# Exposing the Variety of Equilibria in Oligopolistic Electricity Markets with Strategic Storage

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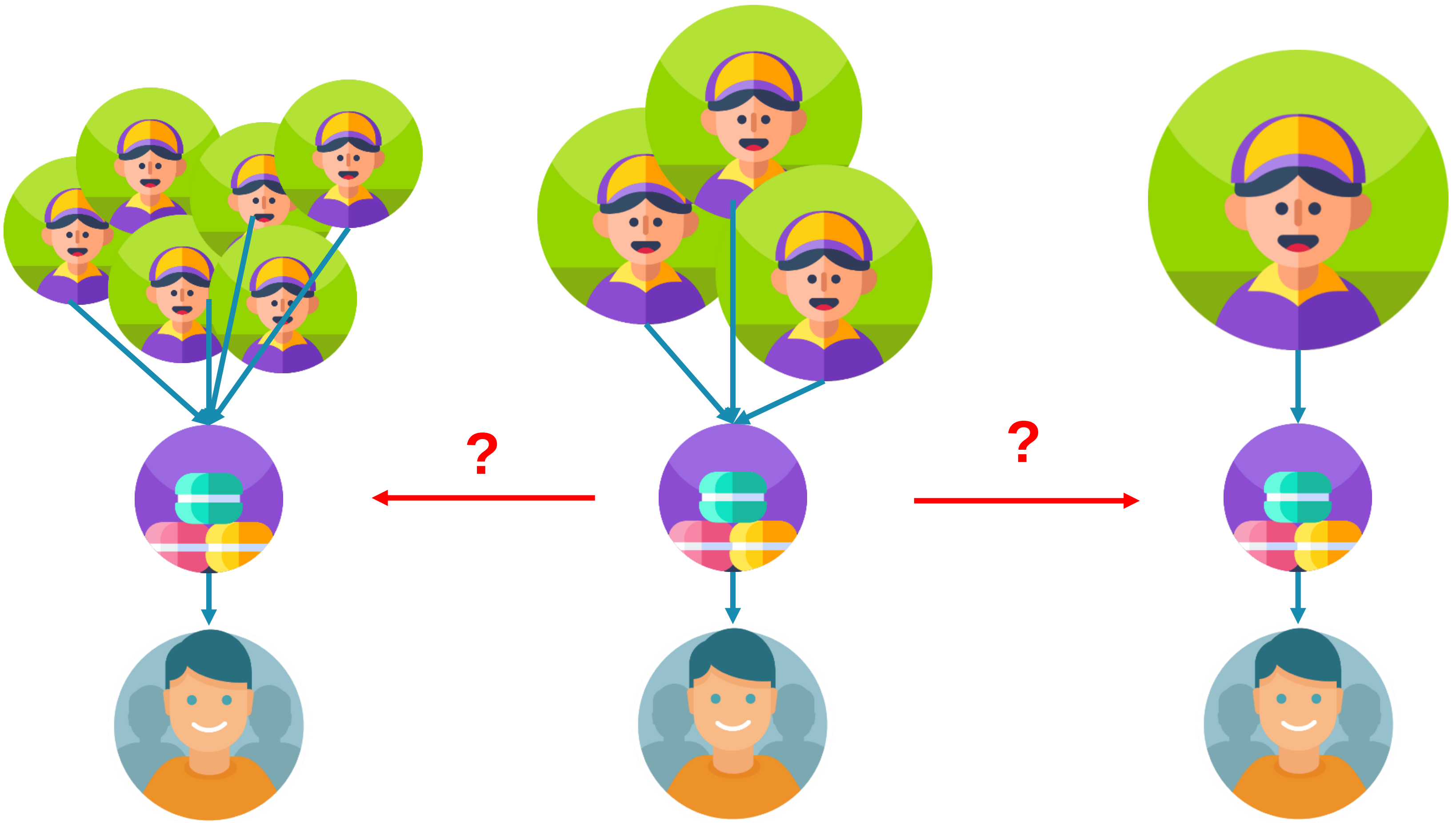
Mihaly Dolanyi[1,2], Kenneth Bruninx[1,2],
& Erik Delarue[1,2]

1: Division of Applied Mechanics & Energy Conversion, KU Leuven
2: EnergyVille, Thor Park 8310, Genk, Belgium

**mech.kuleuven.be/en/tme/research/
energy-systems-integration-modeling**
e-mail: mihaly.dolanyi@kuleuven.be

# Perfect competition, monopoly and oligopoly

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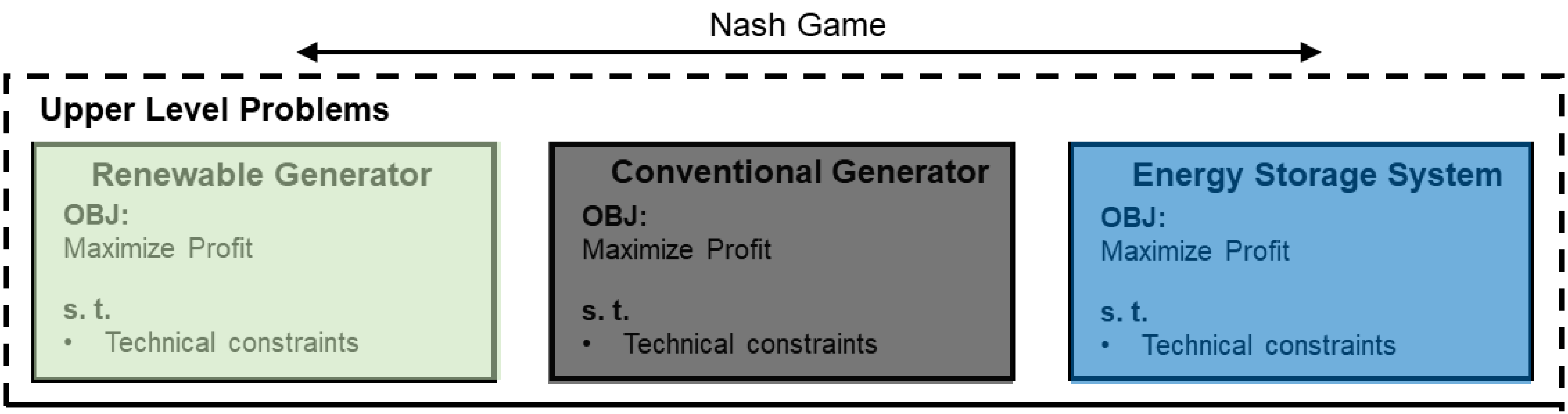
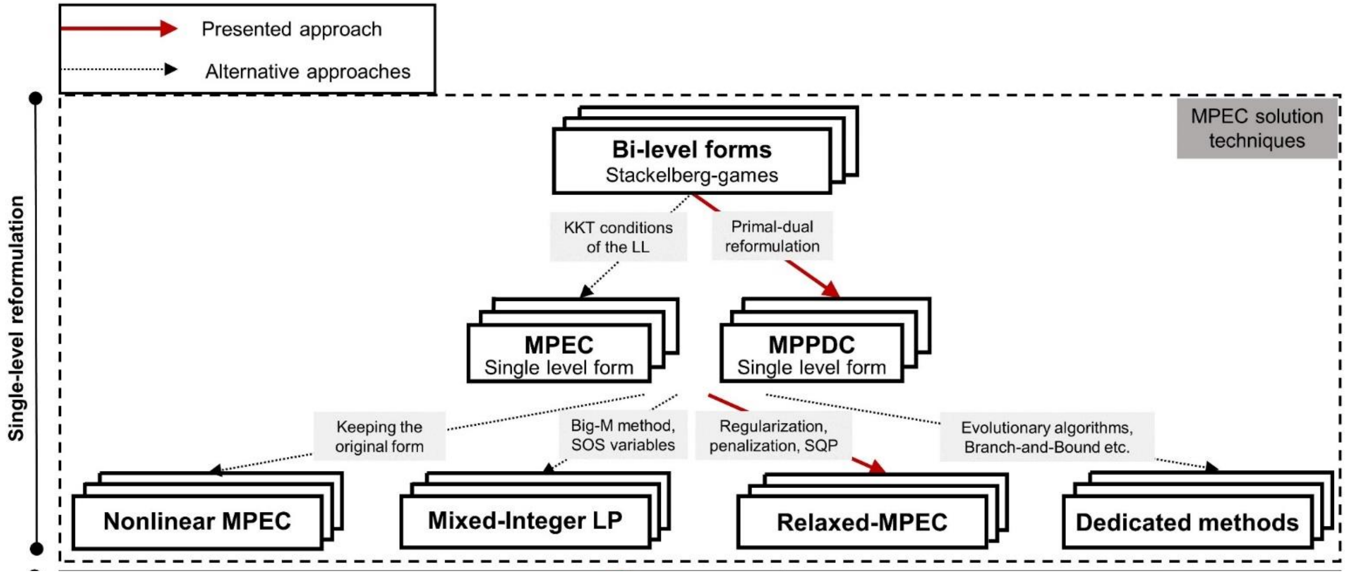
# Motivating example

## Multi-leader-common-follower game as a Generalized-Stackelberg-Nash (GSN) game



# Existing solution strategies



# Input data, parameters

- 8-hour market, hourly clearing
- 500 (MW) Wind, 550 (MW) PV
- ESS: 1600 (MWh), 400 (MW)
- Load: modified, scaled load profile of Belgium*
- 8 GENCO
- Solver: KNITRO
- Language: Julia
- ε1, ε2 were altered from 1 to 100 by 0.5
- Validation by Gauss-Seidel

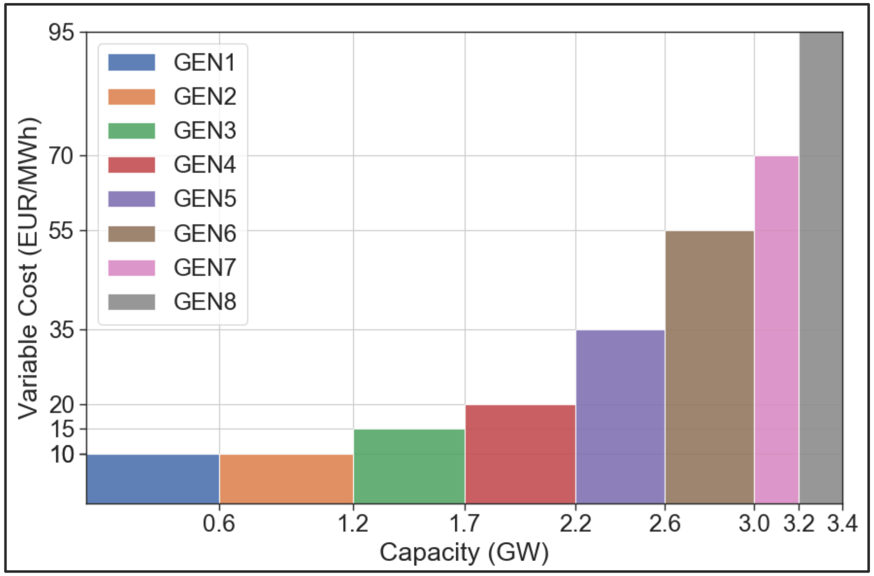



*Fig. 1: Marginal cost curve of the conventional generators.*

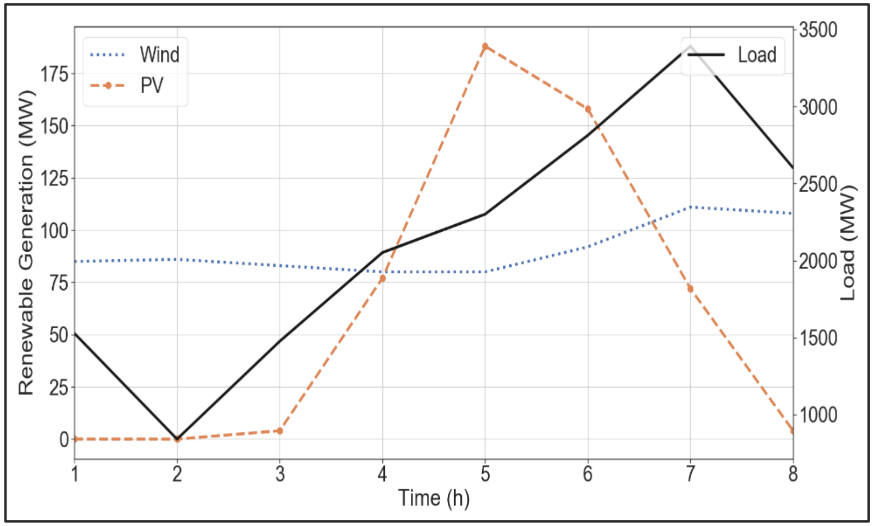



*Fig. 2: Wind, PV generation and load profiles.*

* *Elia NV, "Grid data," Available at http://www.elia.be/en/grid-data, 2018.*

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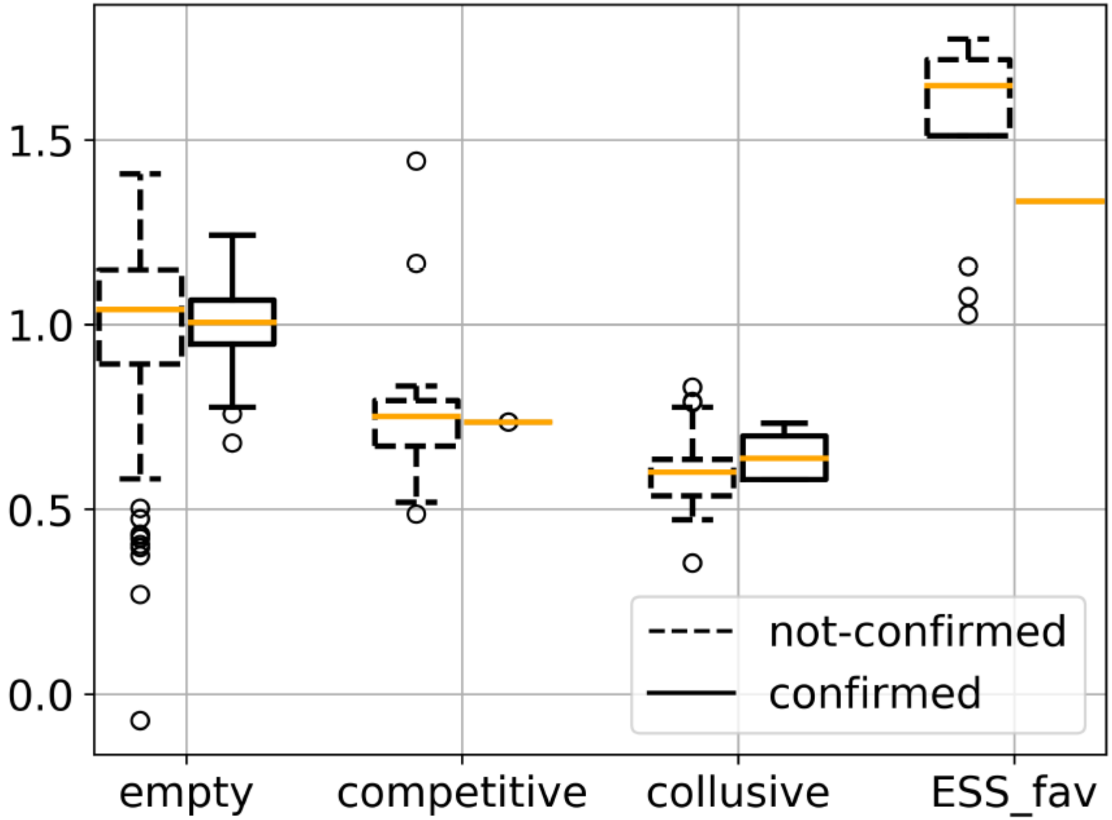
# Range of equilibria for ESS




*Normalized deviations of the ESS profits compared to the empty objective case*

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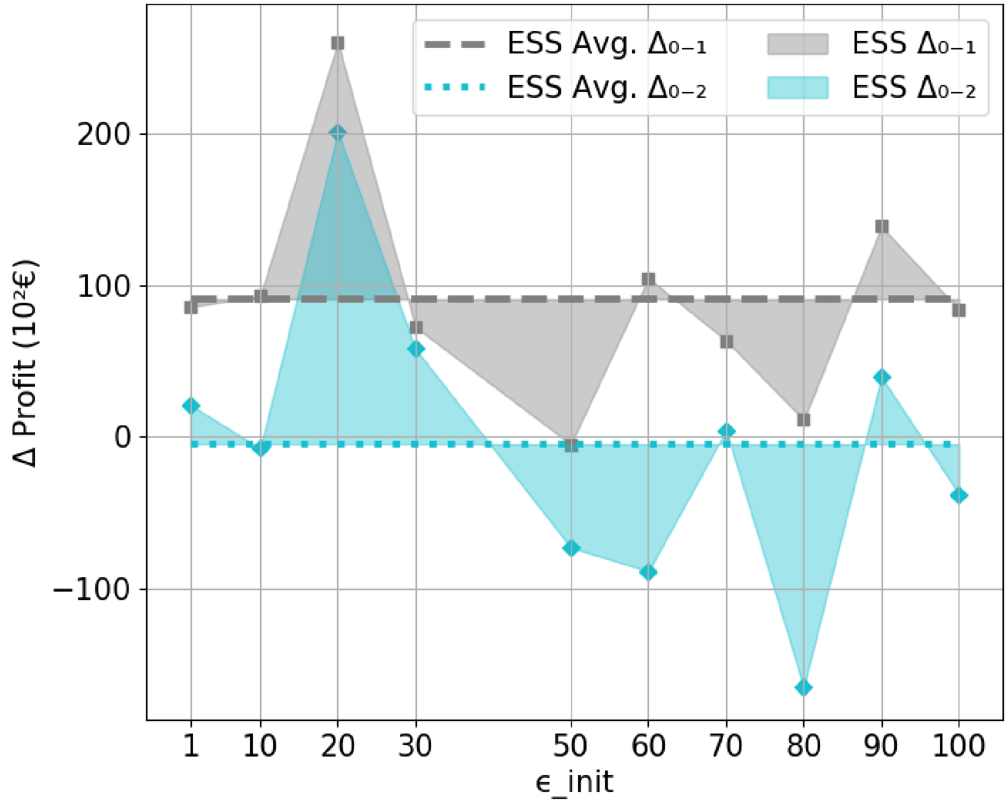
# Systematic trends

| | BASE | $1.5 \cdot CAP_{ESS}$ | $1.5 \cdot CAP_{RG}$ | BOTH |
|---|---|---|---|---|
| ESS | 273.2 / 82.2 (177.6) | 366.3 / 41.2 (266.3) | 272.6 / 24.8 (177.6) | 366.1 / 77.3 (266.3) |
| CG | 638.4 / 93.8 (325.0) | 547.3 / 57.7 (325.0) | 557.7 / 38.8 (325.0) | 451.8 / 115.9 (325.0) |
| RG | 306.5 / 18.8 (213.8) | 290.7 / 34.5 (213.8) | 448.8 / 30.5 (320.8) | 449.2 / 29.6 (320.8) |

*Social welfare, producer surplus and strategic profits in 4 different model settings. Attained value / std (perfect competition equivalent)*

# Case-by-case




*Differences in the ESS's profits between settings, on average (shown by the straight lines), on case-by-case basis (shown by the scatter plot)*

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# Conclusions

- Numerical solutions to an EPEC may exhibit significant variations, remaining present in altered model settings
- Attempts to trigger many of them should be of interest for the modeler
- When studying the average outcomes, the observed trends are more systematic
- Observation made on a case-by-case basis can be fairly misleading

Thank you for your attention!

Questions?

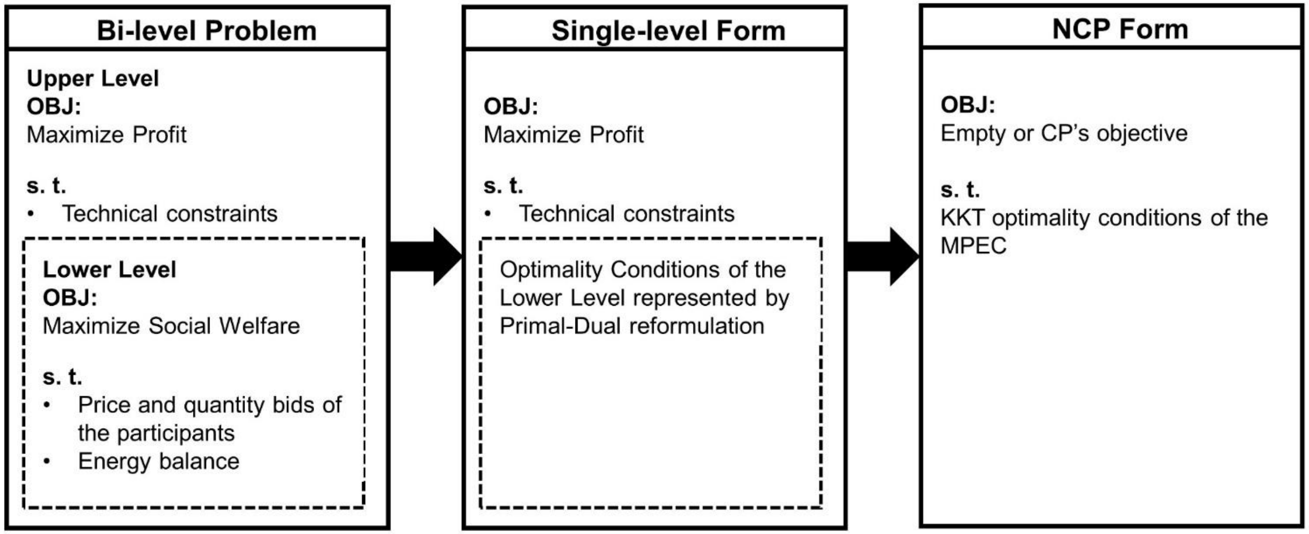
# Back-Up

# Transformation steps

**Central Planner' objective** (*Ruiz et al 2012*)

- Competitive equilibria: Maximize social welfare
- Collusive equilibria: Maximize total profits of all competing firms
- Favor ESS: Maximize storage owner' profit

*Relaxation*



# Contribution

**Sequentially co-regularized NLP formulation for simultaneously solving multi-leader games**

- Efficiently solved for small to middle scale problems
- Using the off-the-shelf non-linear solvers
- Omits the need of using linearization techniques: e.g. big M-method* or parametrization techniques **

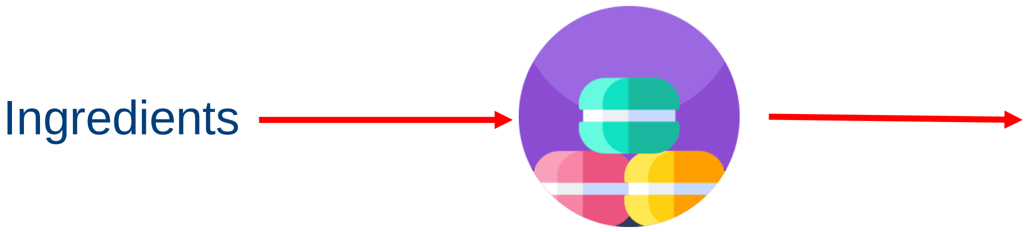
**Exploration of the attainable range of equilibria**

- Altering the regularization parameters and reporting a large set of outcomes
- Studying a range of equilibria triggered by various objectives of an imaginary social planner

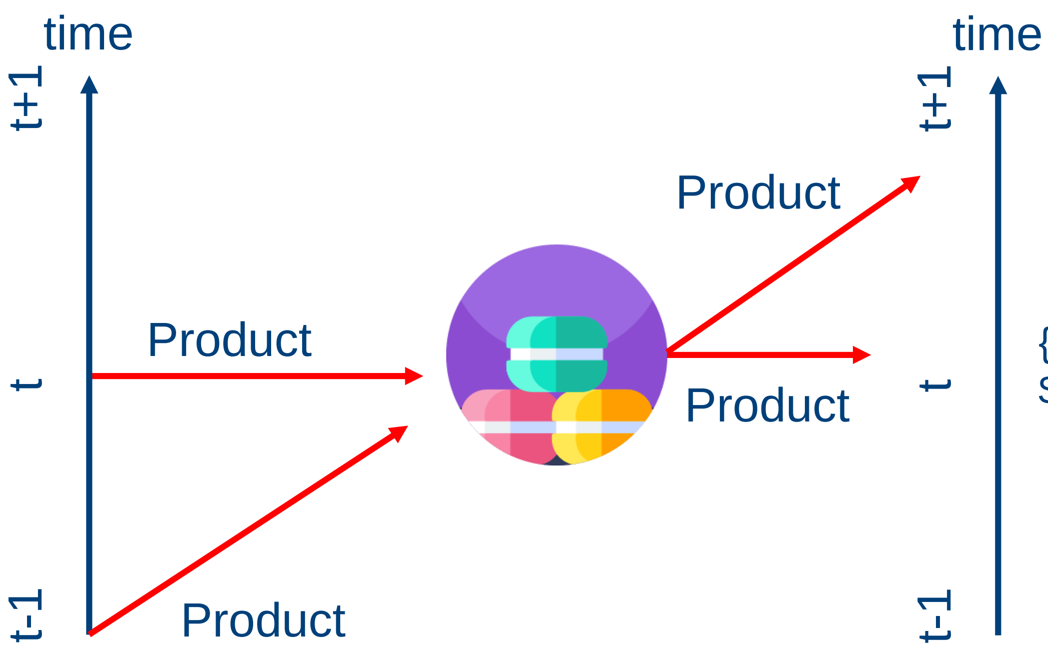
** S. Pineda and J. M. Morales, "Solving Linear Bilevel Problems Using Big-Ms: Not All That Glitters Is Gold,"*

*** C. Ruiz, A. J. Conejo, and Y. Smeers, "Equilibria in an Oligopolistic Electricity Pool With Stepwise Offer Curves," IEEE Transactions on Power Systems, may 2012.*

# Conventional generation vs. storage




{Price = f($ingredients, $operational, $profit)}




{Price = f($product, $operational, $profit, $opportunity, $risk)}

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# Mathematical Formulation: 1st-phase regularization

$$\max_{x^j,y} \mathcal{F}^j(x^j, y) = \lambda^T y + {c^j}^T x^j$$

subject to

$$A^j x^j + By \leq b^j$$

$$x^j \geq 0$$

$$y \in \arg\min_y \left\{ e^T y : C^j x^j + C^{-j} x^{-j} + Dy \geq a,\ y \geq 0 \right\}$$

$$\max_{x^j} \mathcal{F}^j(x^j, \lambda)$$

subject to

$$A^j x^j + By \leq b^j \qquad (\kappa^j)$$

$$x^j \geq 0 \qquad (\omega^j)$$

$$C^j x^j + C^{-j} x^{-j} + Dy \geq a \qquad (\xi^j)$$

$$y \geq 0 \qquad (\zeta^j)$$

$$e^T y - a^T \lambda + \boxed{(C^j x^j)^T \lambda} + (C^{-j} x^{-j})^T \lambda = 0 \qquad (\sigma^j)$$

$$D^T \lambda \leq e \qquad (\pi^j)$$

$$\lambda \geq 0 \qquad (\delta^j)$$

$$\left(C^j x^j\right)^T \lambda - s^j \leq \epsilon_1$$

$$s^j - \left(C^j x^j\right)^T \lambda \leq \epsilon_1$$

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# Mathematical Formulation: 2nd-phase regularization

$$\nabla_{x^j}\mathcal{L}^j_{UL}\left(x^j,\kappa^j,\omega^j,\zeta^j,\sigma^j,\pi^j,\delta^j,y,\lambda\right)=0$$
$$\nabla_{s^j}\mathcal{L}^j_{UL}\left(x^j,\kappa^j,\omega^j,\zeta^j,\sigma^j,\pi^j,\delta^j,y,\lambda\right)=0$$
$$\nabla_{y}\mathcal{L}^j_{UL}\left(x^j,\kappa^j,\omega^j,\zeta^j,\sigma^j,\pi^j,\delta^j,y,\lambda\right)=0$$
$$\nabla_{\lambda}\mathcal{L}^j_{UL}\left(x^j,\kappa^j,\omega^j,\zeta^j,\sigma^j,\pi^j,\delta^j,y,\lambda\right)=0$$
$$\kappa^j\cdot\left(b^j-A^jx^j-By\right)=0$$
$$\omega^j\cdot x^j=0$$
$$\zeta^j\cdot y=0$$
$$\delta^j\cdot\lambda=0$$
$$\xi^j\cdot\left(C^jx^j+C^{-j}x^{-j}+Dy-a\right)=0$$
$$\left[\epsilon_1-\left(C^jx^j\right)^T\lambda+s^j\right]\cdot\underline{\gamma}^j=0$$
$$\left[\epsilon_1+\left(C^jx^j\right)^T\lambda-s^j\right]\cdot\overline{\gamma}^j=0$$
$$\left[\epsilon_1-\left(C^jx^j\right)^T\lambda+s^j\right]\geq 0$$
$$\left[\epsilon_1+\left(C^jx^j\right)^T\lambda-s^j\right]\geq 0$$
$$\left(b^j-Ax^j-By\right)\geq 0$$
$$\left(C^jx^j+C^{-j}x^{-j}+Dy-a\right)\geq 0$$
$$D^T\lambda\leq e$$
$$e^Ty-a^T\lambda+\left(C^jx^j\right)^T\lambda+\left(C^{-j}x^{-j}\right)^T\lambda=0$$
$$\lambda,x^j,y,\kappa^j,\omega^j,\zeta^j,\xi^j,\sigma^j,\delta^j,\overline{\gamma}^j,\underline{\gamma}^j\geq 0$$

$\longrightarrow$

$$\kappa^j\cdot\left(b^j-A^jx^j-By\right)\leq\epsilon_2$$
$$\xi^j\cdot\left(C^jx^j+C^{-j}x^{-j}+Dy-a\right)\leq\epsilon_2$$
$$\omega^j\cdot x^j\leq\epsilon_2$$
$$\delta^j\cdot\lambda\leq\epsilon_2$$
$$\zeta^j\cdot y\leq\epsilon_2$$
$$\left[\epsilon_1-\left(C^jx^j\right)^T\lambda+s^j\right]\cdot\underline{\gamma}^j\leq\epsilon_2$$
$$\left[\epsilon_1+\left(C^jx^j\right)^T\lambda-s^j\right]\cdot\overline{\gamma}^j\leq\epsilon_2$$

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# Future work

- Developing techniques to validate the Nash-equilibrium.
- Exploring the solution space more exhaustively through the adjustment of the social planners objective.
- Studying different volumes of strategic generation and various settings w.r.t the ownership structures.
- Using a more detailed representation for conventional generation (e.g. including ramping constraints).

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# Mathematical Formulation: Agents

$$\max_{c^{DCH}_{l,t}, c^{CH}_{l,t}, \overline{P}^{DCH}_{l,t}, \overline{P}^{CH}_{l,t}} \sum_{t \in T} \lambda_t \cdot (P^{DCH}_{l,t} - P^{CH}_{l,t}) \qquad (1)$$

$$p^{floor} \leq c^{CH}_{l,t} \leq p^{cap} \qquad \forall t \qquad (2)$$

$$p^{floor} \leq c^{DCH}_{l,t} \leq p^{cap} \qquad \forall t \qquad (3)$$

$$0 \leq \overline{P}^{CH}_{l,t} \leq P^{ESS,max}_{l} \qquad \forall t \qquad (4)$$

$$0 \leq \overline{P}^{DCH}_{l,t} \leq P^{ESS,max}_{l} \qquad \forall t \qquad (5)$$

$$0 \leq SoC_{l,t} \leq E^{ESS,max}_{l} \qquad \forall t \qquad (6)$$

$$SoC_{l,t} = SoC_{l,t-1} + \eta^{CH}_{l} E^{CH}_{l,t} - E^{DCH}_{l,t} / \eta^{DCH}_{l} \qquad \forall t \qquad (7)$$

## ESS owner' problem

- Profit of the ESS based on price arbitrage (1)
- Respecting price floor/cap of the market (2-3)
- Technical limits of charging/discharging power (4-6)
- Temporal variation of the stored energy (7)

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# Mathematical Formulation: Agents

$$\max_{c_{i,t}^G, \overline{P}_{i,t}^G} \sum_{t \in T} \lambda_t \cdot P_{i,t}^G - OPEX_i^G \sum_t P_{i,t}^G \quad (8)$$

$$p^{floor} \leq c_{i,t}^G \leq p^{cap} \quad \forall t \quad (9)$$

$$\overline{P}_{i,t}^G = P_i^{G,max} \quad \forall t \quad (10)$$

**Conventional generator' (GENCO) problem**

- Revenue from selling electricity, corrected by operational costs (8)
- Respecting price floor/cap of the market (9)
- Enforces only price-bidding (10)

# Mathematical Formulation: Agents

$$\max_{c^R_{k,t},\overline{P}^R_{k,t}} \sum_{t\in T} \lambda_t \cdot P^R_{k,t} \quad (11)$$

$$c^R_{k,t} = 0 \quad \forall t \quad (12)$$

$$0 \leq \overline{P}^R_{k,t} \leq P^{R,forecast}_k \quad \forall t \quad (13)$$

**Renewable generator' problem**

- Revenue from selling electricity, assuming zero marginal cost (11)
- Respecting price floor/cap of the market (12)
- Enforces only quantity-bidding (13)

# Mathematical Formulation: Market

$$\max_{P^G_{i,t}, P^R_{k,t}, P^{CH}_{l,t}, P^{DCH}_{l,t}, P^D_t} \sum_{t\in T}(P^D_t c^D_t) - \sum_{i\in I, t\in T}(P^G_{i,t} c^G_{i,t}) - \sum_{l\in L, t\in T}(P^{DCH}_{l,t} c^{DCH}_{l,t} - P^{CH}_{l,t} c^{CH}_{l,t}) \quad (14)$$

subject to:

$$P^D_t + \sum_{l\in L}(P^{CH}_{l,t} - P^{DCH}_{l,t}) - \sum_{k\in K} P^R_{k,t} - \sum_{i\in I} P^G_{i,t} = 0 \qquad \forall t \quad (\lambda_t) \qquad (15)$$

$$0 \le P^G_{i,t} \le \overline{P}^G_{i,t} \qquad \forall i, \forall t \qquad (16)$$

$$0 \le P^R_{k,t} \le \overline{P}^R_{k,t} \qquad \forall k, \forall t \qquad (17)$$

$$0 \le P^{CH}_{l,t} \le \overline{P}^{CH}_{l,t} \qquad \forall l, \forall t \qquad (18)$$

$$0 \le P^{DCH}_{l,t} \le \overline{P}^{DCH}_{l,t} \qquad \forall l, \forall t \qquad (19)$$

$$0 \le P^D_t \le \overline{D}_t \qquad \forall t \qquad (20)$$

## Market Clearing (MC)

- Social welfare to be maximized (14)
- Energy balance constraint (15)
- The dispatched quantities are non-negative and limited to the quantity bids of the corresponding agents (16-20)

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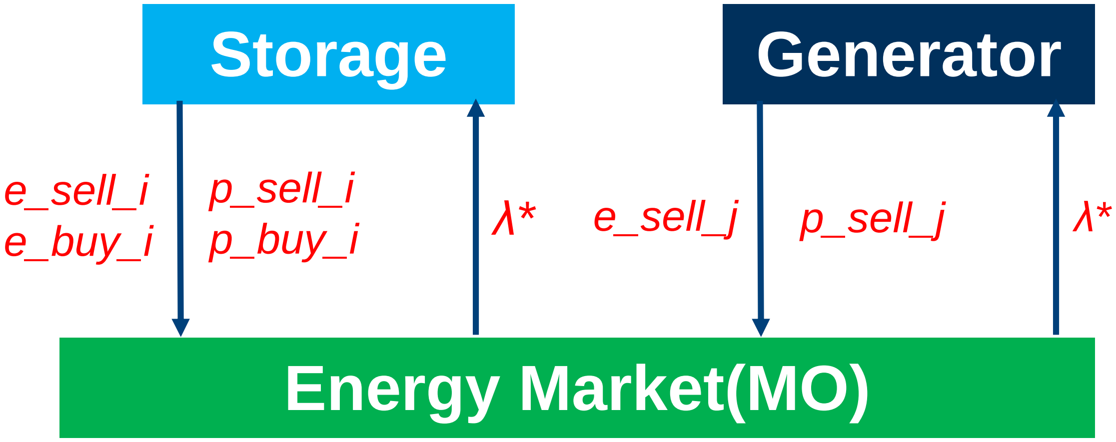
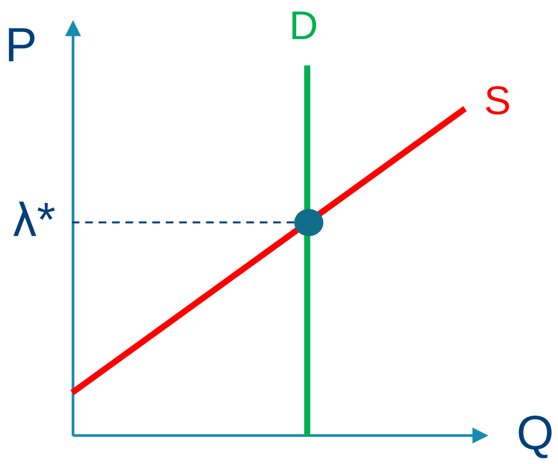
# Context

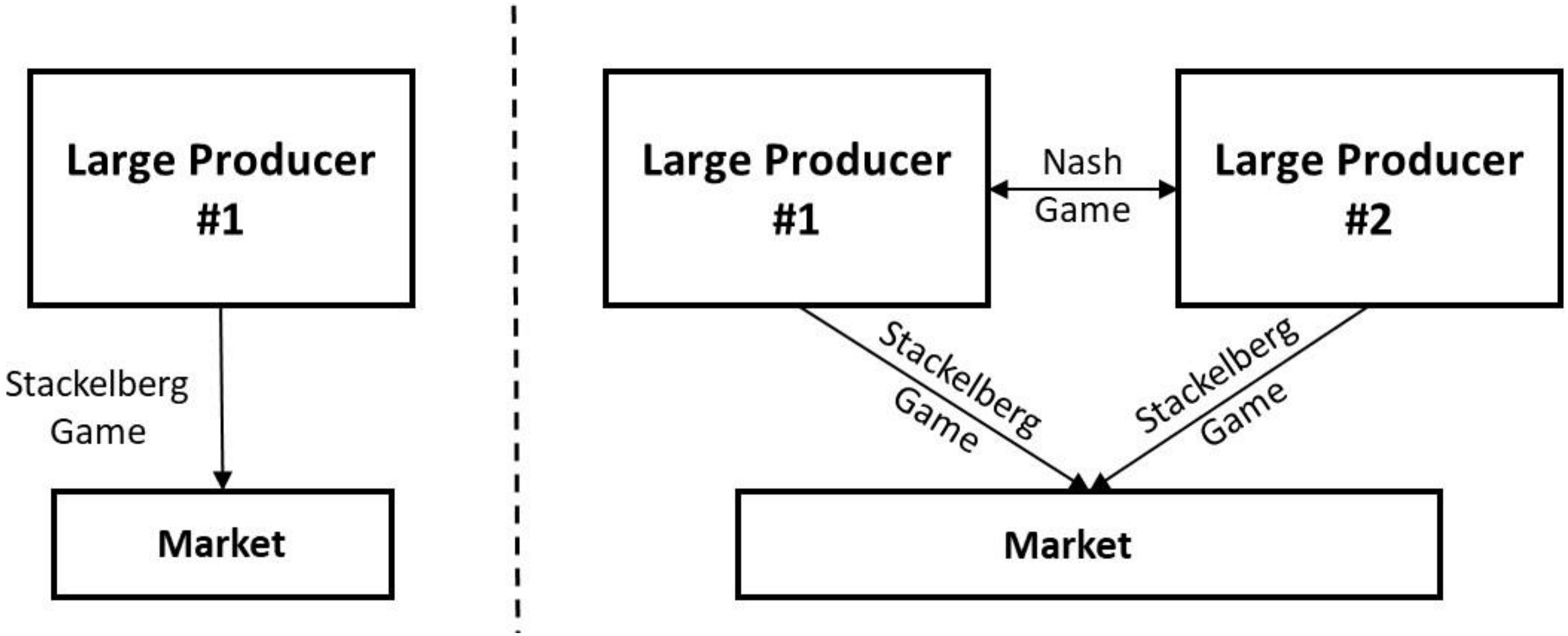


Storage
Generator
e_sell_i
e_buy_i
p_sell_i
p_buy_i
λ*
e_sell_j
p_sell_j
λ*
Energy Market(MO)

P
D
S
λ*
Q

# MPEC vs EPEC



Large Producer #1
Stackelberg Game
Market
Large Producer #1
Nash Game
Large Producer #2
Stackelberg Game
Stackelberg Game
Market

# Primal-Dual reformulation

**Primal Problem**

$$\underset{x}{\text{Minimize}} \quad c^T x$$

s.t.

$$\boxed{\begin{aligned} Ax &\geq b \\ x &\geq 0 \end{aligned}}$$

**Dual Problem**

$$\underset{\mu}{\text{Maximize}} \quad b^T \mu$$

s.t.

$$\boxed{\begin{aligned} A^T \mu &\leq c \\ \mu &\geq 0 \end{aligned}}$$

**Strong Duality Theorem**

$$\boxed{c^T x = b^T \mu}$$

# Is the solution a NE?

A profile of strategies $a \in A$ is a *pure strategy Nash equilibrium* if $a_i$ is a best reply to $a_{-i}$ for each $i$. That is, $a$ is a Nash equilibrium if

$$u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i})$$

*Source: A Brief Introduction to the Basics of Game Theory Matthew O. Jackson, Stanford University*

# Duals of the shared constraints

Can be interpreted as implicit auction of the scarce resource, constraining factor (*Hauppmann and Egerer 2015*)

- If the duals of the common market differ across the UL agents -> GNE -> non-square system ->more difficult to solve/much more interesting
- If they are the same -> NE (facilitates the solution but may not exist)
- Assume an endogenous ratio between the multipliers (*Oggioni et al 2012*)

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