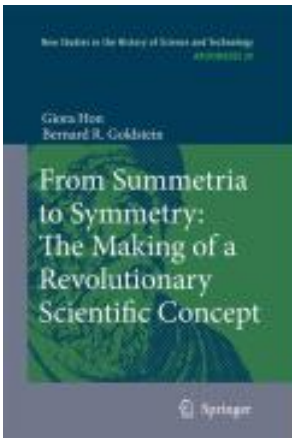
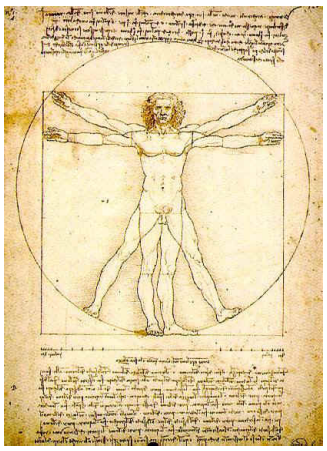
From *Summetria* to Symmetry: The making of a revolutionary scientific concept *Giora Hon and Bernard R. Goldstein.* Archimedes: new studies in the history of science and technology Springer, 2008 (xvi+335) hard cover, ISBN 978-1-4020-8447-8.



The authors of this book present a somewhat controversial and surprising claim: The modern (mathematical) notion of symmetry was not known to men until 1794 when Adrien-Marie Legendre defined an equality of solid angles by symmetry. From his *Éléments de géométrie*, the quote is

> Two equal solid angles which are formed (by the same plane angles) but in reverse order will be called *angles equal by symmetry*, or simply *symmetrical angles*.

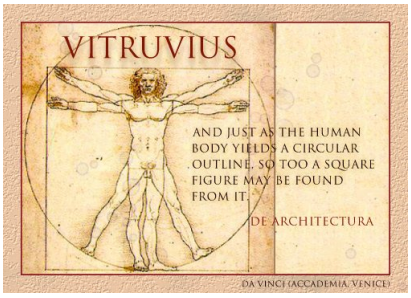
It seems hard to believe that in the long history of science and art, this notion of symmetry did not exist. The authors claim that the concept of symmetry is not innate to the human nature. Otherwise, it would appear in the (scientific) literature of all ages, and that, they prove in this book, is not the case. Not all authors agree with them. So, the statement of their claim and the problem with opposing statements is formulated in the first chapter. Before Legendre, the word symmetry with our present scientific meaning is not used, while after Legendre, it is omnipresent. If Legendre had used another word, we would probably be using that word for what we now call symmetry. The mathematical concept of symmetry was not intrinsic in the use of that word before Legendre, otherwise, Euler and Kant for example who were very close to a definition or tried very hard to define it, would have succeeded in giving a precise formulation, but they never did.


Vitruvius man

Symmetry in its current meaning refers to a mathematical entity which is invariant under certain classes of transformations. These classes of transformations relates symmetry to group theory. But that came only later with Galois. It is essential to realize that symmetry is a mathematical, and not an aesthetical concept.

Of course the word symmetry (or the old Greek *summetria*) does appear in many texts. So what is then the meaning of this word, if not corresponding to our modern mathematical concept? And what should be said about a prominent presence of symmetry in our material world (architecture, human body,...)?

With many citations of the original texts, the authors prove that symmetry before Legendre was mostly used as a general etymological category. For example for the Greek (Plato, Euclid, Achimedes), *summetria* meant something like "proportional". The Euclidian algorithm is formulated in terms of lengths of an interval that are commensurable. Asymmetry thus meant something like "relative prime". Computing the greatest common divisor of two integers is how the Euclidean algorithm is known to most of us. Archimedes, was most interested in "equilibrium", but he never linked this with symmetry.



Of course Vitruvius (1st century BC) is the first candidate to be linked to symmetry. Everybody knows the Vitruvius man, redrawn by many others, among which Da Vinci. The work of Vitruvius has been dominantly influential on aesthetics and especially on architecture till the 18th century. The ancient notion of symmetry explained above, survived, and the medieval translations of Euclid's *Elements* used the Latin "commensurabile" to translate the Greek *summetria*.

Symmetry was not as such an important concept in mathematical writings, but it played an essential role in many texts about aesthetics. There it had the meaning of "well proportioned" and was associated with "beautiful" or "good". That became quite clear in Vitruvius's *De architectura*. Symmetry refers to the fact that all parts of a building, the human body, or even a machine should all be in proper "proportion". Symmetry properties in architecture and in the human body can also be understood as "correspondence between left and right", but it never referred to a specific part. It was a global property of the body or the temple as a whole. This notion persisted. Dürer (1471-1528) e.g. writes his *Vier Bücher von menschlicher Proportion*.

In another chapter, the authors switch to Italian and French architecture, where they explain that the most common translation of symmetry shifts from "proportion" to "correspondence". There is a "correspondence" between different parts of a building and the building as a whole, in which we recognise vaguely the meaning of proportional and of commensurable. The first to use "corresponding" in this context was Leon Battista Alberti



Claude Perrault



The Louvre in Paris by C. Perrault

(1404-1472). Later Sebastiano Serlio (1475-1554) uses both: proportion (as in Vitruvius) and correspondence (as in Alberti) and links both. The same type of shift can be observed in the French texts about architecture.

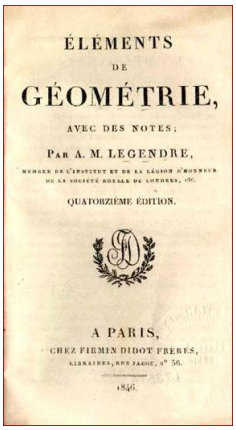
The most significant publication in this respect is the translation of Vitruvius's *De architecture* by Claude Perrault (1613-1688). This contains the foundations of Renaissance with its strict symmetric architecture. Perrault also uses correspondence (*rapport*) to indicate that windows on the left should correspond to windows on the right, the same number and at the same distance, but the symmetry of the windows themselves is not required.



Adrien-Marie Legendre

Denis Diderot (1713-1784) in his *Encyclopédie*, includes a definition of symmetry to mean commensurable, and in an essay of 1751 about architecture and painting, it is clear that he mixes the old Vitruvian meaning and the meaning of Perrault in the sense of a symmetry with respect to a vertical line.

The astronomers of the early modern times: Copernicus (1473-1543) and Galileo (1564-1642), use the word symmetry in traditional Vitruvian sense while Kepler (1571-1630) and Leibniz (1646-1716) hardly use the word. These astronomers were strongly involved with the concept "harmony" and although there is a strong relation with symmetry, the authors of this book keep strictly to their histographical paradigm (only the use of the word symmetry is important) and consider these two concepts as definitely distinct.


ÉLÉMENTS
DE
GÉOMÉTRIE,
AVEC DES NOTES;
PAR A. M. LEGENDRE,
QUATORZIÈME ÉDITION.
A PARIS,
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1846.

In natural history of the period 1738-1815, symmetry was only used as a tool for taxonomy and not elaborated as a concept. In botany, the terms was hardly used, "regular" was more common. In crystallography Haüy considered the symmetry of a crystal as a global property, although he was the first to define axes of symmetry.

Much closer to a definition came Euler (1707-1783) and Kant (1724-1604). Euler was interested in his practical problem (the dual representation of celestial constellations), and did not consider it in a more abstract setting. Kant attempted to undermine Leiniz's theory in favor of Newton's by proving that absolute space has a reality of its own. In his demonstration he comes very close to a definition when he considers the forming of a mirror image, but he calls this image the "incongruent counterpart". However from his further writings, the authors conclude that this concept is for Kant different from what symmetry is to us. Kant eventually failed in his attempt and later moves to metaphysical arguments.

Finally, it is Legendre in his study of three-dimensional geometry, who formulates the definition of symmetrical angles given above. It was actually a problem in Euclid's *Elements* concerning solid angles, brought up by Robert Simson (1687-1768) that brought Legendre to this statement. He was the first to define equality (of solid angles) even though the two bodies can not be superimposed. The step to symmetry in polyhedra and the further development of our modern concept of symmetry was then just a matter of time.

Quite different from Legendre's use in geometry, was the use of symmetry in probability by Laplace and in algebra by Lacroix. Cauchy used the word in geometry and algebra, but did not link the two concepts. Also in physics the term appears and was used by many. However, it had different meanings in different contexts, some of which linked up with older interpretations. The use of the word symmetry in the early 19th century was sometimes confusing and it took a while before group theory entered the arena and bought some structure.

Adhemar Bultheel