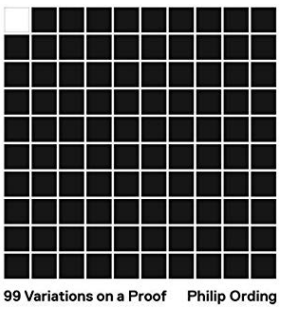
99 Variations on a Proof, *Philip Ording*, Princeton University Press, 2019 (168 p.) isbn: 978-0-691-15883-9.

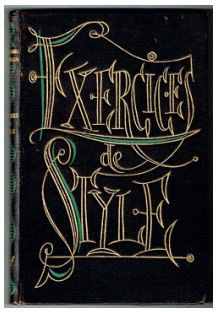




Philip Ording

I have reported on oulipo (**Ou**vroir de **li**ttérature **po**tentielle) before in this Newsletter. This group which is a mixture of French writers and mathematicians was founded in 1960 and they wanted to explore new structures for writing, constraints imposed by symmetry or patterns, almost like mathematics is developed in a strict logic framework, yet looking for elegant and beautiful results.

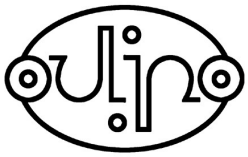
One of the earliest of these experimental writings was *Exercices de Style* (1947) by Raymond Queneau (a poet but also an amateur mathematician), one of the founders of oulipo. A simple story is told and retold, 99 times, about a man getting on the bus where he witnesses a man with a long neck and someone else who are having a row, but each time the story is told in a completely different style.

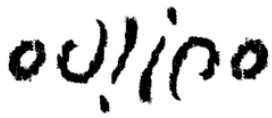


There are several ways to organize symmetry or patterns. Permutation of words or random selection of sentences, or drawing n sentences from n collections, of which the first collection consists of opening sentences, and the nth is a set of closing sentences. In this way $n!$ different stories can be composed. Queneau alo applied this technique, writing 10 sonnets, and you can choose every line from one of the 10 possibilities, and so published it as *Cent Mille Milliards de Poèmes*.

Palindromes are an excellent example of how one can find inspiration in writing something that you can read from left to right or from right to left. Ambigrams are graphical representations of a word or text that looks exactly the same if you turn it upside down. They were also popular at the Martin Gardner meetings.







This idea of *Exercices de Style* has triggered several authors to produce something similar, and it must have inspired Ording also to collect in this book 99 variations of a proof. What is proved here is always the same, but otherwise completely irrelevant: the cubic equation $x^3-6x^2+11x-6 = 2x-2$ has a double solution at $x = 1$ and a third one at $x = 4$.

In fact, there are actually 100 proofs, since proof number 0 is symbolized by the white square on the cover of the book and it is a proof that has been omitted (as we sometimes also read in published mathematical papers: Formulation of the theorem, followed by *Proof*: Omitted. □). Other proofs are more obvious and I am sure you can think of some. A compact proof of just one line for example, or the opposite, a proof where all steps are written out with in the margin an explanation of the rule, property or operation that has been applied, or a graphical proof or using geometric arguments, etc. Also traditional proofs like by contradiction, induction, or exhaustion, but still you may realize soon that to come up with 99 variations is far from a trivial task.

So Ording has been very creative, and he probably has been collecting ideas over several years. There is certainly a selection that that is inspired by several mathematical disciplines: topology, linear algebra, or probability. Then you might think about the way they appear, and that opens

a new box of possibilities: in an email, a blog, a tweet (here presumably sent by Cardano), a newspaper text, using a flow chart, on a blackboard, a pre- or postfix formal language, typeset in LaTeX, in preprint or published form, the messy way of how students sometimes produce a proof at an exam, and even using a music score and one using origami, ...

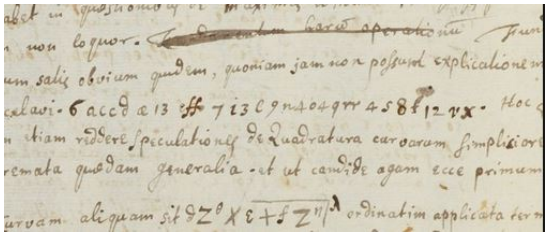
There is also a proof "On the back of an envelope", i.e., short calculation on a little piece of paper, a form of calculation for which Enrico Fermi was famous. There is also an illustration from a book by Cardano in which he explains how to solve a cubic equation, a proof in Babylonian cuneiform symbols and a medieval text. A collaborative proof is inspired by the way people collaborated on the twin prime conjecture in the Polymath project as set up by Yitang Zhang and Terence Tao.




Yitang Zhang


Terence Tao

And there are some fun ones, like a psychedelic (a Mandelbrot-like plot of the attraction basins for the roots of the polynomial equation), mystical, social media, a screenplay, or a nonsense proof in the style of Lewis Carroll, or perhaps in verse, or how proofs are conceived during conversations at coffee breaks, a proof using only one syllabic words (that is typical oulipo, Perec wrote a novel without the letter e appearing once), etc. A paranoid proof refers to the time when mathematicians published their results in riddles so that they could prove later that they had it already if some priority issue did arise. Galileo mailed Kepler the anagram 'smaismrmilmepoetaleumibunenugttauiras' in 1610. Of course the original text was in Latin, but translated its hidden message was "I observed the highest planet to be triple bodied", which refers to the rings of Saturn. However Kepler unscrambled the anagram wrongly and thought that it was about Mars having two moons. But also Newton has sent such an anagram to Leibniz (via Oldenburg) which read '6accdae13eff7i3l9n4o4qrr4s8t12ux' (read 6 a's, 2 c's, 1 d, 1 æ, 13 e's,...) in which he hides the 'method of fluxions' (Newton's calculus). Ording has hidden some extra symmetry for the reader to discover. For example the ancient proof by Babylonian characters is number 16, while there is also a modern proof that is numbered 61.


Newton's anagram

The majority of the proofs take not more than a few lines to half a page. They are given on the odd numbered pages, and on the back, he gives some explanation, perhaps some historical context or just explaining how the "proof" on the other side of the page should be understood.

This is a fun book, and yet it has a lot of information, not essential or high brow mathematics, but still entertaining chit-chat info that is nice to know. It is also not a book that you will read cover to cover. You can pick up the book from time to time and read one or two of the proofs and Ording's comments. Some of the proofs are really smart and witty. Many references are given as notes at the end of the book to justify what has been claimed, and there is also cross referencing when a proof can be compared to another one.

Adhemar Bultheel