KU LEUVEN

DEPARTMENT OF ECONOMICS

Exits and bailouts in a monetary union

Michal KOBIELARZ

FACULTY OF ECONOMICS AND BUSINESS

DISCUSSION PAPER SERIES DPS21.07

MAY 2021

Exits and Bailouts in a Monetary Union*

Michał L. Kobielarz†

This version: May 5, 2021

Abstract

This paper analyzes country bailouts in a monetary union within a framework where sovereign default and exit from the union are two separate decisions. The lack of exit precedent creates uncertainty about the exit cost, which might prevent countries from exiting. The first exit can resolve the uncertainty, which is why the union might bail out a troubled country. As the bailout is meant to prevent an exit from the union, it does not exclude subsequent defaults. The model motivates the occurrence of large fiscal transfers within the Eurozone, and explains why they were insufficient to resolve the debt crisis.

Keywords: monetary union, bailouts, fiscal transfers, exit, sovereign debt.
JEL Classification Numbers: F33, F34, F36, F41.

*For their suggestions and comments, I would like to thank Cristina Arellano, Philippe Bacchetta, Florin Bilbiie, Fernando Broner, Lawrence Christiano, Ferre De Graeve, Alessandro Dovis, Luca Fornaro, Scott Kominers, Enrique Mendoza, Karel Mertens, Wojtek Paczos, Monika Piazzesi, Morten Ravn, Martin Schneider, Ludwig Straub, Harald Uhlig, and seminar participants at the EEA Congress, the Liverpool Workshop in Applied Macro, University of Antwerp, and KU Leuven.

†KU Leuven. E-mail: michal.kobielarz@gmail.com.

1 Introduction

The European debt crisis has challenged the common perception that sovereign debt crises are only a feature of emerging economies. It has also raised the issue of sovereign defaults within a monetary union, and how a default might interact with an exit from the union. At the same time, we have witnessed unprecedented examples of regional country solidarity, where countries within the Eurozone took on the role of lender-of-last-resort to other sovereigns, an international position traditionally occupied by the International Monetary Fund.

It is not only uncommon that the European countries provided financial aid directly (or through EU institutions) to the crisis-hit countries, but the composition and terms of the aid differed substantially from the support typically provided by the IMF (see e.g. Corsetti et al. (2017)). Emergency loans provided by the IMF are actuarially fair, without a fiscal transfer component. Loans extended by the Eurozone countries, in contrast, included substantial implicit transfers as seen in Figure 1, though these varied across beneficiaries.

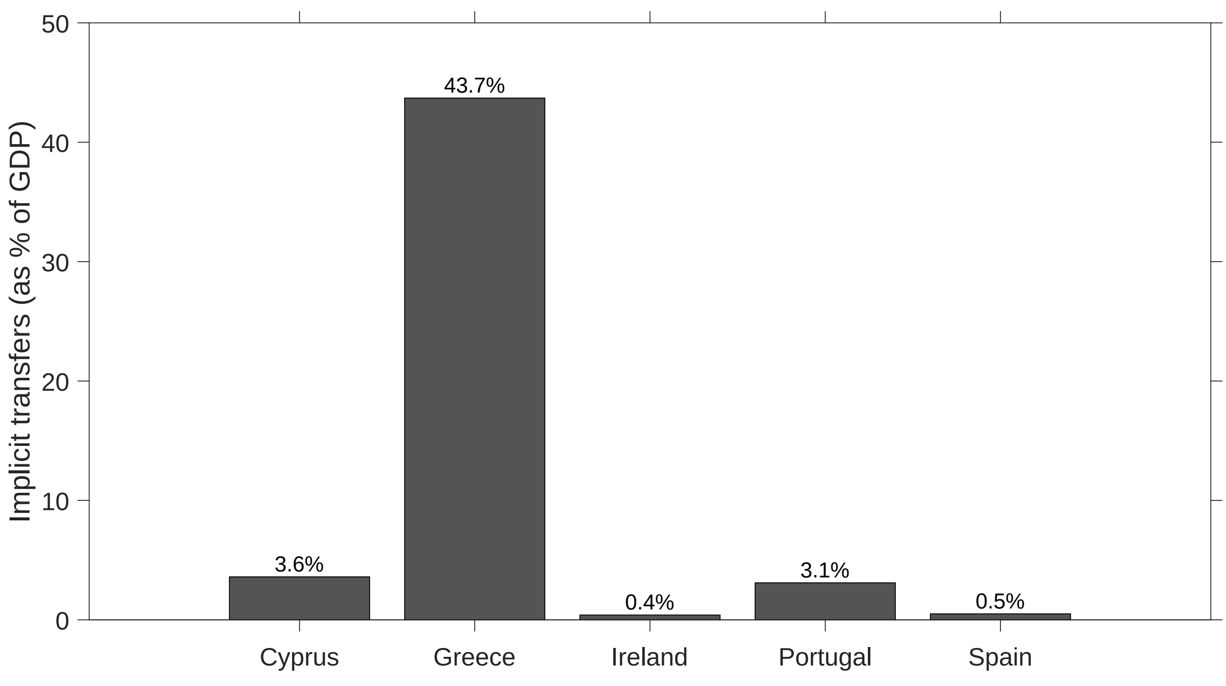


Figure 1: Total discounted value of implicit transfers within the bailout programs (as % of beneficiary's 2010 GDP). Calculations based on Gourinchas et al. (2020).

A common perception in the debate on the Eurozone crisis is that bailouts were necessary to prevent default within a monetary union as default could lead to either exit or substantial spillovers to other countries (see e.g. Ardagna and Caselli (2014), Tirole (2015)). This idea is challenged by the Greek experience, as presented on the timeline in Figure 2. Greece received in total three bailout packages over the course of the crisis. Nevertheless, in March 2012, Greece partially defaulted on bonds held by banks, which is commonly referred to as PSI (private sector involvement). PSI led to spillovers, but did not cause a breakup of the Eurozone. The Greek crisis clearly shows that bailouts do not always prevent default, nor does default automatically imply exit.

Therefore, I set up a model to understand bailouts, and exemplify motivations different from the existing default literature. I explore explicitly the risk of a country exiting the union and a potential breakup of the union. Understanding the nature of the risk and the resulting spillovers makes it also possible to establish how large can a potential bailout be. Furthermore, I allow bailouts to interact with default, to study the dynamic consequences of bailouts for the union members.

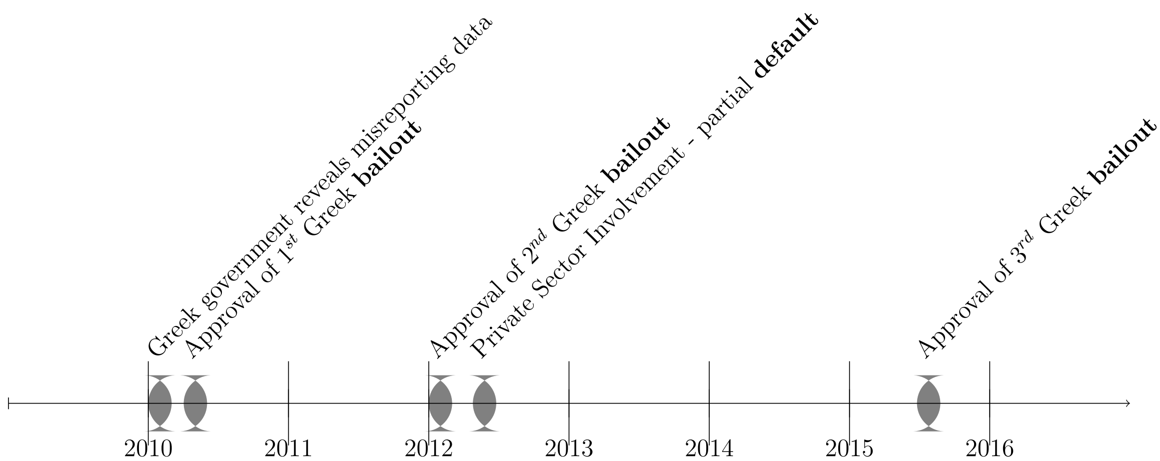


Figure 2: Timeline of the Greek debt crisis. Source: European Stability Mechanism.

I address these issues in a model of a monetary union where membership is a voluntary decision. Each member decides optimally each period whether to remain in the union by comparing the costs and benefits of exiting. On the one hand, exiting the union implies regaining control over monetary policy. On the other hand, exiting may be a disruptive event that brings large economic and political costs. A key feature of the model is the uncertainty related to the cost of exit, which is a consequence of the lack of exit procedures and precedent. The uncertainty will be resolved with the first country exiting. This creates a spillover with substantial long-term consequences. If exiting the union turns out to be accompanied by low cost, the union will unravel over time, as countries hit by bad shocks will choose to exit.

The information revelation associated with the first exit creates an incentive for the other member countries to bail out the potential first exiter. Since the potential exit affects the survival prospects of the whole union, the willingness to save an exiting partner can substantially exceed the marginal value that the union assigns to the country being bailed out. In particular, I show that it might be proportional to the value the other members assign to the existence of the union.

The above findings are consistent with the Eurozone experience. Firstly, out of the countries receiving bailouts, Greece was the only one to receive substantial fiscal transfers (see Fig. 1), a result that goes in line with Greece being the only country that was posing a credible exit threat.[1] Secondly, the transfers received by Greece are above 40% of its 2010 GDP, arguably exceeding the marginal value of Greece to the union, as the country itself is not a substantial financial or trading partner within the EMU. Finally, it also explains why the bailouts were so ill-equipped to deal with issues of long-term debt sustainability; the main aim of the bailouts was preventing an exit, and not necessarily solving the debt crisis.

The model is also extended by embedding the exit decision within the framework of a sovereign default model in the tradition of Eaton and Gersovitz (1981) and Arellano (2008). This allows me to model exit and default as two separate but interrelated decisions. A quantitative analysis of the model captures several salient features of the European bailout experience. Similarly to the timeline presented in figure 2, the first bailout is likely to be followed by further transfers in the following periods, and a substantial fraction of bailout episodes result in a default. Despite the substantial total size of the fiscal transfers, they do not resolve the crisis but merely prevent the bailed out country from exiting.

The possibility of a bailout in case of exit does not increase the average debt level, nor the average unconditional default frequency. These results contradict the concerns raised during the Eurozone Crisis that country bailouts will lead to substantial moral hazard effects. In fact, bailout guarantees slightly reduce default probabilities by eliminating the risk premium associated with a potential exit.

As far as welfare is concerned, the bailouts in the model are optimal from the point of view of the union. The bailed out country in this model, however, does not directly benefit from the bailout, as the fiscal transfer granted is the lowest transfer sufficient to make the

[1] For a quantitative evaluation of the exit risk of different European countries, see Kriwoluzky et al. (2019), De Santis (2015), or Bayer et al. (2018).

country indifferent between exiting and remaining. It benefits indirectly, though, as the bailout guarantee eliminates the exit risk premium on sovereign bonds and, thus reduces borrowing costs. This is especially important in periods of binding nominal rigidities, when additional fiscal space can be used to reduce unemployment. The other member countries benefit from the bailout, as it prevents the first exit, which would bring losses from losing the partner country. More importantly, they benefit from preventing the revelation of the true exit cost, and thus avoiding further exits.

Related literature. My work contributes to three strands of existing literature. The first contribution is to the literature on international bailouts and liquidity assistance in a monetary union, spurred by the policy response to the Eurozone crisis. Tirole (2015) and Gourinchas et al. (2020) explore conditions under which countries within a monetary union may optimally engage in bailouts. Ardagna and Caselli (2014) provide a narrative discussion of the actual Greek bailout arrangements. Dovis and Kirpalani (2020) study the usefulness of fiscal rules in a federation where fiscal transfers between regions are possible. I complement this line of work by providing microfoundations for the externalities that the existing literature takes as given.[2] I also highlight that the sovereign bailouts in the European context were related to a potential exit, and not a default decision. A distinction that has profound consequences but has not been discussed in the bailout literature. I also revisit the moral hazard effects of international bailouts.[3]

The second strand of research that this paper relates to is the large literature studying sovereign default as a strategic decision (Eaton and Gersovitz (1981)). I follow the quantitative tradition of Arellano (2008) and Aguiar and Gopinath (2006). In this literature, my work is the closest to the growing body of studies that use the framework to investigate the dynamics of the Eurozone Crisis (see Aguiar et al. (2015), Farhi and Tirole (2017), Bocola et al. (2019)). The recent work of Na et al. (2018) sheds light on the link between defaults and devaluations by incorporating nominal rigidities in the model and providing a role for exchange rate regimes. They provide an explanation for the high unemployment levels observed in the Eurozone. I extend their work by considering membership in a monetary union and studying not only strategic default, but also strategic exit from the union.

Finally, my research is related to other studies that consider the possibility of exiting

[2] Azzimonti and Quadrini (2018) provide microfoundations for default externalities that could motivate a bailout. Their focus is on studying portfolio choices of international investors, and therefore is not specific to a monetary union.

[3] Other important contributions on country bailouts include i.a. Roch and Uhlig (2018), Fink and Scholl (2016), and Corsetti et al. (2018). Hatchondo et al. (2017) considers the introduction of non-defaultable debt as an alternative policy option.

a monetary union. Fuchs and Lippi (2006) analyze the consequences of the possibility of exit on the optimal monetary policy in a monetary union. They show that the threat of exit generates history-dependent policy rules and time-varying country weights. Kriwoluzky et al. (2019) study default and exit within one framework, but in their model the decisions are exogenous. My model builds on the framework of Eijffinger et al. (2018), who study default and exit as two separate, but related decisions and show that exit rumors might lead to contagion within a monetary union. This paper studies the long-run consequences of exit and introduces international bailouts motivated by an informational friction at the union level.

The remainder of the paper proceeds as follows. Section 2 discusses the monetary union in more detail and analyzes the possibility of bailouts within a union, providing the main results of this paper. Section 3 presents the model of a small open economy in a monetary union. Section 4 is devoted to the quantitative analysis of the model and its implications. Section 5 concludes.

2 Monetary union and bailouts

In this section, I develop a stylized model of a monetary union that rationalizes the occurrence of large fiscal transfers in a monetary union. This model highlights the role of exit from a monetary union as the main incentive to bail out a country and, in particular, the losses associated with the first exit. On top of losing a union member, the first exit reveals information about the cost of exiting the union, which affects the probability of future exits.

2.1 Benefits of a monetary union

Consider a union consisting of $N+1$ symmetric small open economies as in Galí and Monacelli (2008). In every period, each country i derives gains from participating in the monetary union, which depend on the number of fellow countries in the union. Denoting by $M^i(\cdot)$ the discounted flow of benefits country i is enjoying from an infinite membership in a monetary union, it can be expressed as

$$M^i\left(k+1;p^E\right)=\sum_{j\neq i}\sum_{t=0}^{\infty}\beta^t(1-p_j^E)^t m_j^i=\sum_{j\neq i}\frac{m_j^i}{1-\beta\left(1-p_j^E\right)},\tag{1}$$

where $k+1$ is the number of countries currently in the union, m_j^i is the one-period flow utility that country i enjoys due to membership of country j, and the probability p_j^E takes into account the possibility that country j might exit at any period. I allow $k \leq N$, as countries might exit over time, while I assume that $N+1$ is constant over time and represents the initial size of the union.[4]

One argument for a positive m_j is the substantial gain from trade associated with being in a monetary union, as estimated by e.g. Glick and Rose (2016).[5] An indirect way to argue for the existence of the positive m_j is the fact that countries entering the union have given up independent monetary policy in order to join. This has negative welfare consequences, which need to be at least offset by the gains of participation for countries to make a rational entry decision.

Let us assume that the countries are fully symmetric, i.e. the value each country generates for the union is the same and does not differ across the beneficiaries, $m_j^i = m$, and that the unconditional exit probability $p_j^E = p^E$ is equal across countries. Then equation (1) can be simplified to

$$M^i\left(k+1; p^E\right) = \frac{km}{1-\beta\left(1-p^E\right)},$$

and by summing up over all $k+1$ union members, I can calculate the aggregate value of the monetary union, i.e. the sum of the individual values of the union over all its members

$$M\left(k+1; p^E\right) = \sum_{i=1}^{k+1} M^i(k+1; p^E) = \frac{(k+1)km}{1-\beta\left(1-p^E\right)}. \tag{2}$$

2.2 Individual exit decision

The state of the economy of country i is described by $\mathcal{S}_{i,t}$. In this section, I assume that $\mathcal{S}_{i,t}$ is exogenous and follows an IID process, which allows me to obtain analytical results for the stylized model.[6] The state of the monetary union, on the other hand, is denoted by $\mathcal{M}_t$, and it consists of the number of members in the union in a given period $k+1$, and the cost of exiting the union, C.

Let $\tilde{V}_i^U\left(\mathcal{S}_{i,t}; \mathcal{M}_t\right)$ denote the welfare of country i that is a member of the union. Then,

[4] I exclude expansions of the monetary union. Allowing for new membership would not change the analysis of this paper, but would complicate notation.

[5] Their work is especially relevant in the current analysis as they show, using historical data, that the trade losses associated with exit are comparable with the gains from entering the union.

[6] I relax this assumption in the next section, where it includes not only an autoregressive productivity shock, but also public debt, and nominal wages.

$\tilde{V}_i^U(\cdot)$ can be decomposed into

$$\tilde{V}_i^U\left(\mathcal{S}_{i,t};\mathcal{M}_t\right) = V_i^U\left(\mathcal{S}_{i,t}\right) + M^i\left(k+1;p^E\right), \tag{3}$$

where V_i^U denotes the part of the welfare that does not depend on the state of the union.

Similarly, the welfare of country i outside the monetary union is denoted by $V_i^E\left(\mathcal{S}_{i,t}\right)$. I assume that after country i leaves the union, its welfare is independent of the state of the union.

Therefore, the decision of an individual country to exit the monetary union boils down to comparing the difference in expected welfare inside and outside the union with the cost of exiting the union. In particular, country i exits the union whenever

$$V_i^E\left(\mathcal{S}_{i,t}\right) - \tilde{V}_i^U\left(\mathcal{S}_{i,t};\mathcal{M}_t\right) > C, \tag{4}$$

where C is the cost of exiting the union. The cost includes potential disruption resulting from the exit, such as capital flight, the cost of temporary capital controls, trade disruptions, or the cost of setting up a new currency. It might also include political consequences, such as exclusion from other international organizations. I assume that the exit decisions are made sequentially, such that every country takes the decision of the other members as given.[7]

Lemma 2.1 *For every country i the probability of exit p_i^E is a non-increasing function of C.*

The proof of this lemma is provided in the appendix. Lemma 2.1 formalizes the idea that a higher exit cost leads to less exits. Intuitively, for a country to make the exit decision under a higher cost, *ceteris paribus*, it would need to face larger losses from the lack of monetary independence. More precisely, a higher exit cost has not only a direct effect on the decision, but also an indirect one through higher $M^i(\cdot)$, as other countries are also less likely to exit. The two effects amplify each other.

2.3 Optimal bailouts

According to equation (1), whenever a member of a monetary union exits, all the other members suffer a loss from having a smaller union. To facilitate the description of the loss,

[7]This assumption is made to avoid the risk of multiple equilibria. Multiple equilibria could arise if it is optimal for a group of countries to remain within the union if the other members of the group remain, and exit if the other members of the group exit. In reality, it is unlikely to observe the exit decision being taken simultaneously by several members.

let us define $M^{-g}(\cdot)$ as the value of the union aggregated over all its members except for country g. This allows me to calculate the total loss triggered by country g's exit and incurred by the remaining countries

$$M^{-g}\left(k+1;p^{E}\right)-M\left(k;p^{E}\right) = \sum_{i\neq g}\sum_{j\neq i}\frac{m_{j}^{i}}{1-\beta\left(1-p_{j}^{E}\right)}-\sum_{i\neq g}\sum_{j\neq i,g}\frac{m_{j}^{i}}{1-\beta\left(1-p_{j}^{E}\right)}$$
$$= \sum_{i\neq g}\frac{m_{g}^{i}}{1-\beta\left(1-p_{g}^{E}\right)}. \quad (5)$$

The loss is simply a sum of the gains the other countries enjoy from country g participating in the union, discounted properly for the possibility of a future exit. The loss depends only on the economic importance of the country for the union, which is represented by the m_g^i terms, but not on the economic importance of the other countries.[8]

In the special case of fully symmetric m_j^i's, the loss boils down to a fraction $\frac{1}{k}$ of the total value of the union to all the other members prior to the exit

$$M^{-g}\left(k+1;p^{E}\right)-M\left(k;p^{E}\right)=\frac{1}{k}M^{-g}\left(k+1;p^{E}\right).$$

The above result indicates that the other union members can experience a loss from country g exiting the union. Since country g does not take this loss into account when making an exit decision, this decision is not Pareto-optimal. In particular, there is room for a Pareto-optimal transfer, conditional on country g refraining from exit, that corrects for this externality. The conditions for the existence of such a welfare-improving transfer are captured in Proposition 2.2.

Proposition 2.2 *For all states of an economy g, $\mathcal{S}_{g,t}$, and all states of the monetary union, $\mathcal{M}_t$, in which it is optimal for country g to exit the union*

$$V_g^E\left(\mathcal{S}_{g,t}\right)-C\geq\tilde{V}_g^U\left(\mathcal{S}_{g,t};\mathcal{M}_t\right), \quad (6)$$

there exists a utility transfer F_t from the remaining members of the monetary union to country g granted conditionally on country g remaining within the union that can be welfare

[8] This abstracts from the fact that the lower utility from union membership enjoyed by country j, $M^j(k) < M^j(k+1)$, may increase the country's probability of exiting, $p_j^E(C,k+1) \leq p_j^E(C,k)$. A more general version of equation (5) is presented in the appendix.

improving for all union members, if and only if

$$M^{-g}\left(k+1;p^{E}\right)-M\left(k;p^{E}\right)\geq V_{g}^{E}\left(\mathcal{S}_{g,t}\right)-C-\tilde{V}_{g}^{U}\left(\mathcal{S}_{g,t};\mathcal{M}_{t}\right), \tag{7}$$

the union's loss is larger than the exiting country's gain.

The minimal utility transfer that achieves the Pareto-optimal outcome is

$$F_{t}^{min}\equiv V_{g}^{E}\left(\mathcal{S}_{g,t}\right)-C-\tilde{V}_{g}^{U}\left(\mathcal{S}_{g,t};\mathcal{M}_{t}\right). \tag{8}$$

The proof of this and other propositions is provided in the appendix. Inequality (6) guarantees that exit is optimal for country g by requiring the value function under the EXIT regime, corrected by the exit cost C, to be higher than the value function under the UNION regime. Condition (7) requires the loss experienced by the remaining union members to be higher than the net gain obtained by country g. It is the key constraint of the proposition as it is necessary for the optimal decision of country g to differ from the optimal decision of a social planner who would take into account the welfare of all union members. If inequality (7) is violated, then it is optimal for country g to exit, even if it fully internalizes the externality this decision imposes on other countries. Finally, equation (8) specifies the minimal utility transfer that achieves a Pareto-optimal outcome. This transfer makes country g indifferent between exiting and remaining, and assigns all the surplus from country g remaining to the other union members.

2.4 Losses from first exit and optimal bailouts in the presence of uncertainty

The results presented so far implicitly rely on the assumption that an exit by country g does not affect the probability of other countries exiting, p_{j}^{E}. This probability, however, is endogenous and depends crucially on the exit cost, as shown in Lemma 2.1.

Let us consider the case of uncertain exit cost. In particular, I assume that the exit cost can be either low, C^{L}, or high, C^{H}. Let π denote the probability of the low exit cost. All agents in the economy share the same beliefs over the exit cost.

I assume that the uncertainty comes from the lack of exit precedent, and that the first country exiting resolves this uncertainty. In particular, it is useful to consider a monetary union where the expected exit cost is high enough to prevent any country from exiting, and allow for a rare disaster type of event hitting country g, which pushes it towards an exit.[9] Moreover, let us assume that the unconditional exit probability under low cost is country-invariant, i.e. $\forall_j p_j^E(C) = \bar{p}^E$. The loss from the first exit then becomes

$$\begin{aligned} M^{-g}(N+1;0) \;&-\; \mathbb{E}\left[M(N;p^E(C))\right] = \\ &= \sum_{i\neq g}\sum_{j\neq i}\frac{m_j^i}{1-\beta} - \left[(1-\pi)\sum_{i\neq g}\sum_{j\neq i,g}\frac{m_j^i}{1-\beta}\right. \\ &\qquad \left. +\pi\sum_{i\neq g}\sum_{j\neq i,g}\frac{m_j^i}{1-\beta\left(1-p_j^E(C^L)\right)}\right] \\ &= \sum_{i\neq g}\frac{m_g^i}{1-\beta} + \pi\sum_{i\neq g}\sum_{j\neq i,g}\left[\frac{m_j^i}{1-\beta} - \frac{m_j^i}{1-\beta\left(1-\bar{p}^E\right)}\right] \\ &= \sum_{i\neq g}\frac{m_g^i}{1-\beta} + \pi\left[M(N;0) - M(N;\bar{p}^E)\right]. \end{aligned} \tag{9}$$

Comparing the above result with equation (5), it is clear that there is an extra term related to the change of the unconditional exit probability of the remaining countries in case the low exit cost gets revealed. Thus, the loss is a sum of the direct loss from country g exiting and an indirect loss associated with information revelation.[10]

To judge the magnitude of the two loss components, it is useful to consider once more the case of symmetric gains, $m_j^i = m$,

$$\begin{aligned} M^{-g}(N+1;0) \;-\; \mathbb{E}\left[M(N;p^E(C))\right] &= \frac{N^2 m}{1-\beta} - \left[(1-\pi)\frac{N(N-1)m}{1-\beta} + \pi\frac{N(N-1)m}{1-\beta(1-\bar{p}^E)}\right] \\ &= M^{-g}\left(N+1;0\right)\left[\frac{1}{N} + \pi\frac{\beta\bar{p}^E}{\beta\bar{p}^E + (1-\beta)}\frac{N-1}{N}\right]. \end{aligned} \tag{10}$$

The direct loss from country g exiting is represented by the first term in the brackets; losing one of N union partners decreases every country's valuation of the union by a share $1/N$. The second term is connected to the effect of information revelation. With probability π, the revealed cost is low and the union becomes more fragile, which affects the expected gains

[9] The fact that the shock is a rare disaster allows me to reconcile the zero exit probability with an actual exit decision taking place. It allows me also to analyze the exit decision in isolation from other, simultaneous or future, exit decisions.

[10] The analysis presented in equation (9) neglects the individual benefit any union member might experience from learning the true exit cost. This relies on the assumption that all members are myopic about their own exit risk, i.e. all member countries assign a probability zero to themselves facing a negative shock that pushes them to consider exit. This assumption is relaxed in the appendix, where I state the conditions under which union members prefer late resolution of the exit cost uncertainty.

associated with the membership of every single country. The next proposition establishes the result formally.

Proposition 2.3 *For a union consisting of a continuum of small open economies, i.e. $N \to \infty$, where the beliefs about the exit cost are such that*

$$C = \begin{cases} C^L & \textit{with probability } \pi \\ C^H & \textit{with probability } 1-\pi \end{cases} \tag{11}$$

where

$$p^E(C) = \begin{cases} \bar{p}^E & \textit{for } C = C^L \\ 0 & \textit{for } C = \mathbb{E}[C] \equiv \pi C^L + (1-\pi)C^H \\ 0 & \textit{for } C = C^H \end{cases} \tag{12}$$

where $\bar{p}^E > 0$, the loss from a first exit can be represented as

$$\lim_{N\to\infty} \left\{ M^{-g}(N+1;0) - \mathbb{E}\left[M\left(N;p^E(C)\right)\right]\right\} = \pi \frac{\beta \bar{p}^E}{\beta \bar{p}^E + (1-\beta)} M^{-g}(N+1;0). \tag{13}$$

For large values of N, the informational effects can substantially exceed the direct effect of a first exit. As shown in Proposition 2.3, in the extreme case of a union consisting of a continuum of small open economies, the direct effect becomes negligible. Nevertheless, the union still suffers a substantial loss from the first exit, and this loss is associated with information revelation.

To interpret the result even further, in a union of extremely patient members ($\beta \to 1$), the first exit has the potential to eliminate a share π of the value of the union. However, even for more conventional values of β, the informational externality dilutes a substantial part of the value of the union. The size of the effect depends on the probability of exit under low exit cost, and the probability of the exit cost turning out low.

In Proposition 2.2, the maximum bailout available to country g was directly linked to its value to the other countries, i.e. to the sum of m_g^i's. This implies that large fiscal transfers, such as those to Greece shown in Figure 1, would require a high valuation of the receiving country. Such an interpretation might be controversial in the case of the Eurozone, given the limited financial and trade linkages of Greece with the remaining countries.

In the presence of uncertainty, however, the assumption of a high valuation of the beneficiary country is no longer needed. Instead, the bailout decision might be driven by information frictions, as is apparent in equation (10). The following proposition captures the result by describing the optimal bailout decision under uncertainty.

Proposition 2.4 *Consider a monetary union where the exit cost C is uncertain, with the beliefs about the exit cost described by equations (11) and (12), and with no previous history of exits, such that $\mathcal{M}_t = \{N+1, \mathbb{E}[C]\}$. Thus, for all states of an economy g, $\mathcal{S}_t^g$, in which it is optimal for country g to exit the union*

$$V_g^E\left(\mathcal{S}_t^g\right) - \mathbb{E}[C] \geq \tilde{V}_g^U\left(\mathcal{S}_t^g; \mathcal{M}_t\right), \tag{14}$$

there exists a utility transfer F_t from the remaining members of the monetary union to country g granted conditionally on country g, remaining within the union, that can be welfare improving for all union members, if and only if

$$M^{-g}\left(k+1; p^E\left(\mathbb{E}[C]\right)\right) - \mathbb{E}\left[M\left(k; p^E(C)\right)\right] \geq V_g^E\left(\mathcal{S}_t^g\right) - \mathbb{E}[C] - \tilde{V}_g^U\left(\mathcal{S}_t^g; \mathcal{M}_t\right). \tag{15}$$

The minimal utility transfer that achieves a Pareto-optimal outcome is

$$F_t^{min} \equiv V_g^E\left(\mathcal{S}_t^g; \mathcal{M}_t\right) - \mathbb{E}[C] - \tilde{V}_g^U\left(\mathcal{S}_t^g; \mathcal{M}_t\right). \tag{16}$$

In general, Proposition 2.4 is a small modification of Proposition 2.2 with substantial implications. The key difference is that after the first exit, the unconditional exit probability of the remaining countries might change due to the information revelation. How large the expected welfare effect is depends on the beliefs about the exit cost and the relationship between exit cost and exit probabilities. In particular, in the limit case of a union consisting of a continuum of small open economies, in which the assumptions on exit cost beliefs and exit probabilities captured by equations (11) and (12) are satisfied, the welfare effects are of the magnitude of the value of the union to all its members. This means that the expected welfare loss from the first exit is infinitely larger than the size of the exiting economy, implying the availability of unlimited bailout funds, as expressed relative to the size of the exiting small economy.

3 A small open economy in a monetary union

The previous section concentrated on the problem of the monetary union, taking the dynamics of the small open economies as given. This section concentrates on the economy of a small open economy, which is a member of the union. In particular, I provide a relatively rich framework, which lets me distinguish between exiting the monetary union and defaulting on foreign debt.

Let us consider a world with incomplete financial markets, in which a small open economy can issue non-state contingent bonds on international financial markets, as well as make a decision whether to repay its debt or not, in the tradition of Eaton and Gersovitz (1981) and Arellano (2008). The model is extended with nominal frictions in the form of a downward nominal wage rigidity, as introduced by Schmitt-Grohé and Uribe (2016) and Na et al. (2018). Moreover, membership of the monetary union implies a fixed exchange rate. The country chooses each period whether to remain in the union, or exit it in order to regain control of its nominal exchange rate, as first modeled by Eijffinger et al. (2018). A flexible exchange rate policy can alleviate the consequences of the nominal rigidity.

3.1 Domestic economy

The small open economy is populated by a continuum of identical households whose preferences can be described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \tag{17}$$

where c_t is consumption in period t, l_t is labor supply in period t, $u(\cdot)$ is a strictly increasing and strictly concave period utility function, and $\beta \in (0,1)$ is the discount factor. Consumption is a composite index consisting of tradable c^T and non-tradable c^N goods

$$c_t = \left[a \left(c_t^T \right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-a) \left(c_t^N \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{18}$$

where $\varepsilon > 1$ is the elasticity of substitution between the two types of goods, and $a \in (0,1)$ is the weight of tradable goods in total consumption.

Households do not have access to international financial markets. Instead, the government accesses financial markets and provides a transfer to the households T_t. The household's budget constraint takes a simple form, where the households purchase the two types of consumption goods (adjusted by their respective prices, P^T and P^N), and finance those purchases with their endowment of tradable goods $\tilde{y}^T$, the labor income earned in the non-tradable sector $W_t l_t$, government transfer, and firm profits Φ.

$$P_t^T c_t^T + P_t^N c_t^N = P_t^T \tilde{y}_t^T + W_t l_t + T_t + \Phi_t. \tag{19}$$

Households also supply $\bar{l}$ hours of labor inelastically. They might be, however, unable to sell their full labor supply due to a nominal rigidity x_t affecting the non-tradable sector. In particular, the labor employed in equilibrium is equal to

$$\frac{l_t}{\bar{l}} = x_t^{\psi}, \tag{20}$$

where $x \in [\,0, 1\,)$, with $x = 1$ indicating that the nominal rigidity is not binding, and lower values of x represent a stricter nominal rigidity, which comes with a higher slackness in the labor market.

The nominal friction x_t is the result of a downward nominal rigidity in wages as first proposed by Schmitt-Grohé and Uribe (2016). The relationship between the two follows

$$x_t \equiv \frac{W_t^*}{W_t},$$

where W_t^* is the optimal wage set in period t in the absence of nominal rigidities and W_t is the actually set wage. A wedge between the two can arise as the nominal wage is limited from below by $W_t \geq \gamma W_{t-1}$, where γ represents the limit to the one-period downward adjustment of wages.

Non-tradable goods are produced by perfectly competitive firms using labor as their only input

$$y_t^N = F(l_t), \tag{21}$$

where $F(l) = l^{\alpha}$.

Equations (20) and (21) jointly determine the non-tradable output as a function of the nominal friction

$$y_t^N = x_t^{\alpha\psi} \left(\bar{l}\right)^{\alpha}, \tag{22}$$

i.e. the actual non-tradable output is below the potential level of non-tradable output whenever the downward nominal wage rigidity is binding, and the actual output is proportional to the ratio of optimal and actual wages, $x_t^{\alpha\psi}$.

3.2 Government

The government of the small open economy participates in international financial markets on behalf of the households and transfers the available resources to them, or imposes a lump-sum tax on them in case of current account surpluses. In particular, in period t the government issues bonds b_{t+1} at the market price q_t. However, it cannot commit to repay the bonds. Instead the government faces a choice between repaying the bonds and keeping its good financial standing, or defaulting and losing access to the markets.

Let us define $d_t = 1$ if the government defaults in period t, and $d_t = 0$ otherwise. In addition, I will denote the default history of the government, $\mathcal{D}_t$, to be 1 if the government ever defaulted on its debt, and 0 otherwise. Thus, the government budget constraint that determines the transfers to the households, can be written as

$$T_t = (1 - \mathcal{D}_t)\left(q_t b_{t+1} - b_t\right). \tag{23}$$

The above equation shows that whenever the government has no access to financial markets, it cannot make any transfers to the households.

Additionally, default triggers an output loss, $L(y_t^T)$, that persists for every period the government is in default. The tradable good endowment available to the households can be then defined as

$$\tilde{y}_t^T = y_t^T - \mathcal{D}_t L(y_t^T),$$

where y_t^T is the tradable endowment that follows an exogenous $AR(1)$ process.

Next to the repayment/default decision, the government decides each period whether to remain within the monetary union, or exit it. Exit comes with two benefits: an independent exchange rate policy that can alleviate the nominal friction, and a re-denomination of debt into a new domestic currency. The latter, if followed by a devaluation of the new currency, reduces the burden of external debt in the same way a partial default would. The exchange rate policy, as well as the legal and empirical motivation for the re-denomination assumption, are described in detail in Eijffinger et al. (2018). However, with exiting the monetary union, the country also suffers an exit cost, $\tilde{C}$. This cost is a sum of the lost benefits of participating in a monetary union, $M^i(N+1; p^E)$, and the disruption or penalty resulting from an exit, C.

The interaction of the default and exit decisions produces four possible regimes in which the economy could find itself. I refer to the benchmark regime in which the economy starts as the UNION regime. In this regime, the economy has access to international financial markets and remains a member of the monetary union. After defaulting on its external debt, the economy enters the DEFAULT regime (as shown in Figure 3). Similarly, after exiting the monetary union, the country transitions to the EXIT regime. Finally, after both defaulting and exiting the union (either simultaneously, or sequentially), the economy moves

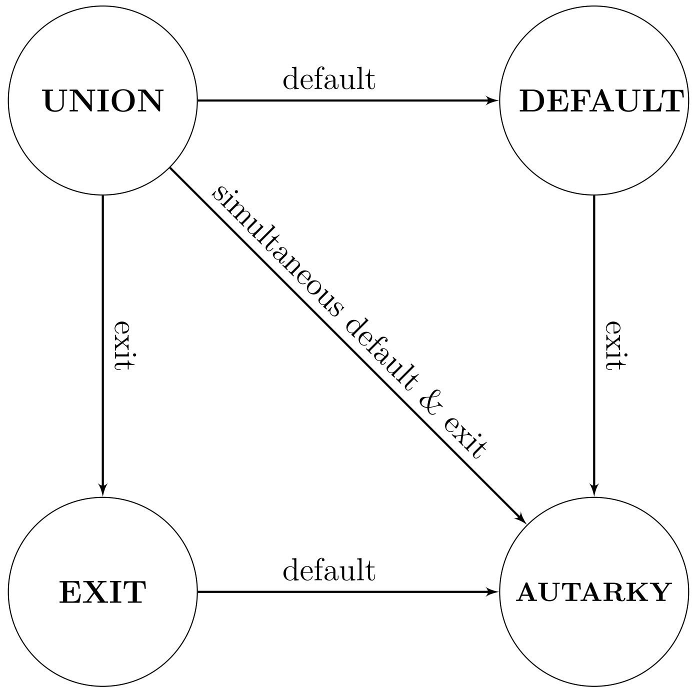


Figure 3: Possible regime switches in the economy

to the AUTARKY regime.[11]

3.3 External lenders

The sovereign bonds are issued on international financial markets. Let us assume that investors on those markets are risk-neutral and have access to a risk-free asset with a constant interest rate r^*. Equilibrium in international financial markets thus implies

$$q_t\left(y_t^T, x_t, b_{t+1}\right) = \frac{\mathbb{E}_t\left[(1-d_{t+1})\left(1-e_{t+1}\frac{\tilde{\epsilon}_t}{\tilde{\epsilon}_{t+1}}\right)\right]}{1+r^*}, \tag{24}$$

where e_t indicates whether exit happens in period t, and $\tilde{\epsilon}_t$ the nominal exchange rate in period t.

The above equation indicates that the higher the expected default probability, the lower the bond price. Similarly, the higher the expected exit probability, or the higher the expected devaluation that follows the exit, the lower the bond price.

[11] A formal definition of the value functions associated with the different regimes can be found in the appendix.

3.4 Default and Exit in equilibrium

Before the discussion of the features of a monetary union, let us establish some general qualitative and quantitative results concerning exit and default. To start off, I consider a special case of the model, to provide some general intuition for the model and the relationship between the state variables and the two binary decisions. I complement the analytical results with numerical examples of default and exit sets, which shows the conditions under which the two decisions can be substitutes.

3.4.1 Analytical results

Let us first concentrate on a special case of the model where the downward nominal wage rigidity is exogenous; it takes on the form $W_t \geq \hat{W}_t$, where $\hat{W}_t$ is independently drawn every period. Moreover, $\hat{W}_t$ and y_t^T are IID.[12] This strong assumption allows me to establish two analytical propositions concerning the default and exit sets, respectively.[13]

Proposition 3.1 *If DEFAULT is the preferred regime for some point in the state space* $(y^0, b^0, \hat{W}^0)$*, then DEFAULT is also the preferred regime for the point* $(y^0, b^1, \hat{W}^0)$*, for any* $\bar{b} \geq b^1 \geq b^0 \geq \underline{b}$.

Proposition 3.1 establishes a property that is typical for this type of models; the default set is an interval along the d_t dimension. In other words, if it is optimal to default under certain conditions, it is also optimal to default under the same conditions but with a higher debt level. For standard Eaton-Gersovitz models, the same is true along the y_t dimension. In this model, however, it does not need to be true, as exiting is an alternative option that might be preferred if the current endowment level is lowered.

An analogous property holds for the exit decision along the x_t dimension, as stated formally in the following proposition.

Proposition 3.2 *If EXIT is the preferred regime for some point in the state space* $(y^0, b^0, \hat{W}^0)$*, then EXIT is also the preferred regime for the point* $(y^0, b^0, \hat{W}^1)$*, for any* $\hat{W}^1 \geq \hat{W}^0$.

The two propositions are an attempt to characterize the default and exit sets, similarly to how the default set is established in models without exit. Numerical explorations of the

[12] The auxiliary variable $\hat{W}_t$ replacing W_{t-1} disconnects the current debt choice from future nominal rigidity issues; the channel is quantitatively weak, but makes formal proofs more complex.

[13] Proofs of both propositions are available in the appendix.

model show that similar propositions along other dimensions do not hold, indicating that the interplay between default and exit decisions is a non-trivial one. This is presented graphically in the next subsection.

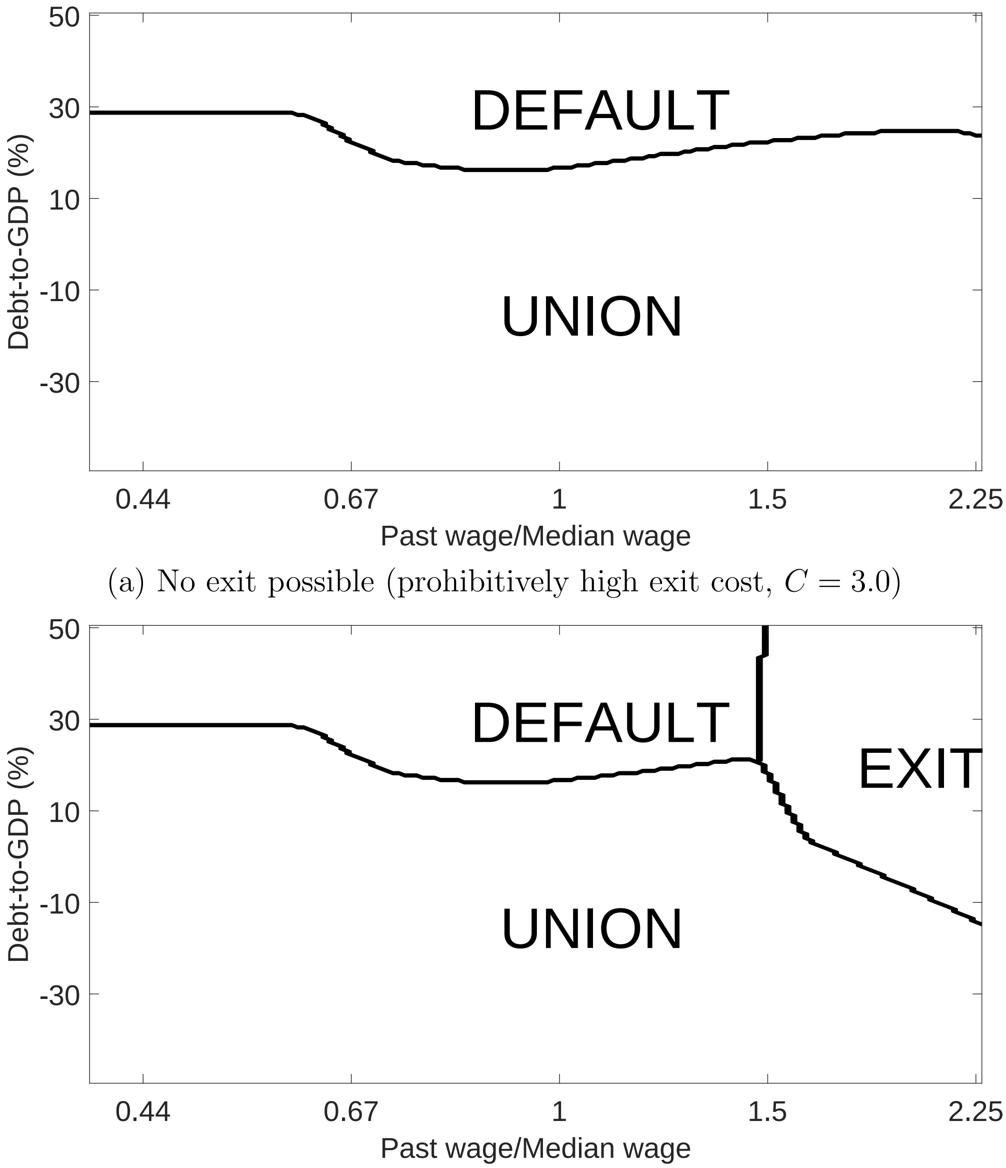
3.4.2 Default and Exit sets

Complementary to the analytical results in the previous subsection, I explore the default and exit sets numerically. Figure 4 provides examples of such sets for a calibrated model. Since the state space is three-dimensional, the figure presents the sets for a given tradable endowment level, allowing the nominal friction and debt levels to vary.

The upper panel (a) of the figure shows a calibration with a high exit cost that effectively prevents the economy from exiting the union. In this case, the only two regimes that are chosen in equilibrium are UNION and DEFAULT, which are similar to the good and bad financial standing in a model without nominal frictions and exits. As expected, default is more likely for higher debt levels. Surprisingly, the threshold debt level at which the country chooses to default changes over the severity of the nominal friction, even though default cannot relieve the nominal friction per se. The intuition behind this result comes from the CES aggregation function in the utility function, which implies that the lower the non-tradable goods production due to a more binding nominal friction, the lower the marginal value of tradable consumption. This affects the trade-off between repaying the debt and defaulting.

The lower panel (b) of Figure 4 presents a calibration with a moderate exit cost. All other parameters are kept the same as in the calibration in the upper panel of the figure. Higher attractiveness of the EXIT regime does not affect the regime choice for low past wages. At this part of the parameter space an exit decision is not very likely. The exit set takes over the upper right corner of the state space in the figure, where the nominal friction becomes important, while debt ranges from moderate to high. When past nominal wages are very high, implying a strongly binding nominal friction, the country prefers to exit, even if it is running a positive net foreign asset position. There is also a part of the state space where default and exit are similarly attractive, when the nominal friction is severe and the debt level is moderately high. The fact that exiting triggers an implicit partial default makes it a good alternative to an outright default, but only up to some level of external debt.

The precise border between default and exit depends on the calibration of the model and the choice of the state space. The intuition behind the trade-offs is common across different calibrations.



(a) No exit possible (prohibitively high exit cost, $C = 3.0$)

(b) Moderate exit cost calibration ($C = 1.5$)

Figure 4: Default and Exit regions as a function of debt b and slackness of the nominal rigidity x (for a given level of tradable output $y^T < \bar{y}$).

4 Quantitative analysis of the model

This section describes the parametrization and numerical strategy used to analyze the effects of bailouts and implicit bailout guarantees on the dynamics of a small open economy (SOE hereafter) within a monetary union.

To distinguish between different channels through which bailouts affect the economy, I consider different levels of the exit cost, which represent monetary unions where exits are likely, unlikely, or even do not occur in equilibrium. I also impose a monetary union history such that (i) there were no exits from the union in the past, and (ii) exit is never considered by two countries at the same time. Analyzing the behavior of an economy without past exits and no expectation of another country exiting this period allows the study of decision making solely based on the SOE's country-specific shocks. Thus, I concentrate on tradable output, past debt decisions and the level of nominal wages in the SOE, and abstract from issues of short-term spillovers or contagion; which are analyzed in detail by Eijffinger et al. (2018).

I first analyze the baseline dynamics of the model without bailouts, and then introduce the possibility of bailouts. Under the bailout scenario, whenever a country would find it optimal to exit the union, a bailout takes place preventing the exit. The transfer is calibrated to make the exiting country indifferent between exiting and remaining in the union. For this experiment, I consider the special case of a union consisting of a continuum of small open economies, with the potential for substantial first exit effects, as specified in Proposition 2.3.

4.1 Functional Forms and Calibration

Table 1 provides the details of the benchmark parametrization of the model. My parametrization largely resembles that of Na et al. (2018). I follow their calibration as EMU countries, which are the natural calibration target, have no modern history of defaults, nor exits from a monetary union.

One of the key parameters of the model is the degree of nominal wage rigidity, γ. Following Na et al. (2018), I fix it at $\gamma = 0.99$. This value implies that nominal wages cannot fall more than four percent per year, an assumption in line with the evidence provided in Schmitt-Grohé and Uribe (2016).

In terms of functional forms, I choose a standard CRRA utility function

$$U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \tag{25}$$

with $\sigma = 2$ and $\beta = 0.85$.[14] The production funtction takes the familiar form

$$y_t^N = h_t^\alpha$$

[14]This value of β is low for standard macroeconomic calibrations, but typical for quantitative sovereign default models.

Table 1: Benchmark Parametrization of the model

Parameter	**Value**	**Description**
γ	0.99	Degree of downward nominal wage rigidity
σ	2	Inverse of intertemporal elasticity of consumption
y^T	1	Steady-state tradable output
$\bar{h}$	1	Labor endowment
a	0.28	Share of tradables
ε	0.44	Elasticity of substitution between tradables and non-tradables
α	0.59	Labor share in the non-traded sector
β	0.85	Quarterly discount factor
r^*	0.01	Quarterly net world interest rate
δ_1	-0.42	Parameters of the output loss functions
δ_2	0.46	
ρ	0.932	Serial correlation of $\ln y_t^T$
σ_y	0.037	Standard deviation of innovation to y_t^T
		Discretization of the state space
n_y	31	Number of tradable output grid points (equally spaced in logs)
n_d	201	Number of debt grid points (equally spaced)
n_w	151	Number of wage grid points (equally spaced in logs)
$[\underline{y}^T, \bar{y}^T]$	[0.65, 1.53]	Grid for tradable output
$[\underline{d}, \bar{d}]$	[-0.5, 0.5]	Grid for external debt
$[\underline{w}, \bar{w}]$	[0.9, 5.15]	Grid for nominal wages

with $\alpha = 0.59$.

I use the Na et al. (2018) OLS estimates of an AR(1) process for the tradable endowment process, i.e. $\rho = 0.932$ and $\sigma_y = 0.037$. The process is discretized, as the tradable endowment is one of the dimensions of the state space.

I specify the output loss function that is relevant in states of default as

$$L(y_t^T) = \max\left(0, \delta_1 y_t^T + \delta_2 (y_t^T)^2\right)$$

and set $\delta_1 = -0.42$ and $\delta_2 = 0.46$. The loss function parameters here differ from the ones in Na et al. (2018), as I exclude the possibility of re-entry. This assumption implies that the country remains in bad financial standing forever after defaulting. I calibrate δ_1 and δ_2 to match the default frequency and average debt level targets of Na et al. (2018). As can be expected without re-entry, the output loss at the steady state level of endowment is lower than the one chosen by the authors.

Another key parameter of the model is the exit cost $\tilde{C}$, specified in this section as a sum of the utility from being in the union $M^i\left(N+1; p^E\right)$, which is lost upon exit, and the dead-weight loss exit cost C. This parameter is difficult to calibrate as no country has ever exited the EMU. Instead of relying on one parameter value, I explore different values of $\tilde{C}$. Whenever not mentioned explicitly, I use $\tilde{C} = 1.6$ as a benchmark value. This value allows for some exits in simulations, albeit with a very low probability.

I solve the model by value function iteration and approximate the equilibrium using a discrete state space by assuming 31 grid points for tradable output, 201 grid points for debt, and 151 grid points for nominal wage. Assuming no re-entry after default simplifies the computations, as the model can be solved sequentially for the different regimes.

4.2 Equilibrium dynamics without bailouts

A key challenge in the calibration exercise is the cost of exit, which is not directly observable as there has been no exit incident until now. Independent of the applied calibration strategy, it is interesting to examine how sensitive the model is to expected exit cost, and what the properties of the probability of exit are with respect to the expected exit cost. In this section I abstract from the uncertainty about the exit cost and the potential of information revelation, as well as from any possibility of bailout.

Figure 5 presents the unconditional exit probability for a number of possible values of the exit cost, ranging from 0.7 to 3.0. The probabilities are estimated based on 10,000 simulation series of 1,000 periods each, for each of the potential values. The figure suggests weak monotonicity of the exit probability on exit cost, similar to the result obtained for a simpler economy in Lemma 2.1. This is a very intuitive result as higher exit cost makes the exit decision less attractive. Additionally, if exit is more costly, the dynamic behavior of the economy may adjust to make exit less frequently needed.[15] The line becomes flat around the exit cost of 2 as there were no exits observed in the simulations beyond the threshold $\tilde{C} = 2.2$.

The simulation results presented in Figure 5 also suggest that the unconditional exit probability is a convex function of the exit cost. Even though this does not necessary hold for all possible setups of the model, there are reasons for the exit probability to behave this way. Abstracting from the effect of exit on sovereign debt decisions, the unconditional exit probability measures the frequency with which the welfare cost of the nominal rigidity exceeds some threshold value. As the welfare cost of the nominal rigidity is proportional to

[15] At this point, there is no distinction made between those two effects.

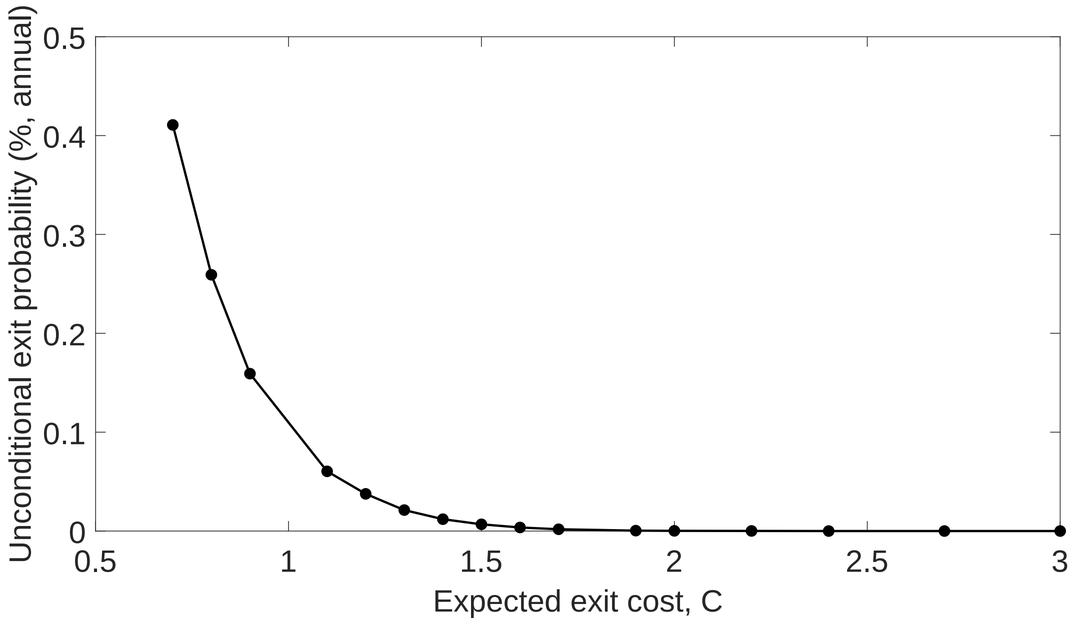


Figure 5: Unconditional exit probability as a function of the exit cost (the case of no uncertainty about $\tilde{C}$).

the severity of the friction, i.e. $\frac{\gamma \cdot W_{t-1}}{W_t^*}$, for many standard (continuous) distributions of the shock, convexity should be guaranteed.[16]

The model without bailouts is equivalent to the benchmark model of Eijffinger et al. (2018); my simulations are fully in line with the dynamics reported there. Thus, I focus on analyzing the effects of bailouts.

4.3 Dynamic effects of bailouts

The previous subsection considered the dynamics of an economy without bailouts, and concentrated on the differences under alternative exit cost parametrizations, thus setting the stage for the analysis of the dynamic effects of bailouts.

Here the focus is now on a monetary union consisting of a continuum of small open economies, with uncertainty about exit cost. As described in Proposition 2.3, the unknown exit cost is important for maintaining the size of the union. Therefore, information revelation may generate substantial welfare losses. Given that the exiting country is infinitesimal, no matter the size of the transfer necessary to prevent the exit, the union finds it optimal to grant the bailout. This can be thought of as the limit case of a monetary union's bailout

[16] Convexity of $p^E(\cdot)$ with respect to $\tilde{C}$ is a relevant feature of the model, as Proposition 2.3 can be generalized by relaxing the assumption in equation (12) and replacing it with convexity.

policy.

Introducing bailouts affects the dynamics of the economy in two ways. Firstly, there is the direct effect of a bailout; whenever the economy chooses to exit, the union bails it out. This happens in the form of a fiscal transfer that makes the country indifferent between staying and exiting. A bailout of this form reduces the current debt level and prevents the economy from exiting.[17]

Figure 6 gives an overview of the actual bailouts taking place in the simulations. Panel (a) of figure 6 shows the distribution of the number of bailout transfers during a single crisis episode. A single bailout transfer is sufficient to keep the country in the union in less than 40% of the crises episodes. At the same time, in almost one-third of the episodes the country requires more than three separate fiscal transfers. The large number of transfers is also reflected in the probability of follow-up bailouts in the periods following the first bailout - presented in panel (c). In almost half of the simulated crises episodes, the second bailout transfer follows in the quarter immediately after the first transfer. This is in line with the perception that the bailout is merely meant to keep the country in the union, instead of resolving the crisis.

The multiple bailouts result in large total transfers as can be seen in panel (b). Over 45% of the crisis episodes result in transfers exceeding 10% of the beneficiaries quarterly GDP, and one in six episodes leads to transfers exceeding 50% of GDP. The large transfers are a reflection of the severity of the nominal friction. When binding, it can lead to substantial involuntary unemployment levels, just as observed in the Eurozone countries.

Despite the large transfers, almost one in four bailout programs is directly followed by a default in the first year after the last bailout transfer, as can be seen in panel (d). Nearly another one in four of the bailed out economies does not default in the first year, but does so in the next nine years. The result highlights the contrast between exit- and default-driven bailouts, where the former are only meant to prevent countries from exiting the union.

The above results are all in line with the bailout experience during the Eurozone Crisis. The country which received the highest implicit transfers as part of the bailout program is Greece, which is arguably also the only Eurozone country realistically considering an EMU exit. The country received three different bailout programs over a period of six years, which were accompanied by several maturity and interest rate adjustments in the meantime - leading to additional implicit fiscal transfers. The total size of the implicit transfers exceeded

[17] The direct effect is only applicable to the states of the world where EXIT is the preferred regime, which constitute a small fraction of the simulated periods in the vast majority of the parametrizations. Nevertheless, the results might be relevant for analyzing the usefulness of the model in replicating the Eurozone Crisis.

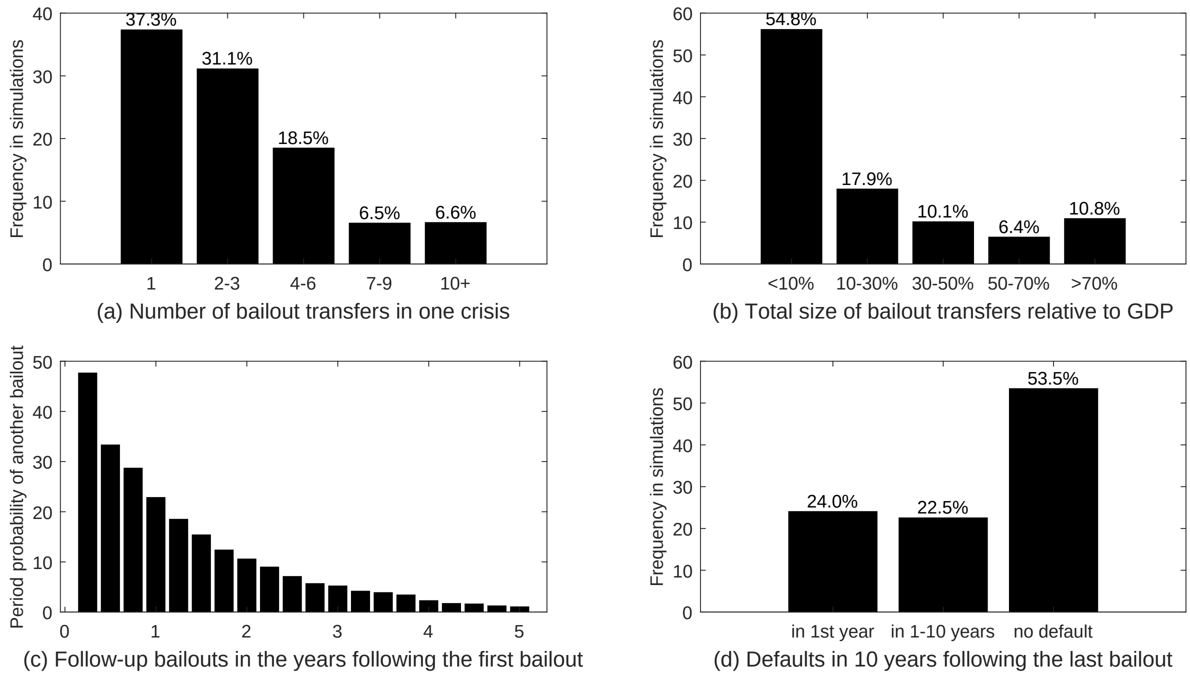


Figure 6: Summary statistics of the bailout programs in the simulations. (a) Number of bailout transfers within one crisis episode. (b) The sum of all bailout transfers received by the country during one crisis episode expressed as a share of its GDP. (c) Probability of the first bailout being followed by further bailouts in the following years. The period of the initial bailout is normalized to $t = 0$. (d) The occurrence of defaults in the ten years directly following the last transfer in the bailout program.

40% of Greece's 2010 GDP, as estimated by Gourinchas et al. (2020). Despite the large transfers, Greece defaulted on private bond holders in 2012. The bailouts did not resolve the crisis, but ensured that Greece remains within the EMU.

Figure 7 presents the average dynamics of the economy around a crisis episode which results in a bailout. The red line presents the simulation results for the economy with bailouts and is plotted against simulations for a benchmark economy without bailouts (blue line). The figure includes twenty periods prior to the bailout, which happens in $t = 0$, and forty periods following the first bailout.

The economy requesting a bailout is in a recession which comes after a strong expansion. The large fall in tradable output leads to nominal wages substantially exceeding their optimal level, i.e. the nominal rigidity is binding. The severity of the friction can be seen in the level of non-tradable consumption, which is around 0.7 - substantially below the full employment level of 1. It can be also seen in the abrupt fall in nominal wages in the benchmark economy, where in approx. 80% of the simulation series the economy exits the union and resets its

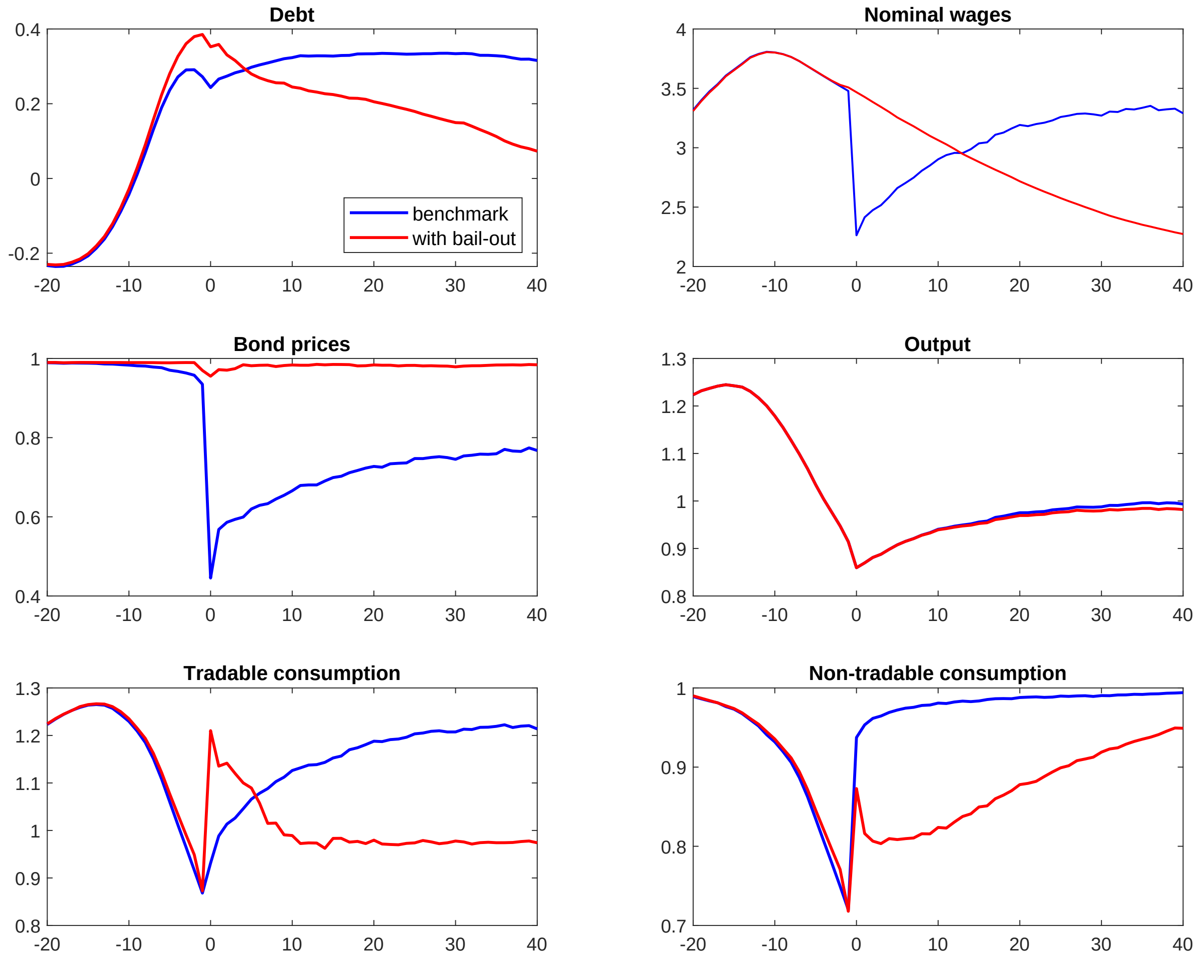


Figure 7: Average dynamics of the economy around a crisis episode with bailouts. The figure contains the simulation series in which a bailout takes place in the bailout economy (red line). Time is normalized such that the first bailout takes place in $t = 0$. Variables are plotted against the benchmark economy with no bailouts (blue line). The sample has been constrained such that the benchmark economy has to be in the UNION regime in period $t = -1$.

wages optimally (through an exchange rate devaluation). The competitive devaluation allows the non-tradable sector to bounce back quickly.

This stands in stark contrast to the bailout economy in which the nominal rigidity is still binding. It is temporarily relaxed by the increase in tradable consumption, which pushes up demand for non-tradable goods and, hence, optimal wages in the non-tradable sector. The higher tradable consumption is an effect of the bailout transfers in period 0 and the following periods for some of the simulations.

The bailout guarantee allows the economy to issue higher debt levels at high bond prices,

Table 2: Summary of 10,000-simulation series ($T = 1,000$) for economies with and without bailouts.

		No bailouts	Unlimited bailouts
	Default probability (annual, %)	1.40	1.33
$\tilde{C} = 1.6$	Exit probability (annual, %)	0.004	0.004
	Average debt-to-GDP (quarterly, %)	0.77	0.78
	Default probability (annual, %)	1.37	1.35
$\tilde{C} = 1.7$	Exit probability (annual, %)	0.002	0.002
	Average debt-to-GDP (quarterly, %)	0.75	0.73
	Default probability (annual, %)	1.38	1.37
$\tilde{C} = 2.0$	Exit probability (annual, %)	0.000	0.000
	Average debt-to-GDP (quarterly, %)	0.71	0.71
	Default probability (annual, %)	1.38	1.38
$\tilde{C} = 3.0$	Exit probability (annual, %)	0	0
	Average debt-to-GDP (quarterly, %)	0.70	0.70

In the table 0.000 indicates probabilities smaller than 0.001, but positive. 0, on the other hand, indicates zero-probability, i.e. an exit did not take place in the simulations.

which reflect the lack of devaluation risk. Thus, the bailout economy increases debt levels before entering a long period of slow austerity. In contrast, the benchmark economy reduces its debt level by exiting and after that increases debt holdings. Such that a year after the bailout period, the benchmark economy records on average higher debt levels. This coincides with the benchmark economy overtaking the bailout economy in tradable consumption. The fall in the debt level of the bailout economy reflects also partially the high default rates, as 13% and 27% of the simulation series observe a default in the bailout economy within one year and five years from the first bailout, respectively. The equivalent numbers for the benchmark economy are just 2% and 7%.

On top of the direct effect, there is also an indirect one. As a bailout prevents an exit, it shields investors from the losses an exit generates, thus eliminating the exit risk premium. Even if exit is rarely chosen by a country, it may be perceived as a possible outcome in the future and affect the current bond prices. The indirect effect will be referred to as the effect

Table 3: Comparison of the timing of default in economies with and without bailouts. Summary of 10,000-simulation series ($T = 1,000$) with $\tilde{C} = 1.6$.

	Fraction of simulation series
$T^{\text{default}}\|_{\text{bailout}} > T^{\text{default}}\|_{\text{no bailout}}$	11.5%
$T^{\text{default}}\|_{\text{bailout}} = T^{\text{default}}\|_{\text{no bailout}}$	82.9%
$T^{\text{default}}\|_{\text{bailout}} < T^{\text{default}}\|_{\text{no bailout}}$	5.6%

of a bailout guarantee, as it does not require a bailout to take place.

During the Eurozone Crisis, some economists and policymakers opposed the idea of European bailouts, arguing that they will create substantial moral hazard. According to those arguments, union members and financial markets would internalize the possibility of bailouts and overborrow, increasing the risk of future defaults. If present in this model, the moral hazard effects should be part of the indirect effect.

Table 2 provides an overview of possible moral hazard effects of exit-driven bailouts. It presents a summary of simulations carried out for different values of the exit cost parameter, $\tilde{C}$. First, I simulate a benchmark economy in which there is no possibility of bailout. I repeat the simulations, keeping the previous shock realizations and introducing unlimited bailouts into the framework. Whenever the economy chooses to exit, it receives a fiscal transfer, making it indifferent between exiting and staying in the union.

For all the parametrizations presented in Table 2 bailouts do not seem to substantially affect the dynamics of the economy. Neither default, nor exit probabilities change significantly. Similarly, the average level of debt remains unchanged.[18]

The unchanged default probabilities hide richer dynamic effects of bailouts, as presented in Table 3. In a vast majority of the simulation series the timing of the first default is not affected by the presence of bailout guarantees. In two-thirds of the remaining series the bailout guarantees postpone a default, while in 5.6% of all series the bailout guarantees speed up a default.

The effect of the bailout guarantees on the timing of the first default suggests that the model may feature moral hazard effects, but that they are on average offset by the elimination

[18] The low average debt levels are a result of lower debt limits in a pegged economy. This quantitative feature of the model was first pointed out by Na et al. (2018). The low average debt levels hide large variations in debt levels over time; debt takes on highly positive and negative values.

of exit risk premia. The resulting lower interest rates on sovereign bonds make it easier for the economy to service its debt, reducing the incentives to default.

5 Conclusion

This paper presents a model of bailouts within a monetary union. In the model, members of a monetary union might decide to bail out a troubled country to prevent it from leaving the union, and not from defaulting. I study the conditions under which international bailouts can be optimal. In particular, the paper shows that the maximum available transfer can be independent of the economic importance of the supported country. An important reason for preventing an exit might be the fear of information revelation when exit cost is uncertain.

I provide microfoundations for spillovers within a monetary union that do not require large cross-border sovereign and private bond holdings. This suggests that the efforts to reform the EMU should not be limited to setting up a banking union, as the spillovers would prevail even with no cross-border portfolio exposure.

The model captures the co-existence of bailouts and defaults in the Eurozone, and gives a potential explanation for the large implicit fiscal transfers received by Greece. Under the interpretation of this model, the financial aid was provided to prevent Greece from exiting the union. This also explains why the other troubled countries did not receive any substantial implicit fiscal transfers as part of their bailout agreements; no other country was expected to exit the union, at least, before the first exit of Greece.

The distinction between default- and exit-driven spillovers and bailouts has further important policy implications. This paper shows that the size of moral hazard effects with bailouts motivated by exit is substantially lower when compared to bailouts preventing default. This finding might be relevant to consider in discussions about permanent financial assistance arrangements in the Eurozone.

This paper also opens several opportunities for future research. The model of default and exit is necessarily simplistic, leaving substantial room for extensions that could give quantitatively reliable predictions. Moreover, the existence of uncertainty about exit cost is assumed as given here; it is meant to reflect the institutional setup of the Eurozone, and not explain it. This leaves open the question of why members of the union would choose to keep the exit cost uncertain, or what the optimal exit cost is. The model of exit shown here can also be used to study the optimal setup of a monetary union, when exit is possible.

References

Aguiar, M., Amador, M., Farhi, E., and Gopinath, G. (2015). Coordination and Crisis in Monetary Unions. *Quarterly Journal of Economics*, XXXIII(2):1727–1779.

Aguiar, M. and Gopinath, G. (2006). Defaultable debt, interest rates and the current account. *Journal of International Economics*, 69(1):64–83.

Ardagna, S. and Caselli, F. (2014). The Political Economy of the Greek Debt Crisis: A Tale of Two Bailouts. *American Economic Journal: Macroeconomics*, 6(4):291–323.

Arellano, C. (2008). Default Risk and Income Fluctuations in Emerging Economies. *American Economic Review*, 98(3):690–712.

Azzimonti, M. and Quadrini, V. (2018). International spillovers and 'ex-ante' efficient bailouts. Working Paper 25011, National Bureau of Economic Research.

Bayer, C., Kim, C., and Kriwoluzky, A. (2018). The term structure of redenomination risk. CEPR Discussion Papers 12965, C.E.P.R. Discussion Papers.

Bocola, L., Bornstein, G., and Dovis, A. (2019). Quantitative sovereign default models and the European debt crisis. *Journal of International Economics*, 118:20 – 30.

Corsetti, G., Erce, A., and Uy, T. (2017). Official Sector Lending Strategies During the Euro Area Crisis. CEPR Discussion Papers 12228, Center for Economic Policy Research.

Corsetti, G., Erce, A., and Uy, T. (2018). Debt sustainability and the terms of official support. Working Paper 2018/116.

De Santis, R. A. (2015). A measure of redenomination risk. ECB Working Paper Series 1785, European Cental Bank.

Dovis, A. and Kirpalani, R. (2020). Fiscal rules, bailouts, and reputation in federal governments. *American Economic Review*, 110(3):860–88.

Eaton, J. and Gersovitz, M. (1981). Debt with Potential Repudiation: Theoretical and Empirical Analysis. *Review of Economic Studies*, 48(2):289–309.

Eijffinger, S. C., Kobielarz, M. L., and Uras, B. R. (2018). Sovereign Default, Exit and Contagion in a Monetary Union. *Journal of International Economics*, 113:1 – 19.

Farhi, E. and Tirole, J. (2017). Deadly Embrace: Sovereign and Financial Balance Sheets Doom Loops. *The Review of Economic Studies*, 85(3):1781–1823.

Fink, F. and Scholl, A. (2016). A quantitative model of sovereign debt, bailouts and conditionality. *Journal of International Economics*, 98:176 – 190.

Fuchs, W. and Lippi, F. (2006). Monetary union with voluntary participation. *The Review of Economic Studies*, 73(2):437–457.

Galí, J. and Monacelli, T. (2008). Optimal monetary and fiscal policy in a currency union. *Journal of International Economics*, 76(1):116–132.

Glick, R. and Rose, A. K. (2016). Currency unions and trade: A post-EMU reassessment. *European Economic Review*, 87:78 – 91.

Gourinchas, P. O., Martin, P., and Messer, T. (2020). The Economics of Sovereign Debt, Bailouts and the Eurozone Crisis. NBER Working Papers 27403, National Bureau of Economic Research, Inc.

Hatchondo, J. C., Martinez, L., and Onder, Y. K. (2017). Non-defaultable debt and sovereign risk. *Journal of International Economics*, 105:217 – 229.

Kriwoluzky, A., Müller, G. J., and Wolf, M. (2019). Exit expectations and debt crises in currency unions. *Journal of International Economics*, 121:103253.

Na, S., Schmitt-Grohé, S., Uribe, M., and Yue, V. (2018). The Twin Ds: Optimal Default and Devaluation. *American Economic Review*, 108(7):1773–1819.

Roch, F. and Uhlig, H. (2018). The dynamics of sovereign debt crises and bailouts. *Journal of International Economics*, 114:1 – 13.

Schmitt-Grohé, S. and Uribe, M. (2016). Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment. *Journal of Political Economy*, 124(5):1466 – 1514.

Tirole, J. (2015). Country Solidarity in Sovereign Crises. *American Economic Review*, 105(8):2333–63.

Appendix A - proofs of propositions in Section 2

Proof of Lemma 2.1 The probability of exit is defined as

$$p_i^E(C) = P\left(V_i^E\left(\mathcal{S}_t\right) - \tilde{V}_i^U\left(\mathcal{S}_t\right) > C + M^i\left(k+1, p^E\right)\right).$$

Let us denote by $F_i(x)$ the cumulative distribution function of the welfare losses from the union rigidity

$$F(x) = P\left(V_i^E\left(\mathcal{S}_t\right) - \tilde{V}_i^U\left(\mathcal{S}_t\right) < x\right).$$

Then,

$$p_i^E(C) = 1 - F_i\left(C + M^i\left(k+1, p^E\right)\right)$$

Since $F_i(\cdot)$ is a cdf, it is non-decreasing. Hence, $1 - F_i(\cdot)$ is a non-increasing function of its argument, $C + M^i\left(k+1, p^E\right)$. This implies that for p_i^E to be a non-increasing function of C, we need $C + M^i\left(k+1, p^E\right)$ to be a non-decreasing function of C. The effect of C on the term of interest is

$$\frac{\partial}{\partial C}\left[C + M^i\left(k+1, p^E\right)\right] = 1 + \sum_{j \neq i} \frac{\partial M^i(k+1, p^E)}{\partial p_j^E} \cdot \frac{\partial p_j^E}{\partial C}.$$

The lemma is trivial if one disregards the effect that the exit cost C has on the value of the union $M^i(\cdot)$ through the exit probability of the other members. The difficulty lies within the feedback loop between the exit probabilities of the union members. In particular, if there exists a bad coordination equilibrium, where all the members believe that the others will exit with high probability and so are also likely to exit, it could also show up here. In Section 2.2, I exclude this equilibrium by assuming sequential decision-making. To introduce sequential decision-making here, I will follow the k-level thinking approach for the members of the union.

1. Let us first consider country i being a level-1 thinker, while all the other countries are level-0 thinkers, implying they do not change their exit probability in response to a change in C. Hence, $\forall_{j \neq i} \frac{\partial p_j^E}{\partial C} = 0$. Then

$$\frac{\partial}{\partial C}\left[C + M^i\left(k+1, p^E\right)\right] = 1 + \sum_{j \neq i} \frac{\partial M^i(k+1, p^E)}{\partial p_j^E} \cdot \frac{\partial p_j^E}{\partial C} = 1,$$

which implies

$$\frac{\partial p_i^E}{\partial C} < 0.$$

2. Let us now consider country i being a level-2 thinker, while all the other members are level-1 thinkers. This means that $\forall_{j \neq i} \frac{\partial p_j^E}{\partial C} < 0$. Moreover, notice that, independent of the level of thinking

$$\forall_{j \neq i} \frac{\partial M^i(k+1, p^E)}{\partial p_j^E} = -\frac{\beta m_j^i}{\left(1 - \beta\left(1 - p_j^E\right)\right)^2} < 0.$$

Then, for the level-2 thinker country i

$$\frac{\partial}{\partial C}\left[C + M^i\left(k+1, p^E\right)\right] = 1 + \sum_{j \neq i} \underbrace{\frac{\partial M^i(k+1, p^E)}{\partial p_j^E}}_{<0} \cdot \underbrace{\frac{\partial p_j^E}{\partial C}}_{<0} > 1.$$

3. Finally, consider that country i is a level-κ thinker, where $\kappa \geq 3$, while all the other countries are a level-$(\kappa - 1)$ thinkers. And assume that for any country j who is a level-$(\kappa - 1)$ thinker $\frac{\partial p_j^E}{\partial C} < 0$, then

$$\frac{\partial}{\partial C}\left[C + M^i\left(k+1, p^E\right)\right] = 1 + \sum_{j \neq i} \underbrace{\frac{\partial M^i(k+1, p^E)}{\partial p_j^E}}_{<0} \cdot \underbrace{\frac{\partial p_j^E}{\partial C}}_{<0} > 1.$$

This proves that the above is true for any $\kappa > 0$.

Hence, for the limit case $\kappa \to \infty$, which approximates a rational expectations equilibrium, it is also true that $C + M^i\left(k+1, p^E\right)$ is a non-decreasing function of C. This, in turn, implies that p_i^E is a non-increasing function of C.

This proof can serve to provide more intuition, as the additional levels of thinking can be viewed as next order effects. The direct effect goes through simply increasing the exit cost for country i, which leads to exit being less likely. The first indirect effect goes through the fact that the other countries also face a higher exit cost, so they are less likely to exit, which means that the value of the union goes up, and increases the threshold for country i to exit. Every next increase in threshold again increases the threshold at which countries exit and so the effect of a higher C gets amplified through its effects on $M^i(\cdot)$.

Proof of Proposition 2.2 For country g to choose exit, inequality (6) must hold. Whenever inequality (6) holds, country g requires a utility transfer equal to $V_g^E(\mathcal{S}_t) - C - \tilde{V}_g^U(\mathcal{S}_t; \mathcal{M}_t)$ to remain in the union. The minimum transfer is defined as F_t^{min} in equation (8).

The maximum amount of the transfer any partner country i is willing to finance is equal to $M^i\left(k+1; p^E\right) - M^i\left(k; p^E\right)$. The sum of all those maximum transfers is

$$\sum_{i \neq g} M^i\left(k+1; p^E\right) - M^i\left(k; p^E\right) = M^{-g}\left(k+1; p^E\right) - M\left(k; p^E\right).$$

Therefore, for a voluntary, exit-preventing bailout to be feasible it must hold that

$$M^{-g}\left(k+1; p^E\right) - M\left(k; p^E\right) \geq F_t^{min}.$$

Since any member country i suffers a loss from the exit of country g, equal to $M^i\left(k+1; p^E\right) - M^i\left(k; p^E\right)$, country i gains from any transfer smaller than that, which prevents country g from exiting.[19]

The utility transfer F_t^{min} conditional on no-exit, makes country g indifferent between exiting and remaining, while it is welfare improving for the other countries, hence it is a Pareto-optimal outcome.

Proof of Proposition 2.3 It follows immediately from taking the limit of equation (10)

$$\begin{aligned}
\lim_{N\to\infty} \quad & \left\{M^{-g}(N+1; 0) - \mathbb{E}\left[M\left(N; p^E(C)\right)\right]\right\} = \\
= \quad & \lim_{N\to\infty} \left\{M^{-g}(N+1; 0)\left[\frac{1}{N} + \pi\frac{\beta p^E}{\beta p^E + (1-\beta)}\frac{N-1}{N}\right]\right\} \\
= \quad & \pi\frac{\beta \bar{p}^E}{\beta \bar{p}^E + (1-\beta)} M^{-g}(N+1; 0),
\end{aligned}$$

as $M^{-g}(\cdot)$ is finite.

Proof of Proposition 2.4 It follows directly from the proof of Proposition 2.2, with the modification of the loss from the first exit under uncertainty as described by equation (9).

[19] I do not explicitly define a rule for sharing the cost of the utility transfer, and I ignore strategic interactions, which might lead to lack of coordination, or free-riding issues.

Appendix B - extensions of the monetary union model in Section 2

B.1 More general calculation of the total loss from exit

In section 2.3, I calculate the total loss triggered by country g's exit assuming that the exit does not alter the exit probability of the other members. This abstracts from the fact that the lower utility from union membership enjoyed by country j, $M^j(k) < M^j(k+1)$, may increase the country's probability of exiting, $p_j^E(C, k+1) \leq p_j^E(C, k)$. The formula below is a more general version of equation (5)

$$\begin{aligned} M^{-g}\left(k+1; p^E\right) \quad - \quad & M\left(k; p^E\right) = \\ = \quad & \sum_{i \neq g} \sum_{j \neq i} \frac{m_j^i}{1-\beta\left(1-p_j^E(C, k+1)\right)} - \sum_{i \neq g} \sum_{j \neq i, g} \frac{m_j^i}{1-\beta\left(1-p_j^E(C, k)\right)} \\ = \quad & \sum_{i \neq g} \frac{m_g^i}{1-\beta\left(1-p_g^E(C, k+1)\right)} + \\ & + \sum_{i \neq g} \sum_{j \neq i, g} \frac{\beta m_j^i\left[p_j^E(C, k)-p_j^E(C, k+1)\right]}{\left[1-\beta\left(1-p_j^E(C, k)\right)\right]\left[1-\beta\left(1-p_j^E(C, k+1)\right)\right]}. \end{aligned}$$

B.2 Optimal bailouts in the presence of individual gains from information revelation

The analysis in section 2.4 relies on the assumption that countries are myopic and do not internalize their individual gains from information revelation. Here, I relax the assumption, and show the conditions under which uncertainty is the preferred outcome, even with fully rational union members.

For convenience, let us focus on the case described by assumption (11) and (12). Any country i, which is a member of the monetary union faces the risk of making a suboptimal regime choice because of the uncertainty surrounding the exit cost. The source of this inefficiency stems from the fact that the country is considering an exit decision evaluating it against the expected exit cost $\mathbb{E}[C]$, whereas the true exit cost is either C^L or C^H.

In particular, a country may either exit inefficiently and discover the exit cost is high, or suboptimaly remain in the union while the exit cost is low. The two cases are presented in the two sets of inequalities below

$$\text{inefficient exit decision}: \quad \mathbb{E}[C] < V_i^E(\mathcal{S}_t) - \tilde{V}_i^U(\mathcal{S}_t; \mathcal{M}_t) < C^H,$$
$$\text{inefficient remain decision}: \quad C^L < V_i^E(\mathcal{S}_t) - \tilde{V}_i^U(\mathcal{S}_t; \mathcal{M}_t) < \mathbb{E}[C].$$

The expected gain from avoiding an inefficient decision consists of two factors: the expected size of the inefficiency and the probability of making the decision. Following assumption (12) the probability of taking an exit decision against the expected exit cost $\mathbb{E}[C]$ is zero. Therefore, inefficient exit decisions do not happen in this framework.

The probability of making an inefficient remain decision can be calculated by comparing the probability of exit under the exit cost C^L against the exit probability under the expected exit cost $\mathbb{E}[C]$. If the true exit cost is C^L, then the exit decision should happen with a probability $\bar{p}^E$. Since it does not happen at all, it implies that a fraction $\bar{p}^E$ of the remain decisions is inefficient. The true exit cost is equal to C^L with probability π, hence, the probability of an inefficient remain decision is $\pi \cdot \bar{p}^E$.

The welfare loss associated with an inefficient remain decision depends on the shape of the value functions $\tilde{V}^U(\cdot)$ and $V^E(\cdot)$, as well as the distribution of the state space $\mathcal{S}_t$. Without making further assumptions, I can limit the size of the inefficiency from above by $\mathbb{E}[C] - C^L$. A loss larger than that would imply that it is optimal to exit also under the expected exit cost $\mathbb{E}[C]$, hence, there would be no inefficient remain decision.

The above analysis implies that a sufficient condition for a country i to prefer for uncertainty to remain unresolved is

$$\pi \frac{\beta \bar{p}^E}{\beta \bar{p}^E + (1-\beta)} M^i(N;0) \quad \geq \quad \frac{\pi \bar{p}^E}{1-\beta}\left(\mathbb{E}[C] - C^L\right),$$

which boils down to

$$M^i(N;0) \geq \frac{1-\beta+\beta\bar{p}^E}{\beta(1-\beta)}\left(\mathbb{E}[C] - C^L\right). \tag{26}$$

The above requires the gross benefit of participating in a monetary union being higher than the spread between expected and low exit cost by a factor $\frac{1-\beta+\beta\bar{p}^E}{\beta(1-\beta)}$.

Whenever condition (26) is satisfied, union members (even if fully rational) prefer a late resolution of uncertainty. The necessary condition for a given parametrization of $\tilde{V}^U(\cdot)$, $V^E(\cdot)$ and a distribution of $\mathcal{S}_t$ is most likely substantially milder than condition (26).

Appendix C (For Online Publication) - Value Functions and Government's Optimization Program

In this section I outline the optimization program of the government in the SOE model of section 3. As long as the economy is in the UNION regime, the government makes two discretionary decisions each period: (i) it decides whether to stay within the monetary union and (ii) it chooses between repaying and defaulting on its external debt. Conditional on exiting the union, the government sets the nominal exchange rate. Conditional on repaying its debt, it issues new external debt to be repaid next period.

The above decisions are made taking the state of the monetary union, $\mathcal{M}_t$, and the state of the country, $\mathcal{S}_t$, as given. The former includes the current size of the union and the beliefs about the exit cost, $\mathcal{M}_t = \left\{k+1, \tilde{C}\right\}$. The state of the country, $\mathcal{S}_t = \{\tilde{y}_t^T, W_{t-1}, b_t, \mathcal{D}_{t-1}, \mathcal{X}_{t-1}\}$, encompasses a number of state variables: the tradable endowment, past nominal wages, and debt level, as well as the history of the country's exit and default decisions.

If the SOE is in the AUTARKY regime at the beginning of period t, its value function takes the form

$$V^A(\mathcal{S}_t) = \max_{c_t, e_t} \left\{ u(c_t) + \beta \mathbb{E}_t \left[V^A(\mathcal{S}_{t+1}) \right] \right\}. \tag{27}$$

subject to (19) and (23). The decision set of the government in the AUTARKY regime is reduced to choosing a nominal exchange rate, such that the nominal rigidity is not binding. I assume that once the economy exits the monetary union, its welfare does not depend on the state of the monetary union. Hence, the value functions of being in the AUTARKY and the EXIT regime do not depend on $\mathcal{M}_t$.

A government of a SOE in the DEFAULT regime cannot choose its nominal exchange rate, unless it chooses to exit the monetary union and enter the AUTARKY regime. The value function of the DEFAULT can be, hence, written as

$$\begin{aligned} V^D(\mathcal{S}_t; \mathcal{M}_t) &= \max_{c_t, e_t} \Big\{ u(c_t) + (1-e_t)\beta \mathbb{E}_t \left[V^D(\mathcal{S}_{t+1}; \mathcal{M}_{t+1}) \right] \\ &\quad + e_t \left(\beta \mathbb{E}_t \left[V^A(\mathcal{S}_{t+1}) \right] - \tilde{C} \right) \Big\}, \end{aligned} \tag{28}$$

subject to (19) and (23). The above takes into account that the SOE might exit the union in period t, by incurring the exit cost $\tilde{C}$, and enter the AUTARKY regime.

If the SOE exited the union in the past, it is in the EXIT regime. The government decisions in this regime include the choice of the nominal exchange rate, the decision whether to repay its past external debt, and, conditional on repayment, the level of the new debt issued. Denoting the value function of being in the EXIT regime as V^E, I can represent the problem of the government as

$$V^E(\mathcal{S}_t) = \max_{c_t, b_{t+1}, \epsilon_t, d_t} \Big\{ u(c_t) + (1 - d_t)\beta \mathbb{E}_t \left[V^E(\mathcal{S}_{t+1}) \right] + d_t \beta \mathbb{E}_t \left[V^A(\mathcal{S}_{t+1}) \right] \Big\}, \tag{29}$$

subject to (19) and (23).

Finally, having defined all the other regimes, I can move to describing the value function of the UNION regime, which I will denote by V^U. The government optimization problem in the UNION regime can be represented as

$$\begin{aligned} V^U(\mathcal{S}_t; \mathcal{M}_t) \quad = \quad & \max_{c_t, b_{t+1}, d_t, e_t} \Big\{ u(c_t) + (1 - d_t)(1 - e_t)\beta \mathbb{E}_t \left[V^U(\mathcal{S}_{t+1}; \mathcal{M}_{t+1}) \right] \\ & + (1 - d_t) e_t \left(\beta \mathbb{E}_t \left[V^E(\mathcal{S}_{t+1}) \right] - \tilde{C} \right) \\ & + d_t (1 - e_t) \beta \mathbb{E}_t \left[V^D(\mathcal{S}_{t+1}; \mathcal{M}_{t+1}) \right] \\ & + d_t e_t \left(\beta \mathbb{E}_t \left[V^A(\mathcal{S}_{t+1}) \right] - \tilde{C} \right) \Big\}, \end{aligned} \tag{30}$$

subject to (19) and (23).

I present the regimes of the model graphically in Figure 3.

Appendix D (For Online Publication) - proofs of propositions in Section 3

Proof of Proposition 3.2

Since exit is the preferred regime for the point $\left(y^0, b^0, \hat{W}^0\right)$, it must follow that

$$\begin{aligned} V^E\left(y^0, b^0, \hat{W}^0\right) &\geq V^A\left(y^0, b^0, \hat{W}^0\right) && (31)\\ V^E\left(y^0, b^0, \hat{W}^0\right) - \tilde{C} &\geq V^D\left(y^0, b^0, \hat{W}^0\right) && (32)\\ V^E\left(y^0, b^0, \hat{W}^0\right) - \tilde{C} &\geq V^U\left(y^0, b^0, \hat{W}^0\right). && (33) \end{aligned}$$

After exiting the monetary union the nominal friction is countered by the flexible exchange rate policy. Therefore, the net present value under the EXIT regime does not depend on the level of the wage constraint

$$V^E\left(y^0, b^0, \hat{W}^0\right) = V^E\left(y^0, b^0\right) = V^E\left(y^0, b^0, \hat{W}^1\right). \quad (34)$$

The same is true for the AUTARKY regime

$$V^A\left(y^0, b^0, \hat{W}^0\right) = V^A\left(y^0\right) = V^A\left(y^0, b^0, \hat{W}^1\right). \quad (35)$$

Bringing together (31), (34) and (35) yields

$$V^E\left(y^0, b^0, \hat{W}^1\right) = V^E\left(y^0, b^0, \hat{W}^0\right) \geq V^A\left(y^0, b^0, \hat{W}^0\right) = V^A\left(y^0, b^0, \hat{W}^1\right). \quad (36)$$

Under the default regime a higher wage constraint reduces current period utility without affecting future periods

$$\begin{aligned} V^D\left(y^0, \hat{W}^0\right) &= u\left(y^0, c^N\left(y^0, \hat{W}^0\right)\right) + \beta E\left[V^D\left(y', \hat{W}'\right)\right]\\ &\geq u\left(y^0, c^N\left(y^0, \hat{W}^1\right)\right) + \beta E\left[V^D\left(y', \hat{W}'\right)\right]\\ &= V^D\left(y^0, \hat{W}^1\right). && (37) \end{aligned}$$

The inequality becomes an equality if and only if $W^* \geq \hat{W}^1$, i.e. even the higher wage constraint is not binding under the considered tradable goods endowment.

Bringing together (32), (34), and (37) yields

$$V^E\left(y^0, b^0, \hat{W}^1\right) - \tilde{C} = V^E\left(y^0, b^0, \hat{W}^0\right) - \tilde{C} \geq V^D\left(y^0, b^0, \hat{W}^0\right) \geq V^D\left(y^0, b^0, \hat{W}^1\right). \quad (38)$$

Similarly, under the union regime a higher wage rigidity reduces current period utility and does not improve the utility in future periods:

$$\begin{aligned} V^U\left(y^0, b^0, \hat{W}^0\right) &= \max_{b'}\left\{u\left(y^0 - b^0 + qb', c^N\left(y^0 - b^0 + qb', \hat{W}^0\right)\right) + \beta E\left[V^U\left(y', b', \hat{W}'\right)\right]\right\} \\ &\geq u\left(y^0 - b^0 + q\tilde{b}, c^N\left(y^0 - b^0 + q\tilde{b}, \hat{W}^0\right)\right) + \beta E\left[V^U\left(y', \tilde{b}, \hat{W}'\right)\right] \\ &\geq u\left(y^0 - b^0 + q\tilde{b}, c^N\left(y^0 - b^0 + q\tilde{b}, \hat{W}^1\right)\right) + \beta E\left[V^U\left(y', \tilde{b}, \hat{W}'\right)\right] \\ &\equiv^{\tilde{b}} \max_{b'}\left\{u\left(y^0 - b^0 + qb', c^N\left(y^0 - b^0 + qb', \hat{W}^1\right)\right) + \beta E\left[V^U\left(y', b', \hat{W}'\right)\right]\right\} \\ &= V^U\left(y^0, b^0, \hat{W}^1\right). \end{aligned} \quad (39)$$

The reasoning is very similar to the case of the default regime, except for the fact that I am explicitly allowing the wage constraint to affect the debt choice. This is necessary, as a higher level of tradable goods consumption alleviates the wage rigidity in the non-tradable sector. Due to the assumed utility function, a higher consumption of tradable goods increases the marginal utility from non-tradable consumption and, hence, raises the relative price of non-tradable goods alleviating the contemporaneous nominal friction.

Bringing together (33), (34), and (39) yields

$$V^E\left(y^0, b^0, \hat{W}^1\right) - \tilde{C} = V^E\left(y^0, b^0, \hat{W}^0\right) - \tilde{C} \geq V^U\left(y^0, b^0, \hat{W}^0\right) \geq V^U\left(y^0, b^0, \hat{W}^1\right). \quad (40)$$

Proof of Proposition 3.1

Since DEFAULT is the preferred regime for the point $\left(y^0, b^0, \hat{W}^0\right)$, it must follow that

$$V^D\left(y^0, b^0, \hat{W}^0\right) \geq V^A\left(y^0, b^0, \hat{W}^0\right) - \tilde{C} \quad (41)$$

$$V^D\left(y^0, b^0, \hat{W}^0\right) \geq V^E\left(y^0, b^0, \hat{W}^0\right) - \tilde{C} \quad (42)$$

$$V^D\left(y^0, b^0, \hat{W}^0\right) \geq V^U\left(y^0, b^0, \hat{W}^0\right). \quad (43)$$

After defaulting the stock of foreign debt becomes an irrelevant variable. Therefore, the net present value under the DEFAULT regime does not depend on the level of debt

$$V^D\left(y^0,b^0,\hat{W}^0\right)=V^D\left(y^0,\hat{W}^0\right)=V^E\left(y^0,b^1,\hat{W}^0\right). \tag{44}$$

The same is true for the AUTARKY regime

$$V^A\left(y^0,b^0,\hat{W}^0\right)=V^A\left(y^0\right)=V^A\left(y^0,b^1,\hat{W}^0\right). \tag{45}$$

Bringing together (41), (44) and (45) yields

$$V^D\left(y^0,b^1,\hat{W}^0\right)=V^D\left(y^0,b^0,\hat{W}^0\right)\geq V^A\left(y^0,b^0,\hat{W}^0\right)=V^A\left(y^0,b^1,\hat{W}^0\right). \tag{46}$$

Under the EXIT regime a higher debt level implies less resources for consumption today, or for future periods.

$$\begin{aligned} V^E\left(y^0,b^0\right) &= \max_{b'}\left\{u\left(y^0-b^0+qb',\bar{c}^N\right)+\beta E\left[V^E\left(y',b'\right)\right]\right\} \\ &\geq u\left(y^0-b^0+q\tilde{b},\bar{c}^N\right)+\beta E\left[V^E\left(y',\tilde{b}\right)\right] \\ &> u\left(y^0-b^1+q\tilde{b},\bar{c}^N\right)+\beta E\left[V^E\left(y',\tilde{b}\right)\right] \\ &\equiv^{\tilde{b}} \max_{b'}\left\{u\left(y^0-b^1+qb',\bar{c}^N\right)+\beta E\left[V^E\left(y',b^1\right)\right]\right\} \\ &= V^E\left(y^0,b^1\right). \end{aligned} \tag{47}$$

Bringing together (42), (44), and (47) yields

$$V^D\left(y^0,b^1,\hat{W}^0\right)=V^D\left(y^0,b^0,\hat{W}^0\right)\geq V^E\left(y^0,b^0,\hat{W}^0\right)-\tilde{C}>V^E\left(y^0,b^1,\hat{W}^0\right)-\tilde{C}. \tag{48}$$

Similarly, under the UNION regime a higher debt level reduces overall welfare

$$\begin{aligned} V^U\left(y^0,b^0,\hat{W}^0\right) &= \max_{b'}\left\{u\left(y^0-b^0+qb',c^N\left(y^0-b^0+qb',\hat{W}^0\right)\right)+\beta E\left[V^U\left(y',b',\hat{W}'\right)\right]\right\} \\ &\geq u\left(y^0-b^0+q\tilde{b},c^N\left(y^0-b^0+q\tilde{b},\hat{W}^0\right)\right)+\beta E\left[V^U\left(y',\tilde{b},\hat{W}'\right)\right] \\ &> u\left(y^0-b^1+q\tilde{b},c^N\left(y^0-b^1+q\tilde{b},\hat{W}^0\right)\right)+\beta E\left[V^U\left(y',\tilde{b},\hat{W}'\right)\right] \\ &\equiv^{\tilde{b}} \max_{b'}\left\{u\left(y^0-b^1+qb',c^N\left(y^0-b^1+qb',\hat{W}^0\right)\right)+\beta E\left[V^U\left(y',b',\hat{W}'\right)\right]\right\} \\ &= V^U\left(y^0,b^1,\hat{W}^0\right). \end{aligned} \tag{49}$$

Bringing together (43), (44), and (49) yields

$$V^D\left(y^0, b^1, \hat{W}^0\right) = V^D\left(y^0, b^0, \hat{W}^0\right) \geq V^U\left(y^0, b^0, \hat{W}^0\right) > V^U\left(y^0, b^1, \hat{W}^0\right). \tag{50}$$