Can we Count on Early Numerical Abilities for Early Probabilistic Reasoning Abilities?

Anne-Sophie Supply [a]*, Wim Van Dooren [a], and Patrick Onghena [a]

[a] *KU Leuven, Leuven, Belgium*

Correspondence concerning this article should be addressed to Anne-Sophie Supply

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Previous research has shown that early numerical abilities are predictive of later mathematical achievement. In line with these previous studies, we investigated whether early numerical abilities are also associated with later probabilistic reasoning abilities. In the present study, we examined children's numerical abilities in the second grade of preschool and their probabilistic reasoning abilities one and two years later. On the one hand, our results indicate that early numerical abilities assessed in the second grade of preschool predict the use of erroneous solving strategies to compare probabilities and create equal probabilities in the third grade of preschool. On the other hand, our results also indicate that the same early numerical abilities predict the use of more advanced or correct strategies when children are in the first grade of primary school. We discuss these seemingly contradicting findings in light of current research on probabilistic reasoning abilities.

Keywords: Early numerical ability; probabilistic reasoning; preschool; primary school

Introduction

In many situations, probabilistic reasoning of children and adults is characterized by misconceptions. In the 1970's, Fischbein pointed towards instruction in search for an explanation for the occurrence of these misconceptions (Fischbein & Gazit, 1984). Fischbein (1975) commented upon education, stating: "Whatever does not conform to strict determinism, whatever is associated with uncertainty, surprise, or randomness is seen as being outside the possibility of a consistent, rational, scientific, explanation" (pp. 124). According to several authors, this focus on causal and deterministic explanations in schools still holds and is also manifested in school mathematics, *possibly impeding children's idea of chance* (Falk et al., 2012; Meletiou-Mavrotheris, 2007). Stohl (2005) explained that many mathematics teachers hold a computational orientation towards mathematics and are often described as having deterministic views. Nonetheless, English and Watson (2016) commented that children in the 21st century are part of a "chance-laden society" (p. 29) and that this should be taken into account during *their first mathematical experiences* in schools.

These first mathematical experiences take place at the preschool level. They seem to be strongly oriented towards stimulating children's early numerical abilities, involving activities such as counting, ranking, and comparing quantities. Such activities are deterministic in nature: two and two sums to four, always. Activities stimulating children's early numerical abilities (i.e., their informal knowledge of number and arithmetic) are assumed to prepare them for later formal mathematics education. Indeed, research has found that early numerical abilities of kindergartners and young primary school children are predictive of their later mathematical performance (De Smedt et al., 2009; Duncan et al., 2007; Nguyen et al., 2016; Passolunghi & Lanfranchi, 2011). Thus, such findings might support curriculum designers to explain the strong focus that is usually put on the development of early numerical abilities in preschool (Nguyen et al., 2016).

Nonetheless, Cirino et al. (2016) pointed out that few studies have focused on the effect of early mathematical abilities on *more advanced mathematical abilities.* Mathematical achievement is often operationalized as performance on general tests which assess a wide range of abilities that are included in primary school curricula (e.g., Jordan et al., 2010; Nguyen et al., 2016; Sasanguie et al., 2013). Other studies operationalized mathematical achievement as performance on tests that focus on one specific component, for example exact arithmetic, which receives a great deal of attention in the primary school mathematics curricula (e.g., Bartelet et al., 2014; Lyons et al., 2014; Sasanguie et al., 2013). Researchers in the aforementioned studies might have simply defined a high mathematical achiever as a child with good grades for (certain topics in) mathematics in school. However, Dede (2010) inspires to be vigilant to more complex abilities that are, today, often not included in primary school curricula but may be (more) helpful in our modern society.

In the present study, we focused particularly on probabilistic reasoning as one of those more complex abilities. Effective probabilistic reasoning helps us in our daily lives (Gal, 2005). Given the strong focus on numerical abilities, it seems relevant to understand how these numerical abilities affect probabilistic reasoning skills before formal instruction. More specifically, we aimed to investigate the effect of early numerical abilities on children's abilities to recognize certainty, compare probabilities, and create equal probabilities. In the literature review, we explained the reasons for this focus.

Literature Review

Probability as a topic is often lacking in countries' primary school curricula (e.g., United States, Belgium, United Kingdom, France; see Langrall, 2018). Langrall (2018) suggested that one of the possible reasons for the absence of probability in many primary school curricula is because probability is regarded as too complex to be introduced in the primary grades. Pratt (2000, p. 602) also spoke of a "sort of folklore, among teachers, that probability is extremely difficult to learn." In the first section below, we described the literature on the development of probabilistic reasoning abilities with a focus on research that is of particular interest for the present study. In a second section, we elaborated on the literature that fuelled our interest to investigate the role of early numerical abilities for probabilistic reasoning abilities.

Probabilistic Reasoning Abilities in Children

Probabilistic reasoning abilities involve the understanding and integration of a range of sub-concepts related to probability (Bryant and Nunes, 2012). Bryant and Nunes (2012) reviewed literature on the development of probabilistic reasoning and identified four cognitive demands for the understanding of probability: understanding randomness, working out the sample space, comparing and quantifying probabilities, and understanding correlation. These cognitive demands are not to be seen as unrelated. For example, to calculate the probability of an event, an understanding of the sample space is also required. As another example, when you correctly identify the event with the highest probability of occurring out of two possible events, you need to know about randomness to understand why it is still possible that the other event could occur. Moreover, each demand requires several insights. For example, in the chapter "quantifying probabilities", Bryant and Nunes (2012) described how children solve probability-related questions (e.g., calculate single probabilities, compare two (or more) probabilities, calculate conditional probabilities). It was beyond our means to investigate the entire range of sub-concepts related to probability. In the present study, we chose to assess three abilities, as they have been studied before in young children: (a) recognizing (un)certainty, (b) comparing probabilities, and (c) creating equal probabilities. In describing the literature on the development of probabilistic reasoning, we focused on findings that are especially relevant for these abilities of interest in this study.

Piaget and Inhelder (1951/1975) were the first to intricately study the development of probabilistic reasoning abilities or what they called "the idea of chance" in children. They conducted several interviews with children and concluded that children's notions about chance can be classified into three developmental phases: preoperational (4 to 7 years); concrete operational (8 to 11 years); and formal operational (from 11 years). They, for example, found that children in the preoperational stage would be unable to grasp the difference between certain and uncertain events. Children in the concrete operational stage would be able to grasp this distinction, but would be unable to effectively use proportional information to compare probabilities. In summary, Piaget and Inhelder concluded that formal operations, which are generally not obtained before the age of 11, are required to properly understand and evaluate probabilities. Their findings might have led to the idea that instruction about probability is only useful for children starting from the age of 11.

However, many researchers criticized drawing such strong conclusions from the work of Piaget and Inhelder (1951/1975). First, the verbally high-demanding tasks used by Piaget and Inhelder were often questioned. Researchers have come to the understanding that "a complete understanding of the concept of probability" involves the understanding of several sub-concepts (e.g., sample space, combinatorics, probability comparison, conditional probability), and that such an understanding can also become apparent in children's behaviour in concrete situations, rather than (only) in their verbalisations. Nevertheless, the work of Piaget and Inhelder, paved the way for an increase in research on both teaching and learning probability. Later research is characterized by studies with great diversity in conceptual topics related to probability (e.g., randomness, sample space, language of chance, independence) and studies with students from different age groups (e.g., college, high school students, elementary students; Falk et al., 2012; Jones & Thornton, 2005). Depending on the sub-concept, research has shown that children might already show meaningful behavior in probabilistic situations *before the age of 11*. For example, Yost et al. (1962) and Davies (1965) suggested that even children as young as 4 and 5 years old are able to make the correct choice in *non-verbal* probabilistic comparison situations and thus concluded that even preoperational children have some understanding of probability. Falk (1980) also observed that when children had to compare probabilities which required ratio comparison, they tended to perform above chance level from the age of 6. Also more recent studies challenged conclusions of Piaget and Inhelder (1951/1975). Denison and Xu (2014) suggested that even preverbal infants are able to use proportional information to make decisions in probabilistic comparison situations. Unfortunately, none of these later studies seemed as influential or were cited as often as the initial work of Piaget and Inhelder.

Second, Fischbein (1999) claimed that in the work of Piaget and Inhelder, there was not enough consideration for the role of instruction. This critique is twofold. First, Fischbein argued that the observation that children are unable to solve a certain problem does not mean that these children are not able to acquire this ability by instruction. Kipman et al. (2019) conducted an intervention that, among other outcomes, targeted the comparison of probabilities and the understanding of probabilistic terms such as "certain" or "possible" in 6- to 12-year old children. They found the intervention to be effective for all participants and noted the largest improvement in the youngest participants, which supports Fischbein's view. Such findings bring into question the absence of probability in many primary school curricula. Second, and more importantly for the present study, Fischbein also criticized Piaget for not taking into account the effect of children's prior knowledge.

Probabilistic Reasoning Abilities and Early Numerical Abilities

Overall, based on the above literature, there seems to be support for the idea that young children may already be able to reason probabilistically in some situations. These findings together with the ideas of Fischbein (1999), raised the question of how young children's prior knowledge today predicts children's probabilistic reasoning abilities. Regarding prior knowledge, it seemed relevant to focus on numerical abilities for several reasons. First, the stimulation of early numerical abilities is prevalent in school. Second, early numerical abilities are shown to be predictive for later mathematical achievement, but this is not yet investigated for probabilistic reasoning. Third, when children are introduced to probability later in school, their probabilistic reasoning abilities would be evaluated based on their capacity to use the right formula *to calculate* the probability of an event. For this calculation, children rely on their numerical abilities. Or, as Laplace (1816, p. 220) puts it "The theory of probabilities is basically just common sense reduced to *calculus*". Lastly, it might also be of interest to investigate how children's early numerical abilities are predictive of their probabilistic reasoning abilities *before* formal instruction in probability starts, assuming that these young children are not actually calculating the probabilities of events they are confronted with.

Little is known about the relationship between numerical abilities and probabilistic reasoning abilities in general or the specific abilities investigated in the present study. Interestingly, several researchers have suggested that counting abilities might foster the use of erroneous counting strategies in probabilistic comparison situations, thus better counting abilities might impede rather than enforce children's judgement (Denison & Xu, 2014; Fontanari et al., 2014). However, this hypothesis has not been explicitly investigated so far. In one study, Ruggeri et al. (2018) investigated the impact of number sense and the Approximate Number System (assessed by the Panamath test) on the performance of 7- and 10- year olds and adults on a probabilistic binary choice task. In binary choice tasks, participants have to choose the best of two sets, each containing desired and undesired elements, to blindly draw from (Falk et al., 2012). Ruggeri et al. (2018) found positive correlations between the numerical and probabilistic reasoning abilities across age groups and within the youngest age group.

Notably, Ruggeri et al. (2018) only assessed children's ability to compare probabilities. In their literature review, Bryant and Nunes (2012) described probability as a complex concept for which we have to fall back on our understanding of different sub-concepts: randomness, sample space, quantification and comparison of probabilities, and correlation. Assessing the full understanding of probability in one study might be a utopian ideal. Nonetheless, findings of Ruggeri et al. (2018) raised the question whether early numerical abilities relate to all aspects of probability in the same way. For example, quantification and comparison of probabilities is directly related to children's proportional reasoning abilities, while this relation is less prominent for other aspects of probability (Bryant & Nunes, 2012).

Given the limited number of studies that are available on numerical abilities in relation to probabilistic reasoning abilities, it may be useful to dwell on similar studies from mathematical domains that are related to probability. For example, some studies suggest that proportional reasoning and numerical abilities are related. Cirino et al. (2016) showed that numerical abilities positively predict proportional reasoning abilities. Primi et al. (2017) suggested that difficulties with rational number concepts (i.e., concepts involving fractions, decimals, and percentages) could be one of the possible reasons for students struggling with probability. Many studies have already shown the associations between numerical abilities and fraction knowledge (see for example Bailey et al., 2014; Steffe & Olive, 2010). Given these findings and given that early numerical abilities have been shown to predict later math achievement more generally, correlations between early numerical abilities and the ability to compare probabilities can be expected.

However, probability is more than just calculus and specifying specific hypotheses about the relation between early numerical abilities and components that differentiate probabilistic reasoning from other forms of mathematical reasoning seems more challenging. Take the probabilistic notions of certainty and uncertainty. The notion of certain events is described by Fischbein and Gazit (1984, p.13) as "one of the basic concepts of the theory of probability." Uncertainty is inherent to random situations and chance events. Perhaps, the ability to distinguish certain from uncertain events is independent of children's early numerical abilities. Perhaps, to recognize certainty and uncertainty, understanding randomness is more important than the ability to quantify and compare probabilities. For example, imagine a child who understands that for uncertainty or randomness, several possible events are required, but who also cannot use proportional reasoning to compare probabilities. This child might succeed in the recognition of a certain event when comparing it to an uncertain one, but fail when comparing the probabilities of two uncertain events.

Moreover, while many studies explored the association between early numerical competence and later mathematical performance, studies that explored the association between numerical abilities and probabilistic reasoning abilities such as the study of Ruggeri et al. (2018) could not be found in other (educational) contexts and age groups. Indeed, the only study that explored the association between numerical abilities and probabilistic reasoning abilities focused on just one time point. Moreover, the study by Ruggeri et al. (2018) had two specific features that might impede its interpretation in terms of an association between children's numerical abilities and children's ability to compare probabilities. First, their binary choice task contained 75% so-called "congruent trials", and 25% "incongruent trials". In congruent trials (e.g., set A: four desired and five undesired elements and set B: three desired and seven undesired elements), participants would be able to correctly identify the best set to blindly draw from by choosing the set containing the highest absolute number of desired elements, regardless of the number of undesired elements. In incongruent trials (e.g., set A: two desired and three undesired elements and set B: three desired and five undesired elements), this absolute number heuristic (for desired elements) would lead participants to the non-optimal set. In this example, they would have to take both the desired and undesired elements into account to identify the best set. Falk et al. (2012) showed that the type of trial (i.e., congruent vs. incongruent) plays a major role in children's performances on binary choice tasks and that younger children are more prone to the absolute number heuristic than older children. Despite Ruggeri et al. (2018) including both types of trials, they did not take the type of trial into account in their analyses to investigate the correlation between performance on the binary choice task and children's numerical skills. Nonetheless, it has been suggested that counting might promote the use of the absolute number heuristic, a popular strategy in young children that would lead to failure on incongruent items but to success in congruent items (Denison & Xu, 2014; Falk et al, 2012; Fontanari et al., 2014). It therefore could be that the positive correlations between numerical abilities and the binary choice task found in Ruggeri et al. (2018) are explained by the ratio of congruent to incongruent items in the binary choice task.

The Current Study

Previous studies reported evidence for associations between early numerical and later mathematical abilities. In these studies, mathematical ability is often operationalized as one very specific mathematical activity (e.g., exact arithmetic) or as a general test aggregating various curricular topics. In the present study, we aimed to look beyond math that is generally taught in school by investigating whether an association exists between early numerical abilities of 4- and 5-year olds in the second grade of preschool (T1) and specific components of their probabilistic reasoning abilities, one year (T2: third grade of preschool) and two years (T3: first grade of primary school) later. As the present study was one of the first to investigate the influence of early numerical abilities on specific probabilistic reasoning abilities, we did not have any hypotheses regarding associations between these components and specific numerical abilities. Moreover, we were interested in the effect of children's prior knowledge related to numerical abilities overall. Therefore, the early numerical abilities measure reflected the range of skills pre-schoolers are confronted with in early educational settings.

Regarding the probabilistic reasoning abilities, we limited ourselves to investigating children's abilities to: a) recognize certainty, b) compare probabilities, and c) create equal probabilities. These aspects were feasible to investigate at this young age,. The last two abilities primarily tap into the cognitive demand that Bryant and

Nunes (2012) describe as *quantifying* probabilities and generating hypotheses about their relation with numerical abilities seems straight forward. Therefore we first described our expectations regarding the relationship between numerical abilities and these two probabilistic reasoning abilities before discussing the ability to recognize certainty.

Concerning the second ability, we expected a positive relationship between early numerical abilities and children's ability to compare probabilities in a binary choice task one and two years later. However, as mentioned above, the composition and types of trials in a binary choice task influence children's performance. Therefore, we examined the relationship between early numerical abilities and the ability to compare probabilities in congruent and incongruent trials separately.

Regarding the third ability, we expected a positive relationship between early numerical abilities and children's ability to create equal probabilities one and two years later. Concerning children's ability to create equal probabilities, Falk and Wilkening (1998) showed that young children often use typical erroneous solving strategies. Children were offered two sets, one complete set with desired and undesired elements and one incomplete set with only one type of elements, and they were asked to add elements of the other type to the incomplete set to equalize the probabilities of both sets. Younger children tend to use a strategy by which they only attend to the numerator of the probabilities: adding the number of elements needed to equalize the numerators of both probabilities. Older children tend to add the number of elements needed to get the same absolute difference between desired and undesired elements in both probabilities. As counting is an important element for these strategies, we also looked into the association between early numerical abilities and the use of these erroneous strategies when creating equal probabilities.

For the first ability, we did not have specific predictions about the association between early numerical abilities and children's later ability to recognize (un)certainty. On the one hand, the ability to recognize (un)certainty might depend on a conceptual insight related to understanding randomness that can be acquired independently of numerical abilities. On the other hand, children might use characteristics of the sample space or still calculate the probabilities to infer certainty. If the latter is the case, it is plausible that children use their numerical abilities in such situation.

As described above, we did not have the same expectations for each probabilistic reasoning ability and its association with early numerical abilities. Consequently, we aimed to investigate whether the associations between numerical abilities and each of the assessed probabilistic reasoning abilities differ. Furthermore, Falk et al. (2012) and Falk and Wilkening (1998) found that children of different ages differ in their approaches to probability problems. Therefore, we aimed to investigate whether the associations between early numerical abilities and the probabilistic reasoning abilities assessed remain the same across development. We examined the probabilistic reasoning abilities in the third grade of preschool and the first grade of primary school to test whether the associations are similar or not.

Method

Participants

The present study is part of a more extensive longitudinal research project on the development of young children's early mathematical competencies called Wis & Co (https://ppw.kuleuven.be/o_en_o/WisenCo). Seventeen schools were selected to assure the representation of children from the whole range of socio-economic backgrounds. Parents of 410 children gave consent for participation at the start of the project. Belgian pupils attend three years of preschool and six years of primary school. For the present study, data were collected from participants attending the second half of their second grade of preschool (T1), the second half of their third grade of preschool (T2; one year later), and the first grade of primary school (T3; two years later). Children were on average 4 years and 10 months at the first moment of testing. Due to drop out (e.g., children moving away, motivational problems, illness) complete data were available for 348 students on all time points.

Procedure

Children individually received a test battery of 30 minutes assessing numerical abilities at T1 (Spring 2017). At T2 (Spring 2018) and T3 (Spring 2019), the same group of children individually received two tasks assessing three probabilistic reasoning abilities, i.e., the ability to a) distinguish a certain from an uncertain event, b) compare probabilities, and c) create equal probabilities. Together these tasks took less than 30 minutes to complete.

Materials

Early Numerical Abilities

Based on recent research, a test battery was developed to assess various components of numerical abilities, which consisted of eight tests previously used in research with children of a similar age range (see Bakker et al., 2018). The test battery consisted of five paper-and-pencil tasks covering verbal counting, object counting, calculation, Arabic numeral recognition, and number order. Additionally, three computerized tasks were administered covering symbolic and non-symbolic number comparison and dot enumeration. Following Wijns et al. (2019) the sum of the Z-scores for performance on each of the tests was used for analyses. Moreover, the high internal consistency (α = .93) of this test battery supported the decision to work with one composite score.

Probabilistic Reasoning Abilities

Binary Choice Task. Supply et al. (2020) developed a computerized item set based on the study of Falk et al. (2012) to assess children's abilities to a) distinguish a certain from an uncertain event and to b) compare two probabilities (discussed below). The item set was presented on animated slides and children were introduced to a blindfolded bird that loves black berries but hates white and green berries. At T2, 29 trials were presented to children in which they had to decide which of two presented boxes was best for the bird to blindly pick from. The presented boxes contained different numbers of white, black, and green berries. At T3, 10 additional items were administered along with the same 29 items assessed at the previous time point. The internal consistency for the complete item set was acceptable (α = .74 at T2 and α = .76 at T3). The complete set of items can also be subdivided in categories.

Recognizing (Un)Certainty. In five items at T2 and seven items (i.e., the same items of T2 and two additional items) at T3 one of the two presented boxes only contained black berries, while the other box contained black and white and/or green berries. These items were developed to investigate if children are able to recognize a box with a certainty of a successful draw (certain box) compared to a box in which a successful draw is not guaranteed (uncertain box). In the first five items (T2 and T3) the "certain box" contained a smaller number of black berries than the "uncertain box". In the two additional items presented at T3 the "certain box" contained a larger number of black berries than the "uncertain box". We referred to this category of items as "certain items”. The internal consistency for this subset of items was acceptable (α = .84 at T2 and α =.75 at T3).

Comparing Probabilities. The remaining items of the binary choice task were developed to investigate children’s ability to compare two probabilities and can be subdivided in two main categories. The first category consisted of 12 items at T2 in which the box with the highest probability of blindly drawing a black berry corresponded with the box with the largest number of black berries. At T3, eight additional items were presented together with the original 12 items. These eight items are congruent to the absolute number heuristic, i.e., deciding by the largest number of desired elements (Falk et al., 2012), and we referred to these items as “the congruent items.” The internal consistency for this subset of items was acceptable (α = .73 at T2 and α = .72 at T3).

The second category of items consisted of 12 items that were presented at T2 and T3 in which the box with the highest probability of blindly drawing a black berry corresponded with the box with smallest number of black berries. These items are incongruent to the absolute number heuristic and will be referred to as “the incongruent items”. The internal consistency for this subset of items was high (α = .91 at T2 and α = .89 at T3).

For a more elaborate description of the procedure of administering and the structure of the item set, see Supply et al. (2020). Note that, based on previously reported limitations, additional items were included at T3 compared to the instrument described in the study of Supply et al. (2020) that was used at T2. The complete item set for the present study can be found in Table 1. An accuracy score was derived by dividing the number of items solved correctly by the total number of items for each category of items and was used for analysis.

Creating Equal Probabilities. One task was developed based on the probability-adjustment task of Falk and Wilkening (1998). During the assessment of this task, children were introduced to different concrete materials (see Figure 1). Children were introduced to two identical birds that love black berries and hate white berries. Each bird has a box with berries and in every trial each bird can blindly draw one berry from its box. The box of the first bird always contained black and white berries, but the box of the second bird only contained white berries. The children were asked to add black berries to the box of the second bird to make it a fair game. Children received nine items. The complete item set for the present study can be found in Table 2.

In their study, Falk and Wilkening (1998) identified several rules that children might use to complete the incomplete box. Children who used the correct proportional rule, would have put the correct number of black berries in the incomplete box. If children used the 'one-dimensional rule' (OD), they would have only taken into account the number of black berries in the complete box and put the same number of black berries in the incomplete box. If children used the 'difference rule' (DIF), they would have calculated the difference between black and white berries in the complete box and added the number of black berries needed to arrive at the same difference in the incomplete box. The use of every rule predicted a different answer. For each child three total scores were calculated: the total number of answers as predicted by the a) correct rule (α = .39 at T2 and α = .62 at T3), b) OD rule (α = .96 at T2 and α = .95 at T3), and c) DIFF rule (α = .53 at T2 and α = .75 at T3).

Statistical Analysis

For the first research question, partial correlation analyses (Cohen et al., 2003) were performed to test the associations between children's early numerical abilities on the one hand and probabilistic reasoning abilities at T2 and T3 on the other hand, controlling for children's age. The age control was included because children within the same grade can differ up to twelve months in age.

William's tests (1959) are *t*-tests that can be used to compare two correlations that are not independent (e.g., correlations based on the same sample) and have one variable in common (Weaver & Wuensch, 2013). Concerning the second research question, Williams's tests were conducted to compare the dependent correlation coefficients with early numerical abilities as the variable in common within time points. Concerning the second research question, Williams's tests were conducted to compare the dependent correlation coefficients between time points. To control the family-wise error rate at the nominal level (α = .05) the Bonferroni-Holm method (Holm, 1979) was applied for each research question.

Results

The means and standard deviations of all variables are shown in Table 3. Table 4 shows for the two time points (third grade of preschool (T2) and first grade of primary school (T3)) the frequency distributions of the total accuracies in percentages on items assessing children's ability to a) distinguish certain from uncertain events, b) compare probabilities in congruent items, c) compare probabilities in incongruent items, and d) create equal probabilities. Partial correlations controlling for age between children's numerical abilities in the second grade of preschool (T1) and their performance on the probability tasks one (T2) and two (T3) years later are reported in Table 5.

First, partial correlations between early numerical abilities assessed at T1 and the probabilistic reasoning abilities assessed at T2 are described. Concerning the binary choice task, no significant partial correlation was found between children's early numerical abilities and their performance on the certain items. A significant partial correlation was found between early numerical abilities and performance on the congruent items, *r* = .26, *p* < .001, but not for performance on the incongruent items. Concerning the task on creating equal probabilities, no significant partial correlation was found between early numerical abilities and children's use of the correct rule. A significant partial correlation between early numerical abilities and children's use of the OD rule was found, *r* = .29, *p* < .001, but not with children's use of the DIF rule.

Second, partial correlations between early numerical abilities assessed at T1 and the components of probabilistic reasoning abilities assessed at T3 are described. Concerning the binary choice task, significant partial correlations were found between early numerical abilities and performance on the certain items, *r* = .19, *p* < .001, congruent items, *r* = .15, *p* = .005, and incongruent items, *r* = .17, *p* = .001. Concerning the task on creating equal probabilities, significant partial correlations were found between children's early numerical abilities and the use of the correct rule, *r* = .16, *p* = .003. No significant correlation was found between early numerical abilities and the use of the OD rule. A significant correlation was also found between early numerical abilities and use of the DIF rule, *r* =.23, *p* < .001.

Williams's tests were conducted to investigate whether correlations between numerical abilities on the one hand and each of the probabilistic reasoning abilities aspects on the other hand differed from each other. All t-values can be found in Table 6. Concerning probabilistic reasoning abilities at T2, results show that the correlation between early numerical abilities and performance on the congruent items of the binary choice task was significantly higher than the correlations between early numerical abilities and performance on the certain items, *t*(345)= 3.24, *p* = .02; *d* = 3.21, incongruent items, *t*(345)= 3.26, *p* = .02; *d* = 3.23, and use of the correct rule, *t*(345)= 3.21, *p* = .02; *d* = 3.18, in the task on creating equal probabilities. Furthermore, the correlation between early numerical abilities and preschoolers' use of the erroneous OD rule when creating equal probabilities was significantly higher than the correlations between early numerical abilities and performance on the certain items, $t(345)= 4.30$, $p < .001$; $d = 4.24$, incongruent items, $t(345)= 4.36$, $p < .001$; $d = 4.30$, of the binary choice task, and use of the correct rule, $t(345)= 3.17$, $p = .03$; $d = 3.14$, in the task on creating equal probabilities. Note that all effect sizes for the reported analyses were found to exceed Cohen's (1988) convention for a large effect ($d \geq .80$).

Concerning probabilistic reasoning abilities assessed at T3, the correlation between early numerical abilities and the use of the OD rule when creating equal probabilities was significantly weaker than the correlations between early numerical abilities and performance on the certain items, $t(345)= 3.38$, $p = .01$; $d = 3.35$, incongruent items, $t(345)= 3.17$, $p = .02$; $d = 3.14$, congruent items, $t(345)= 3.01$, $p = .03$; $d = 2.99$ of the binary choice task, and use of the DIF rule, $t(345)= 3.22$, $p = .02$; $d = 3.19$ when creating equal probabilities. All effect sizes for the reported analyses, were found to exceed Cohen's (1988) convention for a large effect ($d \geq .80$).

Williams's tests were also conducted to investigate whether correlations between numerical abilities on the one hand and each of the probabilistic reasoning abilities assessed at T2 differed from the correlations between early numerical abilities and the corresponding probabilistic reasoning ability assessed at T3. All t-values can be found in Table 7. Concerning the binary choice task, the correlation between early numerical abilities and performance on the certain items at T2 was significantly weaker than the correlation found at T3, $t(345)= 3.95$, $p < .001$; $d = 3.90$. The correlation between early numerical abilities and performance on the congruent items at T2 did not differ significantly from the correlation found at T3. The correlation between early numerical abilities and performance on the incongruent items was significantly weaker than the correlation found at T3, $t(345) = 3.58$, $p = .002$; $d = 3.54$. Concerning the task on creating equal probabilities, the correlation between early numerical abilities on the one hand and the use of the correct rule at T2 or T3 on the other hand did not differ significantly after appliance of the Bonferroni-Holm correction. The correlation between early numerical abilities and the use of the OD rule at T2 was significantly higher than the correlation found at T3, $t(345) = 5.39$, $p < .001$; $d = 5.28$. Finally, the correlation between early numerical abilities and the use of the DIF rule at T2 was significantly weaker than the correlation found at T3, $t(345) = 2.53$, $p = .04$; $d = 2.52$. All effect sizes for the reported analyses, were found to exceed Cohen's (1988) convention for a large effect ($d \geq .80$).

Discussion

A number of studies found evidence that early numerical abilities are predictive of later mathematical achievement (e.g., De Smedt et al., 2009; Duncan et al., 2007; Nguyen et al., 2016; Passolunghi & Lanfranchi, 2011). In these studies, probabilistic reasoning abilities were often not considered as a component of mathematical achievement.

The current study was a first step in grasping the association between early numerical abilities and later probabilistic reasoning abilities. Although our results supported the idea that associations exist between early numerical abilities and later probabilistic reasoning abilities of children, they also showed that this relation is not clear-cut. Early numerical abilities in the second grade of preschool seemed to be only positively associated with performance on congruent comparison items (where only focusing on the number of desired outcomes is sufficient to get the correct answer) and the use of the erroneous one-dimensional (OD) rule in the equal probability creation task one year later. Limiting ourselves to these findings would have led us to conclude that early numerical abilities in the second grade of preschool did not predict stronger probabilistic reasoning abilities in the third grade of preschool, but rather predicted the use of incorrect strategies.

However, a different picture emerges when we considered performance of children on probabilistic reasoning tasks when they are one year older. Positive associations were found between early numerical abilities in the second grade of preschool and performance on all probabilistic reasoning tasks in the first grade of primary school. Moreover, a positive association between early numerical abilities and the use of the erroneous one-dimensional strategy could not be replicated with the same children who are one year older. It is also worth noting that the reported effect sizes can be considered as large (Cohen 1988), suggesting that the differences in correlation were substantial and cannot merely be explained by small differences in correlations that become significant due to the large number of children participating in the present study.

A possible explanation for the results differing depending on the age of the children, might be due to the gradual development of probabilistic reasoning abilities in this age range. Following Piaget and Inhelder (1975), children might move through different developmental stages that impact their understanding of and approach towards probability. Younger children's reasoning ability is often characterized by the use of one-dimensional strategies, which will result in children choosing the set in which the number of desired elements is larger, regardless of the number of undesired elements. Typically, children using a one-dimensional strategy will succeed when solving congruent items, but fail on incongruent items. When children create equal probabilities, the use of the one-dimensional strategy will lead them to add the same number of desired elements in the incomplete set as present in the complete set. The use of these one-dimensional strategies decreases with age and makes way for more "advanced" two-dimensional strategies (the incorrect "difference" strategy and the correct "ratio" strategy) as children move to the next developmental phase. Perhaps early numerical abilities are associated with how easily children move through the different developmental stages: first realizing the importance of the nominator, then realizing the importance of the nominator and denominator, and finally integrating both elements in the correct way to make probabilistic decisions.

The findings of the present study have to be interpreted with caution and cannot be generalized to probabilistic reasoning ability overall. The development of probabilistic reasoning abilities requires more than just the shift from one to two-dimensional thinking in probabilistic situations that are similar to those in the present study. Perhaps, numerical abilities relate differently to other aspects of probabilistic reasoning. On the one hand, some probabilistic reasoning abilities might be even more reliant on early numerical abilities than the probabilistic reasoning abilities that were investigated in the present study (e.g., calculating probabilities, determining the sample space). On the other hand, some probabilistic reasoning abilities might be unrelated to numerical abilities. For example, it might be interesting to investigate whether we use/need/profit from our numerical abilities for the understanding of the independence between events in random situations?

The present study was among the first to investigate associations between early numerical abilities and later probabilistic reasoning abilities and contributed to existing literature on children's development of probabilistic reasoning abilities and on the predictive role of early numerical abilities in several ways. First, many studies investigating the predictive role of early numerical abilities for later mathematical achievement considered just one time-point to assess later mathematical achievement.

However, the current findings seem to suggest that results might depend on the moment of testing. It is possible that associations might be absent in the short-term, but appear in the long-term. Second, the current study supported the idea that probability is a complex concept and that findings about children's probabilistic reasoning abilities are highly dependent on the task at hand (e.g., comparing probabilities vs. creating probabilities) or the composition of items within a task (Bryant & Nunes, 2012; Falk et al., 2012; Falk & Wilkening, 1998). For example, the present results showed that the association between early numerical abilities and the ability to compare probabilities differs depending on the type of item considered (congruent vs. incongruent). Ruggeri et al. (2018) found positive correlations between children's performance on the Panamath test or on a dot enumeration tasks and their overall score on a binary choice task containing 75% congruent trials and 25% incongruent trials. Results of the present study suggested that conclusions of Ruggeri et al. (2018) might have differed if congruent and incongruent trials were considered separately. Third, the present study was one of the few studies that investigates the difference between the ability to compare probabilities and the ability to distinguish a certain from an uncertain event. One could expect that numerical ability in counting or arithmetic is less relevant to distinguish certainty from uncertainty than it is for comparing two probabilities. However, our findings suggested that early numerical ability relates similarly to performance on items assessing the ability to distinguish certainty as to performance on incongruent items. Perhaps, between the third grade of preschool and the first grade of primary school, children realize that undesired elements matter when determining the probability of blindly grabbing a desired element. They might start to count the number of undesired elements and also note when none are present, which has implications for the certainty of an event. In other words, starting to recognize certainty and uncertainty in specific situations might be the first indication of children shifting from one- to two-dimensional thinking. Moreover, this recognition could predict a rapid growth for performance on incongruent items: It might be harder to count the number of undesired elements and integrate them in the correct proportional strategy when both sets contain undesired elements than to note that no undesired elements are present in one of the sets.

Building on our results and interpretation, several limitations of the present study and avenues for future research exist. First, the findings of the current study gave rise to important questions concerning the role of formal education. As mentioned in the introduction, an overemphasis on deterministic thinking in instruction in school was suggested by several authors (Falk et al., 2012; Fischbein, 1975; Meletiou-Mavrotheris, 2007; Stohl, 2005). Although we mentioned the role of instruction earlier on, we were unable to capture the extent to which the children's school curriculum had a deterministic focus. No guidelines exist on how to categorize a curriculum as more or less deterministic. In addition, even if such guidelines existed, we would have also needed to take into account the degree to which the teacher tolerated and understood uncertainty as this might affect the actual instruction of the curriculum that children receive. In short, the present study did not allow us to draw conclusions about the possible effect of a deterministic focus in instruction on the development of probabilistic reasoning. To better understand the effect of a deterministic emphasis in instruction on probabilistic reasoning, an intervention aiming to stimulate probabilistic reasoning should be conducted, comparing children that were exposed to a curriculum with a strong deterministic emphasis to children who received a less "deterministic curriculum". However, this brings us back to the question: "when can we speak of a deterministic curriculum?".

Nonetheless, the current findings did suggest something about the role of instruction. The findings showed that the emergence of an association between early numerical abilities and attending to the denominator in probabilistic situations, *coincides* with children's transition to more formal arithmetic education. One can wonder whether this finding is a coincidence? Would the same relation between early numerical abilities and probabilistic reasoning abilities be found in countries and cultures where formal education starts earlier, later, or not at all? The effect of formal education could be explored in future studies by comparing the relationship between early numerical abilities and later probabilistic reasoning abilities between the oldest children at the end of the third grade of preschool and youngest children at the end of the first grade of primary school. While these preschoolers and first graders in the current study were close in age, only the first graders had experience with formal education.

Second, the current findings also suggested that children are able to make sense of probabilistic situations from a very young age. Although some aspects of probabilistic reasoning (e.g., creating equal probabilities) are still complex for first graders, results suggested that most of the first graders possess the ability to successfully compare probabilities. These findings suggested that some aspects of probability might already be manageable for students in the lower primary grades. This challenges the possible argument of the complexity of probability that might be used by curriculum designers to exclude probability in younger grades (Langrall, 2018). Future intervention studies could investigate whether it is possible to build on children's prior knowledge about probability before instruction and whether numerical abilities play a role in strengthening the concept of probability. Future studies could also look into the association between early numerical and later probabilistic reasoning abilities in older children after they received instruction in probabilistic reasoning.

Third, although the present study did include several probabilistic reasoning abilities, it was impossible to cover all aspects of probabilistic reasoning abilities. It has been argued that formal education, which is often deterministic in nature, might impede children's conceptualization of uncertainty (Fischbein, 1999). However, the present study included probabilistic reasoning ability tasks which children might solve correctly relying solely on their proportional reasoning abilities, not taking into account the role of uncertainty or the probabilistic context. Future studies could investigate whether early numerical abilities also predict children's ability to handle situations which require an understanding of uncertainty and randomness, for example, understanding the independence between repeated die rolls. To understand the role of context, future studies could also investigate whether early numerical abilities predict children's ability differently in proportional comparison situations compared to probabilistic comparison situations. Moreover, it might be possible to enrich the probabilistic reasoning tasks that were included in the present study to allow for a better understanding of the development of two-dimensional thinking. For example, as suggested above, suddenly succeeding on the certainty-uncertainty items might indicate a shift of one- to two-dimensional thinking. This hypothesis might be further investigated by exploring children's reaction to binary choice items in which a successful draw is certain for both sets (i.e., none of the two sets contain undesired elements). It can be expected that two-dimensional thinkers would show hesitation or even refuse to express a choice for one of two sets in such a situation, while one dimensional thinkers might easily decide on one set. Additionally, it might be interesting to include binary choice items with two uncertain sets that have equal probabilities of a successful draw and to investigate performance on these items in relation to numerical abilities. We might observe two groups of two-dimensional thinkers. A first group of two-dimensional thinkers might be able to recognize certainty and hesitate when two certain sets are presented, but still choose one set in situations where the probabilities of two uncertain events are equal. For these children the calculations needed to compare probabilities might still be too complex, which might also be related to poor proportional reasoning abilities or fractional knowledge. A second group of two-dimensional thinkers might be able to recognize certainty and to perform the correct calculation to incorporate the number of desired and undesired elements in the correct proportional strategy. These children might hesitate in choosing a set when two certain sets are presented and also hesitate when two uncertain sets with equal probabilities are presented.

Fourth, future studies could differentiate between specific skills within numerical abilities. The numerical abilities measure in the present study was not conceived and designed for a systematic and deep analysis of specific associations between different numerical abilities and probabilistic reasoning abilities. The numerical tasks that make up the early numerical measure in the present study are intended to be an overall indicator of children's general early numerical abilities. As this is one of the first studies exploring the relationship between early numerical abilities and probabilistic reasoning abilities, it seemed more appropriate to first look into children's general numerical skills. However, research has shown that some early numerical abilities are more important for later mathematical achievement than others (e.g., Bartelet et al., 2014). Future studies could investigate if such findings can be replicated for the probabilistic reasoning abilities investigated in the present study as well as other probabilistic reasoning abilities. From a theoretical viewpoint, it is possible to generate a range of hypotheses. For example, that the absolute number heuristic leads to high scores on congruent probabilistic comparison trials and low scores on incongruent probabilistic comparison trials. It is conceivable that the occurrence of this heuristic is related to young children's non-symbolic number comparison abilities. Likewise, it is conceivable that, for example, number order abilities do not explain individual differences related to the absolute number heuristic. Also, as two-dimensional thinking in probabilistic reasoning is related to taking into account two dimensions (the number of undesired and desired elements), it might be possible that performance on probabilistic tasks is better predicted by numerical tasks that require calculation (e.g., arithmetic problems) in which several terms have to be taken into account. Likewise, one-dimensional thinking in probability might be predicted by children's performance on tasks that require attention to just on term (e.g., counting task, (non-)symbolic number recognition tasks). Moreover, future studies could also aspire to isolate the effect of age/grade from the *ability factor* by including additional time points regarding numerical abilities. For example, one could compare the probabilistic reasoning abilities of "higher achievers" on a (specific) numerical abilities (sub)task in the second grade of preschool, with children who are one year older and were originally "lower achievers" in the second grade of preschool but become as good as "the higher achievers" one year later. This could lead to additional important insights regarding the role of numerical ability for probabilistic reasoning independent of age/grade.

Fifth, whereas the present study suggested that relations between early numerical abilities and probabilistic reasoning abilities exist, intervention studies are needed to gain a deeper understanding of the direction of these relations. The growth in both early numerical abilities and probabilistic reasoning abilities can be compared between different groups receiving business as usual or interventions that aim to stimulate (specific) numerical abilities and probabilistic reasoning abilities.

Conclusion

There is an emphasis on the stimulation of early numerical abilities in preschool to prepare children for formal mathematical education in primary school. The present study showed that early numerical abilities of children are also related to children's ability to recognize (un)certainty, compare probabilities, and create equal probabilities. Initially, these numerical abilities predicted the use of incorrect strategies (i.e. a one-dimensional focus on favourable outcomes), but importantly, they also predicted the use of correct strategies one year later. In sum, the present study suggested that children's early numerical abilities are related to their later probabilistic reasoning abilities, which supported Fischbein's (1999) ideas about the role of prior knowledge.

Acknowledgements: This research was supported by Grant KU Leuven project C16/16/001 "Development and stimulation of core mathematical competencies".

Declaration of interest statement: We wish to confirm that there are no known conflicts of interest related to this publication.

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Table 1.

Items of the Comparison Task With Composition of Berries.

ItemID	Item
Congruent 1	(35:30)>(25:35)
Congruent 2	(5:6)>(3:7)
Congruent 3	(40:5:15)>(30:10:15)
Congruent 4	(6:3:4)>(3:4:4)
Congruent 5	(7:4)>(5:5)
Congruent 6	(35:25)>(30:35)
Congruent 7	(35:15)>(30:25)
Congruent 8	(5:5:1)>(4:4:4)
Congruent 9	(5:5)>(4:7)
Congruent 10	(3:5)>(2:8)
Congruent 11	(30:10:15)>(10:10:5)
Congruent 12	(3:4)>(1:3)
Congruent 13	(8:1:2)>(3:1:1)
Congruent 14	(25:35)>(10:25)
Congruent 15	(4:4)>(2:3)
Congruent 16	(30:15:30)>(5:5:5)
Congruent 17	(4:7)>(1:3)
Congruent 18	(5:4:4)>(1:2:1)
Congruent 19	(5:9)>(1:4)
Congruent 20	(20:45)>(5:25)
Incongruent 1	(15:5)>(20:45)
Incongruent 2	(15:5:5)>(20:25:20)
Incongruent 3	(3:2)>(4:4)
Incongruent 4	(10:10:5)>(15:20:25)
Incongruent 5	(5:1)>(6:5)
Incongruent 6	(3:1)>(4:3:1)
Incongruent 7	(15:5)>(35:20)
Incongruent 8	(4:1)>(8:2:2)
Incongruent 9	(5:1)>(9:4)
Incongruent 10	(20:5)>(45:25)
Incongruent 11	(3:1)>(8:3:2)
Incongruent 12	(15:5)>(45:15:15)
Certain 1	(2:0)>(5:2)
Certain 2	(3:0)>(7:2:1)
Certain 3	(1:0)>(8:2)
Certain 4	(25:0)>(35:5)
Certain 5	(10:0)>(45:10:5)
Certain 6	(5:0)>(2:3)
Certain 7	(15:0)>(5:1)

Note. Every item is described by the numerical composition of berries within each box. The numbers between the first brackets indicate the correct box, or in other words the box with the greater probability for a desired black berry. The second brackets indicate the incorrect box. Within brackets, the first number indicates the count of desired black berries, the second number indicates the count of undesired white berries and if applicable, the third number indicates the count of undesired green berries.

Table 2.

Design of the Task on Creating Equal Probabilities.

	Berries within each box			The number of black berries that would be added to the incomplete box as predicted by rule		
	Complete		Incomplete			
Item	White	Black	White	Correct	OD	DIFF
1	2	1	4	2	1	3
2	3	2	6	4	2	5
3	3	1	6	2	1	4
4	2	6	1	3	6	5
5	4	2	6	3	2	4
6	2	3	4	6	3	5
7	1	2	2	4	2	3
8	1	3	2	6	3	4
9	1	2	3	6	2	4

Table 3.

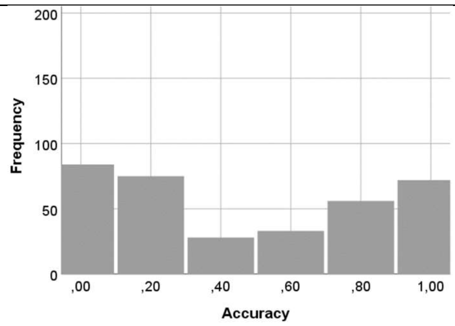
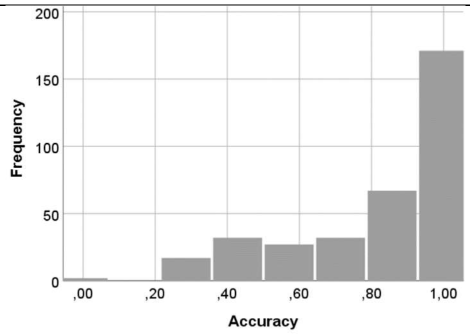
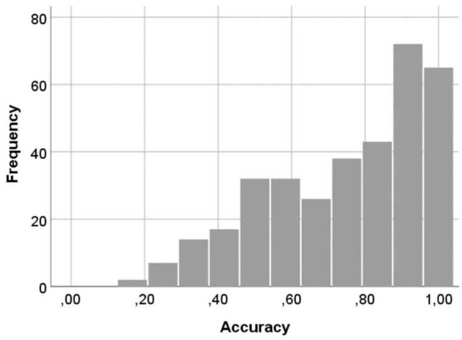
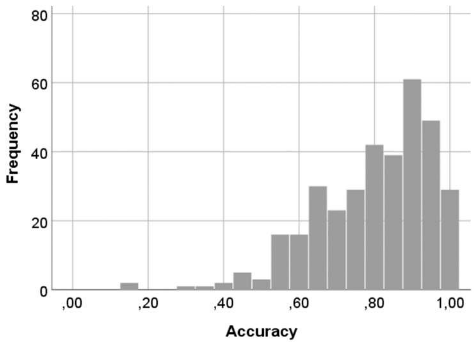
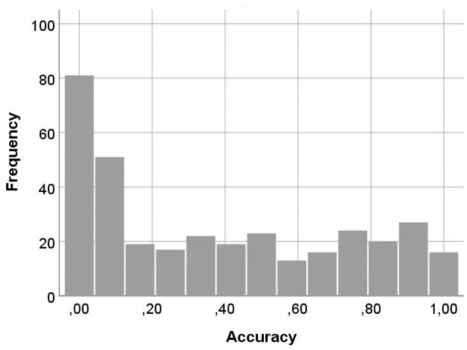
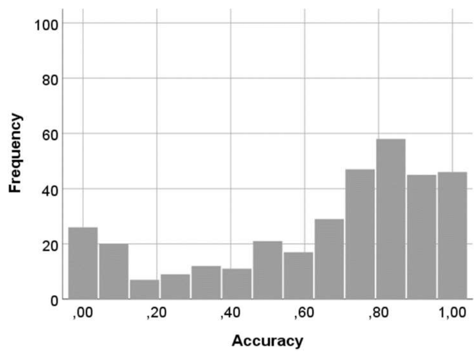
Descriptive Statistics for Age, Composite Scores of Numerical Ability and Accuracy Scores on Probabilistic Reasoning Abilities as well as the Use of the One-Dimensional (OD) and Difference (DIF) Rule When Creating Equal Probabilities.

Measure	M	SD	Range
Age in months T1	58.13	3.42	51 – 64
Numerical ability T1	0.44	5.39	-11.86 – 14.99
Comparing probabilities in certain items T2	.47	.38	.00 – 1
Comparing probabilities in congruent items T2	.75	.22	.17 – 1
Comparing probabilities in incongruent items T2	.38	.34	.00 – 1
Use of correct rule T2	.04	.09	.00 – .44
Use of OD rule T2	.55	.43	.00 – 1
Use of DIF rule T2	.04	.09	.00 – .78
Comparing probabilities in certain items T3	.82	.23	.00 – 1
Comparing probabilities in congruent items T3	.80	.15	.15 – 1
Comparing probabilities in incongruent items T3	.64	.31	.00 – 1
Use of correct rule T3	.08	.13	.00 – .78
Use of OD rule T3	.61	.41	.00 – 1
Use of DIF rule T3	.11	.18	.00 – .89

Note. Values of the probabilistic reasoning parameters are reported in percentages. Depending on the variable in question, a value of 1 indicates that the child gave the correct answers or answers corresponding to a certain rule on all items.

Table 4.

Frequency Distributions of the Total Accuracies in Percentages on Items Assessing Children's Ability to 1) Distinguish Certain From Uncertain Events, 2) Compare Probabilities in Congruent Items, 3) Compare Probabilities in Incongruent Items, 4) Create Equal Probabilities Assessed in the Spring of Their Third Grade in Preschool (T2) and First Grade in Primary School (T3).

Frequency Distribution of Accuracy Scores on	T2	T3
Certain items		
Congruent items		
Incongruent items		

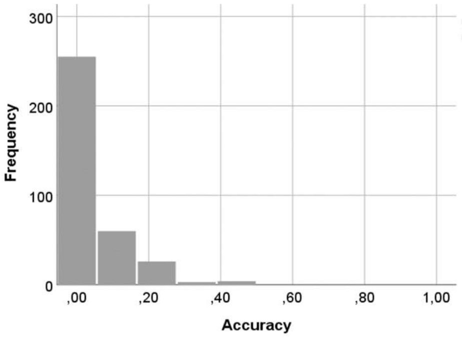
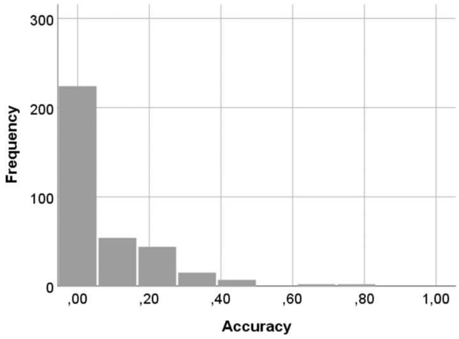
Creating equal probabilities		

Table 5.

Partial Correlations Between the Composite Scores on Numerical Abilities in the Second Grade of Preschool and Performances on the Probabilistic Reasoning Ability Measures in the Third Grade of Preschool (Below Diagonal) and the First Grade of Primary School (Above the Diagonal) While Controlling for Age.

	Binary choice task				Creating equal probabilities		
	Numerical ability	Certain items	Congruent items	Incongruent items	Correct rule	OD rule	DIF rule
Numerical ability	-	.19***a	.15**a	.17***a	.16**a	-.084	.23***a
Certain items	-.05	-	-.28***	.81***	.21***	-.19***	.18***
Congruent items	.26***a	-.59***	-	-.37***	.04	.03	.06
Incongruent items	-.05	.83***	-.63***	-	.26***	-.30***	.25***
Correct rule	.02	.05	-.02	.04	-	-.60***	.36***
OD rule	.29***a	-.13*	.14**	-.13*	-.35***	-	-.71***
DIF rule	.06	.14*	-.05	.16**	.33***	-.34***	-

Note. OD rule refers to the use of an erroneous strategy to create equal probabilities in which children add the same number of black berries in the incomplete set as present in the complete set. The DIF rule refers to the erroneous strategy in which children calculate the difference between black and white berries in the complete box and add the number of black berries needed to arrive at the same difference in the incomplete box.

* $p < .05$

** $p < .01$

*** $p < .001$

[a] Significant after application of Bonferroni-Holm correction

Table 6.

T-values for Williams's Tests for any Pair of Correlations With Early Numerical Ability as the Variable in Common.

r between early numerical and …	Performance on certain items	Performance on congruent items	Performance on incongruent items	Use of correct rule	Use of OD rule	Use of DIF rule
Performance on certain items	-	0.5	0.55	0.48	3.38*	-0.57
Performance on congruent items	-3.24*	-	-0.28	-0.14	3.17*	-1.12
Performance on incongruent items	0.05	3.26*	-	0.22	3.01*	-0.88
Use of correct rule	0.86	3.21*	-0.054	-	2.56	-1.18
Use of OD rule	-4.3*	-0.41	-4.36*	-3.17*	-	-3.22*
Use of DIF rule	-1.5	2.62	-1.59	-0.68	2.69	-

Note. T-values for differences in correlations between early numerical ability and each of the probabilistic reasoning aspects assessed at T2 are presented below the diagonal. T-values for differences in correlations between early numerical ability and each of the probabilistic reasoning aspects assessed at T3 are presented above the diagonal.

* Significant after Bonferroni-Holm correction

Table 7.

T-values for Williams's Tests for Pairs of Correlations Between Early Numerical Ability and Eeach of the Probabilistic Reasoning Aspects at T2 or T3.

r between early numerical and …	Performance on certain items	Performance on congruent items	Performance on incongruent items	Use of correct rule	Use of OD rule	Use of DIF rule
			T3			
T2	-3.95*	1.65	-3.58*	-2.06	5.39*	-2.53*

* Significant after Bonferroni-Holm correction

Figure 1.

Example of an Item Setup