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Abstract

We develop a network trade model with country-sector level input-output linkages. It includes (1) domestic and global value chain linkages between all country-sectors, (2) direct as well as indirect shipments (via other sectors and countries) to a final destination, (3) value added rather than gross trade flows. The model is solved analytically and we use the sectoral World Input Output Database (WIOD) to predict the impact of Brexit for every individual EU country by aggregating up the country-sector effects. In contrast to other studies, we find EU-27 job losses to be substantially higher than hitherto believed as a result of the closely integrated EU network structure. Upstream country-sectors stand to lose more from Brexit due to their network centrality in Europe.

JEL-Codes: C530, D570, F170, F140.

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1. Introduction

Production processes are increasingly fragmented within and across national boundaries. Hence, a full assessment of any type of idiosyncratic shock requires new models that include sector-level value chain linkages (Johnson, 2014; Acemoglu et al., 2012). This paper develops a novel input-output model with country-sector linkages in production. Introducing the country-sector dimension generates a true network model. Thus for any country-sector subject to a trade shock like Brexit, we can measure the loss in value added coming from its direct trade with the UK, as well as the loss in value added coming from the network connections to other EU country-sectors. For example, the Belgian steel sector will suffer from Brexit not just through a reduction in bilateral exports of steel from Belgium to the UK, but also through a reduction of Belgian steel exports used in Germany cars which are subsequently shipped to the UK. Empirically, we find that for most sectors these indirect network effects of the trade shock are large and reinforce the overall impact of a trade shock in an important way.

The academic contribution of this paper is to show that network effects of a trade shock like Brexit cannot be uncovered with a more traditional gravity one sector-model. Existing multi-country, multi-sector models can also not disentangle these indirect effects because their input-output structure does not allow for country-sector sourcing linkages.[3] Introducing country-sector linkages results in indirect effects of a trade shock, which in many cases exceed the direct bilateral gravity effects of the trade shock.

Bringing these network effects to the forefront comes at a price in the modeling strategy. Where other models are general equilibrium in nature, ours is more partial equilibrium and short-term in the sense that we do not consider reallocation effects across sectors and do not assume full employment to materialize after the shock. Instead, we focus on the short-run static effects of a trade shock, and allow for job losses and unemployment at the country-sector level. Clearly, not everyone that loses their job from Brexit will remain unemployed. At least some workers will find other jobs in the same sector or to other sectors. However, this may take time, which is why there is room for a paper that quantifies job losses at at country-sector level even before the reallocation of workers over sectors takes place.

Our model is a love-for-variety model where similar inputs in production can be sourced from different countries between which there is an elasticity of substitution. e.g. German cars can

[3] See Eaton and Kortum, 2002,Caliendo and Parro (2015),Dhingra et al. (2015), Felbermayr et al. (2018) or Yotov et al. (2016).

use Belgian steel as well as Mexican steel and Slovakian steel as an input. Thus trade arises by differences in country of origin-specific demand shifters rather than cost differences. This is different from a Ricardian approach where every input is assumed to be sourced from only one particular country, i.e. the one with the lowest cost. While the Ricardian and love-for-variety assumption empirically equivalent when using sector-level data, in our case it does make a difference. We introduce the love-for-variety assumption[4] assumption at the country-sector level in the model so as to get a network dimension that is absent in a Ricardian model. In a Ricardian framework, German cars source steel, while under Armington, German cars can source Belgian-steel, Mexican-steel and Slovakian-steel. Therefore, under a Ricardian assumption, value added cannot be obtained for a sector like Belgian-steel even when that sector supplies to German cars and the indirect trade flow of Belgian steel to German cars is absent in such a model.

A Ricardian approach therefore results in a gravity model with IO linkages between sectors, but the love-for-variety assumption on the supply side assumption used here results in a network model where IO linkages also give rise to indirect trade flows between country-sectors. Using the love-for-variety assumption, we solve the network model analytically and obtain closed-form solutions that allow for comparative statics on tariff changes in different sectors. We assume technology to be constant and markets to be perfectly competitive.

The model does not deal with firm heterogeneity, which is another price we pay in order to have a model of world trade with network effects across all country-sectors in the world. But a firm-level approach would not be well-suited to trace all the upstream production stages and all the downstream production stages that may get affected by a trade shock. Our network model of sectors in contrast captures all the upstream linkages via the Leontief coefficients which are also present in the WIOD data.

Admittedly, WIOD data have an underlying proportionality assumption embedded in the way the data are constructed. But using firm-level data does not overcome this limitation as constructing firm-level input-output linkages, also requires making assumptions on which inputs are used in which outputs (Vandenbussche and Viegelahn (2018)). In the Brexit application, we introduce symmetric MFN tariffs between the UK and the rest of the EU and consider all other trade relations to remain unchanged. Finally, in our analysis we focus on the value-added share in a country-sector's production and the employment associated with it.

[4]In DSGE/CGE modelling this is referred to as the Armington assumption. But in contrast to a CGE approach, we explicitly solve our love-for-variety model analytically and need not simulate outcomes.

Our sector level approach offers a number of important advantages compared to more aggregate country-level analysis such as Noguera (2012). These advantages are not unique to this study but shared by all studies that use sector-level input-output analysis. First, tariffs vary substantially across sectors, thus a failure to account for tariff heterogeneity across sectors can lead to biased results. Second, trade elasticities differ substantially across sectors, i.e. consumers (and firms) respond differently to price changes in different sectors. Third, there is an increasing availability of sector-level input-output data such as the World Input-Output Database (WIOD) which include services sectors. This is important given that services are increasingly traded as well as embedded in the exports of goods. Disregarding services would therefore miss an important share of global trade. Finally, as the production linkages between two countries typically differ greatly across sectors, our sectoral approach yields a more precise assessment of the indirect effects of a trade shock.

Empirically, we find these indirect effects to be substantial. Indirect effects can be considered at the country-level or at the country-sector level. At the country-level it includes all the network effects between sectors within a country. At the country-sector level, it includes all the network effects both within and across countries. For the EU-27 as a whole the indirect impact of Brexit accounts for 25% of the total impact. But for some countries like Slovenia, indirect effects account for as much as 50% of the overall effects. At the country-sector level, we find the indirect effects on average to account for more than 70% of the impact of a trade shock like Brexit. This shows that incorporating country-sector level linkages into the model is important. These indirect effects also highlight how much traditional single sector gravity models underestimate the impact of trade shocks.

Our sector-level input-output model assumes a Cobb-Douglas specification that nests a CES function both on the production side as well as on the consumption side. On the production side, firms produce output with a Cobb-Douglas technology and fixed expenditure shares on the factor of production (labor) and a composite intermediate good, taking goods and factor prices as given. The composite intermediate good in turn is a Cobb-Douglas combination of intermediate goods from all sectors. Each of these sector-specific intermediate goods is a Constant Elasticity of Substitution (CES) aggregate across all the countries the input can be purchased from.

On the consumption side, final consumers derive utility from an aggregate final good, which is a Cobb-Douglas combination of final goods from different sectors. Every sector-specific final good is a CES aggregate across all countries the good can be purchased from. The CES nests on the production and the consumption side rely on the Armington assumption, i.e. goods produced by different sources are imperfect substitutes simply because of their origin. The Armington assumption closely mimics the input-output data where similar inputs (from the same sector) are purchased from different countries. How much is sourced from each country depends on relative prices, which is a function of the productive efficiency of the supplier and trade costs. In our analysis we focus on the value-added share in a country-sector's production and the employment associated with it.

Our theoretical framework predicts that an increase in import tariffs results in a reduction of production and job losses all along the supply chain. The losses in value added production depend on three parameters, namely the sectoral trade elasticity, the value added shares in production and the Leontief input-output coefficients. While our framework is entirely general, in this paper we calibrate the model to predict the impact of Brexit. The World Input-Output Database (WIOD)[5] provides us with observations on the main variables required for our analysis of the impact of a trade shock like Brexit, i.e. trade flows, value added shares and production input-output linkages. WIOD covers 43 individual countries, including the 28 EU countries, and 56 sectors which allows us to study worldwide production networks. All upstream and downstream sectors can be identified for any sector in the production network, allowing for the construction of input-output linkages at sector level.[6] When calibrating the model, we rely on estimates of sector-level employment elasticities that circulate in the literature. We consider both a "soft" Brexit (the "Norwegian scenario"), where the UK continues to be part of the Single Market but faces increased Non-Tariff Barriers (NTBs), as well as a "hard" Brexit scenario where Most-Favored-Nation (MFN) tariffs between the EU-27 and the UK are put in place in addition to the NTBs.

We focus on the short-term effects of Brexit and do not consider foreign direct investment (FDI) responses to trade policy, which may take longer to materialize. Moreover, we disregard any dynamic effects of Brexit related to investment and innovation, capital mobility and migration. Finally, while our theoretical framework accounts for trade diversion, we ignore any such effects in the model simulation. While this is a limitation, there are a number of studies (e.g. Magee, 2008) that suggest that the trade diversion effects are typically low, compared to the first-order trade effects,

[5] We use the 2016 release of the World-Input-Output Database (WIOD). This sector-level database provides information about the origin and destination of intermediate and final goods and services in 56 sectors (using ISIC Rev. 4 classification) for 43 countries, and a residual rest of the world between the years 2000 and 2014. Dietzenbacher et al. (2013) describes in great detail the procedure that was followed to construct these World Input-Output Tables.

[6] Many firm-level studies with information on firm-level trading are often limited in their geographic scope and typically only include firms from one country without information on who these firms are buying from or selling to (Topalova and Khandelwal, 2011; Amiti and Konings, 2007; Vandenbussche and Viegelahn, 2018).

which is what we focus on in this paper.[7]

The WIOD database is also used by Foster-McGregor and Stehrer (2013), Timmer et al. (2014), Timmer et al. (2015) and others that have investigated inter-sector and international linkages in global value chains albeit to address different questions. Because of its sector-level dimension, WIOD contains more disaggregated information than the data used by Johnson and Noguera (2012) that only include four composite sectors including one service sector. Given our focus on the sector-level dimension, WIOD is thus more appropriate for our purposes as it has 56 sectors including 30 service sectors. While our interest lies in the job losses of trade shocks, our approach differs from Autor et al. (2013), who assess US employment effects of Chinese import penetration at the regional level as they do not consider the input-output linkages between industries. The novelty of our approach is that we consider all the upstream and downstream employment effects of a trade shock.

Another line of work in recent years has gone into identifying the welfare gains and losses from trade policy but has been less about inter-sectoral linkages and intermediates (see Costinot and Rodriguez-Clare, 2013, for an overview). An increasing number of papers in trade also turn to input-output data in the context of trade policy but with a different focus, e.g. Blanchard et al. (2016) who show that countries which are more connected in global value chains have lower tariff protection between them, Dhingra et al. (2017) who evaluate Brexit on UK household income levels and Caliendo et al. (2015) who assess the welfare effects of NAFTA. Blonigen (2016) examined the downstream effects of industrial policy in the steel sector. Finally, several studies in international trade have now shown that gross trade flows do not necessarily reflect the domestic production underlying the trade flow but value added is more appropriate (Koopman et al., 2014; Bernard et al., 2017).[8] In line with these studies we also focus on value added rather than gross export flows.

Our model's predictions indicate that the UK is hit relatively harder than the rest of the EU-27. Brexit reduces economic activity in the UK around three times more than in the EU-27. The UK will experience a drop in value added production as a percentage of GDP of 1.21% under a soft Brexit and up to 4.47% under a hard Brexit scenario.[9] This corresponds to UK job losses of around

[7] Magee (2008) finds that bilateral trade flows are estimated to increase by 82% after countries engage in a regional agreement and this effect is significant across different econometric specifications. On the contrary, the variable capturing trade diversion reduces imports from outside by 2.9% but is not significant across different econometric specifications, suggesting that trade diversion is rather small.

[8] Bernard et al. (2017) empirically show that many products shipped by manufacturing firms are not produced in-house, but are "carry-along trade", i.e. gross export sales are much larger than the domestic production shipped.

[9] GDP data come from WIOD and are the sum of value added in every sector

140 000 jobs in the "soft" Brexit and around 500,000 jobs in the "hard" Brexit scenario.

In contrast to other studies, we find the losses for the EU-27 countries to be much higher than previously thought. The main reason is that our approach incorporates all national and international country-sector-level input-output linkages in both goods and services. Given that EU-27 production networks are closely integrated, tariff changes do not just affect direct bilateral trade flows between any EU-27 country and the UK, but also indirect trade flows via third countries.

For the EU-27 as a whole, the absolute job losses amount to around 280,000 jobs for a soft Brexit and 1,200,00 jobs in case of a hard Brexit, which corresponds to value added losses as a percentage of GDP of 0.38% and 1.54% respectively. The value added and jobs losses turn out to differ substantially across EU-27 member states. One of the main reasons is the difference in sectoral composition. A hard Brexit implies different tariffs across sectors, and therefore the propagation of tariff shocks differs depending on the sectoral composition of the economy. A sector that only has few linkages with other sectors may not affect aggregate output much even when it is subject to high tariffs, as opposed to a sector that is very central in the production network. Our results take the network centrality and the number of sectoral production linkages into account when estimating the loss in value added and jobs caused by Brexit. These results correspond to the notion put forward by Acemoglu et al. (2012) that the network centrality of sectors determines the impact of an aggregate shock through a "cascade effect" in the input-output network.

The remainder of this paper is organized as follows. In Section 2, we develop the theoretical model and obtain an expression for a country-sector's value added production and its determinants on the basis of which we obtain clear predictions on the effects of trade shocks. In Section 3, we explain the methodology and describe the data we use. Section 4 presents the results of the Brexit application. Section 5 compares our results to existing results in the literature and Section 6 concludes.

2. A Sector-Level Input-Output Model of Trade

In this section, we build on the work by Noguera (2012) who considered country-level input-output linkages. In contrast, we build a more disaggregated model at the sector-level on both the consumption and the production side that allows for national and international sector-level input-output linkages. This more disaggregated level of analysis allows us to account for sectoral heterogeneity in input and output responses to tariff changes which we find to be very important. From the theory it becomes clear that the sector-level composition and network structure of a country determine the heterogeneous impact of tariff changes. Empirically we then show that the inclusion of the sector-level dimension corrects for an aggregation bias that would otherwise result in substantial measurement error in the assessment of trade policy effects.

We use superscripts to denote the country-sector of origin and subscripts to denote the country-sector of destination. To facilitate understanding, let us consider the following example. The quantity of intermediate steel from Belgium shipped to the German car industry is denoted by $X_{DE,car}^{BE,steel}$. In general, countries are denoted by i, j and k and sectors by r, s and z.[10] Demand for labor by country k's sector z for example is captured by L_{kz}. Throughout this section, upper-case symbols refer to real quantities, whereas lower-case symbols denote their nominal counterparts.

The model is based on the Armington assumption, i.e. goods produced by different source countries are imperfect substitutes. As a result, within a sector, goods from different countries can coexist in the same destination market, even though their prices may differ as they are determined by the country-sector's marginal production cost and costs of trade with the destination country.[11] Consumers (and firms) have a love-for-variety and prefer to consume positive amounts of each available variety.

2.1. Consumer Demand

The representative consumer in country k derives utility from consuming quantities of an aggregate final good F_k:

$$U_k = F_k = \prod_{s=1}^{S} \left[F_k^s \right]^{\alpha_k^s} \tag{1}$$

which is a Cobb-Douglas combination of quantities F_k^s consumed of final goods from all sectors $s \in S$, with α_k^s the corresponding share in total expenditures. This sector-specific final good is a CES aggregate across all countries the good can be purchased from,

$$F_k^s = \left[\sum_{i=1}^{N} \left(F_k^{is} \right)^{\frac{\sigma_s - 1}{\sigma_s}} \right]^{\frac{\sigma_s}{\sigma_s - 1}} \tag{2}$$

where $\sigma_s > 1$ is the elasticity of substitution (for final goods) between the countries of origin within sector s.[12]

[10] We need at least three symbols in the model to denote countries and sectors because input-output models typically consider three nodes in a supply chain: (1) the supplier of intermediate inputs, (2) the final producer and (3) the consumer.

[11] As in Noguera (2012), production and trade costs are the only determinants of prices in our model. The absence of markups in the model is assumed at sectoral level but does not exclude existence of markups at firm level.

[12] For simplicity, we assume this sector-specific elasticity of substitution to be the same across all countries k.

2.2. Producers

In country k's sector z, output Y^{kz} is produced with a Cobb-Douglas technology that uses as inputs labor L_{kz} and intermediate inputs X_{kz}[13]:

$$Y^{kz} = (L_{kz})^{1-\beta^{kz}} (X_{kz})^{\beta^{kz}} \tag{3}$$

where β^{kz} represents the share of intermediate expenditures in total sales of country k's sector z. The intermediate goods composite X_{kz} is a Cobb-Douglas combination of intermediate goods from all sectors $s \in S$, X^s_{kz}:

$$X_{kz} = \prod_{s=1}^{S} \left[X^s_{kz} \right]^{\gamma^s_{kz}} \tag{4}$$

where X^s_{kz} denotes the real aggregate demand of intermediates from sector s by country k's sector z, and γ^s_{kz} is the corresponding share in total expenditures on inputs. The sector-specific intermediate good X^s_{kz} is a CES aggregate across all countries the input can be purchased from:

$$X^s_{kz} = \left[\sum_{i=1}^{N} \left(X^{is}_{kz} \right)^{\frac{\rho_s - 1}{\rho_s}} \right]^{\frac{\rho_s}{\rho_s - 1}} \tag{5}$$

where $\rho_s > 1$ is the elasticity of substitution (for intermediate goods) between the countries of origin within sector s. Note that this nested Cobb-Douglas-CES structure is similar to that of the consumer demand aggregates.

2.3. Utility and Profit Maximization

Let w_{kz} denote the price of labor in country k's sector z (L_{kz}) and p^{kz} the price of output from kz (Y^{kz}). Given iceberg-type trade barriers, in order to satisfy country j's demand of one unit of kz, kz needs to produce τ^{kz}_j units, with $\tau^{kz}_j > 1$. The price of one unit of kz's output in destination j then equals $p^{kz}_j = \tau^{kz}_j p^{kz}$ accounting for differences in trade costs across destinations j. Note that we typically assume there are no barriers to trade within a country, i.e. $\tau^{kz}_k = 1$.

Firms maximize profits by choosing L_{kz} and X^{is}_{kz} and households maximize utility choosing F^{is}_k subject to their budget which equals $I_k = \sum_{z=1}^{S} w_{kz} L_{kz}$, i.e. their income from supplying labor L_{kz} to each sector z in country k. Firms and households take factor price w_{kz} and goods prices $\tau^{kz}_j p^{kz}$

[13]Following several standard trade models, we only allow for one factor of production. This assumption can be relaxed, for instance by accounting for high- and low-skilled labor.

as given. This results in the optimal nominal counterparts of real demand (which are denoted by a lower-case symbol and that are obtained by multiplying real demand by the corresponding price). Nominal output of kz is represented by $y^{kz} \equiv p^{kz}Y^{kz}$. The CES price index in country k of final goods from sector s equals $P_k^s = \left[\sum_{i=1}^{N} \left(p_k^{is}\right)^{1-\sigma_s} \right]^{\frac{1}{1-\sigma_s}}$. The price of the aggregate intermediate input X_{kz} is given by the Cobb-Douglas price index $PI_{kz} = \prod_{s=1}^{S} (P_k^s)^{\gamma_{kz}^s}$ where P_k^s is the CES price index in country k for intermediate goods from sector s which we assume, for tractability, to be the same as the corresponding price index for final goods (this implies that $\sigma_s = \rho_s$ and that the price of a certain good from sector s is the same whether it is sold as an intermediate or a final good.[14] The (FOB) price[15] of output from kz equals $p^{kz} = (\frac{w_{kz}}{1-\beta^{kz}})^{1-\beta^{kz}} (\frac{PI_{kz}}{\beta^{kz}})^{\beta^{kz}}$. The optimal nominal demands then equal:

$$l_{kz} \equiv w_{kz}L_{kz} = (1-\beta^{kz})y^{kz}$$

$$x_{kz} \equiv PI_{kz}X_{kz} = \beta^{kz}y^{kz}$$

$$x_{kz}^s \equiv P_k^s X_{kz}^s = \gamma_{kz}^s \beta^{kz} y^{kz}$$

$$x_{kz}^{is} \equiv p_k^{is} X_{kz}^{is} = \tau_k^{is} p^{is} X_{kz}^{is} = (\frac{\tau_k^{is} p^{is}}{P_k^s})^{1-\sigma_s} \gamma_{kz}^s \beta^{kz} y^{kz} \tag{6}$$

$$f_k^{is} \equiv p_k^{is} F_k^{is} = \tau_k^{is} p^{is} F_k^{is} = (\frac{\tau_k^{is} p^{is}}{P_k^s})^{1-\sigma_s} \alpha_k^s \sum_{z=1}^{S} (1-\beta^{kz}) y^{kz} \tag{7}$$

2.4. Market Clearing

Let $e_j^{kz} \equiv f_j^{kz} + \sum_{s=1}^{S} x_{js}^{kz}$ denote the nominal gross exports from country-sector kz to (the consumer and producers in) country j. Market clearing requires

$$y^{kz} = \sum_{j=1}^{N} e_j^{kz} \tag{8}$$

Following the same logic as in Anderson and Van Wincoop (2003), we derive gravity equations for final and intermediate goods exports, but now at the sector-level. Denote world nominal output by y^w and country-sector kz's share in world output by $\theta^{kz} \equiv y^{kz}/y^w$. Substituting Equations (6) and

[14]The assumption that firms and consumers share the same price elasticities allows us to substantially simplify the analysis.

[15]The assumption of perfect pass-through inherent to this theoretical framework is a limiting assumption since pass-through depends on firm size with larger firms having lower pass-through rates (Amiti et al., 2014). However, in the WIOD data we have no information on the underlying firm size distribution within a sector.

(7) into Equation (8) allows to solve for prices p^{is}. Substituting these into the price index P_k^s and plugging the resulting expression for P_k^s into (6) and (7) results in the following gravity equations for intermediate and final bilateral exports and equilibrium price indices:

$$x_{js}^{kz} = \frac{y^{kz}\gamma_{js}^{z}\beta^{js}y^{js}}{y^w}(\frac{\tau_j^{kz}}{\Pi^{kz}P_j^z})^{1-\sigma_z} \tag{9}$$

$$f_j^{kz} = \frac{y^{kz}\alpha_j^z\sum_{s=1}^{S}(1-\beta^{js})y^{js}}{y^w}(\frac{\tau_j^{kz}}{\Pi^{kz}P_j^z})^{1-\sigma_z} \tag{10}$$

$$P_j^z = \left[\sum_{k=1}^{N}\theta^{kz}(\frac{\tau_j^{kz}}{\Pi^{kz}})^{1-\sigma_z}\right]^{\frac{1}{1-\sigma_z}}$$

$$\Pi^{kz} = \left[\sum_{j=1}^{N}\phi_j^z(\frac{\tau_j^{kz}}{P_j^z})^{1-\sigma_z}\right]^{\frac{1}{1-\sigma_z}}$$

where $\phi_j^z = \sum_{s=1}^{S}\theta^{js}(\gamma_{js}^z\beta^{js} + \alpha_j^z(1-\beta^{js}))$ is a measure of the importance of goods from sector z for producers and consumers in country j. It takes into account (i) the dependence of producers in all sectors s in country j on intermediates from sector z through $\theta^{js}\gamma_{js}^z\beta^{js}$ and (ii) the importance of goods from sector z in the final demand by households in country j (through α_j^z) and the total income these households earn in all sectors s in j (through $\theta^{js}(1-\beta^{js})$).

Equation (9) relates bilateral intermediate trade between firms in country-sector kz and country-sector js to (i) the economic masses of source and destination relative to the world, (ii) the importance of inputs in the destination's production (β^{js}) and the importance of sector z goods within these inputs (γ_{js}^z), (iii) the bilateral trade costs between countries k and j in sector z (τ_j^{kz}), and (iv) outward and inward multilateral resistance terms (Π^{kz} and P_j^z). Similarly, Equation (10) relates bilateral final goods trade between firms in country-sector kz and the consumers in country j to (i) the economic masses of source (y^{kz}) and destination ($\sum_{s=1}^{S}(1-\beta^{js})y^{js}$)[16] relative to the economic mass of the world (y^w), (ii) the importance of sector z final goods in the destination's consumption (α_j^z), (iii) the bilateral trade costs between countries k and j in sector z (τ_j^{kz}), and (iv) outward and inward multilateral resistance terms (Π^{kz} and P_j^z).

[16] This expression reflects the fact that consumers in country j get their income from supplying labor to all sectors s.

2.5. Input-Output Production Linkages

Dividing both sides of Equation (9) by y^{js} we obtain the technical coefficient a^{kz}_{js} or "dollar's worth of inputs from kz per dollar's worth of output of js":

$$\frac{x^{kz}_{js}}{y^{js}} \equiv a^{kz}_{js} = \frac{y^{kz}\gamma^{z}_{js}\beta^{js}}{y^{w}}\left(\frac{\tau^{kz}_{j}}{\Pi^{kz}P^{z}_{j}}\right)^{1-\sigma_z} \tag{11}$$

Plugging the technical coefficients into the market clearing in condition in (8), we have

$$\begin{aligned} y^{kz} &= \sum_{j=1}^{N}\left(\sum_{s=1}^{S} x^{kz}_{js} + f^{kz}_{j}\right) \\ &= \sum_{j=1}^{N}\sum_{s=1}^{S} a^{kz}_{js} y^{js} + \sum_{j=1}^{N} f^{kz}_{j} \end{aligned}$$

which can be summarized for all countries and sectors as

$$\boldsymbol{Y} = \boldsymbol{A}\boldsymbol{Y} + \sum_{j=1}^{N} \boldsymbol{f_j} \tag{12}$$

where

$$\boldsymbol{Y} = \begin{bmatrix} y^{1,1} \\ y^{1,2} \\ \vdots \\ y^{N,S} \end{bmatrix}; \quad \boldsymbol{A} = \begin{bmatrix} a^{1,1}_{1,1} & a^{1,1}_{1,2} & a^{1,1}_{1,3} & \dots & a^{1,1}_{N,S} \\ a^{1,2}_{1,1} & a^{1,2}_{1,2} & a^{1,2}_{1,3} & \dots & a^{1,2}_{N,S} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a^{N,S}_{1,1} & a^{N,S}_{1,2} & a^{N,S}_{1,3} & \dots & a^{N,S}_{N,S} \end{bmatrix}; \quad \boldsymbol{f_j} = \begin{bmatrix} f^{1,1}_{j} \\ f^{1,2}_{j} \\ \vdots \\ f^{N,S}_{j} \end{bmatrix}$$

where $\boldsymbol{f_j}$ is the $(S * N)$ x 1 vector of country j's final demands and $\boldsymbol{A}$ the $(S^{*}N)$ x $(S^{*}N)$ global bilateral input-output matrix at the sectoral level. The system in Equation (12) can be written as

$$(\mathbb{I} - \boldsymbol{A})\boldsymbol{Y} = \sum_{j=1}^{N} \boldsymbol{f_j} \tag{13}$$

with $\mathbb{I}$ the $(S^{*}N)$ x $(S^{*}N)$ identity matrix. If $\boldsymbol{(\mathbb{I} - A)}$ can be inverted, we can find the solution for nominal output as

$$\boldsymbol{Y} = (\mathbb{I} - \boldsymbol{A})^{-1}\sum_{j=1}^{N} \boldsymbol{f_j} = \boldsymbol{L}\sum_{j=1}^{N} \boldsymbol{f_j} \tag{14}$$

where $\boldsymbol{L}$ is known as the Leontief inverse matrix. Each element L^{kz}_{is} of $\boldsymbol{L}$ is the Leontief coefficient that measures the total of dollars worth of country-sector kz goods required to meet 1 dollar worth of is' final demand. This value combines kz goods used as inputs in is directly as well as kz goods used as inputs in other industries which then also produce inputs for is. Using this, we obtain country k's nominal output in sector z as

$$\begin{aligned} y^{kz} &= \sum_{i=1}^{N}\sum_{s=1}^{S} L_{is}^{kz} \sum_{j=1}^{N} f_j^{is} \\ &= \sum_{i=1}^{N}\sum_{s=1}^{S} L_{is}^{kz} \sum_{j=1}^{N} \left(\frac{y^{is}\alpha_j^s \sum_{r=1}^{S}(1-\beta^{jr})y^{jr}}{y^w} \left(\frac{\tau_j^{is}}{\Pi^{is}P_j^s}\right)^{1-\sigma_s} \right) \end{aligned} \tag{15}$$

where we substituted the gravity relation from Equation (10) for the final value f_j^{is} flowing from country-sector is to the consumer in country j. Finally, we transform this into value added production. For this purpose, we assume that the value added share of a country-sector's production is the part that is generated by its labor. Recalling that in the production function in (3), the value created by country-sector kz after accounting for the intermediates used is captured by the share of labor $1-\beta^{kz}$. Hence, we find the value added embodied in kz's nominal production y^{kz} as $(1-\beta^{kz})y^{kz}$ where $1-\beta^{kz} \equiv v^{kz}$ is the value added to output ratio. The total value added production by kz can thus be written as

$$va^{kz} = v^{kz} \sum_{i=1}^{N}\sum_{s=1}^{S} L_{is}^{kz} \sum_{j=1}^{N} f_j^{is} \tag{16}$$

The extent to which this value added production (and the jobs it represent) is affected by a trade shock is the subject of the next section.

2.6. Evaluating Trade Shocks

In this section, we examine the impact of a trade shock such as Brexit on a country-sector's value added production. Equation (16) shows that an import tariff imposed on a specific good does not only affect the producer of the good, but also the suppliers of goods and services whose output is used as an input in the production of the good. This implies that when the UK imposes a tariff on German cars, the Belgian steel sector which supplies inputs to the German car industry will also be affected, even in the absence of a UK import tariff on Belgian steel. This channel is missing in a traditional gravity approach but can be captured by our sector-level model.

The impact of a trade shock amounts to considering what happens when the variable trade costs

(τ) changes.[17] For example, in the case of a hard Brexit, trade costs go from zero to WTO-levels. For this purpose we now evaluate the new gravity equation in (15) and the total value added in (16) when τ changes. Our interest lies in the change dva^{kz} in country-sector kz's value added production, which we find to equal[18]

$$\begin{aligned} dva^{kz} &= -v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(\sigma_s-1)L_{is}^{kz}\sum_{j=1}^{N}\hat{\tau}_j^{is}\left\{f_j^{is}+\sum_{r=1}^{S}x_{jr}^{is}\right\} \\ &= -v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(\sigma_s-1)L_{is}^{kz}\sum_{j=1}^{N}\hat{\tau}_j^{is}e_j^{is} \end{aligned} \tag{17}$$

from which we can derive the following general result. Rising trade costs reduce bilateral trade flows e_j^{is} between any country-sector is and j. As kz has an interest L_{is}^{kz} in each of these bilateral flows, va^{kz} will decrease as well. The drop depends on the magnitude of the change in relative trade costs $\hat{\tau}_j^{is}$ between is and j and the corresponding trade elasticity σ_s.

In Equation (17), we defined $\hat{\tau}_j^{is} \equiv \frac{d\tau_j^{is}}{\tau_j^{is}} - \frac{d\Pi^{is}}{\Pi^{is}} - \frac{dP_j^s}{P_j^s}$ as the proportionate change in tariffs τ_j^{is} relative to the proportionate changes in the multilateral resistance (MR) terms. When examining trade policy, it is important to take into account that the multilateral resistance (MR) terms will change along with the tariffs. Therefore, Equation (17) not only examines the impact of $\frac{d\tau}{\tau_j^{is}}$ but also that of $\frac{d\Pi^{is}}{\Pi^{is}}$ and $\frac{dP_j^s}{P_j^s}$. As it is relative tariffs that matter rather than absolute tariffs to determine a country's global competitiveness, individual tariff changes should be compared with changes in the average tariff, which is captured by the multilateral resistance terms. Suppose, for instance, that the UK tariff on Belgian goods goes up by 3%. Further suppose for a moment that the UK raises its tariffs on all its other trading partners with 2%, then the "real" or "relative" increases in the BE-UK tariff is only 1% (3% - 2%). In that case, what matters for a country-sector's production change dva^{kz} is the tariff change it faces relative to the tariff change its competitors face.

Under Brexit however, the only countries that are likely to face increased tariffs from the UK are the EU-27, whereas the tariffs the UK imposes on its other trading partners such as the US will not change. This means that US goods will become relatively less expensive for the UK, even though the UK tariffs on US imports do not change. The reason is that Brexit actually decreases (i.e. $\hat{\tau}_{UK}^{US,s} < 0$) the "relative" US-UK trade costs compared to EU-UK trade costs. As a result,

[17] We disregard exchange rate effects on EU-UK trade. Recent work has shown that exchange rate effects may have little effect on trading firms as most importers are also exporters i.e. a depreciation of say the pound would be bad for UK firms' imports but great for their exports (Amiti et al., 2014).

[18] See the Appendix for a detailed derivation.

some trade will be diverted from the EU27-UK to the US-UK. The MR changes $\frac{d\Pi^{is}}{\Pi^{is}}$ and $\frac{dP_j^s}{P_j^s}$ are essential for trade diversion to happen. We can see this by disentangling the change $\hat{\tau}_j^{is}$ into its different components, namely the tariff change and the MR changes:

$$dva^{kz} = \underbrace{-\,v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(\sigma_s-1)L_{is}^{kz}\sum_{j=1}^{N}\frac{d\tau_j^{is}}{\tau_j^{is}}e_j^{is}}_{\text{trade destruction effect}} + \underbrace{v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(\sigma_s-1)L_{is}^{kz}\sum_{j=1}^{N}\left[\frac{d\Pi^{is}}{\Pi^{is}}+\frac{dP_j^s}{P_j^s}\right]e_j^{is}}_{\text{trade diversion effect}} \quad (18)$$

Equation (18) shows that the change in kz's value added production after a change in trade costs τ is a combination of a "trade destruction effect" (-) as a result of higher tariffs and a "trade diversion effect" (+) caused by the change in the multilateral resistance terms.

The "trade destruction effect" measures the drop in va^{kz} that is caused by the reduced trade between any country-sector is and country j. This drop depends on how the output of country-sector kz is used by country i's sector s, as it is the latter sector's exports that will face increased protectionist measures from country j.

The "trade diversion effect", consists of two channels. First, country-sector is will divert some of its exports away from j to alternative destinations that do not impose tariffs on its goods, since these destinations have now become relatively more attractive (i.e. less expensive) for is to export to. This is caused by the increase in is' outward MR term Π^{is}. Second, the fact that j increases the tariffs on its imports will raise the average price in market j which makes the market less competitive, captured by the increase in j's inward MR term P_j^s. As a result, any country i will find it easier to export to country j. Both the first and second channel of trade diversion increase the exports of is and hence its production, which results in an increase in its demand for inputs from country-sector kz, which in turn increases the latter's value added production va^{kz}. Therefore, the "trade diversion effect" will mitigate some of the negative "trade destruction effect" on va^{kz}. The results can be summarized in the following proposition:

Proposition 1: The change in kz's value added production after a trade shock depends on two effects. First, the negative "trade destruction effect" indicates that the loss in va^{kz} depends on kz's connection with each exporting country-sector is. The drop in va^{kz} will be greater, (i) the higher is the trade elasticity in sector s (higher $(\sigma_s - 1)$); (ii) the greater is the increase in protection imposed by j on sector s goods originating in country i (higher $\frac{d\tau_j^{is}}{\tau_j^{is}}$); (iii) the greater is the production interlinkage of kz with is (higher L_{is}^{kz}) and (iv) the stronger is the direct bilateral trade relation in both final and intermediate goods between i and j in sector s. Second, these negative effects will be mitigated through the "trade diversion" channel, as some of kz's production will be used in exports that are diverted to different destinations after the trade shock.

Equation (18) sums up the effects of a trade shock on va^{kz}. It characterizes all the different channels through which a trade shock affects a sector's output. It also shows why the effect of a trade shock such as Brexit can substantially vary by sector, depending on production interlinkages with other sectors as captured by the Leontief coefficients (L), the linkages to exporting sectors (e), the product differentiation in the sector (σ) and the extent of the sector-level tariff change ($\frac{d\tau}{\tau}$).

In the next section, we apply our model to a specific trade shock. For this purpose we turn to the case of Brexit. We calibrate our model using WIOD data to obtain the production and employment effects of Brexit, in which the EU and the UK impose tariffs on each other's goods.

3. Methodology

To calibrate the model developed above we use input-output data for the latest available year 2014 from WIOD that covers 43 countries and 56 sectors. While empirically we consider tariffs imposed by the UK as well as tariffs imposed by the EU-27 for expository simplicity we just discuss the effects of a unilateral UK protection on EU goods since the analysis is entirely symmetric for any EU-27 country. We investigate the impact on kz's production when the UK imposes tariffs on EU goods using Equation (18).

Equation (18) consists of a trade destruction and diversion effect, where the latter derives from the changes in multilateral resistance (MR) terms. These MR terms are not observable, and not controlling for them in gravity estimation is what Baldwin and Taglioni (2006) call "the gold medal of classic gravity model mistakes". Empirically, there are several ways to deal with the issue of MR, see for instance Anderson and Van Wincoop (2003), Baier and Bergstrand (2009) and Novy (2013). Hummels (1999) and Feenstra (2015) suggest to control for MR using directional (exporter and importer) fixed effects in a gravity model based on past data series. However, in our analysis of Brexit, where we calibrate the model to engage in future predictions, the inclusion of fixed effects is not an option. The empirical findings in the literature on the magnitude of the trade diversion effect of import tariffs are ambiguous but its effects appear to be small compared to the trade effects.[19] Indeed, in order to divert trade, new business contacts have to be established, new contracts negotiated and so on, which takes some time to materialize. In our Brexit application,

[19]Using different gravity specifications, Magee (2008) finds estimates of the trade diversion effects of regional agreements to be small and their significance to depend on the specification used. Similarly, Soloaga and Wintersb (2001) find evidence of export diversion in a minority of FTAs, as only 2 out of the 9 FTAs analyzed had substantial trade diversion.Therefore, the trade diversion effects of trade policy are likely to be relatively small.

we therefore concentrate on the short-run effects and restrict Equation (18) to the first term that measures the "trade destruction effect". This is the first-order trade effect, which captures the main effects resulting from the Brexit's tariff changes. The drop in value added production as a result of increased UK trade protection on EU goods (higher $\tau_{UK}^{EU,s}$) under Brexit will thus be approximated by

$$dva^{kz} \approx -v^{kz} \sum_{i \in EU}^{N} \sum_{s=1}^{S} (\sigma_s - 1) \frac{d\tau_{UK}^{EU,s}}{\tau_{UK}^{EU,s}} L_{is}^{kz} e_{UK}^{is}$$

Within this trade destruction effect we now distinguish two different channels of value added loss by decomposing the trade destruction effect of UK protection into "direct" and "indirect" losses. These refer, respectively, to the losses in value added of country-sector kz stemming from direct bilateral trade (in goods and services) with the UK and the value added losses arising through its production linkages with other affected sectors in other EU-27 countries. For any country-sector kz, the loss in va^{kz} can be decomposed into a "direct" and "indirect" loss as follows

$$dva^{kz} \approx \underbrace{-v^{kz} \sum_{s=1}^{S} (\sigma_s - 1) \frac{d\tau_{UK}^{EU,s}}{\tau_{UK}^{EU,s}} L_{ks}^{kz} e_{UK}^{ks}}_{\text{direct loss}} \underbrace{- v^{kz} \sum_{i \in EU \setminus \{k\}}^{N} \sum_{s=1}^{S} (\sigma_s - 1) \frac{d\tau_{UK}^{EU,s}}{\tau_{UK}^{EU,s}} L_{is}^{kz} e_{UK}^{is}}_{\text{indirect loss}} \quad (19)$$

Equation (19) thus captures the effect on va^{kz} of increased UK trade protection on EU-27 goods and services. Similarly, the effects of increased EU-27 protection on UK goods and services can be obtained from equation (19) by simply reversing the country of origin and destination.[20] In Section 4 we present results for the UK and the EU-27 both separately and combined.

3.1. Value Added Production Losses

In order to obtain an estimate of the value added losses, Equation (19) indicates that five key variables are needed. The five determinants in this equation are retrieved from various sources: (i) the the value added share v^{kz}, the Leontief coefficient L_{is}^{kz} and the direct trade flows e_{UK}^{is} are variables from WIOD; (ii) the trade elasticities at sector-level σ_s are obtained from the literature (Imbs and Méjean, 2017) and (iii) the change in trade barriers τ are obtained from potential Brexit scenarios. In order to obtain the job losses corresponding with the loss in value added in production, we turn to Eurostat data on EU-27 and UK sectoral level employment. Using sectoral employment elasticities from the literature (Konings and Murphy, 2006), we obtain the corresponding job losses.

[20] Note that our theoretical framework predicts a loss in UK production even if we only consider trade protection imposed by the UK itself. The main mechanism is that it increases the price of (EU-27) inputs for UK firms and it decreases the demand for UK inputs that are embedded in EU-27 goods and services destined to the UK consumer.

3.1.1. Input-Output Data

The World Input-Output Database (WIOD) contains detailed information on the global value chains of 43 world countries, including an approximation for the rest of the world, and 56 sectors. For calibration purposes we use the data available for the latest available year which is 2014.

For each country-sector, WIOD provides its total production, the inputs it needs from other country-sectors and how much of its output is used by other country-sectors in their production process. The first variable that we obtain from WIOD is the value added share of country-sector kz's production, v^{kz}. This captures the value added, obtained as gross output minus gross intermediate inputs, per unit of gross output. We also obtain the Leontief coefficients, L_{is}^{kz} from WIOD, which are obtained using Equation (14). From WIOD we also obtain the direct trade flows e_j^{is} from country is to country j, by summing exports from is that are destined to country j to satisfy its final and intermediate demand.

3.1.2. Trade Elasticities

A trade elasticity measures the proportionate decrease in demand after a 1% increase in trade costs. Higher UK tariffs and Non-Tariff Barriers (NTBs) will increase the price of EU-27 products in the UK (and vice versa), which will lower UK consumers' demand of EU-27 goods as they substitute away to products of cheaper origin. This is captured by the elasticity of substitution σ_s in sector s, from which the trade elasticity is derived as $\sigma_s - 1$. As a result, the extent to which production decreases after Brexit depends on the trade elasticity.

The literature has shown that trade elasticities vary both across countries and sectors. For example, Imbs and Méjean (2017) find considerable heterogeneity in trade elasticities across countries and sectors. Using product-level export flows, they find the average trade elasticity for EU countries to be -2.98 with a minimum of -2.11 for Germany and a maximum of -4.83 for Greece.[21] Within EU countries, they find trade elasticities to vary across products and across sectors. From their estimates we derive that Germany has an average elasticity across 11 manufacturing sectors of -5.1, with a median of -4.7 and maximum and minimum of -11.1 and -3.2, respectively.[22] Since Imbs and Méjean (2017) do not report estimates for every EU country-sector, whenever we face a missing value we impute the average trade elasticities across countries for which we do observe values at

[21] See Table 4 in Imbs and Méjean (2017).

[22] In our analysis, we use a sectoral aggregation at 2 digit in Nace Rev. 2. We use the nomenclatures tables (RAMON) provided by Eurostat for the correspondence with ISIC Rev. 4 used by Imbs and Méjean (2017).

a sectoral level. This way we obtain trade elasticities for sixteen different manufacturing sectors. For sectors where all information is missing, we simply turn to the most common value for the trade elasticity used in the literature which is -4. This value is at the lower end of all estimates that circulate in the literature. But given that we analyze trade in value added rather than gross flows and that our data are at sector-level and not at product-level, we prefer to use this lower-end estimate of the trade elasticity for sectors where no trade elasticity is available. This renders our results into lower bound estimates.[23] In line with the theory developed above, we assume complete pass-through of tariffs into domestic prices. We point out that our results vary linearly with the trade elasticity i.e. doubling the trade elasticity in every sector, doubles the value added losses from Brexit and as such results depend monotonically on the value of the trade elasticity.

3.1.3. Potential Brexit Scenarios

Equation (19) and the resulting losses in value added hinge on the increase in trade barriers i.e. $\frac{d\tau_{UK}^{EU,s}}{\tau_{UK}^{EU,s}}$. We consider two Brexit scenarios, an optimistic ("soft Brexit") and a pessimistic ("hard Brexit") scenario. In short, in the "soft Brexit" scenario, the UK continues to belong to the EU Single Market and tariffs remain zero, while non-tariff barriers to trade (NTBs) increase by 2.77%.[24] The scenarios are summarized in Table 1. In a "hard" Brexit scenario, the UK leaves the

Table 1: Imposed tariffs and NTBs in both scenarios of Brexit.

	Soft Brexit	Hard Brexit
Tariff	0%	MFN tariff
Non-tariff barrier	2.77%	8.31%

Note: The scenarios are based on Dhingra et al. (2017).

Single Market and trade between the EU-27 countries and the UK is governed by the World Trade

[23]Other trade elasticities estimates in the literature confirm this heterogeneity. Baier and Bergstrand (2001) use trade data to estimate a demand elasticity of -6.43, while Broda et al. (2017) use ten-digit HS data to obtain price elasticities of around -12. A recent paper by Coşar et al. (2016) uses a trade elasticity of -5.66. Ossa (2015) estimates sector level trade elasticities which range between -1.54 and -25.05.

[24]This is similar to the case of Norway whose NTBs with the EU are 2.11% higher than for the EU members. The 2.77% is taken from Dhingra et al. (2017). They compute a weighted average tariff equivalent for the current NTBs on US-EU trade, which amounts to 20.4%. Given that only 54% of this tariff equivalent is reducible, they only take into account an NTB tariff equivalent of ca. 11%. In the optimistic Brexit scenario, Dhingra et al. (2017) assume that the EU-UK trade will be subject to a NTB that is only one quarter of the one on EU-US trade, resulting in a tariff equivalent of 2.77%.

Organization (WTO) rules. This implies an increase in trade tariffs from the current level of 0% to the sectoral "applied tariffs" imposed under the Most Favored Nations (MFN) clause, which differ by sector. These MFN tariffs are the tariffs that are currently imposed on goods traded between the United States and the EU, for instance. In Figure 1, we present the unweighted current MFN tariffs according to WTO rules in the sectors contained in the WIOD database. These are the MFN tariffs from the EU perspective, i.e. those that the EU imposes on imports from abroad. In the “hard Brexit” scenario, we assume EU-UK and UK-EU trade to be subject to an increase in the trade tariffs on goods from 0% to the unweighted average MFN tariff in each sector that ranges from 0% in “Mining and quarrying”, “Forestry” and “Electricity and Gas” to 9.1% in the case of Fishing products. Figure 1 gives an overview of the MFN tariffs that currently apply to trade between members of the WTO. Moreover, we assume that under a “hard Brexit” NTBs rise further to a tariff equivalent of 8.31%.[25]

[25]This corresponds to three quarters of the NTB that applies to EU-US trade, see Dhingra et al. (2017).

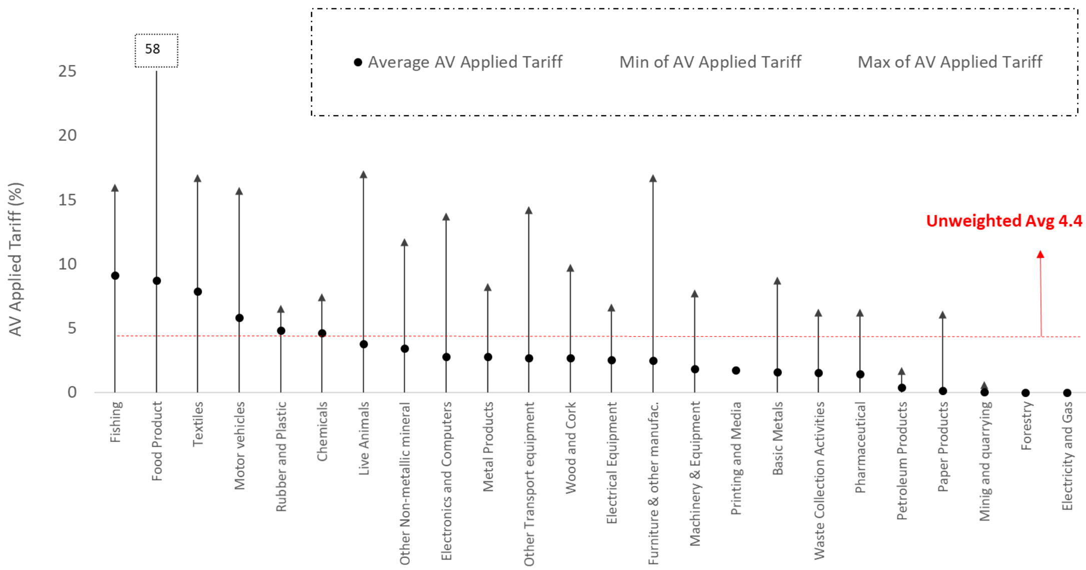

Figure 1: MFN tariffs imposed by the European Union

Note: The upper (lower) bound corresponds to the highest (lowest) tariff imposed within the HS6 classified in a Nace.rev 2 sector. The red dotted line marks the unweighted average tariff of all the HS6 where the European Union reports a tariff to the Most-Favored-Nations (MFNs). Information on the current tariffs applied are collected using the WTO Integrated Data Base (IDB). This database contains information on the applied tariffs at the standard codes of the Harmonized System (HS) for all the WTO Members. In this exercise, we use the Reference and Management of Nomenclatures (RAMON) correspondence tables to classify the equivalent Combined Nomenclature (CN) to the respective CPA 2008 code. In 35 of 5051 HS6 codes considered, the HS6 corresponded to multiply CPA 2008 codes.

3.2. Employment Losses

In order to transform the production losses in job losses, we need an employment elasticity. This elasticity measures the proportionate drop in employment after a 1% decrease in value added production. In our theoretical framework, our production function is characterized by constant return to scale. In theory, Hamermesh (1986) argued that a production function characterized by constant returns to scale is identified by an elasticity of 1. However, this differs from empirical evidence. An employment elasticity measures the proportionate drop in employment after a 1% decrease in value added production. Konings and Murphy (2006) using European firm level data, estimate employment elasticities with respect to value added for manufacturing and non-manufacturing sectors. They find employment elasticities to range between 0.57 and 0.72 in manufacturing sectors and find the average employment elasticity in non-manufacturing sectors to be 0.33.[26] Given our focus on European data, we use the lower bound of these sectoral estimates. This implies that for every 1% drop in domestically produced value added as a result of Brexit, we assume employment to go down by 0.57 % in manufacturing and 0.33% in non-manufacturing sectors. Similar to the trade elasticities, the Brexit results on employment depend linearly on the choice of the employment elasticity. Thus, once we have obtained the relative drop in employment from the decrease in production, we compute the absolute number of jobs lost by multiplying by the country-sector's total employment base.[27]

4. Results

Tables 2 and 3 summarize the economic impact of tariffs imposed by the UK on EU-27 products in terms of value added and employment. Both tables contain information on the potential losses for the EU-27 individual countries and the UK, which are obtained by aggregating all the sector level results at country level.

The fact that the UK can also lose as a result from its own tariffs may at first seem counter-intuitive, but is the direct consequence of taking into account the European production network which makes our analysis unique. To see this consider the following example. When German car manufacturers use UK insurance as an inputs, then an import tariff on German cars by the UK will not only decrease the demand for German cars in the UK, but it will also decrease German demand for UK insurance. This illustrates how the UK can lose production and value added from its own tariffs. Note that the same mechanism applies to the EU-27 when it imposes import tariffs on UK goods and services.

In the Tables 2 and 3 we distinguish between a "soft" and a "hard" Brexit, respectively. Columns (1) and (2) in both tables show the *absolute* numbers, in terms of millions of dollars and thousand of people, that would hypothetically be lost in a "soft" Brexit scenario. Column (1) shows the direct losses for each EU-27 country from lower direct bilateral trade with the UK, while column (2) shows the losses resulting from reduced indirect trade with the UK via "other" EU-27 countries. Column (3) sums total losses. Column (4) normalizes the losses by country size by expressing it as

[26] An employment elasticity of 1 would imply that wages do not adjust and stay constant. An employment elasticity below 1 suggests that wages adjust somewhat but are not fully flexible since that would imply an observed employment elasticity of 0.

[27] Throughout the analysis, we assume that any job lost in the UK is not going to move to the EU-27 and vice versa.

a percentage of the total value added (and employment) of the country.[28] The remaining columns do the same but for a "hard" Brexit scenario. While in the tables we aggregate the sector-level effects at the country-level, our analysis is carried out entirely at sectoral level.

When the UK imposes tariffs, some sectors will be more affected than others. Table 4 lists for every country the most affected sector in terms of value added and employment. This sector can differ depending on whether we express losses in terms of value added or employment. The reason is that the value added contribution per worker can differ substantially across sectors, which means that the same drop in value added might lead to different employment effects in different sectors.

When the EU-27 imposes tariffs against the UK, we also expect losses. Tables 5, 6 and 7 summarize the economic impact of such tariffs. The EU-27 will lose from its own tariffs imposed on the UK because lower exports for the UK to Europe will also contain lower demand for EU-27 intermediate inputs.

The overall effect of Brexit, where we sum over all the losses from tariffs installed by both trading partners are shown in Tables 8 and 9. The results show that the UK is hit *relatively* harder than the rest of the EU-27 in both the "soft" and "hard" Brexit scenario. In either case, Brexit reduces economic activity in the UK three times more than in the EU-27. The UK will experience a drop in value added production as a percentage of GDP of 1.21% under a "soft" Brexit and up to 4.47% under a "hard" Brexit scenario. This corresponds to UK job losses of 139,860 jobs in the "soft" Brexit and 526,830 jobs in the "hard" Brexit scenario. For the EU-27, the *absolute* job losses are larger, with the numbers of EU-27 jobs lost from Brexit varying between 284,440 jobs and 1,209,470 jobs *respectively* which corresponds to value added losses as a percentage of GDP of 0.38% for the "soft" and 1.54% for the "hard" Brexit. The losses in value added and jobs differ substantially across EU-27 member states. EU-27 member states that lose most are countries with close historical ties to the UK (e.g. Ireland, Malta) and small open economies (e.g. Belgium and the Netherlands).

The sectors which overall will be most affected under a "hard" Brexit are shown in Table 10, which is the result of both the direct and indirect trade effects. These most affected sectors can differ from other Brexit studies because we have accounted for input-output linkages between goods and services sectors. The most affected sector in Germany is "Motor Vehicles" (in value added), while for the UK (GBR), it is "Wholesale Trade" that is most affected by Brexit in value added. In terms of job losses, Germany has most job losses in "Machinery and Equipment", while the UK has most job losses in the service sector "Administrative and support activities".

[28] Total Value Added (TVA) for each country is obtained using the WIOD database.

5. Discussion

This section compares our results with those found by other papers that investigate the potential impact of Brexit. Emerson et al. (2017) summarize the results of six existing papers.[29] These studies each consider an optimistic and a pessimistic Brexit scenario that correspond closely to our "soft" and "hard" Brexit scenarios. For the UK, our results appear to be in line with what others find.[30] However, for the EU-27 our results diverge a lot from existing studies. The negative impact that we find for the EU-27 is much stronger with losses being approximately three times as high as previously thought. Previous studies all find larger *absolute* losses for the UK than for the EU-27 as a whole, whereas we find the *absolute* loss in value added production for the EU-27 to be 1.7 times larger than the UK losses. The most important reason for the larger absolute losses for the EU-27 that we find here is the fact that we have accounted for country-sector level input-output linkages in production and services which for the EU-27 turn out to be very important as production networks and value chains are closely integrated. Thus, accounting for country-sector level input-output linkages, as we do in our analysis, gives rise to indirect trade effects whereby local goods and services are shipped as intermediate inputs to "third" EU countries to finally end up in the UK. These indirect trade flows also affect local jobs and production under Brexit (e.g. decreased Belgian steel production due to reduced German car exports to the UK). Empirically they account on average for one fourth (25%) of the overall effect when measured at the country-level, although their relative importance differs across EU-27 countries which can be seen in Table 2 and 3.[31] When measured at the country-sector level, the average indirect effect rises to 70% e.g. for Belgian steel this would mean that only 30% of the Brexit effect comes from direct trade with the UK and 70% of the change in value added in Belgian steel comes from the network effects via other Belgian sectors and foreign country-sectors using Belgian steel which is then shipped to the UK.

The inclusion of the indirect trade effects in our analysis is the main reason why our approach yields greater estimated losses for Brexit since they add to the effects of direct trade captured in earlier literature. Another reason is that the WIOD data that we use includes information on all the services sectors. Since many services are intermediates and embedded in manufactured goods,

[29]See Ottaviano et al. (2014), Aichele and Felbermayr (2015), the OECD study by Kierzenkowski et al. (2016), Rojas-Romagosa (2016), Booth et al. (2015) and HMTreasury (2016).

[30]See, for instance, Rojas-Romagosa (2016), Aichele and Felbermayr (2015), Booth et al. (2015) and Ottaviano et al. (2014).

[31]Tables 2 and 3 display the calibrated losses in terms of value added and employment associated with a tariff imposed by the UK on EU-27 goods and services where we distinguish between direct and indirect effects, see columns (1), (2), (5) and (6).

the inclusion of services reinforces the negative impact on local jobs resulting from indirect trade effects via “third countries”(e.g. decreased Belgian car insurance sales due to reduced German car exports to the UK). The larger impact of the Brexit trade shock in this paper demonstrates the importance of including the sector-level dimension. It also provides a measure of the potential bias in studies that ignore the sector dimension and that are based on a more aggregated country-level analysis. While for expository clarity we present results at the country-level, we point out that all our results are obtained at sector-level.

In this paper we only focus on the static effects of a trade shock and do not include dynamic effects such as access to foreign markets, firm investment and innovation, capital mobility or accumulation and migration. Clearly if Brexit would trigger more European FDI into the UK in order to avoid the import tariffs, this could mitigate some of the negative trade effects for the UK that we find. If on the other hand many multinationals leave the UK, because it is no longer an attractive FDI destination to get access to the EU Single Market, foreign investment previously flowing into the UK may now be diverted to the other EU-27 member states, which would aggravate the losses for the UK. This makes it difficult to predict whether the more dynamic longer-run aspects of Brexit would aggravate or mitigate the negative trade effects that we report here. What we do know is that the trade effects are first order in magnitude and are likely to account for the main part of the Brexit impact, while the dynamic effects, while potentially important, are only of a second order nature.[32] In terms of timing, we assume all effects to occur immediately after Brexit happens. Put differently, without mitigating measures, this paper finds Brexit to result in a UK loss in value added of 4.47% of GDP.[33] This means that UK economy is expected to be 4.47% smaller in the event of Brexit than in the counterfactual scenario where it would stay in the single market. Without mitigating measures, this drop in UK value added would be a permanent one whereby the size of the UK economy would be permanently lower than what it would otherwise have been.

However, in reality it can take some time for this effect to materialize. Especially, non-tariff barriers (NTBs) can have a lagged effect.[34]

6. Conclusion

This paper offers a new approach to evaluate trade policy in the presence of global value networks. We first build a network trade model with worldwide country-sector level input-output linkages

[32] See for example Kierzenkowski et al. (2016), HMTreasury (2016), Dhingra et al. (2017).
[33] For GDP we also use the data in WIOD.
[34] Jung (2012) estimates that for NTBs an adjustment period of 10 to 12 years could be in order.

in production that allows for a more complete assessment of trade policy shocks. A key insight from the model is that import tariffs affects both direct bilateral trade between countries but also indirect bilateral trade, via "third" countries. The model shows that the network structure of a country and itscountry-sectoral input-output linkages in national and international value chains play an important role in the total impact of tariff changes.

Next, we calibrate our model to predict the short-run trade impact of Brexit in terms of value added and employment for each of the 28 individual EU countries involved. The data that we use for this purpose is the sectoral World Input Output Database (WIOD) featuring 56 sectors which include 30 services sectors. To calibrate the model we complement WIOD data with trade and employment elasticities obtained from the literature.

In contrast to other studies, we find the losses for the EU-27 countries to be much higher than previously thought. The main reason is that our approach incorporates all national and international country-sector level input-output linkages in both goods and services. Given that EU-27 production networks are closely integrated, tariff changes do not just affect direct bilateral trade flows between any EU-27 country and the UK, but also indirect trade flows via third countries. These indirect trade effects substantially reinforce the trade destruction effects of Brexit for all EU-27 countries and reinforce the negative impact previously reported by studies that only consider the direct trade effects.

The losses from Brexit in value added and jobs differ substantially across EU-27 member states. We find the propagation of a tariff shock to depend on the sectoral composition and on the input-output linkages between sectors. When a particular sector, facing tariffs for its output abroad, has only a few linkages with other sectors, this may not affect aggregate output of the country as a whole much as opposed to a sector that is very central in the production network of its own country and of others. The network centrality of sectors and the number and intensity of sectoral production linkages within and across EU countries proves important when estimating the loss in value added and jobs caused by a trade shock such as Brexit.

These findings thus give support to the idea that it is the network centrality of sectors that determines the impact of an aggregate shock through a "cascade effect" in the input-output network (Acemoglu et al. (2012)). This network approach is very different from Autor et al. (2013), who study the local employment effects in the US of trade liberalization with China. In contrast to our analysis, theirs does not account for downstream effects. They only consider the first round employment losses from Chinese import competition. In contrast, this paper also accounts for all the downstream effects that a shift in trade policy can bring about, thus providing a more complete estimation of the overall employment effects brought about by a shift in trade policy.

In sum, our sector-level input-output approach clearly shows that the EU-27 stands to lose considerably more from Brexit than hitherto believed. The main reason is the closely integrated European production networks in both goods and services that we account for in this paper.

7. References

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8. Tables

Table 2: Loss in Value Added (VA) as a result of UK protection against the EU-27

Country	Soft Brexit				Hard Brexit			
	Via domestic sectors	**Via other EU-27 countries**	**Total loss**		**Via domestic sectors**	**Via other EU-27 countries**	**Total loss**	
	(million $)	(million $)	(million $)	(% of total VA)	(million $)	(million $)	(million $)	(% of total VA)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
AUT	-554	-370	-924	-0.23%	-2159	-1571	-3730	-0.92%
BEL	-2156	-536	-2692	-0.54%	-8662	-2284	-10946	-2.18%
BGR	-76	-41	-117	-0.22%	-299	-175	-473	-0.90%
CYP	-31	-32	-63	-0.29%	-103	-104	-207	-0.95%
CZE	-615	-277	-893	-0.45%	-2542	-1203	-3745	-1.89%
DEU	-12021	-2305	-14326	-0.40%	-49640	-9851	-59492	-1.64%
DNK	-1076	-190	-1266	-0.40%	-4138	-780	-4918	-1.55%
ESP	-2126	-456	-2582	-0.20%	-9236	-1982	-11219	-0.86%
EST	-40	-22	-63	-0.25%	-148	-88	-236	-0.95%
FIN	-423	-158	-581	-0.23%	-1491	-649	-2140	-0.86%
FRA	-6599	-1166	-7765	-0.29%	-25704	-5093	-30797	-1.16%
GRC	-175	-39	-214	-0.10%	-598	-162	-760	-0.35%
HRV	-59	-29	-88	-0.17%	-208	-122	-330	-0.65%
HUN	-362	-154	-516	-0.41%	-1429	-676	-2105	-1.66%
IRL	-2766	-149	-2915	-1.23%	-12300	-605	-12905	-5.46%
ITA	-4500	-887	-5388	-0.27%	-19436	-3862	-23298	-1.17%
LTU	-111	-36	-147	-0.32%	-459	-151	-611	-1.32%
LUX	-133	-107	-240	-0.39%	-422	-420	-842	-1.38%
LVA	-63	-21	-83	-0.29%	-229	-85	-314	-1.09%
MLT	-142	-6	-149	-1.52%	-434	-25	-460	-4.69%
NLD	-3873	-1238	-5111	-0.62%	-14578	-5047	-19625	-2.37%
POL	-1470	-501	-1972	-0.39%	-5883	-2178	-8060	-1.57%
PRT	-437	-100	-536	-0.25%	-1919	-440	-2359	-1.09%
ROU	-254	-138	-392	-0.21%	-1079	-593	-1672	-0.89%
SVK	-378	-116	-494	-0.51%	-1341	-498	-1839	-1.89%
SVN	-58	-48	-107	-0.24%	-221	-206	-427	-0.94%
SWE	-1228	-378	-1605	-0.30%	-4487	-1569	-6056	-1.14%
EU-27			**-51227**	**-0.35%**			**-209567**	**-1.44%**
GBR		**-1484**	**-1484**	**-0.05%**		**-6337**	**-6337**	**-0.23%**

Note: See the Appendix for a list of the country name abbreviations.

Table 3: Loss in Employment (EMP) as a result of UK protection against the EU-27

Country	Soft Brexit				Hard Brexit			
	Via domestic sectors	**Via other EU-27 countries**	**Total loss**		**Via domestic sectors**	**Via other EU-27 countries**	**Total loss**	
	(1000 pers)	(1000 pers)	(1000 pers)	(% of total EMP)	(1000 pers)	(1000 pers)	(1000 pers)	(% of total EMP)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
AUT	-2.36	-1.48	-3.84	-0.09%	-9.52	-6.38	-15.91	-0.37%
BEL	-7.61	-1.77	-9.39	-0.21%	-32.02	-7.65	-39.66	-0.87%
BGR	-2.43	-1.33	-3.75	-0.11%	-10.53	-6.25	-16.78	-0.49%
CYP	-0.17	-0.16	-0.33	-0.09%	-0.63	-0.52	-1.15	-0.32%
CZE	-7.26	-3.21	-10.47	-0.20%	-30.60	-14.00	-44.61	-0.87%
DEU	-54.07	-10.49	-64.56	-0.15%	-228.23	-45.40	-273.63	-0.64%
DNK	-3.34	-0.51	-3.85	-0.14%	-13.69	-2.23	-15.91	-0.57%
ESP	-12.50	-2.46	-14.96	-0.08%	-56.00	-10.82	-66.83	-0.37%
EST	-0.42	-0.21	-0.63	-0.10%	-1.67	-0.84	-2.51	-0.41%
FIN	-1.58	-0.62	-2.20	-0.09%	-5.72	-2.58	-8.31	-0.33%
FRA	-27.38	-4.73	-32.11	-0.12%	-110.88	-21.05	-131.93	-0.48%
GRC	-1.06	-0.27	-1.33	-0.03%	-4.00	-1.19	-5.19	-0.13%
HRV	-0.77	-0.41	-1.19	-0.08%	-2.88	-1.77	-4.65	-0.30%
HUN	-4.83	-1.98	-6.82	-0.16%	-20.09	-8.80	-28.90	-0.69%
IRL	-10.24	-0.40	-10.64	-0.55%	-45.46	-1.71	-47.17	-2.43%
ITA	-24.82	-4.74	-29.55	-0.12%	-111.55	-20.85	-132.40	-0.54%
LTU	-1.17	-0.39	-1.55	-0.12%	-5.32	-1.73	-7.05	-0.54%
LUX	-0.22	-0.20	-0.42	-0.12%	-0.70	-0.79	-1.50	-0.42%
LVA	-0.09	-0.03	-0.12	-0.03%	-0.29	-0.11	-0.40	-0.10%
MLT	-0.51	-0.03	-0.53	-0.37%	-1.57	-0.10	-1.68	-1.16%
NLD	-13.19	-3.92	-17.12	-0.20%	-51.15	-16.30	-67.45	-0.77%
POL	-20.33	-6.43	-26.76	-0.17%	-87.56	-28.64	-116.20	-0.74%
PRT	-5.06	-0.96	-6.02	-0.13%	-24.09	-4.43	-28.52	-0.63%
ROU	-6.10	-2.81	-8.91	-0.10%	-28.59	-12.83	-41.43	-0.48%
SVK	-2.65	-1.14	-3.80	-0.17%	-10.03	-4.95	-14.98	-0.67%
SVN	-0.51	-0.44	-0.96	-0.10%	-2.02	-1.90	-3.92	-0.42%
SWE	-3.57	-1.14	-4.71	-0.11%	-13.61	-4.80	-18.42	-0.41%
EU-27			**-266.50**	**-0.14%**			**-1137.06**	**-0.58%**
GBR		**-6.02**	**-6.02**	**-0.02%**		**-26.09**	**-26.09**	**-0.08%**

Note: See the Appendix for a list of the country name abbreviations.

Note: Employment data in Eurostat is missing for some sectors in the following countries: Estonia, Latvia, Lithuania, Luxembourg, Malta and Sweden. Therefore, the presented employment results for these countries will likely underestimate the true impact.

Table 4: Most affected sector across countries: UK protection against the EU-27 (Hard Brexit Scenario)

Country	**Sector Nace Rev.2**			
	Value Added (VA)		**Employment (EMP)**	
	(1)	(2)	(3)	(4)
AUT	Machinery & Equipment	C28	Metal products	C25
BEL	Food Product	C10-C12	Food Product	C10-C12
BGR	Textiles	C13-C15	Live Animals	A01
CYP	Financial Services	K64	Administrative and support act.	N
CZE	Electronics and Computers	C26	Metal products	C25
DEU	Motor vehicles	C29	Machinery & Equipment	C28
DNK	Food Product	C10-C12	Food Product	C10-C12
ESP	Food Product	C10-C12	Live Animals	A01
EST	Wood and Cork	C16	Textiles	C13-C15
FIN	Paper Products	C17	Machinery & Equipment	C28
FRA	Administrative and support act.	N	Administrative and support act.	N
GBR	Wholesale trade	G46	Administrative and support act.	N
GRC	Water transport	H50	Live Animals	A01
HRV	Other services	R_S	Metal products	C25
HUN	Electronics and Computers	C26	Electronics and Computers	C26
IRL	Food Product	C10-C12	Live Animals	A01
ITA	Textiles	C13-C15	Textiles	C13-C15
LTU	Petroleum Products	C19	Textiles	C13-C15
LUX	Financial Services	K64	Administrative and support act.	N
LVA	Wood and Cork	C16	Administrative and support act.	N
MLT	Other services	R_S	Other services	R_S
NLD	Wholesale trade	G46	Administrative and support act.	N
POL	Wholesale trade	G46	Live Animals	A01
PRT	Textiles	C13-C15	Textiles	C13-C15
ROU	Textiles	C13-C15	Textiles	C13-C15
SVK	Real Estate	L68	Metal products	C25
SVN	Metal products	C25	Metal products	C25
SWE	Machinery & Equipment	C28	Machinery & Equipment	C28

Note: See the Appendix for a list of the country name abbreviations and sector codes.

Note: The most affected sector can differ depending on whether we look in terms of value added or employment. The reason is that the value added contribution per worker can differ dramatically across sectors, which means that the same drop in value added might lead to different employment effects in different sectors.

Note: Employment data in Eurostat is missing for some sectors in the following countries: Estonia, Latvia, Lithuania, Luxembourg, Malta and Sweden. Therefore, the presented employment results for these countries will likely underestimate the true impact.

Table 5: Loss in Value Added (VA) as a result of EU-27 protection against the UK

Country	**via GBR**			
	Soft Brexit		**Hard Brexit**	
	(million $)	(% of total VA)	(million $)	(% of total VA)
	(1)	(2)	(3)	(4)
AUT	-71	-0.017%	-286	-0.071%
BEL	-207	-0.041%	-836	-0.167%
BGR	-10	-0.018%	-38	-0.072%
CYP	-4	-0.018%	-15	-0.068%
CZE	-60	-0.030%	-240	-0.121%
DEU	-1038	-0.029%	-4207	-0.116%
DNK	-96	-0.030%	-365	-0.115%
ESP	-168	-0.013%	-683	-0.052%
EST	-5	-0.022%	-21	-0.085%
FIN	-52	-0.021%	-208	-0.084%
FRA	-611	-0.023%	-2392	-0.090%
GRC	-19	-0.009%	-72	-0.033%
HRV	-6	-0.012%	-25	-0.048%
HUN	-38	-0.030%	-151	-0.119%
IRL	-162	-0.069%	-670	-0.283%
ITA	-326	-0.016%	-1301	-0.065%
LTU	-10	-0.022%	-42	-0.092%
LUX	-20	-0.033%	-77	-0.126%
LVA	-7	-0.026%	-29	-0.099%
MLT	-4	-0.045%	-16	-0.166%
NLD	-492	-0.059%	-1897	-0.229%
POL	-138	-0.027%	-557	-0.109%
PRT	-34	-0.016%	-135	-0.063%
ROU	-26	-0.014%	-104	-0.055%
SVK	-26	-0.027%	-100	-0.103%
SVN	-8	-0.019%	-34	-0.075%
SWE	-136	-0.026%	-540	-0.101%
EU-27	**-3777**	**-0.026%**	**-15042**	**-0.103%**
GBR	**-32528**	**-1.159%**	**-119161**	**-4.246%**

Note: See the Appendix for a list of the country name abbreviations.

Table 6: Loss in Employment (EMP) as a result of EU-27 protection against the UK

Country	**via GBR**			
	Soft Brexit		**Hard Brexit**	
	(1000 pers) (1)	(% of total EMP) (2)	(1000 pers) (3)	(% of total EMP) (4)
AUT	-0.27	-0.006%	-1.11	-0.03%
BEL	-0.67	-0.015%	-2.72	-0.06%
BGR	-0.27	-0.008%	-1.11	-0.03%
CYP	-0.02	-0.005%	-0.07	-0.02%
CZE	-0.67	-0.013%	-2.70	-0.05%
DEU	-4.50	-0.011%	-18.29	-0.04%
DNK	-0.26	-0.009%	-0.99	-0.04%
ESP	-0.88	-0.005%	-3.58	-0.02%
EST	-0.05	-0.009%	-0.20	-0.03%
FIN	-0.19	-0.008%	-0.77	-0.03%
FRA	-2.40	-0.009%	-9.39	-0.03%
GRC	-0.10	-0.002%	-0.38	-0.01%
HRV	-0.08	-0.005%	-0.32	-0.02%
HUN	-0.46	-0.011%	-1.85	-0.04%
IRL	-0.68	-0.035%	-3.17	-0.16%
ITA	-1.67	-0.007%	-6.74	-0.03%
LTU	-0.09	-0.007%	-0.38	-0.03%
LUX	-0.04	-0.010%	-0.14	-0.04%
LVA	-0.01	-0.003%	-0.04	-0.01%
MLT	-0.02	-0.012%	-0.07	-0.05%
NLD	-1.48	-0.017%	-5.75	-0.07%
POL	-1.66	-0.011%	-6.75	-0.04%
PRT	-0.30	-0.007%	-1.20	-0.03%
ROU	-0.48	-0.006%	-2.00	-0.02%
SVK	-0.21	-0.009%	-0.81	-0.04%
SVN	-0.07	-0.008%	-0.30	-0.03%
SWE	-0.39	-0.009%	-1.55	-0.03%
EU-27	-17.94	-0.009%	-72.41	-0.04%
GBR	-133.85	-0.435%	-500.74	-1.63%

Note: See the Appendix for a list of the country name abbreviations.

Note: Employment data in Eurostat is missing for some sectors in the following countries: Estonia, Latvia, Lithuania, Luxembourg, Malta and Sweden. Therefore, the presented employment results for these countries will likely underestimate the true impact.

Table 7: Most affected sector across countries: EU-27 protection against the UK (Hard Brexit Scenario)

Country	Sector Nace Rev.2			
	Value Added (VA)		**Employment (EMP)**	
	(1)	(2)	(3)	(4)
AUT	Wholesale trade	G46	Metal products	C25
BEL	Chemicals	C20	Administrative and support act.	N
BGR	Wholesale trade	G46	Live Animals	A01
CYP	Financial Services	K64	Administrative and support act.	N
CZE	Motor vehicles	C29	Metal products	C25
DEU	Chemicals	C20	Administrative and support act.	N
DNK	Mining and quarrying	B	Administrative and support act.	N
ESP	Chemicals	C20	Administrative and support act.	N
EST	Wood and Cork	C16	Metal products	C25
FIN	Paper Products	C17	Paper Products	C17
FRA	Administrative and support act.	N	Administrative and support act.	N
GBR	Administrative and support act.	N	Administrative and support act.	N
GRC	Water transport	H50	Wholesale trade	G46
HRV	Wholesale trade	G46	Metal products	C25
HUN	Motor vehicles	C29	Motor vehicles	C29
IRL	Live Animals	A01	Live Animals	A01
ITA	Administrative and support act.	N	Administrative and support act.	N
LTU	Chemicals	C20	Live Animals	A01
LUX	Financial Services	K64	Administrative and support act.	N
LVA	Wood and Cork	C16	Administrative and support act.	N
MLT	Financial Services	K64	Financial Services	K64
NLD	Wholesale trade	G46	Administrative and support act.	N
POL	Wholesale trade	G46	Retail trade	G47
PRT	Wholesale trade	G46	Administrative and support act.	N
ROU	Land & Pipeline transport	H49	Live Animals	A01
SVK	Real Estate	L68	Metal products	C25
SVN	Metal products	C25	Metal products	C25
SWE	Chemicals	C20	Administrative and support act.	N

Note: See the Appendix for a list of the country name abbreviations and sector codes.

Note: The most affected sector can differ depending on whether we look in terms of value added or employment. The reason is that the value added contribution per worker can differ dramatically across sectors, which means that the same drop in value added might lead to different employment effects in different sectors.

Note: Employment data in Eurostat is missing for some sectors in the following countries: Estonia, Latvia, Lithuania, Luxembourg, Malta and Sweden. Therefore, the presented employment results for these countries will likely underestimate the true impact.

Table 8: Total loss in Value Added from Brexit

Country	Soft Brexit		Hard Brexit	
	(million $)	(% of total VA)	(million $)	(% of total VA)
	(1)	(2)	(3)	(4)
AUT	-995	-0.25%	-4016	-0.99%
BEL	-2899	-0.58%	-11782	-2.35%
BGR	-127	-0.24%	-512	-0.97%
CYP	-67	-0.31%	-222	-1.02%
CZE	-952	-0.48%	-3985	-2.01%
DEU	-15364	-0.42%	-63699	-1.76%
DNK	-1362	-0.43%	-5283	-1.67%
ESP	-2749	-0.21%	-11902	-0.91%
EST	-68	-0.28%	-257	-1.04%
FIN	-633	-0.25%	-2348	-0.95%
FRA	-8376	-0.32%	-33190	-1.25%
GRC	-233	-0.11%	-831	-0.38%
HRV	-94	-0.18%	-355	-0.69%
HUN	-554	-0.44%	-2256	-1.78%
IRL	-3077	-1.30%	-13575	-5.74%
ITA	-5713	-0.29%	-24599	-1.23%
LTU	-157	-0.34%	-653	-1.42%
LUX	-260	-0.43%	-919	-1.51%
LVA	-91	-0.31%	-343	-1.19%
MLT	-153	-1.56%	-476	-4.86%
NLD	-5604	-0.68%	-21523	-2.59%
POL	-2110	-0.41%	-8618	-1.68%
PRT	-570	-0.26%	-2494	-1.16%
ROU	-418	-0.22%	-1775	-0.95%
SVK	-520	-0.53%	-1939	-1.99%
SVN	-115	-0.25%	-461	-1.02%
SWE	-1742	-0.33%	-6596	-1.24%
EU-27	**-55004**	**-0.38%**	**-224609**	**-1.54%**
GBR	**-34012**	**-1.21%**	**-125497**	**-4.47%**

Note: See the Appendix for a list of the country name abbreviations.

Table 9: Total loss in Employment from Brexit

Country	**Soft Brexit**		**Hard Brexit**	
	(1000 pers) (1)	(% of total EMP) (2)	(1000 pers) (3)	(% of total EMP) (4)
AUT	-4.12	-0.10%	-17.02	-0.40%
BEL	-10.06	-0.22%	-42.39	-0.93%
BGR	-4.02	-0.12%	-17.89	-0.52%
CYP	-0.35	-0.10%	-1.22	-0.34%
CZE	-11.14	-0.22%	-47.31	-0.93%
DEU	-69.06	-0.16%	-291.93	-0.68%
DNK	-4.11	-0.15%	-16.90	-0.61%
ESP	-15.84	-0.09%	-70.41	-0.39%
EST	-0.69	-0.11%	-2.71	-0.45%
FIN	-2.39	-0.10%	-9.08	-0.36%
FRA	-34.50	-0.13%	-141.32	-0.52%
GRC	-1.42	-0.04%	-5.57	-0.14%
HRV	-1.27	-0.08%	-4.97	-0.32%
HUN	-7.28	-0.17%	-30.75	-0.73%
IRL	-11.32	-0.58%	-50.33	-2.59%
ITA	-31.23	-0.13%	-139.14	-0.57%
LTU	-1.64	-0.12%	-7.43	-0.56%
LUX	-0.45	-0.13%	-1.63	-0.46%
LVA	-0.13	-0.03%	-0.44	-0.11%
MLT	-0.55	-0.38%	-1.75	-1.21%
NLD	-18.60	-0.21%	-73.20	-0.84%
POL	-28.42	-0.18%	-122.95	-0.78%
PRT	-6.32	-0.14%	-29.72	-0.66%
ROU	-9.39	-0.11%	-43.43	-0.50%
SVK	-4.00	-0.18%	-15.79	-0.71%
SVN	-1.03	-0.11%	-4.22	-0.45%
SWE	-5.10	-0.11%	-19.97	-0.45%
EU-27	**-284.44**	**-0.15%**	**-1209.47**	**-0.62%**
GBR	**-139.86**	**-0.45%**	**-526.83**	**-1.71%**

Note: See the Appendix for a list of the country name abbreviations.

Note: Employment data in Eurostat is missing for some sectors in the following countries: Estonia, Latvia, Lithuania, Luxembourg, Malta and Sweden. Therefore, the presented employment results for these countries will likely underestimate the true impact.

Table 10: Most affected sector across countries: Brexit ("Hard" Brexit Scenario)

Country	Sector Nace Rev.2			
	Value Added (VA)		**Employment (EMP)**	
	(1)	(2)	(3)	(4)
AUT	Machinery & Equipment	C28	Metal products	C25
BEL	Food Product	C10-C12	Food Product	C10-C12
BGR	Textiles	C13-C15	Live Animals	A01
CYP	Financial Services	K64	Administrative and support act.	N
CZE	Electronics and Computers	C26	Metal products	C25
DEU	Motor vehicles	C29	Machinery & Equipment	C28
DNK	Mining and quarrying	B	Food Product	C10-C12
ESP	Food Product	C10-C12	Live Animals	A01
EST	Wood and Cork	C16	Wood and Cork	C16
FIN	Paper Products	C17	Administrative and support act.	N
FRA	Administrative and support act.	N	Administrative and support act.	N
GBR	Administrative and support act.	N	Administrative and support act.	N
GRC	Water transport	H50	Live Animals	A01
HRV	Other services	R_S	Metal products	C25
HUN	Electronics and Computers	C26	Electronics and Computers	C26
IRL	Food Product	C10-C12	Live Animals	A01
ITA	Textiles	C13-C15	Textiles	C13-C15
LTU	Petroleum Products	C19	Textiles	C13-C15
LUX	Financial Services	K64	Administrative and support act.	N
LVA	Wood and Cork	C16	Administrative and support act.	N
MLT	Other services	R_S	Other services	R_S
NLD	Wholesale trade	G46	Administrative and support act.	N
POL	Wholesale trade	G46	Live Animals	A01
PRT	Textiles	C13-C15	Textiles	C13-C15
ROU	Textiles	C13-C15	Textiles	C13-C15
SVK	Real Estate	L68	Metal products	C25
SVN	Metal products	C25	Metal products	C25
SWE	Petroleum Products	C19	Machinery & Equipment	C28

Note: See the Appendix for a list of the country name abbreviations and sector codes.

Note: The most affected sector can differ depending on whether we look in terms of value added or employment. The reason is that the value added contribution per worker can differ dramatically across sectors, which means that the same drop in value added might lead to different employment effects in different sectors.

Note: Employment data in Eurostat is missing for some sectors in the following countries: Estonia, Latvia, Lithuania, Luxembourg, Malta and Sweden. Therefore, the presented employment results for these countries will likely underestimate the true impact.

9. Appendix

9.1. Abbreviations

Table 11: Countries and ISO-3 Codes

Country Name	**Code (ISO-3)**	**Country Name**	**Code (ISO-3)**
Austria	AUT	Hungary	HUN
Belgium	BEL	Ireland	IRL
Bulgaria	BGR	Italy	ITA
Cyprus	CYP	Lithuania	LTU
Czech Republic	CZE	Luxembourg	LUX
Germany	DEU	Latvia	LVA
Denmark	DNK	Malta	MLT
Spain	ESP	Netherlands	NLD
Estonia	EST	Poland	POL
Finland	FIN	Portugal	PRT
France	FRA	Romania	ROU
United Kingdom	GBR	Slovakia	SVK
Greece	GRC	Slovenia	SVN
Croatia	HRV	Sweden	SWE

Table 12: Nace Rev. 2 Codes & Labels

Goods		Services	
Nace Rev.2	**Sector Legend (Short)**	**Nace Rev.2**	**Sector Legend (Short)**
A01	Live Animals	F	Construction
A02	Forestry	G45	Wholesale and retail trade
A03	Fishing	G46	Wholesale trade
B	Mining and quarrying	G47	Retail trade
C10-C12	Food Product	H49	Land & Pipeline transport
C13-C15	Textiles	H50	Water transport
C16	Wood and Cork	H51	Air transport
C17	Paper Products	H52	Warehousing
C18	Printing and Media	H53	Postal
C19	Petroleum Products	I	Accommodation & Food serv.
C20	Chemicals	J58	Publishing Act.
C21	Pharmaceutical	J59_J60	Media Production
C22	Rubber and Plastic	J61	Telecom
C23	Other Non-metallic mineral	J62_J63	Computer Programming, consultancy
C24	Basic Metals	K64	Financial Services
C25	Metal products	K65	Insurance
C26	Electronics and Computers	K66	Auxiliary Financial Serv.
C27	Electrical Equipment	L68	Real Estate
C28	Machinery & Equipment	M69_M70	Legal and Accounting
C29	Motor vehicles	M71	Architectural and engineering act.
C30	Transport equipment	M72	Scientific Research
C31_C32	Furniture & other manufac.	M73	Advertising and market research
C33	Installation of machinery	M74_M75	Other professional activities
D35	Electricity & Gas	N	Administrative and support act.
E36	Water Collection Activities	O84	Public admin and defence
E37-E39	Waste Collection Activities	P85	Education
		Q	Health
		R_S	Other services

9.2. Derivations

Equation (17) can be found as follows. From Equation (16), we find $\mathrm{d}va^{kz}$ as

$$\mathrm{d}va^{kz} = \underbrace{v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}L_{is}^{kz}\sum_{j=1}^{N}\mathrm{d}f_j^{is}}_{\text{final trade effect}} + \underbrace{v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}\mathrm{d}L_{is}^{kz}\sum_{j=1}^{N}f_j^{is}}_{\text{intermediate trade effect}} \tag{20}$$

Next, we apply the following rule to Equation (20): Differentiating $L^{-1}L = \mathbb{I}$ yields $L^{-1}\mathrm{d}L + \mathrm{d}L^{-1}L = 0$ from which it follows that $\mathrm{d}L = -L\mathrm{d}L^{-1}L$. Given that $L = [\mathbb{I} - A]^{-1}$, we have that $\mathrm{d}L^{-1} = -\mathrm{d}A$ and hence $\mathrm{d}L = L\mathrm{d}AL$, from which it is straightforward to obtain the individual elements $\mathrm{d}L_{is}^{kz}$. Hence, we obtain

$$\begin{aligned}
\mathrm{d}va^{kz} &= v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(1-\sigma_s)L_{is}^{kz}\sum_{j=1}^{N}\hat{\tau}_j^{is}f_j^{is} + v^{kz}\sum_{h=1}^{N}\sum_{r=1}^{S}\sum_{h'=1}^{N}\sum_{r'=1}^{S}(1-\sigma_r)L_{hr}^{kz}a_{h'r'}^{hr}\hat{\tau}_{h'}^{hr}\sum_{i=1}^{N}\sum_{s=1}^{S}L_{is}^{h'r'}\sum_{j=1}^{N}f_j^{is} \\
&= v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(1-\sigma_s)L_{is}^{kz}\sum_{j=1}^{N}\hat{\tau}_j^{is}f_j^{is} + v^{kz}\sum_{h=1}^{N}\sum_{r=1}^{S}\sum_{h'=1}^{N}\sum_{r'=1}^{S}(1-\sigma_r)L_{hr}^{kz}a_{h'r'}^{hr}y^{h'r'}\hat{\tau}_{h'}^{hr} \\
&= v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(1-\sigma_s)L_{is}^{kz}\sum_{j=1}^{N}\hat{\tau}_j^{is}f_j^{is} + v^{kz}\sum_{h=1}^{N}\sum_{r=1}^{S}\sum_{h'=1}^{N}\sum_{r'=1}^{S}(1-\sigma_r)L_{hr}^{kz}x_{h'r'}^{hr}\hat{\tau}_{h'}^{hr} \\
&= v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(1-\sigma_s)L_{is}^{kz}\sum_{j=1}^{N}\hat{\tau}_j^{is}f_j^{is} + v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}\sum_{j=1}^{N}\sum_{r=1}^{S}(1-\sigma_s)L_{is}^{kz}x_{jr}^{is}\hat{\tau}_j^{is} \\
&= -v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(\sigma_s-1)L_{is}^{kz}\sum_{j=1}^{N}\hat{\tau}_j^{is}\left\{f_j^{is}+\sum_{r=1}^{S}x_{jr}^{is}\right\} \\
&= -v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(\sigma_s-1)L_{is}^{kz}\sum_{j=1}^{N}\hat{\tau}_j^{is}e_j^{is}
\end{aligned}$$

where we defined $\hat{\tau}_j^{is} \equiv \frac{\mathrm{d}\tau_j^{is}}{\tau_j^{is}} - \frac{\mathrm{d}\Pi^{is}}{\Pi^{is}} - \frac{\mathrm{d}P_j^s}{P_j^s}$ as the proportionate change in τ_j^{is} net of the proportionate changes in the multilateral resistance (MR) terms.