Covid-19 and optimal urban transport policy

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Abstract

Covid-19 has important implications for public transport operations. Increased teleworking and the perceived infection risk on public transport vehicles have drastically reduced demand in many cities. In this paper, we use a simple model to study the effect of these changes on second-best optimal pricing and frequency provision, assuming that car use is under-priced. A numerical application reflecting the public transport situation in Brussel is provided. Results include the following. First, more telework and the increased perceived crowding cost associated with the infection risk have opposite effects on the fare, so that it may be optimal not to change the fare at all. Optimal frequency is likely to decline. Second, surprisingly, holding the fare and frequency constant at their pre-Covid second-best optimal values, more telework reduces the public transport deficit. Third, extending the model to allow for passengers with different vulnerability towards Covid-19, allowing fare and frequency differentiation implies that vulnerable users will face higher fares only if their risk perception is sufficiently higher than that of the non-vulnerable, and car use is not too much under-priced. Occupancy rates will be lower for the vulnerable passengers. Fourth, the numerical results for Brussels show that telework combined with a high perceived infection risk for workers may yield a welfare optimum whereby commuters do not use public transport at all. Offering a low frequency suffices to deal with the demand by school children and students. Lastly, reserved capacity for the vulnerable users and stimuli for walking and biking to school may be useful policies to deal with the crowding risk.

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1. Introduction

The Covid-19 crisis has drastically affected individuals' way of life around the world. Moreover, it has implications in all major sectors of the economy. In this paper, we focus on the effects of the virus for public transport. The virus has important effects on both the demand and supply sides of the market. On the demand side, some passengers may be reluctant to use public transport because the perceived infection risk is higher (both the in-vehicle risks and the risk of contacts in stations) in comparison with a trip by car; others may find public transport less attractive because of the requirement to wear a mouth mask. Moreover, the large shift in teleworking has reduced the total demand for commuting trips, including trips by public transport. On the supply side, social distancing has affected the real capacity of public transport vehicles, and operators have faced extra costs trying to keep public transport use as safe as possible.

In many countries the total number of transport trips went down due to telework, but on top of that the market share of public transport dropped drastically[1]. In this paper, we study the effect of teleworking and the high perceived infection risk on public transport vehicles on the socially optimal pricing and frequency of public transport trips. To do so, we first present a simple graphical analysis, using linear cost and demand functions that allows us to develop some intuition for the effect of the corona virus on the socially optimal fare when frequency cannot be changed in the short-run (and assuming a homogeneous population). We then study more formally the role of teleworking, the perceived infection risk and the restricted capacity of public transport vehicles on fares and frequencies. The pandemic implies an objective risk of transmitting the infection to others on public transport vehicles. Apart from this objective externality, it also increases the perceived infection risk for passengers. Note that one of the problems in isolating the objective infection transmission in public transport vehicles is to distinguish it from the infection transmission at other places. This is the reason why we will base our policies on the perceived risk by potential passengers, and not on the objective infection risk: it is the perceived risk that determines their behavior and therefore their ultimate demand for public transport.

[1] Unsurprisingly, the pandemic also resulted in much more biking.

The model we use is a traditional model of public transport decision-making (early versions of such models are, among many others, Mohring (1972), Jansson (1983), and Frankena (1983)) that captures the interaction with the competing car mode and allows to introduce telework and infection risk in a straightforward way. Moreover, the model allows for different types of passengers that may differ in their vulnerability to a Covid-19 infection (for example, younger versus older passengers). We numerically illustrate the model using data for the city of Brussels.

Our findings include the following. First, unsurprisingly, an increase in telework reduces the demand for both transport modes. If the public transport firm could flexibly adapt its policies, the efficient second-best response would be to reduce both the fare and the frequency. The increase in the external crowding cost due to the (perceived) infection risk reduces public transport demand and increases car use. We show that it plausibly increases the optimal fare; the effect on optimal frequency depends on the relative importance of higher frequency on demand. Noting that telework and the infection risk have opposite effects on the fare implies that in practice it may be optimal for the firm not to change the fare at all. Second, holding the fare and frequency constant at their pre-Covid second-best optimal values we find that, surprisingly, more telework reduces the public transport deficit. Holding the fare and frequency constant at their pre-Covid optimal values, the effects of a higher perception risk is ambiguous in general. If car use is strongly under-priced, higher risk perception reduces the deficit. Third, extending the model to allow for passengers with different vulnerability towards Covid-19 we show, allowing fare and frequency differentiation implies that lower occupancy rates per vehicle for the vulnerable are socially optimal. They will face higher fares only if their risk perception is sufficiently higher than that of the non-vulnerable and car use is not too much under-priced. Fourth, more restrictive social distancing on wagons for the vulnerable raises the fare for vulnerable passengers, and it implies that more wagons are reserved for the vulnerable. Lastly, a separating equilibrium implies that fares for trips in the wagons intended for the vulnerable are sufficiently higher than fares in the wagons for the non-vulnerable to prevent non-vulnerable passengers to use wagons reserved for the vulnerable.

The application to Brussels yields additional insights. The model considers the peak period and allows commuters to choose between car and public transport use; however, students and school children can only use public transport. In the baseline, a very low fare and a high frequency are justified to address un-priced car congestion. Whenever telework reduces the demand for trips by at least 25% and the perceived crowding externality becomes higher due to Covid19, we find that from a welfare perspective the best option is to decrease the frequency. This also reduces the public transport deficit. We do find that this outcome may imply that all work trips switch to car use, an outcome that may be difficult to accept by policy-makers. However, it performs much better from a welfare viewpoint than continuing to offer high frequency and keeping a fair share of public transport trips to work. As telework may continue to be more important in the future, even after the Covid pandemic, car congestion in the peak period will also be less important and one may opt for an equilibrium with a much smaller frequency of public transport. Lastly, the numerical exercise suggests that reserving part of the available capacity for vulnerable users and stimuli for walking and biking to school may be useful policies to deal with the crowding risk.

The structure of the paper is the following. In section 2 we start with a brief literature review. In the next section we present a graphical analysis showing what the main implications of the virus are for the socially optimal fare, holding frequencies fixed. We then develop a formal model in Section 4 to study the effect of telework and the risk of being infected on second-best optimal fare and frequency, given under-priced car use. In Section 5 the model is extended to allow for passengers that differ in vulnerability with respect to the virus and we ask whether and under what conditions discriminatory policies are warranted. A numerical model, calibrated using data for the city of Brussels, is used in Section 6 to illustrate the effects of a number of Covid19-policies. A final section concludes.

2. Literature review

This paper introduces some of the implications of the Covid-19-pandemic into standard models of optimal pricing and frequency in public transport. It therefore relates to the literature on modeling public transport decisions and the (scarce) literature dealing with the consequences of the pandemic for the behavior of potential passengers.

The behavior of potential passengers depends among others on the perceived risk of being infected on public transport vehicles. There is now convincing evidence on the general transmission of the Corona-virus. For example, Chu et al. (2020) and Leung et al. (2020) review the evidence on the role of distance, face masks and the duration of contacts with infected persons for the spreading of the disease. However, less is known about the infection risk and the transmission of the virus within the public transport system. The first scarce evidence is becoming available on the factors that determine the risks of infection on board of public transport vehicles and in bus, rail and metro stations. An early study focused on the infection of a non-covid disease in the metro of New York (Cooley et al., 2011) and London (Goscé and

Johansson (2017). More recent studies do focus on Covid-19. These include analyses of the infection transmission in the NY metro (Harris, 2020) and a study of train infections in China (Hu et al. 2020).

Krishnakumari and Cats (2020) used interesting simulation experiments on the Washington metro using smart card data that trace the itinerary of a metro-user. Smart card data allow to reconstruct the contact network reflecting the passengers one potentially gets in contact with during the course of his/her public transport journey. Moreover, this type of data allows to estimate probabilities to get infected, using connections in the contact graph based on crowding estimates. Combining the capacity of the metro system (1700 passengers per train) with the actual number of passengers on each train, they found an average of 1200 interactions per passenger. Assuming that persons within a distance of less than 1 meter get infected by an infected person and persons at a distance of more than 1,5 meter do not get infected, they show that a few primary infections are sufficient to have the whole population infected within a week[2]. However, when the number of passengers is restricted to 20% of the capacity, and the passengers respect the social distance, the risk of infections is very low.

There is overwhelming evidence that demand has indeed declined drastically for public transport systems around the world. Evidence is available from the Google Mobility Data and from private mobility firms such as Moovit. Moreover, recent papers document the decline in detail for specific countries (see Jelenius and Cebecauer (2020), Didi and Dei (2020), and Arrelana, Marques and Cantillo (2020)). The decline was partly due to government policy in response to the pandemic. For example, many governments strongly encouraged telework and required wearing masks on public transport vehicles. But consistent with the observation above, partly the demand reduction was also due to the perception of potential passengers that public transport implied a higher risk of infection than the use of other modes. This happened despite the safety measures taken: on the supply side, the social distancing rules that were put in place by many governments reduced the capacity available and lowered the objective infection risk.

The main challenges for public transport from an economic point of view are easily summarized (also see Tirachini and Cats (2020)). First, what is the effect of teleworking on the demand side of the public transport market? Second, how large should the capacity of public transport systems be to allow passengers to make trips under safe conditions. These include the

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[2] Of course, this is only a simulation study, and reality in some metro cities seems to be less problematic. An example is Sweden, where mouth masks were not obligatory and where Covid-19 was not spreading as fast as would be predicted by the study of the Washington metro.

role of physical distancing and of mouth masks, contact tracing and enforcement. Third, how to regulate access to public transport: do we count on fare policies or on permit or reservation systems? Lastly, how to manage the short-run financial implications of the crisis for the public transport sector, and what are the implications for the longer term. In this paper, we explicitly deal with several of these issues.

Our paper builds upon the extensive literature on optimal pricing and supply characteristics of public transport firms, initiated by Mohring (1972). He studied socially optimal fare and frequency policies, emphasizing the existence of economies of density in scheduled public transport services. He showed that constant returns to scale in operational costs and randomly arriving passengers at bus stations imply that the marginal cost of frequency adjustment is compensated by the marginal benefit of waiting time reduction. Marginal cost pricing leads to a zero fare, and operating costs have to be fully recovered by subsidies.

The model has been extended to capture, among others, the role of crowding, second-best considerations due to under-priced car use, the implications for government subsidies, and how to optimize additional decision variables (including the number of vehicles, the size of vehicles, and the number of stops). Crowding implies that additional passengers impose an external cost on other users (see, among many others, Jansson, 1980; Jara-Díaz and Gschwender, 2003; Haywood and Koning, 2015) so that, in very crowded public transport systems, welfare-optimal fares may be positive and quite large. If car use is under-priced due to absence of congestion tolls, low public transport fares can be defended as a standard second-best policy, reducing the social costs of congestion (see, for example, Parry and Small, 2009; Anderson, 2014; De Borger and Proost, 2015). Unsurprisingly, studies that jointly optimize prices for road and public transport use in the presence of road congestion and crowding in public transport find quite high optimal public transport fares (see, for example, Basso and Silva, 2014; de Palma, Kilani and Proost, 2014). Subsidies to public transport are socially costly because they have to be financed by distortionary taxes. The implications of optimal fares and frequencies for public transport subsidies have been scrutinized by Frankena (1980, 1983), De Borger and Wouters (1998), and Parry and Small (2009), among others. The efficient public transport subsidy strongly depends on the cross-price elasticities between car and public transport use, whether or not car use is correctly priced, and the cost of public funds (Proost and Van Dender (2008)). Moreover, distributional motives may lead to low fare policies that require large subsidies.

3. Covid-19 and optimal fares: a simple graphical approach

In this section, we use a simple graphical approach to start the discussion of the effect of Covid-19 for optimal public transport policies. We start from a baseline pre-Covid second-best equilibrium where the relevant public transport policy variables (fare, frequency) have been optimized. Then we introduce an unexpected Covid-19 shock and analyze how this affects the public transport system and how optimal transport policy needs to be adapted. Of course, our simplified approach needs to be extended and completed by an analytical and numerical model in the next sections.

We use a comparative statics framework. In the initial pre-Corona equilibrium, there is no congestion toll on road use, and the fare and the frequency in public transport are second-best optimal. In the equilibrium after the Covid19-shock, we assume that a given share of the population is infected. It is unclear how important transmission of infections via public transport use precisely is. However, we do know that the fear of being infected increases potential passengers' aversion to crowding, leading to higher discomfort and to a decrease in the modal share of public transport. Moreover, the Covid-crisis has strongly increases telework, reducing the total number of peak-period trips.

3.1. The optimal public transport system before the Covid-19 shock

Consider a homogeneous corridor where all passengers want to move from an Origin to a Destination. Figure 1 represents the demand for road transport and the demand for public transport in the peak period as a function of their generalized cost, holding the total number of trips constant. Both modes are assumed to be perfect substitutes, so that in an equilibrium the generalized prices are equal. For car use the generalized cost consists of the money cost plus the time cost. The average user cost (*AC*) is increasing with car use and, since we use linear functions for simplicity, the marginal social cost (*MSC*) has twice the slope of the *AC*-function. For public transport, the average cost equals the fare plus the access cost, the waiting time cost and the in-vehicle cost. The latter is increasing in the number of passengers because of crowding discomfort. The slope of the *MSC* is again twice the slope of the *AC*. The slope of the *AC* of public transport depends on the frequency offered: higher frequency decreases the waiting time cost as well as the cost of crowding discomfort, reducing the slope of the *AC*-curve. Note that the optimal fare in the absence of road congestion would be equal to the discomfort externality (the difference between *MSC* and *AC*); fare revenues would cover a large part of the frequency costs.

The first-best outcome is found where the two marginal social cost curves intersect. The first-best fare and road toll would give an optimal modal allocation indicated by *Z* on the horizontal axis. Charging only the marginal external cost in public transport, and not pricing road use, would give a modal allocation indicated by *Z'*. Of course, this is suboptimal from a second-best viewpoint: un-tolled road use requires the optimal second-best fare to be below the marginal social cost of public transport use; the lower fare (and the higher frequency) attract car users and reduce un-paid congestion. The second-best optimal modal distribution is indicated by point *A* in Figure 1. Of course, the optimal fare depends on the extent of car congestion and the price elasticity of the total number of trips (here set to 0). Finally, note that the optimal second-best public transport policy requires a larger subsidy: the total fare revenues cover no longer most of the variable costs of public transport (including the costs of rolling stock and personnel).

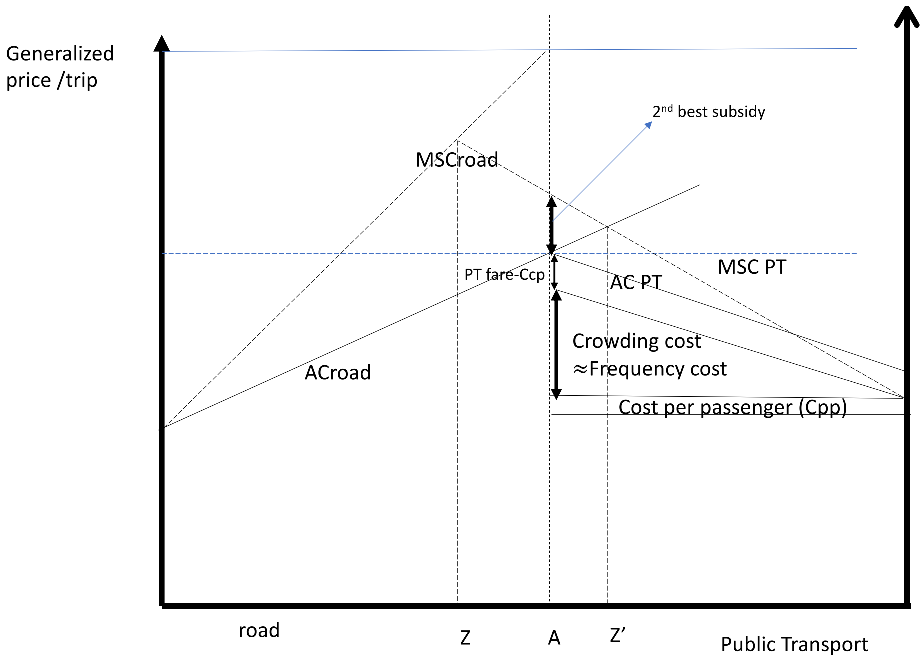

Figure 1. The transport system before Covid19

3.2. The effect of the Covid-19 shock

The magnitude of the Covid-19 shock will depend on the amount of telework imposed by the government, on the confinement measures imposed by the public transport operator and on the perceived risk of infection. As mentioned before, we rely not on the objective risk measure (which passengers do not know) but on the revealed increased discomfort of crowding that is a function of the perceived risk of infection.

Keeping the capacity, the frequency and the fare of public transport constant, the Covid-19 shock implies two effects in Figure 2 :

a) Telework reduces the total number of trips by *A'P'*: this reduction is distributed over road users and public transport users. It will also reduce the market share of public transport when the slope of the average cost function of road use is higher than that of public transport use. The decline in road use implies less congestion.

b) The higher perceived infection risk for Covid19 in public transport translates in a larger discomfort for users: the average cost *AC* of public transport rotates upwards. This implies that users will substitute part of their public transport trips by road trips, and road congestion will increase: many people prefer car use above a crowded public transport vehicle. We end up with a lower market share for public transport and a level of car use (*OA''*) that is higher than when there would only be telework (*OA'*).

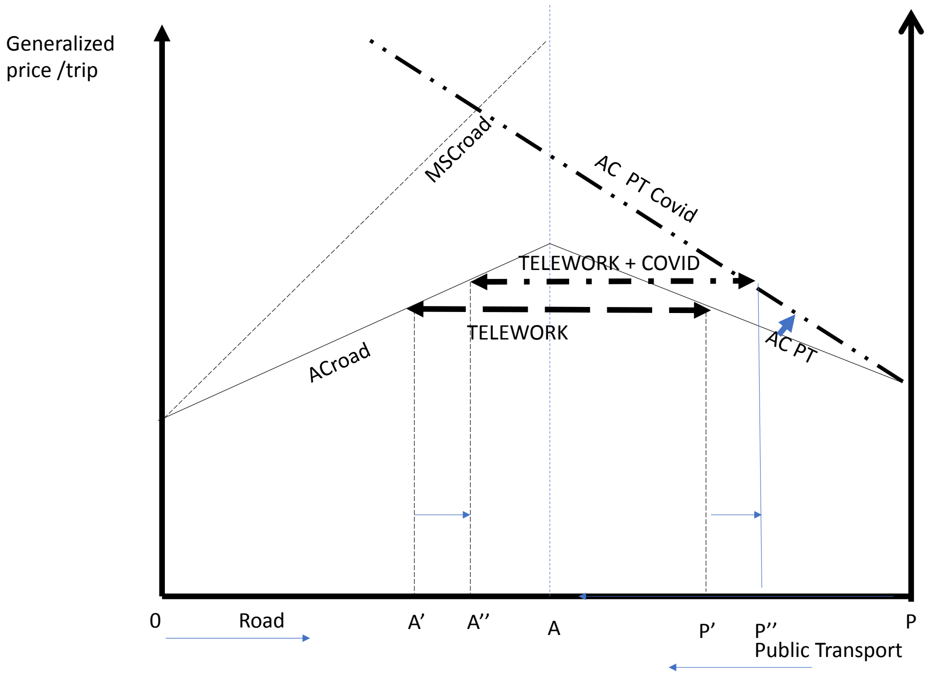

Figure 2. Effect of Telework and Covid-19 on the transport market

Of course, telework and the fear for a Covid infection will have second-order effects on the total number of trips not captured in Figure 2. Telework will decrease the generalized cost of trips because road congestion and crowding in public transport will decrease. This leads to a small compensation of the downward demand shock. The perceived Covid infection risk will increase the cost of public transport use and, by modal substitution to road use, it will also increase the generalized cost of all trips. This may re-enforce the reduction of the total number of trips induced by telework.

In Figure 2, we kept the frequency and the fare of public transport constant at the initial second best equilibrium but, of course, both the optimal fare and the optimal frequency may need to be adapted, although a priori it is not clear in which direction. The optimal fare may increase due to the increased discomfort of public transport use and the reduction in road congestion after the increase in telework, but it may have to decline when demand for public transport drops. On the one hand, optimal frequency may increase to tackle the increased discomfort of crowding; on the other hand, there will be fewer public transport users so that frequency declines. To study the effects of Covid-induced telework and the increased costs of crowding, a more formal analysis is warranted, to which we turn in the next section.

4. The effect of infection risk and telework on public transport policies: a basic model

Two major implications of the Covid-19 pandemic for public transport are the large increase in telework – a policy explicitly encouraged and even imposed by many governments – and the risk of being infected when using public transport vehicles. The perceived infection risk raises the marginal external cost of crowding in public transport.

In this section, we use a simplified version of the model discussed in De Borger and Proost (2015) to study the effect of more telework and the effect of a higher perceived risk by public transport passengers on optimal transport policies. The model describes behavior in the peak period. It is assumed that people can commute by car or they can go by public transport. The road is congestible, and a toll may be in place for car use. Public transport is subject to crowding; the cost of crowding increases due to the risk of infection.

4.1. The model

To keep the analytics tractable without affecting the main insights, we make a number of strong assumptions. First, we assume that car and public transport use are perfect substitutes; this much simplifies the derivations. Total demand for peak-period trips is assumed to be linear. Denoting the generalized price by P, the inverse demand function relates the generalized price to the total number of trips. We denote public transport trips by Y, car trips are denoted X. The inverse demand function is given as

$$P(X+Y) = a - b(X+Y). \tag{1}$$

The generalized user costs of public transport and car use are formulated as, respectively:

$$g^Y(Y, p, f) = p + \alpha\left(\frac{1}{f}\right) + \beta\frac{Y}{f} \tag{2}$$

$$g^X(X, \tau°) = \tau° + vX \tag{3}$$

The generalized cost of public transport consists of the fare (denoted *p*), the cost of waiting time (*f* is the frequency, α is half the value of waiting time), and the cost of crowding (β is the crowding parameter). The generalized cost of car use is a fixed monetary user cost inclusive of a possible toll plus the congestion cost; the slope of the congestion function is *v*.

Given our assumptions a user equilibrium requires:

$$\begin{aligned} P(Y+X) &= g^Y(Y, p, f) \\ P(Y+X) &= g^X(X, \tau°) \end{aligned} \tag{4}$$

Using the demand and generalized costs defined before, and solving the equilibrium conditions gives the reduced-form demand functions, relating demand for public transport and for car use to the public transport policy variables (*p, f*) and to exogenous parameters. We find:

$$\begin{aligned} Y^r(p, \tau°, f) &= \frac{1}{\Delta}\left\{(b+v)\left(a - p - \frac{\alpha}{f}\right) - b(a - \tau°)\right\} \\ X^r(p, \tau°, f) &= \frac{1}{\Delta}\left\{\left(b + \frac{\beta}{f}\right)(a - \tau°) - b\left(a - p - \frac{\alpha}{f}\right)\right\} \end{aligned} \tag{5}$$

$$\text{where } \Delta = \frac{\beta}{f}(v+b) + vb > 0. \tag{6}$$

We then have the following fare and frequency effects on the demand for public transport and car use:

$$\frac{dY}{dp} = \frac{-(b+v)}{\Delta} < 0; \quad \frac{dX}{dp} = \frac{b}{\Delta} > 0. \tag{7}$$

$$\frac{dY}{df} = \frac{(b+v)(\alpha+\beta Y)}{f^2\Delta} > 0; \quad \frac{dX}{df} = \frac{-b(\alpha+\beta Y)}{f^2\Delta} < 0 \tag{8}$$

A higher fare reduces public transport demand and raises car use. Higher frequency raises public transport demand and reduces car use.

We determine the fare and frequency that maximizes the following social welfare function:

$$\underset{p,f}{Max} \quad W(p,f) = \int_0^{X+Y} P(z)dz - g^Y(.)Y - g^X(.)X + \left[pY - c_0 f - c_Y Y\right] + \tau^\circ X$$

The first three terms together define net consumer surplus (gross consumer surplus minus de generalized costs) of public and private transport trips. The term between square brackets captures the net revenues of the public transport operator. It equals the fare revenues minus the operating costs of providing the service. The operating costs are specified in the simplest possible manner: the cost is linear in its two main determinants, the frequency and the number of passengers. Each extra bus trip offered by the public transport firm implies an extra cost, assumed to be constant. Moreover, there is a small marginal cost of carrying extra passengers. The final term in the social welfare function consists of the road toll revenues (we normalize other monetary costs of car use to zero – so that τ° can be directly interpreted as the toll -- without affecting the qualitative results).

Using equality between generalized price and generalized cost in equilibrium, the first-order conditions can be written as

$$\begin{aligned} \frac{\partial W}{\partial p} &= p - c_Y - \frac{\beta}{f}Y - (\tau^\circ - \nu X)\left(\frac{b}{b+\nu}\right) = 0 \\ \frac{\partial W}{\partial f} &= -c_0 + \frac{Y}{f^2}(\alpha + \beta Y) = 0 \end{aligned} \tag{9}$$

Rearranging yields:

$$\begin{aligned} p &= c_Y + \frac{\beta}{f}Y + (\tau^\circ - \nu X)\left(\frac{b}{b+\nu}\right) \\ f &= \sqrt{\frac{(\alpha + \beta Y)Y}{c_0}} \end{aligned} \tag{10}$$

The socially optimal fare equals the marginal social cost corrected for suboptimal pricing of car use. The frequency rule is the standard square root expression familiar from the public transport literature surveyed in the introduction.

Note that this second-best optimum, assuming under-priced car use, necessarily produces a public transport deficit. To see this, note that operating profit for the public transport firm is:

$$\pi = (p - c_Y)Y - c_0 f \; .$$

Using the first-order conditions (9) in this expression, we have:

$$\pi = \left[\frac{\beta}{f}Y + (\tau^\circ - \nu X)\left(\frac{b}{b+\nu}\right)\right]Y - \frac{Y}{f}(\alpha + \beta Y)$$

Working out we find:

$$\pi = \left[(\tau^{\circ} - \nu X)\left(\frac{b}{b+\nu} \right) - \frac{\alpha}{f} \right] Y \,. \tag{11}$$

If car use is priced below marginal external cost this expression is necessarily negative, so that a deficit occurs: $\pi < 0$.

4.2. The effect of telework and infection risk on fare and frequency

We are interested in the effect of exogenous changes in telework and a higher perceived infection risk on the policy variables of the public transport sector. We view more telework that is exogenously imposed by the government as an exogenous downward shift in demand; more precisely, we interpret an exogenous increase in telework as a downward shift in the intercept of the inverse demand function, the demand parameter '*a*'. Covid has also raised the crowding cost: a higher perceived infection risk can be interpreted as an increase in the crowding parameter β.

To solve for the effect of telework and infection risk on the optimal fare and frequency, first note that the effects of an exogenous change in *a* on the demand for public transport and car use are obtained by differentiating the reduced-form demand functions given in (5) above. We find:

$$\frac{dY}{da} = \frac{v}{\Delta} > 0; \quad \frac{dX}{da} = \frac{1}{\Delta}\frac{\beta}{f} > 0\,. \tag{12}$$

$$\frac{dY}{d\beta} = -\frac{Y(v+b)}{f\Delta} < 0; \quad \frac{dX}{d\beta} = \frac{\left(a - \tau^{\circ} - (v+b)X\right)}{f\Delta} = \frac{bY}{f\Delta} > 0 \tag{13}$$

To arrive at the final result in (13) we used the equilibrium conditions (4). Unsurprisingly, an exogenous increase in teleworking (a decline in *a*) reduces the demand for both modes of transport. A higher Covid risk reduces public transport demand, while it increases car use.

Totally differentiating the first-order conditions of the welfare optimization problem and using the demand effects just derived, we show in Appendix that, in general, the effects of telework and infection risk on the optimal fare and frequency are all ambiguous. This is not surprising. For example, telework reduces the demand for public transport as well as car use. The first-order condition for the optimal fare suggests that less public transport reduces the optimal fare, while less car traffic (and hence less congestion and a smaller distortion on the car market) raises the optimal fare. Moreover, telework affects frequency which has itself an effect on the optimal fare. The lower demand for public transport suggests reducing frequency, but doing so may raise infection risk which implies higher optimal frequency.

Although the effects of telework and infection risk are ambiguous in general, in Appendix A we show that the first-order effects (more precisely, the effect on the fare at given frequency, and the effect on frequency at given fare) are as follows:

$$\begin{aligned} &\frac{dp}{da} > 0; \quad \frac{df}{da} > 0 \\ &\frac{dp}{d\beta} > 0; \quad \frac{df}{d\beta} < 0 \text{ or } > 0 \end{aligned} \tag{14}$$

Noting that telework is interpreted as a decline in 'a', these first-order effects suggest that telework reduces both the optimal fare and the optimal frequency. Telework reduces both public transport demand (and this is an argument to lower the fare and the frequency) but it also reduces the demand for car trips (this reduces congestion and raises the optimal public transport fare, a standard second-best argument). Clearly, the first effect on the fare dominates the second. A higher infection risk raises the optimal fare, and it can raise or reduce the optimal frequency. The reason is that offering more frequency offers safer travel, holding the number of passengers fixed; however, it also increases demand, which reduces safety. As shown in Appendix A, a higher risk of infection will increase the optimal frequency if the infection risk only mildly reduces demand. If demand strongly declines, a higher infection risk reduces the optimal frequency.

Proposition 1. The effects of telework and the perceived infection risk

a. **Telework reduces the demand for both transport modes. A higher perceived infection risk reduces public transport demand and increases car use.**
b. **The effect of telework on the socially optimal fare and frequency is ambiguous in general. Plausibly, however, it reduces both the fare and the frequency.**
c. **A higher perceived infection risk plausibly increases the optimal fare. If the perceived risk of being infected with Covid-19 strongly reduces demand then the optimal frequency declines.**

These findings have a simple practical policy implication. Two major consequences of Covid-19 are the increase in telework (reducing the optimal fare) and the increase in crowding costs (increasing the optimal fare). As these two effects have opposite signs the joint effect on fares may be very small, and the associated welfare gain may be limited. Given the cost of adapting fares, it may be optimal not to change fares in response to the Covid pandemic at all. Under plausible conditions, is does make sense to offer lower frequency in response to the joint increase in telework and infection risk.

4.3. Implications of telework and infection risk for the public transport deficit

Policy-makers are concerned about the implications of telework and the infection risk on the public transport deficit. We can make the following observations.

First, trivially, if current behavior of the firm is not socially optimal and the firm does not adapt its fare-frequency policy in response to the changes in telework and perceived infection risk, then the implications for the deficit depend on whether or not the fare *p* exceeds the marginal cost of an extra passenger c_Y. If it does, the deficit will increase when telework increases; the same holds for an increase in infection risk. If the fare falls short of the marginal cost, then telework and infection risk reduce the deficit.

Second, suppose the pre-Covid fare and frequency policy was second-best socially optimal and assume the public transport firm holds the fare and the frequency constant when telework increases. For example, it may be politically as well as commercially unattractive to adapt fare and frequency during the Covid-crisis. In that case, surprisingly, more telework in fact reduces the deficit. To see this, differentiating (11) with respect to *a* gives

$$\frac{\partial \pi}{\partial a} = \left[(\tau^\circ - \nu X)\left(\frac{b}{b+v}\right) - \frac{\alpha}{f} \right] \frac{\partial Y}{\partial a} - \frac{Yvb}{b+v} \frac{\partial X}{\partial a} .$$

The term between brackets is negative when car use is under-priced, and we showed before that $\frac{\partial Y}{\partial a} > 0; \frac{\partial X}{\partial a} > 0$. As a consequence, we have $\frac{\partial \pi}{\partial a} < 0$. More telework (a reduction in *a*) raises profit or, more precisely, it reduces the deficit. Intuitively, the second-best optimal fare is below the marginal production cost of extra passengers, so that fewer passengers reduce the deficit.

Similarly, we can determine the effect of a higher perceived infection risk on the deficit, holding fare and frequency constant at the second-best optimal levels prior to the Covid pandemic. We find

$$\frac{\partial \pi}{\partial \beta} = \left[(\tau^\circ - \nu X)\left(\frac{b}{b+v}\right) - \frac{\alpha}{f} \right] \frac{\partial Y}{\partial \beta} - \frac{Yvb}{b+v} \frac{\partial X}{\partial \beta}$$

Rewriting gives

$$\frac{\partial \pi}{\partial \beta} = \left(\frac{b}{b+v}\right) \left[\tau^\circ \frac{\partial Y}{\partial \beta} - v\left(X \frac{\partial Y}{\partial \beta} + Y \frac{\partial X}{\partial \beta} \right) \right] - \frac{\alpha}{f} \frac{\partial Y}{\partial \beta}$$

The sign of this expression is ambiguous. If the toll is small then – using earlier results (13) – we easily show that the effect of Covid-19 is, again surprisingly, to reduce the deficit.

Third, suppose that the firm has implemented the socially optimal fare-frequency policy prior to changes in telework and infection risk, but now assume that the firm optimally the frequency downward while keeping the fare fixed. This is what has been proposed in several countries: if due to the Covid-crisis demand for public transport drastically declined, why not reduce the frequency? In Appendix A we then show that, assuming the frequency decline itself does not much affect car use, more telework again unambiguously reduces the deficit.

Proposition 2. The effects of telework and the perceived infection risk on the public transport deficit.

a. **Holding the fare and frequency constant at their pre-Covid optimal values, more telework reduces (rather than increases) the public transport deficit.**
b. **If the firm optimally adapts frequency but holds the fare fixed, more telework again reduces the deficit.**
c. **Holding the fare and frequency constant at their pre-Covid optimal values, the effects of a higher perception risk is ambiguous. If car use is strongly under-priced, higher risk perception reduces the deficit.**

5. Distinguishing vulnerable and less vulnerable passengers

It is straightforward to extend the model for two types of commuters that may differ in vulnerability with respect to Covid-19. Let us assume the morning commute consists of workers and students. Assume students are less vulnerable to the illness (call them the non-vulnerable, indexed *N*) than workers (the vulnerable, indexed *V*). We first assume that all passengers use the same public transport vehicles. Next we allow different vehicles to be used for the two types of passengers.

5.1. Extending the model: two types of passengers

Suppose students have no access to a car; they either go by public transport, or they walk or bike to school. The workers (the vulnerable) have the option to go by car or public transport. The two types of public transport users have different generalized costs of public transport use:

$$g^{YV} = p + \alpha\left(\frac{1}{f}\right) + \beta^{V}\,\frac{(Y^{V} + Y^{N})}{f}$$

$$g^{YN} = p + \alpha\left(\frac{1}{f}\right) + \beta^{N}\,\frac{(Y^{V} + Y^{N})}{f}$$

Differentiating the total generalized cost of all public transport trips, $g^{YV}Y^{V} + g^{YN}Y^{N}$, with respect to Y^{V} and Y^{N} we find that the marginal external cost of crowding due to an extra passenger is independent of the passenger type; in both cases, it equals

$$\frac{\beta^{V}}{f}Y^{V} + \frac{\beta^{N}}{f}\bar{Y}^{N}.$$

As before, the cost of car use depends on the number of vulnerable users, viz. workers driving to work by car:

$$g^{X} = \tau^{\circ} + \nu X^{V}$$

The inverse demand for public transport trips by the non-vulnerable students is given by:

$$P^{N}(Y^{N}) = a - b^{N}Y^{N}.$$

As before, we assume car and public transport use are perfect substitutes for the workers, so that inverse demand reads:

$$P^{V}(Y^{V} + X^{V}) = a - b(Y^{V} + X^{V})$$

Note that -- to save on notation but without implications for the main results -- we assume the same intercept but a different slope for the inverse demand functions of the two types.

Equilibrium now requires the following three conditions:

$$\begin{aligned} & p + \alpha\left(\frac{1}{f}\right) + \beta^{V}\,\frac{(Y^{V} + Y^{N})}{f} = a - b(Y^{V} + X^{V}) \\ & \tau^{\circ} + vX^{V} = a - b(Y^{V} + X^{V}) \\ & p + \alpha\left(\frac{1}{f}\right) + \beta^{N}\,\frac{(Y^{V} + Y^{N})}{f} = a - b^{N}Y^{N} \end{aligned} \tag{15}$$

Solving this system yields the reduced-form demand functions Y^{V}, Y^{N}, X^{V}; they are complex functions of the parameters. However, it is easy to show that

$$\frac{dY^V}{dp}=\frac{(v+b)\left(\frac{\beta^V}{f}-b^N-\frac{\beta^N}{f}\right)}{\Sigma};\qquad \frac{dY^N}{dp}=\frac{(v+b)\left(\frac{\beta^N}{f}-\frac{\beta^V}{f}\right)-vb}{\Sigma}<0$$

$$\frac{d(Y^V+Y^N)}{dp}=\frac{-\left[(v+b)b^N+vb\right]}{\Sigma}<0$$

$$\frac{dX}{dp}=\frac{b\left(\frac{\beta^N}{f}+b^N-\frac{\beta^V}{f}\right)}{\Sigma}$$

Here $\Sigma=\left[vb\left(b^N+\frac{\beta^N}{f}\right)+b^N(v+b)\frac{\beta^V}{f}\right]>0$. Note that a higher fare unambiguously reduces the demand for public transport by the non-vulnerable; it also unambiguously reduce total public transport demand. However, the sign of the effects on public transport and road use by the vulnerable are ambiguous in general. Provided $\frac{\beta^V}{f}<b^N+\frac{\beta^N}{f}$ we have the expected signs: public transport demand declines and road use increases.

To find the optimal fare and frequency we solve

$$\begin{aligned}\underset{p,f}{Max}\quad &\int_0^{X^V+Y^V} P(z)dz+\int_0^{Y^N} P(q)dq-g^{YV}(.)Y^V-g^{XV}(.)X^V-g^{YN}Y^N\\ &+\left[p(Y^V+Y^N)-c_0 f-c_Y(Y^V+Y^N)+\tau^{\circ}X^V\right]\end{aligned}$$

The first-order condition for the fare p is:

$$\left(p-c_Y-\frac{\beta^V Y^V}{f}-\frac{\beta^N Y^N}{f}\right)\frac{\partial(Y^V+Y^N)}{\partial p}+\left(\tau^{\circ}-vX\right)\frac{\partial X}{\partial p}=0$$

Rearranging we find, using earlier expressions:

$$p=c_Y+\frac{\beta^V Y^V}{f}+\frac{\beta^N Y^N}{f}+\left(\tau^{\circ}-vX\right)\frac{b\left(b^N+\frac{\beta^N}{f}-\frac{\beta^V}{f}\right)}{vb+b^N(v+b)}. \qquad (16)$$

The fare equals the marginal production cost plus the marginal external cost of an extra passenger, corrected for the inefficient pricing of road use. Assuming that car use is under-priced and that a higher public transport fare increases road use the optimal fare is less than the marginal social cost of an extra passenger.

The optimal frequency is

$$f=\sqrt{\frac{(\alpha+\beta^V Y^V+\beta^N Y^N)(Y^V+Y^N)}{c_0}}\,. \qquad (17)$$

The frequency rule takes account of infection risk for both passenger types.

5.2. Allowing discrimination between vulnerable and non-vulnerable users

To study the effect of differences in vulnerability on fares and frequency, we consider the possibility of discriminating between types. Note that fare discrimination between different types of passengers based on differences in vulnerability does not make much sense, except when different vehicles can be reserved for the different types. The intuitive reason is that, as shown above, the marginal external crowding cost each type imposes on others is equal for both types. Consequently, in first-best fares will be equal for all passengers. Of course, in second-best with under-priced road use a small fare difference between the vulnerable and non-vulnerable will result due to the different response of car users to the two fares, but not due to differences in vulnerability themselves.

Fare differentiation does make sense if one can separate the two types by offering each type different vehicles. Suppose, for example, that the public transport firm operates separate busses for the two types, and assume passenger type can easily be established. Denote the frequencies of the buses reserved for types *V*, *N* by f^V, f^N. Since passengers are now confronted (making abstraction of interaction at stations…) only with their own types, generalized costs are:

$$g^{YV} = p^V + \alpha\left(\frac{1}{f^V}\right) + \beta^V \frac{Y^V}{f^V}$$

$$g^{YN} = p^N + \alpha\left(\frac{1}{f^N}\right) + \beta^N \frac{Y^N}{f^N}$$

Equilibrium requires:

$$p^V + \alpha\left(\frac{1}{f^V}\right) + \beta^V \frac{Y^V}{f^V} = a - b(Y^V + X^V)$$

$$\tau^\circ + vX^V = a - b(Y^V + X^V)$$

$$p^N + \alpha\left(\frac{1}{f^N}\right) + \beta^N \frac{Y^N}{f^N} = a - b^N Y^N$$

We immediately find the price and frequency effects of demand:

$$\frac{dY^V}{dp^V}=\frac{-(b+v)}{\Delta^V}<0;\quad \frac{dX^V}{dp^V}=\frac{b}{\Delta^V}>0\,.$$

$$\frac{dY^V}{df^V}=\frac{(b+v)(\alpha+\beta Y^V)}{(f^V)^2\Delta^V}>0;\quad \frac{dX^V}{df^V}=\frac{-b(\alpha+\beta Y^V)}{(f^V)^2\Delta^V}<0$$

$$\frac{dY^N}{dp^N}=-\frac{1}{b^N+\frac{\beta^N}{f^N}}<0;\ \frac{dY^N}{df^N}=-\frac{\alpha+\beta^N}{(f^N)^2\left(b^N+\frac{\beta^N}{f^N}\right)}<0\,.$$

Here $\Delta^V=\frac{\beta^V}{f^V}\left(v+b\right)+vb>0$.

We determine optimal policies by solving

$$\underset{p^N,p^V,f^N,f^V}{Max}\quad \int_0^{X^V+Y^V}P(z)dz+\int_0^{Y^N}P(q)dq-g^{YV}(.)Y^V-g^{XV}(.)X^V-g^{YN}Y^N$$
$$+\left[p^VY^V+p^NY^N-c^Vf^V-c^Nf^N-c_Y(Y^V+Y^N)+\tau^\circ X^V\right]$$

First-order conditions for fares and frequencies lead to the following rules:

$$p^V=c_Y+\frac{\beta^VY^V}{f^V}+(\tau^\circ-vX)\frac{b}{b+v};\quad f^V=\sqrt{\frac{(\alpha+\beta^VY^V)Y^V}{c^V}}$$
$$p^N=c_Y+\frac{\beta^NY^N}{f^N};\qquad f^N=\sqrt{\frac{(\alpha+\beta^NY^N)Y^N}{c^N}}$$

Whether the vulnerable pay more than the non-vulnerable is a priori unclear. On the one hand, at given demand and frequency their marginal crowding cost is higher, raising the optimal fare. On the other hand, one easily shows that the optimal occupancy rate is declining in the infection risk:

$$\frac{\partial(\frac{Y^i}{f^i})}{\partial\beta^i}<0;\ i=V,N\ .$$

Moreover, competition with car use implies a lower public transport fare if car use is under-priced. Therefore, the vulnerable will face higher fares than the non-vulnerable only if the difference in infection risk is sufficiently large.

Proposition 3. Price and frequency discrimination

- **a. Fare differences between different types of passengers based on differences in vulnerability are unwarranted unless for each type separate vehicles can be used.**
- **b. Allowing fare and frequency differentiation implies lower occupancy rates per vehicle for the vulnerable. They will face higher fares only if their risk perception**

is sufficiently higher than that of the non-vulnerable and car use is not too much under-priced.

5.3. Covid infection risk and the optimal composition of trains

Consider a similar but slightly different aspect of the implications of Covid-19 for public transport, which is potentially important mainly for rail and metro service. Specifically, in this section we focus on a given peak period train or metro trip and consider the problem of reserving train wagons to different passenger types. The idea is to separate vulnerable from non-vulnerable passengers by offering different wagons at different prices for the same trip. The idea of separating vulnerable and non-vulnerable passengers is advocated as a general principle in the Covid-literature by several authors (see, for example, Acemoglu et al. (2020) and Gollier (2020)). Vulnerable passengers are then protected in two ways. By using different wagons they are protected from the interaction with non-vulnerable passengers. Moreover, the wagons reserved for vulnerable passengers restrict the number of passengers per wagon.

There are two ways to make sure passengers use separate wagons according to their type. First, if the distinction between types is based on observable characteristics (for example, if the vulnerable are defined as passengers older than 65, or the non-vulnerable are defined as students) the separation can be implemented at low cost by a simple passport control. Second, if the distinction is based on unobservable characteristics or on passengers' own perception, appropriate price discrimination can induce a separating equilibrium whereby neither type has an incentive to use a vehicle reserved for the other type.

In an equilibrium where passengers use wagons according to their type, the generalized costs for the vulnerable and non-vulnerable are given by, respectively:

$$g^V = p^V + \beta^V \frac{y^V}{Q^V}; \quad g^N = p^N + \beta^N \frac{y^N}{Q^N} .$$

Here the y's are the number of passengers of a given type on a given peak-period trip. The Q's are the number of wagons allocated to the two types.

We assume the decision-maker decides on the number of wagons to reserve for the vulnerable and non-vulnerable users and the fares to be charged for admission to each wagon type. It faces two constraints. In the peak period there is a limited number of wagons available, determined by the overall capacity of available rolling stock. Moreover, the wagons reserved for the vulnerable have limited capacity due to stricter social distancing rules than the vehicles reserved for the non-vulnerable. The problem is to:

$$\underset{p^V, p^N, Q^V, Q^N}{Max} \int_0^{y^V} P^V(z)dz + \int_0^{y^N} P^N(z)dz - g^{yV}(.)Y^V - g^{yN}(.)Y^N$$

$$+\left[p^V Y^V + p^N Y^N) - c_Y(Y^V + Y^N) \right] - r^V Q^V - r^N Q^N$$

$$s.t. \quad Q^V + Q^N \leq Cap \quad (\lambda)$$

$$\frac{y^V}{Q^V} \leq Z \qquad (\delta)$$

The first constraint is a capacity restriction on the available wagons. The second restricts the occupation rate on the wagons for the vulnerable due to social distancing. The *r*'s are the costs the firm incurs for operating wagons of each type.

First consider the solution when separating the two types can be based on observable indicators. We find, under the reasonable assuming that both the overall capacity constraint and the occupancy restriction for the wagons reserved for the vulnerable are binding at the optimum:

$$p^V = c + \left(\frac{\beta^V y^V}{Q^V} + \frac{\delta}{Q^V} \right); \qquad p^N = c + \frac{\beta^N y^N}{Q^N}$$

$$Q^V = \sqrt{\frac{\left(\beta^V y^V + \delta\right) y^V}{r^V + \lambda}}; \qquad Q^N = \sqrt{\frac{\beta^N (y^N)^2}{r^N + \lambda}}.$$

The optimal fare for use of the wagon reserved for the non-vulnerable is the marginal social cost, consisting of the production cost *c* plus the marginal external inconvenience cost. The fare for the vulnerable has an extra component because of the occupation restriction due to social distancing. Whether the fare is higher or lower for the vulnerable or the non-vulnerable is a priori unclear: vulnerable passengers can be assumed to perceive higher inconvenience at the margin ($\beta^V > \beta^N$) and they are charged for the capacity constraint due to social distancing, but the occupation rate of their wagons is lower.

Looking at the optimal allocation of the available peak period wagons, we note that the overall capacity constraint restricts the number of both wagon types. More importantly, the restriction on the occupation rate of the wagons reserved for the vulnerable raises the number of wagons allocated to the vulnerable passengers. Whether more than half of the wagons is reserved for the vulnerable depends on the number of vulnerable versus non-vulnerable passengers, and on the severity of the social distancing rules implemented in the wagons reserved for the vulnerable.

Of course, price discrimination is not necessary. The solution to the following problem gives an optimal equilibrium without price discrimination:

$$\begin{aligned}
&\underset{p,Q^V,Q^N}{Max} \quad \int_0^{y^V} P(z)dz + \int_0^{y^N} P(z)dz - g^{yV}(.)Y^V - g^{yN}(.)Y^N \\
&\qquad + \left[p(Y^V + Y^N) - c_Y(Y^V + Y^N) \right] - r^V Q^V - r^N Q^N \\
&\qquad s.t. \quad Q^V + Q^N \le Cap \quad (\lambda) \\
&\qquad\qquad \frac{y^V}{Q^V} \le Z \qquad\qquad (\delta)
\end{aligned}$$

We find

$$p = c + s\left(\frac{\beta^V y^V}{Q^V} + \frac{\delta}{Q^V} \right) + (1-s)\frac{\beta^N y^N}{Q^N}$$

$$Q^V = \sqrt{\frac{\left(\beta^V y^V + \delta\right) y^V}{r^V + \lambda - (1-s)(T)\frac{\partial y^V}{\partial Q^V}}}; \qquad Q^N = \sqrt{\frac{\beta^N (y^N)^2}{r^N + \lambda + s(T)\frac{\partial y^N}{\partial Q^N}}}$$

where

$$s = \frac{\frac{\partial y^V}{\partial p}}{\frac{\partial y^V}{\partial p} + \frac{\partial y^N}{\partial p}}; \quad T = \frac{\beta^V y^V}{Q^V} + \frac{\delta}{Q^V} - \frac{\beta^N y^N}{Q^N}.$$

The optimal fare is the marginal production cost plus a weighted sum of the marginal external inconvenience costs, including the correction for the constraint on the wagon capacity for the vulnerable travelers. Assuming that $T>0$, the larger is T the larger will be the fraction of wagons reserved for the vulnerable.

Finally, if the separation of vulnerable from non-vulnerable passengers is not based on observable characteristics but passengers self-select the wagons to enter, fares and capacity allocation should be such that neither type has an incentive to use a wagon reserved for the other type. This requires

$$p^V + \beta^V \frac{y^V}{Q^V} \le p^N + \beta^V \frac{y^N}{Q^N} \to p^V - p^N \le \beta^V \left(\frac{y^N}{Q^N} - \frac{y^V}{Q^V} \right)$$

$$p^N + \beta^N \frac{y^N}{Q^N} \le p^V + \beta^N \frac{y^V}{Q^V} \to p^V - p^N \ge \beta^N \left(\frac{y^N}{Q^N} - \frac{y^V}{Q^V} \right)$$

If the above-described solution satisfies these constraints we have a separating equilibrium. Note that, since the number of passengers in the wagons for the non-vulnerable will be higher than for the vulnerable, satisfying the constraints implies that the vulnerable will face a higher fare in a separating equilibrium. We will necessarily have $p^V > p^N$.

Proposition 4. Optimal allocation of capacity for the vulnerable versus non-vulnerable

a. **More restrictive social distancing on wagons for the vulnerable raises the fare for vulnerable passengers, and it implies that more wagons are reserved for the vulnerable.**
b. **A separating equilibrium implies that fares for trips in the wagons intended for the vulnerable are sufficiently higher than fares in the wagons for the non-vulnerable to prevent non-vulnerable passengers to use wagons reserved for the vulnerable.**

6. Application to public transport in Brussels

In this section, we use data for Brussels, a city of 1 million inhabitants, to illustrate some of the implications of Covid-19 for public transport. The numerical model is calibrated to reflect the local transport situation in Brussels prior to the Corona-crisis. As explained in more detail below, the numerical model is not a literal translation of the theory presented above. For example, the theory allowed marginal changes in the frequency; in the numerical model, we (realistically) restrict frequencies to integer values. Moreover, the application takes into account the institutional setting; this implies, among others, that most peak-period public transport passengers have subscriptions that are financed by employers and/or the government.

6.1 Description of the case study

We distinguish between two types of peak period trips: the trips to school (schoolchildren) or to the university (students), and the trips to work and for other purposes. For school trips, we assume there is no modal choice between public transport and car use; children can only forego the trip, or they can walk or bike to their destination. For the other (mainly work) trips we do allow the choice between road use and public transport use; the use of other modes is not explicitly represented. For the other trips, the user costs for public transport and car use are equal in equilibrium; they are considered as perfect substitutes.

The public transport data come from the local public transport firm. Information on car use was provided by the Brussels Regional government. We represent the relative trip levels for a typical day in the week in pre-Corona times in Table 1. We normalize the total number of trips at 100 and use rounded figures; our main interest is in showing the broad effects of telework and Covid-19 on fares, frequencies etc. As mentioned, we distinguish between public transport trips that are substitutable by car trips, and bus or train trips where the car is not a viable alternative. The former trips are mainly work trips, the latter consist of school trips by children and students. Based on the information provided, about 40% of public transport trips in the peak period are school trips. For the journey to work to Brussels, slightly less than 30% of all trips are by public transport, the remainder are car trips. The total trip volume for non-school trips (car and public transport) amounts to 210.

Relative number of peak trips/day	Substitutable trips (road, public transport): non-school (mainly work) trips		Non-substitutable trips: school trips	Total public transport
	Road	Public transport	Public transport	
Pre-Covid	150	60	40	100

Table 1. Trip distribution in the pre-Corona equilibrium

We will consider school children as less vulnerable than workers commuting to work. In our analysis we will therefore let the aversion to crowding (due to the perceived risk of infection) because of the Corona-pandemic increase only for the trips to work and not for the school trips, see below.

6.2 Model calibration

The calibration of the demand and generalized cost functions is explained in detail in Appendix B. We calibrate the demand functions for substitutable work trips as linear functions with price elasticity -0.4. We further assumed a value of time for substitutable work trips of 10€/hour and 5€/hour for non-substitutable school trips. As there is no toll in the baseline, the external costs are not charged for road use, so that the generalized price is the user cost that consists of money costs (fuel, wear and tear) and time costs. For public transport, the average cost consists of the fare, the time costs associated with walking and waiting at the station or bus stop, and the in-vehicle time.

We interpret the baseline equilibrium as a second-best equilibrium. The second-best fare (see the theoretical section) implies charging for the external cost of crowding, but it also implies a subsidy to correct for the absence of a road toll on car use. This gives (see Appendix B) a second-best fare in the baseline very close to zero. Note that this is entirely consistent with the observed fares. Most public transport users in Brussels have a monthly or annual pass that in fact makes the marginal price of an extra trip zero. Moreover, the cost of the pass is in the huge majority of cases paid by the government (for school trips) or by the employer. As a reasonable approximation, we therefore set the second-best baseline fare equal to zero. The implied subsidy to public transport is a second-best subsidy shifting users from road to public transport. For work trips, the subsidy is often justified to correct for the absence of road pricing. For school trips, the zero fare in the baseline is often motivated by income distribution concerns. Do note that the zero fare remains second-best because it leads to a combined (road and public transport) equilibrium trip cost that is too low compared to the first-best optimum.

Based on the information provided in Appendix B, we give in Table 2 the composition of the generalized costs of car and public transport use in the baseline equilibrium.

Car use	**Average Cost (€/trip)**	**Public transport (non-school)**	**Average Cost (€/trip)**	**Public transport (school)**	Average Cost (€/trip)
Vehicle cost	0.66	**Walk**	1.14	**Walk**	0.57
Min time cost	1.83	**Wait time**	0.5	**Wait time**	0.25
Congestion	1.5	**In-vehicle**	1	**In-vehicle**	0.5
		Crowding	1.36	**Crowding**	0.68
Toll	0	**Fare**	0	**Fare**	0
Total generalized cost	4	**Total generalized cost**	4	**Total generalized cost**	2

Table 2. Composition of generalized costs in the baseline equilibrium

Figure 3 represents the baseline (pre-Covid) equilibrium graphically.

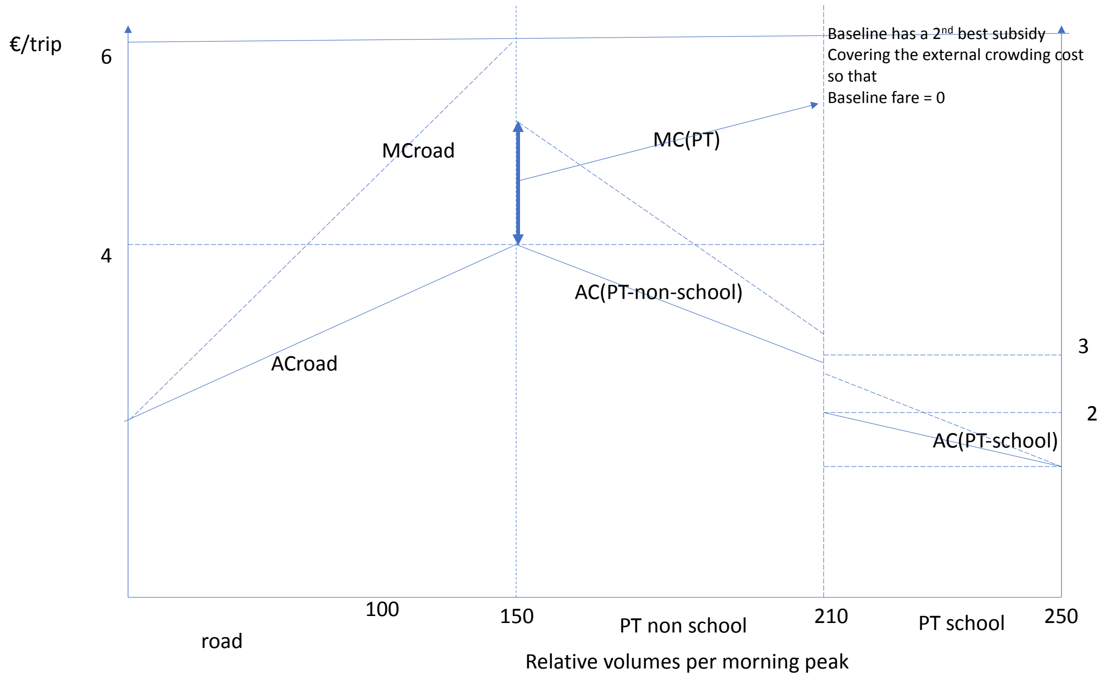

Figure 3. The baseline equilibrium for Brussels

6.3 Model simulations and results

We test three different policies. We first study the effects of Covid19 (changes in telework and perceived infection risk) on ridership of public transport and its implications for the optimal frequency. Next we look into the potential effect of reserving capacity for the more vulnerable. Finally, we consider the effects of policies that promote the use of other transport modes for trips to school.

Note that we do not investigate variations in the public transport fare. The reason is twofold. First, as argued in the theoretical sections, telework and a high perceived infection risk have opposite effects on the optimal fare, with the joint effect likely to be very small. Second, fare adaptations are politically difficult in the short-run. We therefore prefer to simulate some of the policies that are potentially easier to implement.

Studying the effects on ridership, frequency and the deficit

We simulate results for 3 cases of lower demand for work trips (telework) and for trips to school: demand is reduced by 25%, 50% and 75%, respectively. In each case, the demand function for work trips is recalibrated so as to reproduce a decrease of trips by the relevant percentage, keeping the same slope at baseline fare and frequency conditions. We also consider three levels for the crowding aversion factor, capturing different levels of the perceived infection risk. The index in the baseline is set at 100, and increases by 50% and by 200% are implemented in the model.

Results are summarized in Table 3. Two preliminary remarks are useful to properly interpret the figures in the table. First, note that we restrict, unlike in the theory, frequency to integer figures. We therefore report simulation results for different values of frequency, identifying the optimal integer value by evaluating the welfare index. Second, although the meaning of the other columns is self-explanatory, this may not be the case for the deficit reported in the final column. We calculated the deficit taking into account the institutional setting in which public transport operates. The costs include the cost of offering frequency ($c_0.f$ in the notation of our theoretical model) and the variable cost of carrying passengers ($c_Y.Y$). The revenues are computed in function of the current institutional system. Employers pay for a public transport subscription of their employees, so that employees travel for free. Employers pay the cost of frequency and the variable passenger cost in proportion to their use (which was 60% in our baseline). For school trips, children and students benefit from a subscription at a very low price. For practical purposes, it is fair to say that the government pays for all school trips. To arrive at the deficit figures reported in the final column of Table 3, we first calculated the deficit in absolute terms. For example, in the baseline, the deficit amounted to 40% of variable costs. We then normalized the absolute baseline deficit to equal 100.

n°	Relative demand (telework)	Relative Crowding aversion parameter	**PT-trips work**	PT-trips school	Fare PT work	Freq-uency	**Gen. Price Car trip**	**Welfare**	**Crowding index**	**Deficit In absolute Terms**
1 (baseline)	1	100	**60**	40	0	10	**4**	**891**	**100**	**100**
2	1	150	**36**	40	0	10	**4,2**	**860**	**76**	**129**
3	1	150	**50**	40	0	12	**4,1**	**850**	**74**	**131**
4	1	150	**13**	40	0	7	**4,4**	**863**	**75**	**131**
5	0,75	100	**13**	30	0	5	**4,0**	**507**	**72**	**111**
6	0,75	100	**0**	30	0	3	**4,1**	**517**	**100**	**75**
7	0,75	150	**28**	30	0	10	**3,8**	**472**	**58**	**125**
8	0,75	150	**0**	30	0	3	**4,1**	**517**	**100**	**75**
9	0,75	200	**0**	30	0	3	**4,1**	**517**	**100**	**75**
10	0,5	100	**6**	20	0	6	**3,6**	**222**	**44**	**109**
11	0,5	100	**0**	20	0	3	**3,6**	**250**	**67**	**73**
12	0,5	150	**20**	20	0	10	**3,5**	**178**	**40**	**147**
13	0,5	150	**0**	20	0	2	**3,6**	**252**	**100**	**50**
14	0,5	200	**0**	20	0	3	**3,6**	**252**	**100**	**50**
15	0,25	150	**12**	10	0	10	**3,1**	**-24**	**22**	**107**
16	0,25	150	**0**	10	0	2	**3,2**	**78**	**50**	**48**

Table 3. The effect of increases in telework and in the perceived infection risk: simulation results for Brussels (the bold columns represent simulation results, other columns are assumptions).

The first line in Table 3 represents the baseline. In the next three lines we keep the same total demand for work and school trips and investigate the effect of a higher perceived infection risk on the optimal frequency. We specifically increase the discomfort factor for work trips by 50%. The result in line 2 suggests that, keeping frequency at its baseline value of 10 busses per hour leads, as expected, to a strong decline in the use of public transport for work trips (36 instead of 60). Crowding declines, whereas congestion on the road increases: the generalized cost for car use increases from 4 to 4,2 €/trip. The deficit increases because of the decrease in ridership by employees.

One may be tempted to address the higher discomfort in public transport use and the increased road congestion by increasing the frequency. This is illustrated in line 3, where an increase of the frequency from 10 to 12 vehicles per hour is able to attract more public transport users for the trip to work than in line 2, and to decrease the level of car congestion. However, augmenting frequency is welfare-reducing (welfare declines from 860 to 854); moreover, it raises the deficit. It turns out that, in line with the theory (see Proposition 1), the optimal response to a higher perceived infection risk is to reduce frequency to 7 buses per hour. This is illustrated in line 4; welfare increases to 863. Clearly, the effect of the higher perceived infection risk is to strongly reduce demand, allowing a lower frequency to be offered.

Lines 5 to 14 in Table 3 consider the effect of demand reductions due to the government's effort (or even requirement) to increase telework. First we discuss lines 10 to 14, where we look at the implications of a 50% demand reduction relative to the baseline. Interestingly, we find (see the comparison of lines 10 and 11) that, if there were no increase in crowding discomfort, it is optimal not to have any work trips by public transport at all. Line 10 (a frequency of 6 per hour and an index of 6 trips to work) generates lower welfare than the optimum given in line 11 (a frequency of 3 per hour but no more public transport work trips). The main reason is that telework reduces the demand for road use and leads to a decline in congestion; addressing congestion by encouraging public transport use for work trips is then costly in terms of the extra frequency offered; it is not efficient. The lower frequency in line 11 decreases the deficit (73 instead of 109). All this under the assumption one can save the costs of frequency, which may be economically and politically difficult in the short term.

In lines 12 to 14 we add a higher Covid-inspired discomfort for public transport trips to work to the 50% demand reduction due to telework. Comparing lines 12 and 13 we see that workers avoid public transport (their demand drops to zero), and we find the optimal frequency dropping to 2 (line 13); this leads to a higher welfare level than keeping the baseline frequency of 10 and keeping some commuters to use public transport (line 12). Offering a frequency of 2 per hour is optimal; it serves to accommodate the demand for school trips. For an even higher Covid-inspired discomfort, one has similar results (line 14).

It is argued that once the Covid pandemic is over, telework will be more common than in pre-Corona times, but less pronounced than during the pandemic. To simulate this case, we look at a demand reduction of only 25%, see lines 5-9. If the perceived infection risk drops to its baseline value, there are two options for public transport. This is illustrated in lines 5 and 6. One option is to have an internal solution with some work trips going by public transport, see line 5. The frequency is kept relatively high at 5 per hour (but much below the baseline value of 10). The alternative is a corner solution where public transport is only used for trips to school (and an optimal frequency of 3). We find that the equilibrium with low frequency is welfare optimal. The main reason is again that road congestion has decreased so that the main motivation to have low fares and high frequencies disappears. Combining high levels of telework with higher perceived infection risk (lines 7 to 9) shows that, as before, it becomes optimal to have no more work trips by public transport and a low frequency at 3 vehicles per hour. All this holds, of course, assuming one can save the full costs of high frequencies.

The main (and politically) controversial message from these results is that, if more telework becomes the norm and the perceived infection risk does not return to pre-Covid times, from a welfare point of view it becomes difficult to justify substantial public transport use for commuting trips.

Reserving capacity for the more vulnerable

In Section 5.3, it was shown that reserving a larger part of the capacity for the vulnerable may be a useful policy. We illustrate this idea by showing the effects of such a reservation system in Table 4. In practice, reserving capacity for the vulnerable should not be too difficult to organize. For example, it can be implemented by asking the users of the reserved spaces to show their subscriptions. Another option is to use a small entrance fee for the reserved spaces.

In line 1, we start from "full sharing" of capacity among the vulnerable and the non-vulnerable. We assume 50% telework and a relative crowding parameter of 50% more than the baseline, and we set the frequency at 10. This line corresponds to line 12 in Table 3. In this full sharing case, there are 20 trips to work (and 20 trips to school). As trips to work are more vulnerable to crowding, we then simulate what happens when one reserves more than 50% of the capacity of each train for the trips to work. This will have two beneficial effects: the vulnerable suffer less from crowding, and the average trip cost to work declines because more work trips will take public transport.

n°	Relative demand	Frequency	Relative crowding aversion	Share Capacity Vulnerable	Share Capacity Non-Vulnerable	**AC Car and PT**	**Trips to work by car**	**Trips to work by PT**
1	0,5	10	150%	Full sharing		**3,46**	**96**	**20**
2	0,5	10	150%	0,7	0,3	**3,39**	**90**	**26**
3	0,5	10	150%	0,8	0,2	**3,37**	**88**	**28**
4	0,5	10	150%	0,9	0,1	**3,34**	**85**	**31**

Table 4. Effects of reserving public transport capacity for the vulnerable (the bold columns represent simulation results, other columns are assumptions).

Results are in lines 2 to 4. Reserving progressively 70%, 80% and 90% of the capacity for work trips (lines 2,3 and 4), one attracts more commuters to public transport; this reduces the average trip costs to work for both modes. The smaller capacity available for the fixed number of trips to school leads (not shown in the table) to a small increase in average cost because crowding is less valued in school trips.

Promoting the use of other transport modes for the trips to school

The Covid-pandemic makes the vulnerable users substitute public transport trips by car trips. One of the problems is that public transport school trips take away an important part of the capacity of public transport, so that commuters face more crowding. Moreover, increasing the fares for trips to school is politically difficult. However, what can easily be done is to encourage the use of other modes (walk, (e) bike, (e) step) by subsidies and better facilities.

In Table 5 we illustrate the effects of a policy that manages to make 10 of the 20 pupils or students switch to other modes.

n°	Relative demand	Frequency	Relative crowding	PT Trips to School	**PT trips to work**	**AC Road &PT**	**Welfare**	**Deficit**
1	0,5	10	150%	20	20	3,455	178	120
2	0,5	10	150%	10	27	3,396	181	64

Table 5. Effects of exogenous changes in the modal choice for school trips

We start in line 1 from the case with a frequency of 10 and 50% telework and teleschool; this again corresponds to line 12 in Table 3. In line 2 we assume that 10 trips to school opt for another mode (walking, step, bike). Keeping the frequency constant, this allows to have more trips to work via public transport; these extra users value the decrease in crowding (as they are vulnerable users) leading to a decrease in the average cost AC of trips to work. The result is a small increase in welfare (181 instead of 178). There is also a decrease in the deficit because there are now more public transport trips to work and, therefore, employers contribute more to revenues.

In the simple exercise above we shifted exogenously the modal share for school trips; we kept the total number of school trips constant except for teleschool obligations. Of course also trips to work could shift to other modes than car and public transport; this would also be an efficient policy to cope with larger infection risks.

7. Conclusions

We study the effect of telework and perceived Covid-19 infection risk for public transport policies within the framework of a simple model to determine second-best optimal fare and frequencies for a public transport operator when the competitive mode (car use) does not pay for the full marginal external cost. An increase in telework reduces the demand for both transport modes. If the public transport firm could flexibly adapt its policies the efficient second-best response would be to reduce both the fare and the frequency. The increase in the external crowding cost due to the perceived Corona-infection risk reduces public transport demand and increases car use. We found that under plausibly conditions the optimal fare increases; the effect on optimal frequency depends on the relative importance of higher frequency on demand. Interestingly, as telework and the infection risk have opposite effects on the fare in practice the welfare improvement of fare changes may be very small; for all practical purposes it may be optimal for the firm not to change the fare at all. Second, assuming that pre-Covid fare and frequency were second-best optimal, holding the fare and frequency constant at these values implies that, surprisingly, more telework reduces the public transport deficit. The effect of a higher perception risk is ambiguous in general but, if car use is strongly under-priced, the higher risk perception reduces the deficit as well. Third, extending the model to allow for passengers with different vulnerability towards Covid-19, we show that fare differences between different types of passengers are unwarranted unless for each type separate vehicles can be used. Allowing fare and frequency differentiation implies that lower occupancy rates per vehicle for the vulnerable are socially optimal. They will face higher fares only if their risk perception is sufficiently higher than that of the non-vulnerable and car use is not too much under-priced. Fourth, more restrictive social distancing on wagons for the vulnerable raises the fare for vulnerable passengers, and it implies that more wagons are reserved for the vulnerable. Lastly, a separating equilibrium implies that fares for trips in the wagons intended for the vulnerable are sufficiently higher than fares in the wagons for the non-vulnerable to prevent non-vulnerable passengers to use wagons reserved for the vulnerable.

In the application to Brussels we assumed that trips to work can chose between car and public transport use, but school trips can only use public transport. In the baseline, a very low fare and a high frequency are justified to address un-priced car congestion. Whenever telework reduces the demand for trips by at least 25% and the perceived crowding externality becomes higher, there are two policy options. The first is to keep frequency relatively high and keep a fair share of public transport use for trips to work. The second option is a corner solution:

strongly decrease the frequency, and have all trips to work switch to car use. The second option is difficult in the short-term but was found to be clearly superior in welfare terms; moreover, it decreases the deficit. As telework may continue to be more important in the future, even after the Covid pandemic, car congestion in the peak period is likely to be less important. One may opt for an equilibrium with a much smaller frequency of public transport. We also found that a reservation of capacity for the vulnerable users and a modal shift to soft modes could produce important efficiency gains.

Many caveats apply to the model and the numerical illustration to Brussels. The first is that we use perceived infection risk as basis for agents' behaviour. Of course also the objective risk of infection transmission matters, but we do not know how the two types of risks correlate. If the objective risk is much higher than the perceived risk, the priorities in terms of fare and frequency will be very different. The second caveat is that we represent the behaviour of only two categories of agents (vulnerable and less vulnerable users) that have each different options to travel. Reality offers much more diversity in types of agents and in types of transport modes available to the two types of agents. Biking, steps and simple walking saw their market share increase and this extra margin of adaptation softens the public transport versus car paradox we studied in this paper.

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Appendix A: the effect of telework and infection risk on fare, frequency and the deficit

Denoting the social welfare function by W the first-order condition with respect to the fare can be written as

$$\frac{\partial W}{\partial p} = (p - c_Y - \frac{\beta}{f}Y)\frac{\partial Y}{\partial p} + (\tau^\circ - \nu X)\frac{\partial X}{\partial p} = 0$$

Using the price effects calculated in the main body of the paper this can be rewritten as

$$\frac{\partial W}{\partial p} = -\frac{1}{\Delta}\left[(b+v)(p - c_Y - \frac{\beta}{f}Y) - b(\tau^\circ - \nu X)\right] = 0 \qquad \text{(A1)}$$

The first-order condition with respect to frequency reads:

$$\frac{\partial W}{\partial f} = -c_0 + \frac{Y}{f^2}(\alpha + \beta Y) = 0 \qquad \text{(A2)}$$

Totally differentiating system (A1)-(A2) this system gives, in matrix notation:

$$\begin{bmatrix} \frac{\partial^2 W}{\partial p^2} & \frac{\partial^2 W}{\partial p \partial f} \\ \frac{\partial^2 W}{\partial f \partial p} & \frac{\partial^2 W}{\partial f^2} \end{bmatrix} \begin{bmatrix} dp \\ df \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 W}{\partial p \partial a} da - \frac{\partial^2 W}{\partial p \partial \beta} d\beta \\ -\frac{\partial^2 W}{\partial f \partial a} da - \frac{\partial^2 W}{\partial f \partial \beta} d\beta \end{bmatrix}$$

Solving by Cramer's rule, we find the effects of telework and infection risk on fare and frequency in general as:

$$\begin{aligned} \frac{dp}{da} &= \frac{1}{\Omega}\left[-\frac{\partial^2 W}{\partial p \partial a}\frac{\partial^2 W}{\partial f^2} + \frac{\partial^2 W}{\partial f \partial a}\frac{\partial^2 W}{\partial f \partial p}\right] \\ \frac{df}{da} &= \frac{1}{\Omega}\left[-\frac{\partial^2 W}{\partial f \partial a}\frac{\partial^2 W}{\partial p^2} + \frac{\partial^2 W}{\partial p \partial a}\frac{\partial^2 W}{\partial f \partial p}\right] \\ \frac{dp}{d\beta} &= \frac{1}{\Omega}\left[-\frac{\partial^2 W}{\partial p \partial \beta}\frac{\partial^2 W}{\partial f^2} + \frac{\partial^2 W}{\partial f \partial \beta}\frac{\partial^2 W}{\partial f \partial p}\right] \\ \frac{df}{d\beta} &= \frac{1}{\Omega}\left[-\frac{\partial^2 W}{\partial f \partial \beta}\frac{\partial^2 W}{\partial p^2} + \frac{\partial^2 W}{\partial p \partial \beta}\frac{\partial^2 W}{\partial f \partial p}\right] \end{aligned} \qquad \text{(A3)}$$

where $\frac{\partial^2 W}{\partial p^2} < 0, \frac{\partial^2 W}{\partial p^2} < 0,$ and $\Omega > 0$ by the second-order condition for welfare maximization.

Extensive algebra further shows, using earlier results:

$$\frac{\partial^2 W}{\partial f\,\partial p} = \frac{\partial^2 W}{\partial p \partial f} = \frac{(\alpha + 2\beta Y)\dfrac{\partial Y}{\partial p}}{f^2} < 0$$

$$\frac{\partial^2 W}{\partial p \partial a} = \frac{1}{\Delta^2}\frac{\beta v^2}{f} > 0$$

$$\frac{\partial^2 W}{\partial f\,\partial a} = \frac{(\alpha + 2\beta Y)\dfrac{\partial Y}{\partial a}}{f^2} > 0$$

$$\frac{\partial^2 W}{\partial p \partial \beta} = \frac{1}{\Delta^2}\frac{Ybv^2}{f} > 0$$

$$\frac{\partial^2 W}{\partial f\,\partial \beta} = \frac{Y^2 + (\alpha + 2\beta Y)\dfrac{\partial Y}{\partial \beta}}{f^2}$$

All terms except the last one can be unambiguously signed. Using these expressions to evaluate the effect of telework and infection risk on fare and frequency we see that all these effects are ambiguous in general. This is not surprising. For example, teleworking affects the demand for both public and private transport demand, and these demand changes affect the optimal fare in opposite directions; moreover, telework affects the optimal frequency which in turn affects the optimal fare.

To get some intuition, it is instructive to consider the effect of telework and infection risk on the optimal fare, holding frequency constant. We then find:

$$\frac{dp}{da} = -\frac{\dfrac{\partial^2 W}{\partial p \partial a}}{\dfrac{\partial^2 W}{\partial p^2}} = -\frac{\dfrac{1}{\Delta^2}\dfrac{\beta v^2}{f}}{\dfrac{\partial^2 W}{\partial p^2}} > 0$$

$$\frac{dp}{d\beta} = -\frac{\dfrac{\partial^2 W}{\partial p \partial \beta}}{\dfrac{\partial^2 W}{\partial p^2}} = -\frac{\dfrac{1}{\Delta^2}\dfrac{Ybv^2}{f}}{\dfrac{\partial^2 W}{\partial p^2}} > 0$$

Noting the telework is a decline in *a*, this suggests that – holding frequency constant -- telework reduces the optimal fare. The effect of the reduction on public transport demand dominates the opposite effect of the reduction in car use. A higher infection risk raises the optimal fare, as it amounts to an increase in the perceived cost of crowding.

Similarly, let us see what the effect is of telework and infection risk on frequency at given fares. We find:

$$\frac{df}{da} = -\frac{\dfrac{\partial^2 W}{\partial f \partial a}}{\dfrac{\partial^2 W}{\partial f^2}} = -\frac{\dfrac{(\alpha + 2\beta Y)\dfrac{\partial Y}{\partial a}}{f^2}}{\dfrac{\partial^2 W}{\partial f^2}} > 0$$

$$\frac{df}{d\beta} = -\frac{\dfrac{\partial^2 W}{\partial f \partial \beta}}{\dfrac{\partial^2 W}{\partial p^2}} > or < 0$$

More telework reduces the optimal frequency. Note that this result assumes that the infection risk coefficient β remains constant. A higher perceived risk of infection itself can raise or reduce the optimal frequency. The reason is that offering more frequency offers safer travel, holding the number of passengers fixed; however, it also increases demand, which reduces safety. A higher risk of infection will increase the optimal frequency if the infection risk only mildly reduces demand. If demand strongly declines, a higher infection risk reduces the optimal frequency.

Finally, consider the effect of telework and infection risk on the public transport deficit. Suppose that the firm has implemented the socially optimal fare-frequency policy prior to changes in telework and infection risk, and that it optimally adapts the fare and the frequency in response to these changes. We then easily show:

$$\frac{d\pi}{da} = \left[(\tau^\circ - \nu X)\left(\frac{b}{b+v}\right) - \frac{\alpha}{f}\right]\left[\frac{\partial Y}{\partial p}\frac{dp}{da} + \frac{\partial Y}{\partial f}\frac{df}{da} + \frac{\partial Y}{\partial a}\right] - \frac{Yvb}{b+v}\left[\frac{\partial X}{\partial p}\frac{dp}{da} + \frac{\partial X}{\partial f}\frac{df}{da} + \frac{\partial X}{\partial a}\right]$$

$$\frac{d\pi}{d\beta} = \left[(\tau^\circ - \nu X)\left(\frac{b}{b+v}\right) - \frac{\alpha}{f}\right]\left[\frac{\partial Y}{\partial p}\frac{dp}{d\beta} + \frac{\partial Y}{\partial f}\frac{df}{d\beta} + \frac{\partial Y}{\partial \beta}\right] - \frac{Yvb}{b+v}\left[\frac{\partial X}{\partial p}\frac{dp}{d\beta} + \frac{\partial X}{\partial f}\frac{df}{d\beta} + \frac{\partial X}{\partial \beta}\right]$$

These effects are of course ambiguous in general.

However, suppose the firm only adapts the frequency and holds the fare constant. Then

$$\frac{d\pi}{da} = \left[(\tau^\circ - \nu X)\left(\frac{b}{b+v}\right) - \frac{\alpha}{f}\right]\left[\frac{\partial Y}{\partial f}\frac{df}{da} + \frac{\partial Y}{\partial a}\right] - \frac{Yvb}{b+v}\left[\frac{\partial X}{\partial f}\frac{df}{da} + \frac{\partial X}{\partial a}\right]$$

$$\frac{d\pi}{d\beta}=\left[(\tau^{\circ}-\nu X)\left(\frac{b}{b+\nu}\right)-\frac{\alpha}{f}\right]\left[\frac{\partial Y}{\partial f}\frac{df}{d\beta}+\frac{\partial Y}{\partial\beta}\right]$$
$$-\frac{Y\nu b}{b+\nu}\left[\frac{\partial X}{\partial f}\frac{df}{d\beta}+\frac{\partial X}{\partial\beta}\right]$$

Assuming the frequency effect on car use is quite small, more telework unambiguously raises profit (or reduces the deficit).

Appendix B: calibrating the example for Brussels

We use data from the statistical yearbook of the Brussels public transport authority (STIB, 2019) for the year 2018. In Brussels, 75% of public transport trips are by metro or tram; general subscriptions (mainly commuters to work) count for 65% and trips to school for 22% of all trips. The major part of the employee subscriptions are paid by the employer and the government; by far the largest fraction of school subscriptions is also paid by subsidies. The marginal price for both these trips is therefore almost 0. We neglect the 13% of the trips that are paid by other types of tickets.

The average cost of a public transport trip in Brussels equals 1.41 €.We reckon that a trip in the peak costs 50% more than in the off-peak; 66% of the trips are in the peak period. This brings the cost of a trip in the peak to 1.59 €. We further assume that there is a variable cost per passenger of 0.11 € so that the cost of offering peak frequency equals per passenger 1.48 €.

1. Demand and generalized costs for work trips

The generalized cost of public transport work trips

The generalized price consists of the fare, plus the cost of the walking time, the waiting time, and the in-vehicle time. Moreover, we capture the discomfort of crowding in the specification of the cost of the in-vehicle time.

The fare in the baseline equilibrium is set to zero. There are two reasons for doing so. First, the huge majority of trips in the peak period is by students and workers that have a subsidized (by the government in the case of students, by the employer and the government jointly in the case of workers) pass that makes the effective marginal price zero. Second, we assume that the initial pre-Corona equilibrium is a second-best equilibrium with underpriced car use due to the absence of a toll on road use. A second-best fare that neither corrects for crowding nor for road congestion boils down to a fare very close to zero, see below.

The public transport data suggest that the average trip takes 21,64 minutes (most public transport trips cover a distance less than 3 km) and consists of 8,64 minutes walking time, 3 minutes waiting time, and 10 minutes trip time in the public transport vehicle itself. To arrive at the average waiting time, we used the information from the Brussels' public transport operator MIVB that the frequency on the main lines if 10 services per hour, or a vehicle every 6 minutes. Assuming passengers arrive at random, this gives an average waiting time of 3 minutes. The MIVB further targets a commercial speed of 30km/h, so that a 3 km trip takes 10 min. The VOT for work trips is valued 10€/h (cfr. Bartley et al., 2019).

Lastly, to account for the discomfort of crowding in the baseline second-best equilibrium, we applied a discomfort parameter for in-vehicle time, following de Palma, Kilani and Proost (2015) and Tirachini, Henscher and Rose (2013). However, we used a lower value than in these studies, because Brussels has not the same extreme crowding as Paris or London. Our analysis used a crowding multiplier for the value of time VOT of 1.06.

The computation of the generalized price of public transport is explained in Table A1.

In € per PT trip in peak in baseline	Value in €	Minutes	Comments
Money price	0		Cheap subscription, so marginal price=0
Walk time	1,44	8,64	(7,92min/60min) (VOT=10 euro per hour)
Waiting time (1/(2f))	0,5	3	(1/(2f))VOT = (1/20) 10
In-vehicle time	1	6	3 km à 30km/h x 10 euro per hour
Discomfort factor (discomfort Beta . Y/f)(in-vehicle time).VOT	1,06		Factor +106% for in vehicle time in peak period,
TOTAL	4,00		

Table A1. Computation generalized cost public transport (work trips)

<u>The generalized price of car use for work trips</u>

We treat car and public transport trips as perfect substitutes for work trips. This is a strong assumption, but it much simplifies both the theoretical and the numerical analysis. Moreover, it is less unrealistic than it seems once one realizes that crowding in public transport was accounted for, and that both waiting times and in-vehicle time are taken into account in calculating the generalized price of public transport, see above.

Given the assumption of perfect substitutability, in equilibrium we will have that generalized prices of car and public transport use are equal. This information is used to calibrate the generalized price for car use. Given an average car trip of 4 km on Brussels territory the generalized price for car use has to equal 4 €/trip (the generalized price of public transport) when there are 150 (expressed as index, where total public transport use is 100) vehicles on the road. There is no congestion toll.

The money cost of car use was determined using a cost per kilometer of 0.165 euro. This gives (0.165 €/km)(4km)= 0.66 €. To obtain equal generalized prices in equilibrium we calibrated the congestion function accordingly. We assumed a maximal speed in the Brussels area of 30 km/h and an observed speed 60% lower, i.e., of 12 km/h. This gives a time cost at maximal speed (30km/h) = (4/30)10 = 1.33 €. The time cost at the observed volume of 150 is then 12 km/h = (4/12)10=3.33 €. This ultimately gives an average cost of 0.66 + (3.33/150)150 = 4€/trip.

Note that the average cost of car use is therefore 0.66 + (3.33/150)X, where X is the traffic volume.

Calibrating the demand function for work trips

The total number of trips for non-school purposes in the baseline equilibrium is (in index form, see above) Y+X = 150+60=210. The generalized price is 4€/trip. Using a price elasticity of -0.4, this gives the demand function

X+Y=294 -21 P.

In inverse form this reads P=14-0.0476 (X+Y).

2. Demand and generalized costs for school trips

A similar procedure is followed for school trips. However, for school traffic we assume that car use is not an option. The only available substitutes are biking and walking, two substitutes without external costs.

Using the same procedure as for work trips, but now assuming a VOT for trip to school at 5 €/h (cfr. Bartley et al., 2019), we have the components of the generalized price for school trips as given in Table A2.

In € per PT trip in peak in baseline	Value in €	In minutes	Comments
Money price	0		Cheap subscription, so marginal price=0
Walk time	0.72	8.64	(8.64 min/60min) 5€ VOT = 0.72
Waiting time (1/(2f))	0.25	3	0.5 (1/(2f)) VOT = 0.5 (1/10) 5=0.25
In Vehicle time	0.50	6	3 km à 30km/h x VOT
Discomfort factor (discomfort Beta . Y/f)(in-vehicle time).VOT	0.53		Factor +1.06% for in vehicle time in peak period.
TOTAL	2.00		

Table A2. Computation generalized cost public transport (school trips)

As before, we assume a generalized price elasticity of -0.4. Since the baseline school trip volume, denoted Y^S is 40 (in index form) this gives the demand function

$Y^S = 56 - 8\ (GP^Y)$

Lastly, we calibrate the cost of frequency. We know that in a second best fare case, where cars are not priced for the external congestion cost, the traditional optimal frequency formula continues to apply. So $f = \sqrt{\frac{(\alpha + \beta Y)Y}{c_0}}$. Setting $f=10$ and using the other values already calibrated, we find

$$c_0 = \frac{(2.5+6.8)40+(5+13.6)60}{10^2} = 14.88$$

For a total frequency of 10, we obtain 148.8 € and normalized for 100 passengers, we obtain 1.488 which is close to the cost data in the statistical book of the STIB (STIB, 2019).

3. The baseline as a second-best optimum

Remember that we want the baseline equilibrium to be as close as possible to a second-best optimum. For car use, it is easy to find the marginal external congestion cost *MECC*. At the baseline equilibrium, it equals the additional delay caused to the other drivers, calculated at 2€. For public transport use, the only external cost of an extra trip (for given frequency) is the crowding discomfort. Using the values computed above, we obtain 1.59 € (1.06+0.53).

Using the expression for the optimal second-best fare, and assuming the toll is 0, we obtain 0.26 euro for the second best fare. Note that the first best fare would be larger than 2 €.

Given the low second-best fare, and given the institutional system of subscriptions paid by the employer, we will work with a de facto zero fare for public transport use in the baseline equilibrium.