Minimizing the number of cancellations at the time of a severe lack of postanesthesia care unit beds or nurses

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Abstract

A deficiency of the postanesthesia care unit (PACU) beds or nurses may cause delays in the operating rooms (ORs) and increase the number of cancellations. Some disruptions like the COVID-19 pandemic may cause this deficiency. This paper investigates two integrated OR and PACU scheduling problems; one with few PACU beds, and the other with few PACU nurses. For each problem, a mathematical model and a matheuristic are proposed for minimizing the number of cancellations. To the best of our knowledge, it is the first study that investigates the implications of a severe lack of the PACU beds or nurses on the number of cancellations. The matheuristics hybridize the decomposition of each instance into some small-sized sub-instances with a variable neighborhood search algorithm. The main advantages of these methods are their flexibility to incorporate many problem details (such as a step-wise demand for the PACU nurses) and to solve any large-scale problem. Numerical results for a data set with 22 ORs show that with an increasingly severe lack of PACU capacity there is progressively greater benefit of the matheuristics than their initial solutions. Moreover, these results show the influence of the overtime and the recovery in ORs on improving the situation.

Keywords: Operating room scheduling, Post-anesthesia care unit, Bed, Nurse scheduling, Cancellation.

1. Introduction

The post-anesthesia care unit (PACU) is an immediate downstream unit for the operating rooms (ORs), in which patients are recovering after their surgeries. The PACU beds and nurses are two main resources in this unit. A deficiency in the PACU capacity may cause patients to be blocked in the ORs after their surgeries, and consequently the successor surgeries may be delayed or cancelled.

In terms of financial issues, it is not reasonable for a PACU to be a bottleneck in a hospital. However, because of some limitations in hospitals or some unexpected disruptions, it can happen. For example, at a hospital in Iran, both ORs and the PACU were located in one floor of a building, and there was a space restriction for the PACU. Also, at the time of renovation of the PACU, the number of available beds may decrease markedly. As an example of a disruption, Metzler et al. (2015) reported a large disruption for the PACU capacity of a major academic medical center in Boston, Massachusetts, USA, in Jan 2014. A sudden pipe burst resulted in the

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complete loss of a 66-bed combined preoperative and PACU. The 43 ORs were undamaged. Using the free capacity of other locations, some beds were gradually provided for the patient recovery during the week immediately following the crisis. This resulted in cancellations of 91%, 45.1%, 28.7%, 21.4%, and 15.3% of the scheduled surgical cases, respectively, for the five working days. As another example of a disruption, in case of outbreaks of infectious diseases, explosions, earthquakes, etc., some part of the PACU nursing staff may be unavailable (e.g., used to assure full intensive care unit (ICU) capacity). During the COVID-19 pandemic, some hospitals were forced to assign a part of the PACU capacity to patients overflowed from the ICU (Jarzebowski 2020), and PACU beds must be distanced reducing capacity (Dexter et al. 2020).

The PACU nursing is different from the OR, ward, etc. nursing, and needs special training. When patients are in ORs, the anesthesiologists recover the patients (Thenuwara et al. 2018). Some OR nurses pass the required training for the recovery of patients and assist. Surgery residents may also assist. Therefore, it is not easy to mitigate a lack of the PACU nurses by on-call nurses or other units' staff, and it may be impossible in severe unforeseen situations.

This study focuses on the surgery scheduling at the time of a short-term functional shortage of PACU beds or nurses to care for the day's surgical cases. The objective is to minimize the number of surgical cases cancellations. The elective surgeries are only considered, assuming that there is a dedicated capacity for the non-elective ones; the focus of study is the sequencing of cases within ORs dedicated to elective cases. The two following situations are investigated:

- few PACU beds and adequate PACU nurses;
- few PACU nurses and adequate PACU beds.

Two factors may decrease the implications of a lack of the PACU beds or nurses. First, patients can be recovered while blocking ORs. The recovery of patients can be interrupted and continued in the PACU whenever there is a free capacity in this unit. Second, OR staff and PACU nurses can be asked to work overtime as much as possible. We investigate the impact of these factors on decreasing the number of cancellations.

The integrated OR and PACU scheduling can be seen as a two-stage hybrid flow shop scheduling (HFS) problem. The two-stage HFS problem has been shown to be strongly NP-hard for the number of tardy jobs (Bulfin and M'Hallah 2003). Moreover, we have proved the NP-hardness of the surgery scheduling problem with one OR, one PACU bed, sufficient PACU nurses, and the possibility of recovery in ORs in Part A of a supplementary document.

The problems investigated in this paper generally happen in disrupted situations where quick decision making is needed. Therefore, we apply fast matheuristics for solving the problems. However, the mathematical model of the real problems with many ORs cannot be solved optimally in an acceptable time, but small-sized problems are solvable. Based on this fact, the matheuristic approaches decompose the problem into some small-sized sub-problems, and solve them separately. However, the way of decomposing the problem affects the resulting solution. Therefore, different ways of decomposing the problem are considered in these matheuristics using a variable neighborhood search algorithm (VNS) (Mladenović and Hansen 1997). For simplifying the solution approach and making it as fast as possible, we consider surgery and recovery times to be deterministic. Nevertheless, we suggest static rescheduling at the time of significant changes for surgery or recovery times. More details about this rescheduling can be found in Part B of the supplementary document.

The paper is organized as follows. In Section 2, the literature is reviewed, and in Section 3, the problems are described and their mathematical formulations are presented. Then in Section 4, matheuristics are proposed for solving the large-scale instances. In Section 5, the numerical results are reported. Finally, in Section 6, the conclusion and some suggestions for future research are stated.

2. Literature review

The literature on the integrated OR and PACU scheduling is far less extensive than that for OR scheduling. Dexter et al. (2001) studied the PACU nurse scheduling problem with the objective of minimizing the percentage of days with delay in the ORs. Each PACU nurse schedule was represented by the numbers of nurses working each of multiple shifts, differing with each other by their start times. The authors showed that the percentage of workdays with delay could be reduced from 56% to 24%. In our study, we use the same concept of nurse scheduling, but with different objectives and problem characteristics.

Dexter et al. (2006) expanded upon the method for determining the nurse schedule in the PACU when the ratio of the PACU nurses to patients for some patient types differs from 1 nurse: 2 patients for at least one part of their PACU stay (e.g., if tracheal intubation is required). In the current article, a stepwise demand for the PACU nurses is considered for each patient according to which the recovery of each patient has two phases: in the first phase, the ratio is 1 nurse per patient, and for the rest it is 1 nurse per 2 patients.

Khorasanian et al. (2018) reported some disruptions that may happen concerning the capacity of the PACU, and analyzed the chance of obtaining at least one delay in ORs at the time of the shortage of the PACU beds or nurses.

Similarly to the current paper, Jebali et al. (2006), Fei et al. (2010), Augusto et al. (2010), and Khlif Hachicha et al. (2018) assumed that patients can be recovered in the ORs while the PACU is not available. Nevertheless, these papers limited their consideration to the PACU beds, not to the nurses. In addition, the papers did not investigate sufficiently large disruptions that may cause the cancellation of some surgical cases.

Khorasanian and Moslehi (2017) investigated a two-stage flow shop scheduling problem with blocking, multi-task flexibility of the first stage, and preemption for minimizing the makespan. It has been inspired by the features of the OR scheduling problem with one OR (first stage) and one PACU bed (second stage). The OR has been considered a multi-task flexible stage because the recovery can also be done in it.

Unlike the current study, Latorre-Núñez et al. (2016) and Bam et al. (2017) did not consider the recovery in ORs: they delayed the start times of the surgeries in their model in order to avoid the patients to be blocked in the ORs when a free PACU bed is not available. Xiang et al. (2015) solved the integrated surgery and nurse scheduling problem considering all three stages of the pre-operative, intra-operative, and post-operative (including the PACU) with an ant colony optimization approach. The surgery process had to be postponed (similarly to Latorre-Núñez et al. (2016)) when any resource in this three-stage surgery flow was not available.

In another point of view, Hsu et al. (2003), Marcon and Dexter (2007), Cardoen et al. (2009b; Cardoen, Demeulemeester, and Beliën 2009a), and Fairley et al. (2019) minimized the peak number of patients in the PACU, constrained by not having any recovery in ORs.

Some studies have not considered the blocking time in the OR as a part of the recovery time (Lee and Yih 2014; Yin, Zhou, and Lu 2016; Bai, Storer, and Tonkay 2017; Pham and Klinkert 2008). Consequently, the recovery time of a patient in the PACU is assumed not to decrease if that patient has been blocked for a while in the OR after the surgery. However, in some countries like Japan, most hospitals (83.9%) have no PACU and patients regularly recover entirely in ORs (Sento et al. 2017).

The main contributions of this paper to the literature are stated in the following:

- It is the first study that addresses the implications of a severe lack of PACU beds on the number of cancellations.
- Almost all previous studies about integrated OR and PACU scheduling only considered the PACU beds, not the PACU nurses. The PACU nurses add some special characteristics to the problem such as a variable demand for PACU nurses during the recovery of each patient and a limited shift length for each PACU nurse that is less than the opening hours of the PACU.
- Fast matheuristics are presented which are capable of solving large-scale problems.

3. Problem description and formulation

Consider a situation in which patients have been assigned to a specific date of surgery. All patients can reasonably expect to receive care on the planned date. With the normal capacity of the PACU, which is called the baseline capacity, delay in admission of patients from OR to the PACU would rarely occur. However, there is an unexpected, large disruption before that day causing the capacity of the PACU beds and/or nurses available to care for OR patients to be less than the baseline capacity. This disruption is large enough to have some surgical cases cancelled and/or to cause patients to be blocked in ORs. The two following problems are modeled and analyzed in this study:

- SS_PB: Surgery scheduling considering few PACU beds and adequate PACU nurses for minimizing the number of cancellations;
- SS_PN: Surgery scheduling and PACU nurse scheduling considering few PACU nurses and adequate PACU beds for minimizing the number of cancellations.

In our models, only elective patients are considered. In addition, each OR represents a single surgeon's list of patients (Samudra et al. 2017; Gul et al. 2011; Santibáñez, Begen, and Atkins 2007; van Essen et al. 2012). High productivity surgical suites have most operating rooms each with a single surgeon, as shown by Sulecki et al. (Sulecki et al. 2012). Therefore, each patient has surgery in only one predetermined OR which is called that patient's original OR. The other ORs are named non-original ORs for that patient.

There are two durations, namely "surgery time" and "recovery time". We refer to the "surgery time" of a patient as the sum of the durations of all tasks in the OR such as anesthesia and surgery, except the clean-up. The "recovery time" denotes the duration of the recovery needed after the surgery which is generally done in the PACU. However, the recovery of patients can also be done in ORs by eligible OR staff, especially when there is not enough PACU capacity. The recovery of each patient can be started immediately after the surgery in the same OR. This recovery can be interrupted and continued in a PACU bed or a non-original OR. Unlike the surgery that must be done by a prespecified surgeon, the recovery of each patient can be done by any recovery expert such as available nurses in the PACU and anesthesiologists in the ORs. The PACU beds are also shared among all ORs. The total recovery time for each patient is the same regardless of the location. We have given an example in Part C of the supplementary document showing the advantage of recovering a patient in an OR even when a PACU bed is available for the patient.

Between the finish time of the surgery and the finish time of the recovery, each patient is transported at most once. We inferred this assumption based on the regular patient flow in the hospitals. The possibility of more such transportations makes the model more complex and does not make sense in terms of safety issues for patients. Therefore, the recovery of patients can be done: (i) completely in the PACU, or (ii) completely in the original OR or a non-original OR, or (iii) a part in the original OR and the rest in the PACU or a non-original OR. In addition, the recovery of at most one patient at a time can be done in an OR in which there is no surgery being performed.

Each OR is opened at a specific time and is closed at its "maximum allowable opening time". For example, if the shift length of an OR is 8 h and at most 2 h overtime is possible, then the maximum allowable opening time for that OR is 10 h, assuming it is opened at time zero. The PACU is also opened at time zero, and is available until its maximum allowable opening time. A case is cancelled if its surgery and/or recovery cannot be completed with the available resources in the ORs and PACU.

After the exit of each patient from his/her original OR, there is a clean-up time with a fixed value. However, after the recovery of each patient in one of his/her non-original ORs, no considerable clean-up time is considered in that OR. In addition, the clean-up time after the departure of the last patient from an OR is not included in the available time of the ORs.

The symbols used in the mathematical formulations are defined below:

Indexes and sets

h	Stages, h = 1, 2, with 1 being the ORs, and 2 the PACU
$j,k \in \hat{J}$	Patients (surgical cases), $j=1,...,J$, with $\hat{J}$ being the set of all patients, and J being the total number of patients assigned to all ORs
$\hat{J}_i$	Set of patients assigned to OR *i*, with J_i being the number of these patients
i	ORs, $i=1,...,I$, with I being the number of ORs
b	PACU beds, $b=1,...,B$, with B being the number of PACU beds
f	PACU nurses' shifts, $f=1,...,F$, with F being the number of shifts
l	PACU nurses, $l=1,...,L$, with L being the number of PACU nurses
t	Time periods, $t=0,...,T-1$, with T being the number of time units in a day

Parameters

$p_{h,j}$	Processing time of the *h*-th task of Patient *j*
$p_h^{i,(r)}$	*r*-th smallest $p_{h,j}$ among all patients assigned to OR *i*,
$x_{i,j}$	1, if OR *i* is the original OR for Patient *j*, otherwise 0
s_i^O	Regular shift length of OR *i* (without considering overtime)
s^P	Regular shift length (opening hours) of the PACU
s^N	Regular shift length for the PACU nurses
o_i^O	Maximum possible overtime for OR staff in OR *i*
o^N	Maximum possible overtime for the PACU nurses
$a_{f,t}$	1, if shift *f* for the PACU nurses includes the time period of *t*, otherwise 0
γ_j	The duration of the first phase of the recovery (with a 1:1 ratio of the number of nurses to patients) of Patient *j*
clt	Clean-up time between the departure of a patient from the original OR and the next surgery
σ_i^O	The earliest start time of a surgery/recovery in OR *i*
σ_b^P	The earliest start time of a recovery in the PACU bed b
M	A very big positive value

Variables

Z_j	1, if Patient *j* is canceled; otherwise, 0
$D_{1,j}$	Departure time of Patient *j* from his/her original OR
$D_{2,j}$	Departure time of Patient *j* from the system, or equivalently, completion time of his/her recovery
$Y_{j,k}$	1, if Patient *j* precedes Patient *k*, otherwise 0
α_j	A portion of the recovery of Patient *j* which is done in his/her original OR
$X_{i,j}^{O}$	1, if a portion of the recovery of Patient *j* is done in OR *i* which is non-original for this patient, otherwise 0
$X_{b,j}^{P}$	1, if a portion of the recovery of Patient *j* is done in the PACU bed *b*; otherwise 0
Q_j	1, if a portion of the recovery of Patient *j* is done in the PACU; otherwise 0
$E_{t,j}^{(1)}$	1, if, in time period *t*, the first part of the recovery of Patient *j* is done in the PACU, otherwise 0
$E_{t,j}^{(2)}$	1, if, in time period *t*, the second part of the recovery of Patient *j* is done in the PACU, otherwise 0
ρ_f	Number of PACU nurses with Shift *f*
U_t	Number of PACU nurses available in time period *t*
Q'_t	1, if $\sum_{j\in J}(E_{t,j}^{(1)}+E_{t,j}^{(2)})>0$, otherwise 0

3.1. A mathematical model for the SS_PB problem

In order to evaluate the impact of a lack of the number of PACU beds on the number of cancellations, the SS_PB problem in which there are few PACU beds but adequate PACU nurses is investigated. The mathematical formulations of this problem are stated in the following. The objective function stated in (1) minimizes the number of cancellations.

$$\text{Min } obj=\sum_{j=1}^{J} Z_j \tag{1}$$

Constraint set (2) determines a lower bound for the departure time of each patient from the OR. It avoids a zero-departure time for the first patient of each OR.

$$D_{1,j}\ge\sum_{i=1}^{I} x_{i,j}\sigma_i^{O}+p_{1,j}+\alpha_j p_{2,j},\ j=1,\dots,J \tag{2}$$

Constraint set (3) demonstrates the relationship between the departure time from the OR and the completion time of the recovery for each patient.

$$D_{2,j}-D_{1,j}=(1-\alpha_j)p_{2,j},\ j=1,\dots,J \tag{3}$$

According to constraint sets (4) and (5), each patient can be assigned to at most one non-original OR for the recovery.

$$\sum_{i=1}^{I} X^{O}_{i,j} \leq 1,\ j=1,\ldots,J \tag{4}$$

$$\sum_{i=1}^{I} x_{i,j} X^{O}_{i,j} = 0,\ j=1,\ldots,J \tag{5}$$

The four following sets of constraints are needed to avoid interferences of different tasks in the ORs:

- Constraint set (6) avoids the interference of two surgeries in each OR
- Constraint set (7) does the same job for a recovery in a non-original OR and a surgery coming after it in the same OR
- Constraint set (8) applies for a surgery and a recovery in a non-original OR coming after that surgery in the same OR
- Constraint set (9) considers two recoveries in a non-original OR

$$D_{1,j} + M(2 + Y_{j,k} - x_{i,j} - x_{i,k}) \geq D_{1,k} + clt + p_{1,j} + \alpha_j p_{2,j}, i=1,\ldots,I,\ j,k=1,\ldots,J \tag{6}$$

$$D_{1,j} + M(2 + Y_{j,k} - x_{i,j} - X^{O}_{i,k}) \geq D_{2,k} + p_{1,j} + \alpha_j p_{2,j}, i=1,\ldots,I,\ j,k=1,\ldots,J \tag{7}$$

$$D_{2,j} + M(2 + Y_{j,k} - X^{O}_{i,j} - x_{i,k}) \geq D_{1,k} + clt + (1-\alpha_j) p_{2,j}, i=1,\ldots,I,\ j,k=1,\ldots,J \tag{8}$$

$$D_{2,j} + M(2 + Y_{j,k} - X^{O}_{i,j} - X^{O}_{i,k}) \geq D_{2,k} + (1-\alpha_j) p_{2,j}, i=1,\ldots,I,\ j,k=1,\ldots,J \tag{9}$$

Each patient is assigned to a PACU bed in (10). It does not mean that each patient will definitely go to the PACU, but if so, he/she will be served in this specific bed.

$$\sum_{b} X^{P}_{b,j} = 1,\ j=1,\ldots,J \tag{10}$$

If $1-\alpha_j > 0$ and $\sum_i X^{O}_{i,j} = 0$, then a portion of the recovery of Patient j will definitely be done in the PACU. According to constraints (11), we have $Q_j = 1$, if a portion of the recovery of Patient j is performed in the PACU.

$$(1-\alpha_j) - \sum_i X^{O}_{i,j} \leq MQ_j,\ j=1,\ldots,J \tag{11}$$

Constraint set (12) ensures the non-interference of patients in the PACU beds.

$$D_{2,j} + M(3 + Y_{j,k} - X^{P}_{b,j} - X^{P}_{b,k} - Q_j) \geq D_{2,k} + (1-\alpha_j) p_{2,j}, b=1,\ldots,B,\ j,k=1,\ldots,J \tag{12}$$

The constraint sets (13), (14) and (15), respectively, ensure that the departure time of a non-cancelled case from the original OR, a non-original OR, and the PACU must not exceed the corresponding maximum allowable opening time.

$$D_{1,j} - \sum_i (s_i^O + o_i^O) x_{i,j} \leq MZ_j, \; j = 1, ..., J \tag{13}$$

$$D_{2,j} - \sum_i (s_i^O + o_i^O) X_{i,j}^O \leq MZ_j + M(1 - \sum_i X_{i,j}^O), \; j = 1, ..., J \tag{14}$$

$$D_{2,j} - s^P \leq MZ_j + M(1 - Q_j), \; j = 1, ..., J \tag{15}$$

The expressions (16)-(21) state the type and range defined for the variables.

$$0 \leq \alpha_j \leq 1, \; j = 1, ..., J \tag{16}$$

$$Y_{j,k} \in \{0,1\}, \; j, k = 1, ..., J \tag{17}$$

$$X_{i,j}^O \in \{0,1\}, i = 1, ..., I, \; j = 1, ..., J \tag{18}$$

$$X_{b,j}^P \in \{0,1\}, b = 1, ..., B, \; j = 1, ..., J \tag{19}$$

$$Q_j, Z_j \in \{0,1\}, \; j = 1, ..., J \tag{20}$$

$$D_{1,j}, D_{2,j} \geq 0, \; j = 1, ..., J \tag{21}$$

A modified form of this model should be considered in case of having no PACU bed, in which the constraint set $Q_j = 0, \; j = 1, ..., J$ is added, and (10), (12), (15), and (19) are omitted.

3.2. A mathematical model for the SS_PN problem

A mathematical model is presented for the SS_PN problem in this section. It is assumed that each PACU nurse can care for all types of patients in the unit. The number of available PACU nurses is known before the start of the day. There are some predefined shifts during the day for PACU nurses that differ from one another in terms of their start times. There are no preferences for the available PACU nurses about their shifts. In the solution obtained for the problem, it should be decided how many nurses are assigned to each shift.

Patients are more likely to incur medical difficulties as they begin to emerge from anesthesia than later in their recovery (Epstein et al. 2014). For the initial part of the recovery with a specific duration, it is necessary to have one nurse caring exclusively for that patient. After that initial part of the recovery, patients can be monitored by a nurse who is simultaneously watching another similar patient (Haret, Kneeland, and Ho 2012). Furthermore, at least two nurses are needed whenever there is at least one patient in the PACU (Dexter, Epstein, and Penning 2001).

The PACU nurses and/or OR staff may be able to stay longer than their regular shift length because of the disruption, the amount of which for each staff is limited and known. For example, with the shift length of a PACU nurse being equal to 8 hours (h), he/she can work for two more hours, i.e., a total of 10 h a day.

In addition to constraints (1)-(9), (11), (13)-(15), (16)-(18), (20), (21), the model of the SS_PN includes the following constraints.

The duration of the recovery of Patient j, $j \in J$ in the PACU is equal to $\sum_t (E_{t,j}^{(1)} + E_{t,j}^{(2)})$. According to (22), if part of the recovery of Patient j, $\forall j$ is done in the PACU (i.e., $Q_j = 1$), then it would be equal to $(1-\alpha_j)p_{2,j}$. Otherwise, based on (23), it must be equal to zero.

$$\sum_t (E_{t,j}^{(1)} + E_{t,j}^{(2)}) + M(1-Q_j) \geq (1-\alpha_j)p_{2,j},\ j = 1,\ldots,J \tag{22}$$

$$\sum_t (E_{t,j}^{(1)} + E_{t,j}^{(2)}) \leq MQ_j,\ j = 1,\ldots,J \tag{23}$$

According to constraints (24) and (25), $E_{t,j}^{(1)}$, $j \in J$ is equal to one if $\gamma_j > \alpha_j p_{2,j}$ and $t \in \{D_{1,j},\ldots,D_{1,j} - \alpha_j p_{2,j} + \gamma_j - 1\}$ (i.e., the start and end times of the first part of the recovery in the PACU are respectively $D_{1,j}$ and $D_{1,j} - \alpha_j p_{2,j} + \gamma_j$), and otherwise it is equal to zero (i.e., the whole of the first part of the recovery is done in the OR).

$$tE_{t,j}^{(1)} + M(1-E_{t,j}^{(1)}) \geq D_{1,j}, t = 0,\ldots,T-1,\ j = 1,\ldots,J \tag{24}$$

$$tE_{t,j}^{(1)} \leq D_{1,j} - \alpha_j p_{2,j} + \gamma_j - 1, t = 0,\ldots,T-1,\ j = 1,\ldots,J \tag{25}$$

Constraint sets (26), (27), and (28) determine the value of $E_{t,j}^{(2)}, \forall t, j$. According to these constraints, $E_{t,j}^{(2)}$, $j \in J$ is equal to one if $\max(D_{1,j} - \alpha_j p_{2,j} + \gamma_j, D_{1,j}) < D_{1,j} + (1-\alpha_j)p_{2,j} - 1$ and $t \in \{\max(D_{1,j} - \alpha_j p_{2,j} + \gamma_j, D_{1,j}),\ldots,D_{1,j} + (1-\alpha_j)p_{2,j} - 1\}$, and otherwise it is equal to zero.

$$tE_{t,j}^{(2)} + M(1-E_{t,j}^{(2)}) \geq D_{1,j} - \alpha_j p_{2,j} + \gamma_j, t = 0,\ldots,T-1,\ j = 1,\ldots,J \tag{26}$$

$$tE_{t,j}^{(2)} + M(1-E_{t,j}^{(2)}) \geq D_{1,j}, t = 0,\ldots,T-1,\ j = 1,\ldots,J \tag{27}$$

$$tE_{t,j}^{(2)} \leq D_{1,j} + p_{2,j}(1-\alpha_j) - 1, t = 0,\ldots,T-1,\ j = 1,\ldots,J \tag{28}$$

Constraint set (29) ensures that the number of patients recovered in the PACU in each time period is not greater than the number of PACU beds.

$$\sum_{j \in J} (E_{t,j}^{(1)} + E_{t,j}^{(2)}) \leq B, t = 0,\ldots,T-1 \tag{29}$$

According to (30), the total number of PACU nurses assigned to different shifts is equal to the number of available PACU nurses.

$$\sum_f \rho_f = L \tag{30}$$

Constraint set (31) determines the number of PACU nurses available in each time period for each nurse schedule.

$$\sum_f a_{f,t}\rho_f = U_t, t = 0,...,T-1 \tag{31}$$

Constraint set (32) satisfies the condition whereby each nurse can take care of one and two patients at a time respectively in the first and second parts of the recovery of patients.

$$U_t \geq \sum_j E_{t,j}^{(1)} + 0.5\sum_j E_{t,j}^{(2)}, t = 0,...,T-1 \tag{32}$$

According to constraints (33) and (34), if, in time period *t*, the recovery of a patient is done in the PACU, at least two nurses must be available in the PACU during this period.

$$\sum_j (E_{t,j}^{(1)} + E_{t,j}^{(2)}) \leq MQ'_t, t = 0,...,T-1 \tag{33}$$

$$U_t + M(1-Q'_t) \geq 2, t = 0,...,T-1 \tag{34}$$

The expressions (35)-(37) state the type and range defined for the variables.

$$E_{t,j}^{(1)}, E_{t,j}^{(2)} \in \{0,1\}, t = 0,...,T-1, j = 1,..., J \tag{35}$$

$$Q'_t \in \{0,1\}, t = 0,...,T-1 \tag{36}$$

$$\rho_f, U_t \in \{0,1,..., L\}, f = 1,..., F, t = 0,...,T-1 \tag{37}$$

For incorporating the overtime for the PACU nurses, the shift length for each nurse would become equal to $s^N + o^N$, and the values of $a_{f,t}$ should change accordingly.

The Part D of the supplementary document describes examples for the optimal solutions of the SS_PB and SS_PN problems.

4. Matheuristics

In this section, a matheuristic is presented for each of the SS_PB and SS_PN problems. The main idea behind these methods is to decompose a large-scale problem into some small-sized solvable sub-problems, and to find a solution for the problem by aggregating the solutions obtained for all sub-problems. In these methods, different ways of decomposing each problem to sub-problems are searched by a modified VNS algorithm for obtaining a high-quality solution. For simplicity, we call sub-problem(s) as sub(s) from now on.

In the following, the symbols used for characterizing the matheuristics are defined.

Indexes

v	Subs, $v = 1,...,V$, with V being the number of subs

Parameters

$I^{sub,ini}$	The maximum number of ORs that can be assigned to a sub in the initial solution
tl^{sub}	Time limit for solving each sub of an instance
tl^{total}	Time limit for solving an instance

Variables

$obj(\pi)$	Number of cancellations for the solution π
$obj_v(\pi)$	Number of cancellations for sub v in solution π
I_v^{sub}	Number of ORs assigned to the v-th sub
B_v^{sub}	Number of PACU beds assigned to the v-th sub
L_v^{sub}	Number of PACU nurses assigned to the v-th sub
$\bar{U}_{t,v}$	Number of nurses available in time period t considering the solutions obtained for subs 1 to v
$\bar{U}'_{t,v}$	Number of free nurses available in time period t considering the solutions obtained for subs 1 to v
θ_i	Index value for OR i

Mathematical operators

$\lfloor x \rfloor$	The largest integer number which is not greater than x
$\lceil x \rceil$	The smallest integer number which is not smaller than x

4.1. Method for the SS_PB

A matheuristic, called DV^B, is proposed in this section for solving the SS_PB. In this approach, a solution is generated for each instance of the problem in two steps. First, the instance is decomposed into some small-sized subs. This decomposition is done simply by partitioning the available ORs and PACU beds into some subs. Fig. 1 shows a decomposition of an instance with 10 ORs and 3 PACU beds to three subs. In the second step, the objective function of this decomposition is calculated. The objective function of a decomposition is equal to the sum of the objective functions of all subs solved independently from each other.

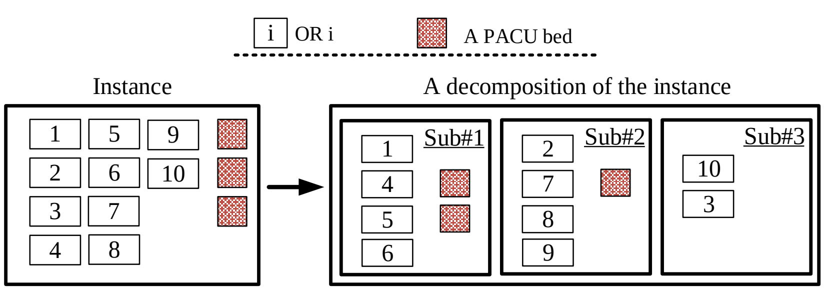

Fig. 1. An example decomposition for the SS_PB problem

In the DV^B, different ways of decomposing the instance are investigated in a VNS framework in order to find a good heuristic solution. For this purpose, first an initial solution is generated and is considered as the current solution. Then, the current solution is improved in a loop with three subsequent phases of "shaking", "local search", and "neighborhood change". This loop is repeated until a time limit is met or a solution with no cancellations is obtained. In the shaking phase, a neighbor is generated using the w-th neighborhood structure, and the local search phase tries to improve it subsequently. The initial value of w is considered equal to 1. In the neighborhood change phase, if $obj(\pi_{loc}) \le obj(\pi_{cur})$, then $\pi_{cur} = \pi_{loc}$. In addition, if $obj(\pi_{loc}) < obj(\pi_{cur})$, then $w = 1$, and otherwise, $w = w + 1$ (if $w > w^{\max}$, then $w = 1$).

To generate an initial solution, a heuristic approach is proposed in Algorithm 1. In this heuristic, the number of subs is determined considering the value of the maximum number of ORs that can be assigned to each sub in the initial solution (i.e., $I^{sub,ini}$, that is a parameter and should be tuned). Then, ORs are assigned to these subs according to a non-increasing order of an index value defined for each of them. The index value of each OR is equal to the least amount of the total recovery times of that OR's patients that must be performed in the PACU or a non-original OR. The algorithm of the initial solution generation for the SS_PB is stated in the following.

Algorithm 1. Initial solution for the SS_PB problem

Step 1: The number of subs is equal to:

$$V = \lceil I / I^{sub,ini} \rceil \tag{38}$$

Step 2: Calculate the $\theta_i, \forall i$:

$$\theta_i = \sum_{j \in \hat{J}_i} p_{2,j} - (s_i^O + o_i^O - \sum_{j \in \hat{J}_i} p_{1,j}) \tag{39}$$

Step 3: Sort the ORs according to the non-increasing order of θ_i values, and assign them to the subs one by one in this order, so that the 1st OR is assigned to sub 1, …, the V-th OR to sub V, the (V + 1)-th OR to sub 1, and so on.

Step 4: Assign the PACU beds to the subs one by one so that the 1st bed is assigned to sub 1, …, the V-th bed to sub V, the (V + 1)-th bed to sub 1, and so on. Stop.

The solution obtained by this algorithm is called the current solution (π_{cur}). Three neighborhood structures are used in the DV^B:

- OR-Insert: an OR is removed from a sub and is assigned to another sub;
- OR-Swap: two ORs not assigned to the same sub are swapped;
- Bed-Insert: one PACU bed is removed from a sub with non-zero PACU beds and is assigned to another one.

An example about these neighborhood structures has been provided in Part E of the supplementary document.

In the local search phase, first, two subs in the output solution of the shaking phase, i.e. π_{nei}, are selected randomly. Then, each two ORs assigned to these subs are swapped. The best neighbor is selected as the output of this phase and is called π_{loc}.

4.2. Method for the SS_PN

The matheuristic presented for the SS_PN is called DV^N. This method is similar to the DV^B but includes a few changes because of the focus on PACU nurses instead of PACU beds.

The initial solution of the DV^N is generated by the following algorithm.

Algorithm 2. Initial solution for the SS_PN problem

Step 1: Calculate the number of subs using (38), and the $\theta_i, \forall i$ using (39).

Step 2: Sort the ORs according to the non-increasing order of θ_i values, and assign them to the subs one by one in this order, so that the 1st OR is assigned to sub 1, …, the V-th OR to sub V, the (V + 1)-th OR to sub 1, and so on.

Step 3: First, assign two PACU nurses to sub 1. Then, assign the PACU nurses left one by one to the next subs so that the 3rd PACU nurse is assigned to sub 2, the 4th PACU nurse to sub 3, …, the (V + 1)-th PACU nurse to sub V, the (V + 2)-th PACU nurse to sub 1, and so on. Stop.

In general, a ratio of 1.5 to 2 has been suggested for the number of PACU beds to the number of ORs (Metzler et al. 2015; Haret, Kneeland, and Ho 2012). Because there is no PACU bed limitation for the SS_PN, the number of PACU beds in each sub is considered to amount to twice its number of ORs.

The solution obtained by this algorithm is considered as the current solution. Then, a loop with two phases of “shaking” and “neighborhood change” is performed for improving the current solution. There is no local search phase in this approach because it was very time-consuming, and did not improve the performance of the solution approach.

As described in the problem description, there should be at least 2 PACU nurses when there is a patient in the PACU. Then, if we solve subs independently, we may have at least 2 PACU nurses in a sub during a period of time, but we do not consider them for the other subs at the same period of time. Subs are solved one after another. After solving each sub, we will know the start time and finish time of its PACU nurses and their free capacity. Therefore, we can use this information for improving the solution of the next sub we solve. The following algorithm explains the procedure of calculating the objective function of a decomposition for the SS_PN.

Algorithm 3: Objective calculation of a decomposition π for the SS_PN

Step 1: Re-number subs so that the lower the number of the sub, the larger is the number of its PACU nurses, and let $v=1$, and $\bar{U}_{t,0}, \bar{U}'_{t,0} = 0, \forall t$.

Step 2: Solve a modified version of the model of SS_PN for sub v in which two constraints of (32) and (34) are replaced with respectively (40) and (41).

$$U_t + \bar{U}'_{t,v-1} \geq \sum_j E^{(1)}_{t,j} + 0.5 \sum_j E^{(2)}_{t,j}, \forall t \tag{40}$$

$$U_t + \bar{U}_{t,v-1} + M(1-Q'_t) \geq 2, \forall t \tag{41}$$

Step 3: Calculate $\bar{U}_{t,v}, \bar{U}'_{t,v}, \forall t$ using (42) and (43):

$$\bar{U}_{t,v} = \bar{U}_{t,v-1} + U_t, \forall t, v \tag{42}$$

$$\bar{U}'_{t,v} = \bar{U}'_{t,v-1} + U_t - (\sum_j E^{(1)}_{t,j} + 0.5 \sum_j E^{(2)}_{t,j}), \forall t, v \tag{43}$$

Step 4: If $v < V$, then $v = v+1$; otherwise, let $obj(\pi) = \sum_v obj_v(\pi)$, and stop.

The shaking phase for the DV^N is similar to that of DV^B but with “nurse-insert” instead of “bed-insert”. The “nurse-insert” neighborhood structure always generates a neighbor in which there is at least one sub with at least 2 PACU nurses. We know that the initial solution satisfies this condition. According to this neighborhood structure, one PACU nurse is removed from a sub v_1 with non-zero PACU nurses and is assigned to another sub of v_2. If there is not at least one sub with at least 2 PACU nurses in the neighbor generated, we can conclude that the sub v_1 has had the maximum number of PACU nurses (equals to 2) among all subs. Therefore, for satisfying our condition, we repeat the same movement; i.e., one PACU nurse is removed from the sub v_1 and is assigned to the sub v_2.

Because the number of ORs changes after applying the “OR-insert”, we should update the number of PACU beds assigned to the subs according to the new number of ORs assigned to them. The number of PACU beds must be equal to two times the number of ORs.

5. Numerical experiments

In this section, the matheuristics presented for the SS_PB and SS_PN problems are evaluated. The models of all instances have been solved by CPLEX 12.8. All algorithms have been coded in C#. All evaluations have been performed on a PC with Intel(R) Xeon(R) CPU E5-2620 0 @ 2.00 GHz, and 32 GB RAM.

5.1. Experimental design

We have generated random instances based on the characteristics of the data of the university hospital in Leuven (Belgium) that was described in detail by Samudra et al. (2017). There were 13 disciplines and 22 ORs for elective surgeries. The mean and standard deviation of the surgery times for each discipline can be found in the aforementioned paper. The number of ORs and the regular shift length of each OR for each discipline have been reported in Part F of the supplementary document. In order to have homogeneous instances, we have considered a fixed number of ORs for each discipline which was roughly calculated proportional to its average number of arrivals reported in Samudra et al. (2017).

Because there is no information about the recovery times in Samudra et al. (2017), the recovery time of each surgical case is generated based on a lognormal distribution with a mean equal to its surgery time minus 10 minutes (min), and a standard deviation of 15 min (Jebali, Hadj Alouane, and Ladet 2006; Latorre-Núñez et al. 2016). The clean-up time between surgeries in the OR is set as a fixed value of 0.5 h (Dexter et al. 2005).

Moreover, all ORs and the PACU are opened at time zero, which is 8:00 AM. The regular opening hours of the PACU is considered equal to 16 h (i.e., s^P = 16 h). Therefore, the PACU is closed at 12:00 AM (midnight).

For each OR, surgical cases are randomly generated and assigned to it until the next one does not fit. The surgery time of each surgery case is obtained using a lognormal distribution with parameters calculated based on the mean and standard deviation of the surgery times in the corresponding discipline. The sum of the surgery times and the clean-up times of the surgical cases assigned to an OR must be less than or equal to its regular shift length (Marcon and Dexter 2007).

5.2. Results of the solution method for the SS_PB problem

In order to tune the parameter $I^{sub,ini}$ for the DV^B, two random instances were solved in three repetitions for different combinations of $B \in \{0,4,8,12,16,20\}$ and $I^{sub,ini} \in \{2,3,4,5,6,7,8\}$. The time limit parameters were $tl^{total} = I^2$ and $tl^{sub} = 0.2(I_v^{sub})^2, \forall v$. Fig. 2 displays the average number of cancellations for different values of $I^{sub,ini}$, where the best value has been achieved by

$I^{sub,ini} = 5$. This best value depends on the allowed time limit, and significantly larger time limits may increase it.

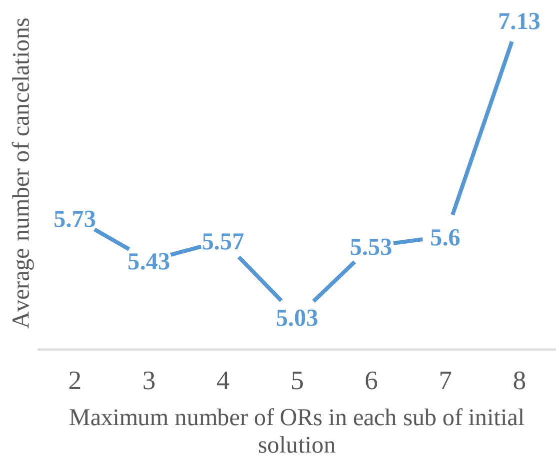

Fig. 2. Tuning the $I^{sub,ini}$ for the DV^{B} method

The performance of the DV^{B} was investigated for the following three scenarios:

- no recovery in ORs and no overtime for the ORs;
- recovery in ORs is allowable but no overtime for the ORs;
- recovery in ORs is allowable, and 2 h overtime can be used in each OR.

For each scenario, an identical set of 10 instances were solved using the DV^{B} in three repetitions considering $B \in \{0, 2, 4, ..., 20\}$, $I^{sub,ini} = 5$, $tl^{total} = I^{2}$ and $tl^{sub} = 0.2(I_{v}^{sub})^{2}, \forall v$. Because the mathematical model for the SS_PB by default considers the recovery in ORs, for checking the first scenario, we should slightly modify this model. For this purpose, the two following constraints are added to this model: $\alpha_{j} = 0, j = 1, ..., J$, and $X_{i,j}^{O} = 0, i = 1, ..., I, j = 1, ..., J$.

Table 1 shows the results of the DV^{B} for the scenario of the no recovery in ORs and no overtime for the ORs. In this scenario, there is no need to solve instances with zero PACU beds because there is no place for the recovery and all patients must be cancelled. In this table, the average number of cancellations and the average execution time for both the initial solution of the DV^{B} (Algorithm 1) and DV^{B}'s solution have been reported. The DV^{B}'s solution has 1.57 cancellations less than its initial solution on average. The fewer the numbers of PACU beds, the greater the difference from the initial solution. Because no cancellations occurred for any of the 10 instances with 18 PACU beds, we removed the row with 20 PACU beds.

Table 1. Results of the DV^B for the scenario of the no recovery in ORs and no overtime for the ORs

Num. PACU beds	Initial solution: Ave. num. canc.	Initial solution: Ave. exe. time (s)	DV^B: Ave. num. canc.	DV^B: Ave. exe. time (s)
2	42.10	5.22	38.43	484.00
4	29.63	7.19	27.00	484.00
6	19.90	7.95	18.07	484.00
8	13.37	8.58	11.37	484.00
10	6.90	9.19	5.77	484.00
12	3.50	4.96	2.27	467.65
14	1.70	3.40	0.63	289.99
16	0.50	1.66	0.03	70.35
18	0.10	1.12	0.00	2.36

In Table 2, the results of the DV^B for the scenario of the allowable recovery in ORs and no overtime for the ORs have been displayed. Comparing this table with Table 1, we can obviously notice the significant effect (p-value<0.01) of the recovery in ORs on decreasing the number of cancellations. For example, with 10 PACU beds, number of cancellations decreases from 5.77 to 1.30 in average.

Table 2. Results of the DV^B for the scenario of the allowable recovery in ORs and no overtime

Num. PACU beds	Initial solution: Ave. num. canc.	Initial solution: Ave. exe. time (s)	DV^B: Ave. num. canc.	DV^B: Ave. exe. time (s)
0	15.00	12.12	13.17	484.00
2	11.90	9.85	10.23	484.00
4	9.00	8.08	7.73	484.00
6	6.37	8.19	5.13	484.00
8	4.60	9.33	3.27	484.00
10	2.57	9.50	1.30	449.89
12	1.40	6.62	0.60	329.52
14	0.70	6.72	0.20	179.42
16	0.20	4.51	0.00	44.37

Table 3 gives the results of the DV^B for the scenario of the allowable recovery in ORs and 2 h overtime for the ORs. It can be seen in this table that the overtime in the ORs along with the allowable recovery in ORs can highly be influential (p-value<0.01) in decreasing the number of cancellations. For example, with 10 PACU beds, 2 h overtime in ORs has resulted in an average decrease of 1.3 in comparison with Table 2, and therefore no cancellations.

Fig. 3 depicts a comparison between different scenarios in terms of the average number of cancelations obtained by the DV^B for different number of PACU beds.

Table 3. Results of the DV^B for the scenario of the allowable recovery in ORs and 2 h overtime for each OR

	Initial solution		DV^B	
Num. PACU beds	Ave. num. canc.	Ave. exe. time (s)	Ave. num. canc.	Ave. exe. time (s)
0	8.60	9.47	7.43	484.00
2	5.90	7.54	4.73	484.00
4	3.17	6.21	2.00	473.78
6	1.67	5.49	0.60	327.04
8	0.80	3.38	0.07	99.59
10	0.00	1.40	0.00	1.40

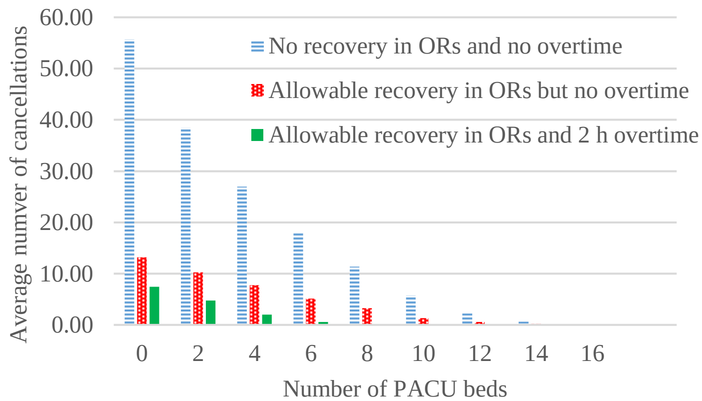

Fig. 3. Average number of cancellations obtained by the DV^B for different scenarios

5.3. Results of the solution method for the SS_PN problem

This section investigates the implications of a lack of the PACU nurses assuming that PACU beds are not restricting resources. For this purpose, it is assumed that there are adequately many PACU beds, namely twice the number of ORs which is equal to 44 (Metzler et al. 2015; Haret, Kneeland, and Ho 2012). Assume that the regular shift length of each PACU nurse is equal to 8 h. Moreover, there are 5 shift types for PACU nurses with different start times of 8:00 AM, 10:00 AM, 12:00 PM, 02:00 PM, and 04:00 PM. Moreover, the duration of the first phase of the recovery with a 1:1 ratio of the number of nurses to patients for each patient is considered equal to 15 min, i.e. $\gamma_j = 15$ min, $\forall j = 1,\ldots,J$ (Haret, Kneeland, and Ho 2012). Each time unit is considered equal to 15 min. All time values (in hours) are transformed to this unit by multiplying them by 4, and then rounding them up.

For tuning the parameter of $I^{sub,ini}$ for the DV^N, two random instances were solved in three repetitions for different combinations of $L \in \{0,2,4,6,8,10,12\}$ and $I^{sub,ini} \in \{2,3,4,5,6,7,8\}$. The time limit parameters were $tl^{total} = 2I^2$ and $tl^{sub} = (I_v^{sub})^2, \forall v$. Fig. 4 shows the average number of cancellations for different values of $I^{sub,ini}$, where the minimum value has been obtained by $I^{sub,ini} = 5$.

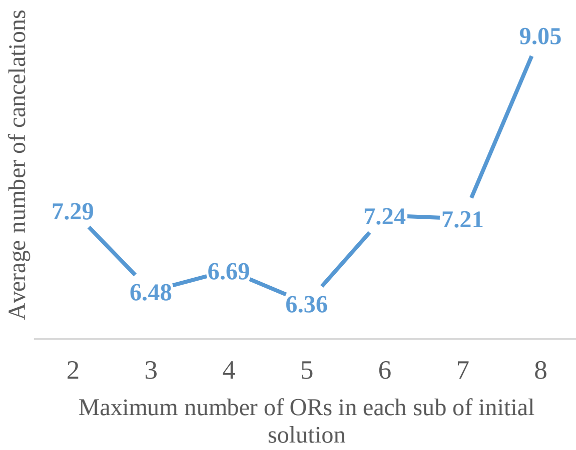

Fig. 4. Tuning the $I^{sub,ini}$ for the DV^N

The performance of the DV^N was investigated for the three following scenarios:

- no overtime for the ORs and PACU nurses;
- 2 h overtime for each OR but no overtime for the PACU nurses;
- no overtime for the ORs but 2 h overtime for each PACU nurse.

In all these scenarios, the recovery in the ORs is allowable. For each scenario, an identical set of 10 instances were solved using the DV^N in three repetitions considering $L \in \{0, 2, 4, 6, 8, 10, 12\}$, $I^{sub,ini} = 5$, $tl^{total} = 2I^2$ and $tl^{sub} = (I_v^{sub})^2, \forall v$.

Table 4 reports the results of the DV^N for the scenario of the no overtime for the ORs and the PACU nurses. It can be seen in this table that with 12 nurses no cancellation has occurred for our 10 instances at the time of having no overtime.

Table 4. Results of the DV^N with no overtime for the ORs and PACU nurses

	Initial solution		DV^N	
Num. PACU nurses	Ave. num. canc.	Ave. exe. time (s)	Ave. num. canc.	Ave. exe. time (s)
0	14.70	39.05	12.80	968.00
2	11.87	68.63	10.47	968.00
4	10.33	73.42	8.13	968.00
6	8.07	80.85	6.17	968.00
8	6.00	74.67	4.53	968.00
10	3.43	64.73	1.63	897.64
12	0.60	37.69	0.00	102.25

Table 5 provides the results of the DV^N for the scenario of 2 h overtime for each OR and no overtime for the PACU nurses. It can be seen in the table that overtime in the ORs can be significantly (p-value<0.01) helpful for alleviating the implications of having few PACU nurses. For example, with 6 PACU nurses, the average number of cancellations has decreased from 6.17 to 1.03.

Table 5. Results of the DV^N with 2 h overtime for each OR and no overtime for the PACU nurses

	Initial solution		DV^N	
Num. PACU nurses	Ave. num. canc.	Ave. exe. time (s)	Ave. num. canc.	Ave. exe. time (s)
0	8.20	18.99	7.20	968.00
2	6.70	58.67	5.03	968.00
4	4.13	56.00	2.93	968.00
6	1.97	49.30	1.03	799.77
8	0.60	37.38	0.03	166.74
10	0.20	31.53	0.00	58.22

Table 6 displays the results of the DV^N for the scenario of 2 h overtime for each PACU nurse. In comparison with the scenario with no overtime (Table 4), the number of cancellations has decreased considerably (p-value<0.01). For example, with 6 PACU nurses, the average number of cancellations has decreased from 6.17 to 3.20. This improvement is less than that obtained by 2 h overtime in the ORs (Table 5). However, we should consider that the results of Table 5 have been obtained with a total of 44 h overtime in the ORs, but those of Table 6 have been achieved by two times the number of PACU nurses (e.g., 12 h for instances with 6 PACU nurses).

Table 6. Results of the DV^N with 2 h overtime for each PACU nurse

	Initial solution		DV^N	
Num. PACU nurses	Ave. num. canc.	Ave. exe. time (s)	Ave. num. canc.	Ave. exe. time (s)
0	14.70	39.09	12.70	968.00
2	11.00	57.96	9.60	968.00
4	8.20	62.73	6.57	968.00
6	4.73	60.45	3.20	968.00
8	2.33	57.67	1.37	836.87
10	1.67	50.90	0.17	361.17
12	0.20	32.86	0.00	44.18

Fig. 5 compares different scenarios in terms of the average number of cancelations obtained by DV^N at the time of having different number of PACU nurses. The detailed results can be found in https://github.com/danial-khorasanian/OR-PACU.

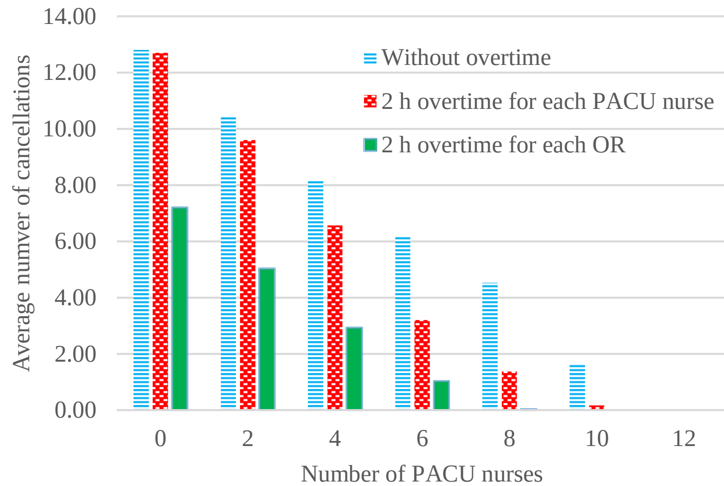

Fig. 5. Average number of cancellations obtained by the DV^N for different scenarios

6. Conclusion

This paper investigated the implications of a severe lack of the PACU beds or the PACU nurses on the number of surgical case cancellations in ORs. Two variants of the integrated OR and PACU scheduling problem were considered, one with few PACU beds, and another with few PACU nurses. For each problem, we proposed a mathematical model and a matheuristic for minimizing the number of case cancellations. In each of these matheuristics, different ways of decomposing an instance into some small-sized sub-instances were investigated by a VNS algorithm. Numerical results showed the effectiveness of this hybridization in reducing the cancellations in comparison with the initial solution.

The models assumed the possibility of recovery in ORs and overtime for ORs and PACU nurses for mitigating the lack of the PACU capacity. Numerical experiments showed that the recovery in ORs and overtime (either for ORs or PACU nurses) can considerably decrease the number of cancellations. For example, for a surgical suite with 22 ORs and 10 PACU beds, the recovery in ORs decreased the average number of cancellations from 5.77 to 1.30. Then, having 2 h overtime in ORs additional to the recovery in ORs resulted in no cancellations for this situation. Also, for the same surgical suite with 22 ORs but with 6 PACU nurses and an adequate number of PACU beds, there were 6.17 cancellations on average, but 2 h overtime in each OR and by each nurse resulted in 1.03 and 3.20 cancellations respectively.

Future research can focus on different directions. First, the interaction of the ICUs and wards with the PACU can affect the capacity of the PACU. Incorporating these units in the problem in the future research may result in more realistic results. Related, we assumed that the capacity of the PACU nurses is known at the start of a day. Interaction with ICU and wards can be not only patients but also personnel. Finally, we have suggested a procedure for calculating a lower bound on the number of cancellations for the SS_PB problem in Part G of the supplementary document.

It may be inspiring for future research about developing more effective lower bounds and maybe exact approaches for both SS_PB and SS_PN problems.

References

Augusto, Vincent, Xiaolan Xie, and Viviana Perdomo. 2010. "Operating Theatre Scheduling with Patient Recovery in Both Operating Rooms and Recovery Beds." *Computers & Industrial Engineering* 58 (2): 231–238.

Bai, Miao, Robert H Storer, and Gregory L Tonkay. 2017. "A Sample Gradient-Based Algorithm for a Multiple-OR and PACU Surgery Scheduling Problem." *IISE Transactions* 49 (4): 367–380.

Bam, Maya, Brian T Denton, Mark P Van Oyen, and Mark E Cowen. 2017. "Surgery Scheduling with Recovery Resources." *IISE Transactions* 49 (10): 942–955.

Bulfin, R L, and R M'Hallah. 2003. "Minimizing the Weighted Number of Tardy Jobs on a Two-Machine Flow Shop." *Computers and Operations Research* 30 (12): 1887–1900.

Cardoen, B, E Demeulemeester, and J Beliën. 2009a. "Sequencing Surgical Cases in a Day-Care Environment: An Exact Branch-and-Price Approach." *Computers and Operations Research* 36 (9): 2660–2669.

Cardoen, B, Erik Demeulemeester, and Jeroen Beliën. 2009b. "Optimizing a Multiple Objective Surgical Case Sequencing Problem." *International Journal of Production Economics* 119 (2): 354–366.

Dexter, F., M. Elhakim, R.W. Loftus, M.S. Seering, and R.H. Epstein. 2020. "Strategies for Daily Operating Room Management of Ambulatory Surgery Centers Following Resolution of the Acute Phase of the COVID-19 Pandemic." *Journal of Clinical Anesthesia* 64: 109854.

Dexter, F, R H Epstein, E Marcon, and J Ledolter. 2005. "Estimating the Incidence of Prolonged Turnover Times and Delays by Time of Day." *Anesthesiology* 102 (6): 1242–1248.

Dexter, F, R H Epstein, and D H Penning. 2001. "Statistical Analysis of Postanesthesia Care Unit Staffing at a Surgical Suite with Frequent Delays in Admission from the Operating Room - A Case Study." *Anesthesia and Analgesia* 92 (4): 947–949.

Dexter, F, R E Wachtel, and R H Epstein. 2006. "Impact of Average Patient Acuity on Staffing of the Phase I PACU." *Journal of Perianesthesia Nursing* 21 (5): 303–310.

Epstein, R H, F Dexter, M G Lopez, and J M Ehrenfeld. 2014. "Anesthesiologist Staffing Considerations Consequent to the Temporal Distribution of Hypoxemic Episodes in the Postanesthesia Care Unit." *Anesthesia and Analgesia* 119: 1322–1333.

Fairley, M, D Scheinker, and M L Brandeau. 2019. "Improving the Efficiency of the Operating Room Environment with an Optimization and Machine Learning Model." *Health Care Management Science* 22 (4): 756–767.

Fei, H, N Meskens, and C Chu. 2010. "A Planning and Scheduling Problem for an Operating Theatre Using an Open Scheduling Strategy." *Computers and Industrial Engineering* 58 (2): 221–230.

Gul, S, B T Denton, J W Fowler, and T Huschka. 2011. "Bi-Criteria Scheduling of Surgical Services for an Outpatient Procedure Center." *Production and Operations Management* 20 (3): 406–417.

Haret, Denisa, Misty Kneeland, and Edmund Ho. 2012. "Chapter 14: Postanesthesia Care Units." In *Operating Room Design Manual*. American Society of Anesthesiologists.

Hsu, Vernon Ning, Renato de Matta, and Chung-Yee Lee. 2003. "Scheduling Patients in an Ambulatory Surgical Center." *Naval Research Logistics (NRL)* 50 (3): 218–238.

Jarzebowski, Mary. 2020. "Creating an Intensive Care Unit From a Postanesthesia Care Unit for the COVID-19 Surge at the Veterans Affairs Ann Arbor Healthcare System." *Federal Practitioner* 37 (8). Frontline Medical Communications: 348.

Jebali, I, A B Hadj Alouane, and P Ladet. 2006. "Operating Room Scheduling." *International Journal of Production Economics* 99: 52–62.

Khlif Hachicha, H, and F Zeghal Mansour. 2018. "Two-MILP Models for Scheduling Elective Surgeries within a Private Healthcare Facility." *Health Care Management Science* 21 (3): 376–392.

Khorasanian, D., and G. Moslehi. 2017. "Two-Machine Flow Shop Scheduling Problem with Blocking, Multi-Task Flexibility of the First Machine, and Preemption." *Computers and Operations Research* 79: 94-108.

Khorasanian, D, F Dexter, and G Moslehi. 2018. "Analyses of the Phase I Postanesthesia Care Unit Baseline Capacity and Effect of Disruptions to Its Beds or Nurse Availability on Operating Room Workflow." *International Journal of Planning and Scheduling* 2 (4): 350–372.

Latorre-Núñez, Guillermo, Armin Lüer-Villagra, Vladimir Marianov, Carlos Obreque, Francisco Ramis, and Liliana Neriz. 2016. "Scheduling Operating Rooms with Consideration of All Resources, Post Anesthesia Beds and Emergency Surgeries." *Computers & Industrial Engineering* 97: 248–257.

Lee, S, and Y Yih. 2014. "Reducing Patient-Flow Delays in Surgical Suites through Determining Start-Times of Surgical Cases." *European Journal of Operational Research* 238 (2): 620–629.

Marcon, E, and F Dexter. 2007. "An Observational Study of Surgeons' Sequencing of Cases and Its Impact on Postanesthesia Care Unit and Holding Area Staffing Requirements at Hospitals." *Anesthesia and Analgesia* 105 (1): 119–126.

Metzler, E C, B S Kodali, R D Urman, H L Flanagan, M S Rego, and J C Vacanti. 2015. "Strategies to Maintain Operating Room Functionality Following the Complete Loss of the Recovery Room Due to an Internal Disaster." *American Journal of Disaster Medicine* 10 (1): 5–12.

Mladenović, N, and P Hansen. 1997. "Variable Neighborhood Search." *Computers and Operations Research* 24 (11): 1097–1100.

Pham, D N, and A Klinkert. 2008. "Surgical Case Scheduling as a Generalized Job Shop Scheduling Problem." *European Journal of Operational Research* 185 (3): 1011–1025.

Samudra, M, E Demeulemeester, B Cardoen, N Vansteenkiste, and F E Rademakers. 2017. "Due Time Driven Surgery Scheduling." *Health Care Management Science* 20 (3): 326–352.

Santibáñez, P, M Begen, and D Atkins. 2007. "Surgical Block Scheduling in a System of Hospitals: An Application to Resource and Wait List Management in a British Columbia Health Authority." *Health Care Management Science* 10 (3): 269–282.

Sento, Yoshiki, Toshiyasu Suzuki, Yasuyuki Suzuki, David A Scott, and Kazuya Sobue. 2017. "The Past, Present and Future of the Postanesthesia Care Unit (PACU) in Japan." *Journal of Anesthesia* 31 (4): 601–607.

Sulecki, L., F. Dexter, A. Zura, L. Saager, and R.H. Epstein. 2012. "Lack of Value of Scheduling Processes to Move Cases from a Heavily Used Main Campus to Other Facilities within a Health Care System." *Anesthesia and Analgesia* 115 (2): 395–401.

Thenuwara, K.N., T. Yoshimura, Y. Nakata, and F. Dexter. 2018. "Time to Recovery after General Anesthesia at Hospitals with and without a Phase I Post-Anesthesia Care Unit: A Historical Cohort Study." *Canadian Journal of Anesthesia* 65 (12): 1296–1302.

van Essen, J Theresia, Johann L Hurink, Woutske Hartholt, and Bernd J van den Akker. 2012. "Decision Support System for the Operating Room Rescheduling Problem." *Health Care Management Science* 15 (4): 355–372.

Xiang, Wei, Jiao Yin, and Gino Lim. 2015. "A Short-Term Operating Room Surgery Scheduling Problem Integrating Multiple Nurses Roster Constraints." *Artificial Intelligence in Medicine* 63 (2): 91–106.

Yin, Meng, Bing-hai Zhou, and Zhi-qiang Lu. 2016. "An Improved Lagrangian Relaxation Heuristic for the Scheduling Problem of Operating Theatres." *Computers & Industrial Engineering* 101: 490–503.