - \alpha_6\boldsymbol{C}_r)\ddot{\boldsymbol{q}}_{r,t} \tag{17b}$$

Within each time step of the Newmark-Beta method, $\boldsymbol{K}_{cr}$ and $\boldsymbol{F}_{cr}$ in equation (17b) is unknown. Therefore, an iterative algorithm is needed to solve the model. For the augmented Lagrange multiplier method, the dual loop contact algorithm can be employed, and it is adapted to be compatible with the reduced model, which is shown in Algorithm 1.

Algorithm 1: Dual loop contact algorithm with reduced model in Newmark-Beta method

INPUT structural FE matrices $\boldsymbol{M}$, $\boldsymbol{K}$, $\boldsymbol{C}$, loads $\boldsymbol{F}$, collection of contact element matrices $\{\boldsymbol{K}_{ce}\}$
GENERATE ROM by BSOK and interface reduction, $\boldsymbol{M}_r$ $\boldsymbol{K}_r$ $\boldsymbol{C}_r$ $\boldsymbol{F}_r$ and $\boldsymbol{V}_t$
SET initial active contact set, initial LMs and $\boldsymbol{u}_{r0}$, $\boldsymbol{v}_{r0}$
LOOP over time steps n
    LOOP over iterations i = 1,…, convergence of active contact set (outer loop)
        Assemble contact stiffness matrix $\boldsymbol{K}_c$ according to active contact set and reduce to $\boldsymbol{K}_{cr}$
        LOOP over augmentations j =1,…, convergence of contact tractions (inner loop)
            IF first time step
                Compute $\boldsymbol{a}_{r0}$
            END IF
            Compute reduced displacements $\boldsymbol{q}_{rn}$
            Compute contact forces due to penetration according to $\boldsymbol{V}_r\boldsymbol{q}_{rn}$ and update LMs
        END LOOP
        Update active contact set according to LMs
    END LOOP
END LOOP

In this algorithm, the initialization should be set for the contact stiffness and Lagrange multipliers, as well as the initial displacement, velocity and acceleration. The contact stiffness is dependent on the active set of the contact elements. For the initial step, it can be set as that all the contact elements are active and contribute to the contact stiffness matrix. During the iteration of the outer loop in each time step, the contact stiffness is updated based on the previous outer loop step. For the Lagrange multipliers, it will be computed to find a convergent value for each inner loop, and they will be reset for each outer loop. As tested, resetting them to zero for each outer loop is a safe choice for convergence. The initial values of the displacement and velocity can be both set as zero, and the initial acceleration should be computed from the external loads. The reduced model is used to compute the reduced displacements by the Newmark-Beta method, and the reduced displacements are projected back to the original coordinates for the computation of the contact forces and the judgement of the active set of contact elements.

The computation of this algorithm can be regarded as two parts, namely the offline phase related to the generation of the ROM and the online phase related to the computation by the dual loop contact algorithm and the time integration. It should be noted that the computation of the online phase can be divided into three main parts, namely updating the reduced contact stiffness, computing the ROM for reduced displacements and computing the contact forces. The computational cost for the online phase is the one we most care about, and each of these should be efficient. To update the reduced contact stiffness matrix, the active set of contact elements is used to indicate which contact elements contribute its element matrix to the global contact stiffness matrix. To have a fewer number of operations during this update, the contact stiffness matrix of the previous loop is saved and the different active contact elements of the current loop is changed in the contact stiffness matrix. For the computation of the dynamic responses, this part is where the ROM works. For the computing of the contact forces, it is theoretically computed based on each element. But to have a more efficient computation, the relevant element-wise computation can be vectorized and some auxiliary matrices can be computed once in advance to avoid redundant computation.

5 Numerical example

To illustrate the performance of the proposed MOR method for time domain computation, a proof-of-concept single bolt joint model is used, as shown in Figure 2. One bolt is used to connect the two identical plates with a size of $150\times40\times8\text{mm}^3$,which are made of aluminum. The bolt is a standard M10 bolt made from Steel. Since the nut is modelled as stiffly connected to the bolt, the selection of its size is simplified to be the same as the bolt head. Two identical washers are used and are located between the bolt and the plate, as well as between the nut and the plate. The size of the washers are selected to suit the M10 bolt. The nut and washers are also made from Steel. The material properties of these two materials used are given in table 1. All the components in this structure are modelled by 3D hexahedral elements, which can be seen in Figure 2, resulting in 16065 DOFs in total without consideration of the boundary condition. There are five pairs of contact surfaces between the bolt, the two washers, the nut and the two connected plates. The contact elements are based on the interpolation formulation of the elements involved in the contact and use the same isoparametric shape functions for the 2D contact surfaces. This results in 3888 DOFs involved in contact. The model is fully clamped at the left end. The external excitation is regarded as a concentrated force and applied vertically at the right end, shown as the black arrow in Figure 2. Since the solid bolt model is used, the preloads are applied around the middle surface of the bolt stud in the lumped way, which is uniformly distributed. The penalty factors are constant during the computation. The penalty factors for the normal and tangential directions are selected to be 200 times the Young's modulus value of the softer material in each contact pair.

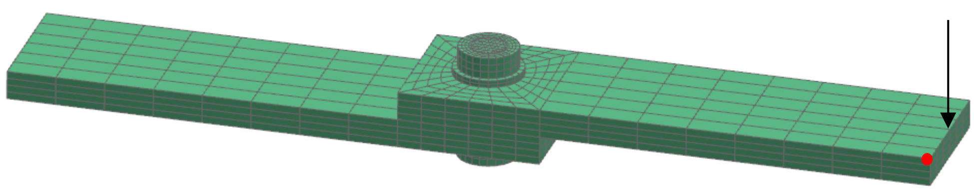

Figure 2: The schematic of the single bolt joint structure and the mesh distribution

Table 1: Used material properties

Material	Young's modulus	Possion ratio	Density
Steel	210e9GPa	0.30	7800kg/m^3
Aluminum	67e9GPa	0.33	2700kg/m^3

First, a constant external excitation force of 10N is applied to the structure. The corresponding time domain dynamic response is calculated using the original full model (FOM) and the reduced model (ROM). The ROM here is generated by the proposed MOR method. The order for the generation of the block second order Krylov subspace is set to one. The meanings of two 'order' should be distinguished. The first one is related to the iterations computed for the bases of the Krylov subspace, and the second indicates the format of the aimed system equation. The dimensions for both the CCM and JIM are chosen to be 100. It should be noted that the collection of CCM and JIM should be orthogonalized to avoid the ill conditioned problem.

Figure 3 shows the vertical displacement at the same position as the force location, calculated by the FOM and the ROM. The results from the ROM are in good agreement with the results from the FOM, only having an average relative error of about 0.35%. The computational cost is reduced from 1559.0s for the FOM to 95.6s for the ROM, which corresponds to about 93.9% time reduction. The details of the online phase computation are given in Table 2. As introduced in Section 4, there are three main parts in the online phase computation. Due to the vectorization of the computation and the pre-computation of the auxiliary matrices, the computation of the contact stiffness update and the reduction and computation of the contact forces are much more efficient, which only take a tiny portion of the total computational time. Therefore, the computation of the displacement costs the most time. Since the MOR method mainly acts on reducing the model dimension, the computation of the displacements is highly accelerated. Because the contact stiffness update and the reduction and computation of the contact forces are both based on the original coordinates, the time spent on these operations is not reduced. Moreover, since the results of these two parts are also in the original coordinates, extra matrix manipulations are needed to project them to the reduced coordinates, which results in more time consumption. But because the dimension is highly reduced, this increase is small compared to the entire computational time of the FOM, and the total computational cost is still highly reduced. It might be noticed that summation of these parts is not exactly equal to the total computational time. This is because the codes between the three main functions is not counted, whose computational costs can be ignored.

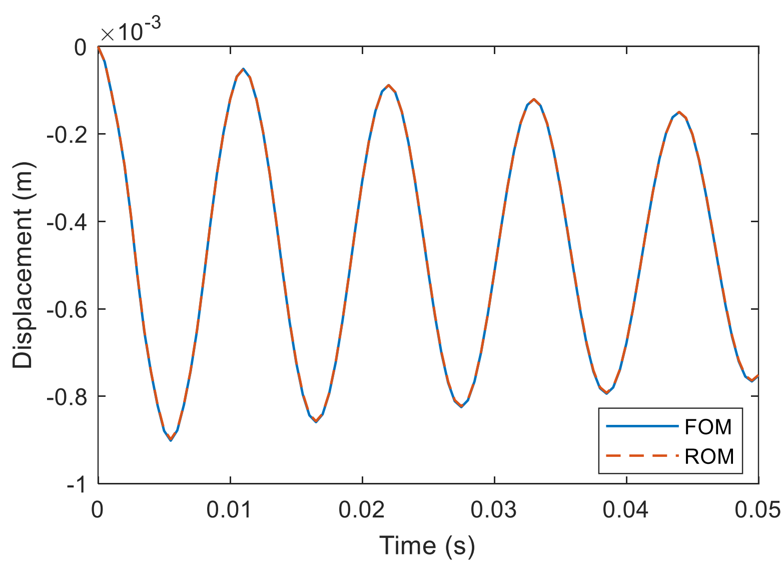

Figure 3: Vertical displacement at the force location, computed by the FOM and the ROM

Table 2: Computation costs of the online phase for both FOM and ROM

	Contact stiffness update and reduction	Computation of displacements	Computation of contact forces	Total
FOM	0.6s	1551.9s	5.4s	1559.0s
ROM	19.8s	22.4s	51.1s	95.6s

The accuracy of a ROM can be highly dependent on the level of dimension reduction, and this analysis can be found in many researches [14, 22]. Moreover, the selection of the components in the reduction basis can also significantly influence the accuracy of the ROM, even for the same obtained reduced dimension. For the reduction of the contact surfaces, CCM and JIM represent different types of characteristic info for the contact interfaces. The selection of the number of each of them should be independent. To illustrate their influence on the accuracy of ROMs with the same reduced dimension, the total dimension is set to be 200, and the number of each type is varied. Figure 4 illustrates the evolution of the accuracy for this change in the number of CCM and JIM. From this figure, we can observe that when the CCM and JIM have the same number, the highest accuracy is achieved. Increasing or decreasing any of them increases the error, however, it is still acceptable in a considerable range. This may be understood as that the free interface and the contact forced interface are equally considered, which may both happen when the structure deforms. Also, CCM and JIM are computed from Krylov subspace method with the same order, rather than the modal methods that may lead to difference behaviors due to selected eigenfrequencies although in the same dimension. It is worth noting that when only CCM are used, the ROM can still capture the properties of the FOM with a relatively low error, but when only JIM are used, the ROM fails to represent the FOM, as also shown in Figure 5. This is might because that JIM enforces the contact force equilibrium at the contact surfaces, where the free interface condition is not considered that could happen at contact surfaces during the structure vibration. Therefore, it makes the structure much stiffer and then cannot approximate the original full model.

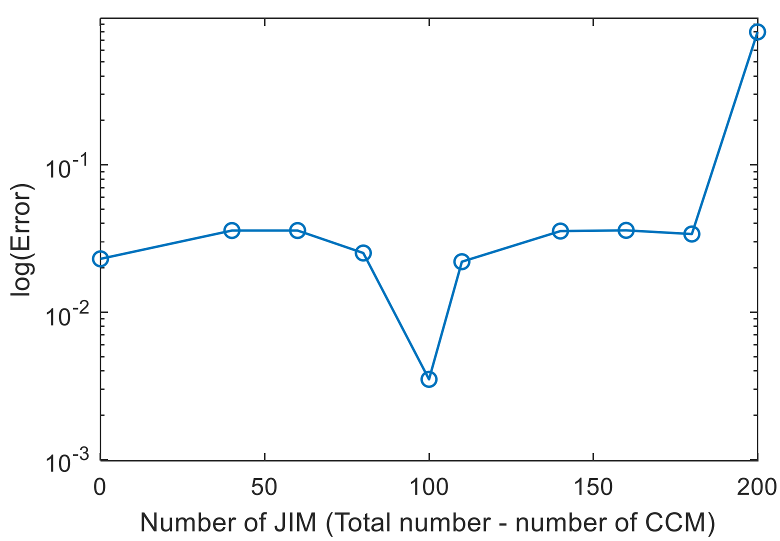

Figure 4: The accuracy of the ROM for the difference selection of CCM and JIM with the constant total number of 200

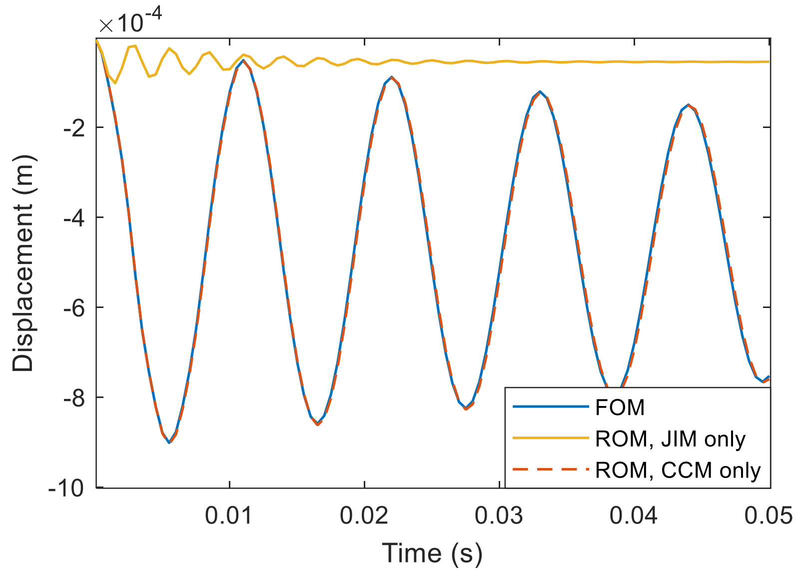

Figure 5: Vertical displacement at the force location computed by the FOM, ROM with only CCM and ROM with only JIM for the interface reduction

Since the final dimension of the ROM depends on both the reduced dimension from the interface reduction and the order for the computation of the block second order Krylov subspace, there is a trade between these two factors on the dimension of the reduced model. For the BSOK method, the higher order it is, the more accuracy should be gained. But the reduced dimension will be inevitably increased especially for the multi-input case, which is usually the integer multiples of the number of inputs. Thus, to set the higher order to compute the Krylov subspace and keep the reduced dimension roughly unchanged, the number of CCM and JIM should be selected less so that can achieve more interface reduction. The relation between this pair is clearer than the previous one, since the error source of the proposed MOR method is mainly from the interface reduction, as shown in Table 3. Therefore, to achieve better accuracy, more CCM and JIM should be selected rather than the higher order. The order one of the block second order Krylov subspace can already generate an accurate ROM.

Table 3: The error source of ROM

	ROM with interface reduction	ROM without interface reduction
Error	0.35%	7.8e-4

Since both the BSOK and the CCM & JIM only depend on the location of the contact DOFs, the generated ROM can also be used for different excitations without any modification. To demonstrate this, the previously constant external excitation force is replaced by a periodic excitation force with the frequency of 50Hz, and the acting position is moved to one end point at the free end, shown as the red point in Figure 2. Actually, since this external excitation takes a very limited part in the inputs of the system compared to the contact contribution, the only one column in the Boolean input matrix corresponding to the external excitation is neglected. The dynamic responses to this excitation computed by the FOM and the ROM are shown in Figure 6. The average relative error between the two curves in the figure is about 4.35%, which is still an acceptable result.

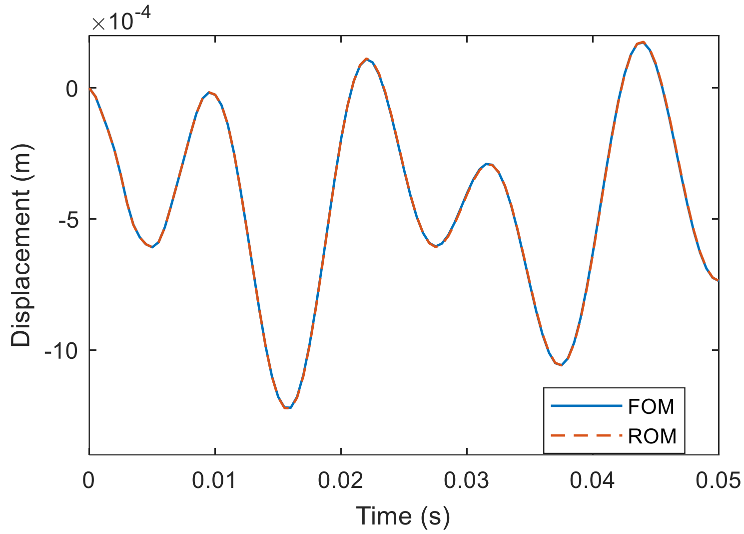

Figure 6: Vibration signal of the vertical displacement under the periodic excitation at the end point of the free end computed by FOM and ROM

6 Conclusions

A model order reduction method for the dynamic simulation of bolt joints was proposed and presented in this paper. The augmented Lagrange multiplier method was used to model the contact contribution, and introduced the unknown contact stiffness and contact forces into the equation of motion. Treating the contact contribution as the low rank change in the system, an artificial input matrix was introduced mainly for the contact contribution. Then, the equation of motion was rewritten into a multi-input system. This way, only locations of the contact were considered and the unknown values of contact forces were avoided for the model order reduction. The BSOK method was employed to reduce this multi-input system by constructing the reduction basis and generating the reduced model. Since the reduction level of the BSOK method depends on the number of inputs, namely the number of DOFs at the contact surfaces, to achieve more model reduction, the interface reduction was conducted. JIM was used to extend the CCM to achieve an accurate interface reduction for the contact surfaces. This way, the contact force equilibrium at the contact surfaces are also enforced such that the free and forced condition were both considered. The interface reduction was conducted before the BSOK was applied to the entire system. Apart from the MOR method to reduce the model dimension, the computation algorithm for the time domain dynamic simulation was modified to be compatible with the proposed reduced model. The dual loop contact algorithm for the convergence computation was adapted to be compatible with the reduced model and integrated into each time step of the Newmark-Beta method for the time integration. The method was applied to a proof-of-concept model. The comparison of the results computed from FOM and ROM illustrated the good performance of the proposed method, achieving high level time reduction and also keeping a relatively high accuracy. The selection of CCM and JIM influences the accuracy of the interface reduction and therefore the accuracy of the ROM. The highest accuracy can be achieved when the same number is used for both, while the ROM failed to approximate the original full model when only JIM was used. Since the MOR creation only depends on the DOF location, the proposed ROM can also be used for different excitations.

In next steps, more detailed friction models will be introduced into the model to test the compatibility and the performance of the proposed method. As mentioned in Section 3.1, due to the special treatment for the contact contribution, it should be feasible to directly apply for different contact models. Moreover, considering the computational time reduction achieved shown in Table 2, the high-level reduction is achieved through the overall computation. However, it is noteworthy that the computation of contact forces costs a considerable part in the calculation of ROM. The nonlinear contact forces are evaluated element-wise and are based on the original coordinates of the full dimension. For the case where the contact area takes a much larger area and much more elements, the computation time could be significantly increased. Therefore, further reducing the computation of these nonlinear contact forces could be important. The hyper reduction specifically for the contact problem will be investigated.

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