KU LEUVEN

DEPARTMENT OF ECONOMICS

Role of quality differentiation on firm size distributions

William CONNELL and Hylke VANDENBUSSCHE

FACULTY OF ECONOMICS AND BUSINESS

DISCUSSION PAPER SERIES DPS20.16

SEPTEMBER 2020

Role of Quality Differentiation on Firm Size Distributions

William Connell, Hylke Vandenbussche[1]

Abstract

The dispersion of firm size distributions vary strongly across sectors. Until now, sectoral variations in intra-industry heterogeneity were attributed to sectoral differences in the exogenous dispersion of the productivity of firms, differences in sectoral horizontal differentiation, sectoral trade openness and country characteristics. In this paper, we build on this result by additionally examining the role of the sectoral scope for quality differentiation. Our theoretical and empirical findings reveal that whenever there is room for quality differentiation, the role of large firms is even stronger and inequality in firm size is exacerbated.

JEL Classification: L11, F10, F13, F14, F23,

Keywords: Size distribution of Firms, Power Law Distribution, Quality, Firm Level Analysis, Trade Openness

[1]University of Leuven (KU Leuven) - Department of Economics, Naamsestraat 69, B-3000 Leuven, Belgium (corresponding author: william.connell-garcia@ec.europa.eu).

The views expressed in this paper are those of the authors and do not necessarily reflect the views of any other institution with which one of the authors is affiliated. All remaining errors are our own.

William Connell thanks KU Leuven for the hospitality during my Ph.D where this paper was developed.

1. Introduction

Firm size distributions vary strongly across sectors. Throughout this paper, we are going to argue that this aggregate outcome is the result of decisions of heterogeneous firms operating in sectors with different underlying characteristics. Early studies evaluating decisions of heterogeneous firms with aggregate effects within a sector, such as Bernard et al. (2003) and Melitz (2003), identified firms engaging in export activity as a small group of firms characterised by firm-specific supply factors such as high productivity and size. While these studies identify important characteristics of exporting firms, the role of demand factors was largely neglected. Following these early papers, vertical differentiation in terms of product qualities has been identified as an important determinant to understand the competitiveness of firms. Manova and Zhang (2012) and Kugler and Verhoogen (2011) show that within a sector firms offering higher quality products are larger, more productive, use more expensive inputs, charge higher prices, earn greater revenues, export more and enter a higher number of foreign markets when compared to firms offering low quality products.[2] In addition to the vertical differentiation attributes, Di Comite et al. (2014) find evidence that horizontal attributes should be taken into consideration when evaluating the international trade performance of firms. Complementing the previous study, Aw-Roberts et al. (2018) suggest that the horizontal dimension explains around 50% of the variation in export revenue within firms operating in the same sector.

While these studies concentrate on explaining the economic outcomes of firms based on different firm characteristics within a sector, the aggregate effect of firms' decisions operating in heterogeneous sectors has received less attention. The particular aggregate outcome that we explore in this paper is the variation in the degree of intra-industry firm size heterogeneity. Consequently, the aim of this paper is to identify different elements that explain sectoral differences in intra-industry heterogeneity. In other words, differences in the dispersion of sectoral firm size distributions. Throughout this paper, firm size

[2] In an application of French wine, Crozet et al. (2011) find evidence that higher quality wine producers are more likely to export and, in doing so, they export more quantity and charge a higher price for their bottles of wine.

heterogeneity will be characterised by power law coefficients. These estimated coefficients, apart from being theoretically identified, are metrics that summarise the degree of right-skewness of particular firm size distributions. In this respect, the lower the estimated absolute power law coefficient, the stronger the firm size heterogeneity.

Until now, sectoral variations in intra-industry heterogeneity have been attributed to sectoral differences in the exogenous dispersion of the productivity of firms, differences in sectoral horizontal differentiation, sectoral trade openness and country characteristics. In this paper, we build on this result by additionally examining the role of the sectoral scope for quality differentiation. This channel is based on the idea that in sectors where consumers have a higher taste for quality and are willing to pay a higher price for higher quality goods, firms have an incentive to use higher quality inputs and sell higher quality products, at least for those firms that can afford to invest in quality. Consequently, we argue that the higher the reward of firms for increasing its quality output, the higher the firm size heterogeneity. In other words, the lower the absolute power law coefficient of firm size distributions.

We build on a model developed by Kugler and Verhoogen (2011), which we augment with the insights from Di Giovanni et al. (2011) and Di Giovanni and Levchenko (2012). Our contribution consists of explicitly solving for the sectoral determinants, explaining the power law coefficient of firms size distributions. Assuming that the capability and input quality of a firm are complements in generating output quality,[3] our theoretical framework shows that higher sectoral scope for quality differentiation leads to a higher use of quality inputs by highly productive firms, which then produce higher-quality varieties. The higher quality output results in higher firm revenues and increases the aggregate firm

[3]This assumption suggests that increasing quality does not require an upgrade in the fixed costs. In contrast, Sutton (2007) , in his study on the effect of globalisation in creating a lower bound of quality below which firms cannot sell their products, used the alternative assumption that firms could upgrade their quality by spending on fixed and sunks costs (R&D). Contrary to Sutton (2007) , who concentrates on the theoretical effect of globalisation on small firms, this paper looks theoretically and empirically at the effect of different sectoral characteristics on the right tail of the firm size distribution by evaluating the power law coefficient of distributions of medium and large firms.

size heterogeneity. The latter effect is enhanced once we open up to trade as some firms are able to export to foreign markets. Consequently, the main prediction is that, *ceteris paribus*, firm size distributions are more skewed in industries where the scope for quality differentiation is higher.

To test the main prediction of the theoretical framework, we estimate power law coefficients of firm size distributions for a large number of country-sector pairs across different European countries. These coefficients are obtained using different measures of firms size, i.e. "Employment" and "Total Assets". In order to test the main predictions of our theoretical framework, the estimated power law coefficients are then regressed on various variables measuring the scope for vertical differentiation within an industry. Controlling for other sectoral characteristics identified in the theoretical framework, our findings clearly show that whenever there is room for quality differentiation, firm size distributions are more skewed. In other words, the role of large firms is even stronger and inequality in terms of firm size is exacerbated.

Understanding the determinants of the aggregate firm size heterogeneity is important given its economic implications. Costinot et al. (2011) identified intra-industry firm heterogeneity as a key determinant of comparative advantage at the industry level. Prior to that, Helpman et al. (2004) showed that firm heterogeneity in a sector determines the extent of Foreign Direct Investment (FDI) versus exports abroad. More recently, Bonfiglioli et al. (2017) suggested that firm size heterogeneity is important to understand income inequalities, given that large firms are observed to pay higher wages. Moreover, Gabaix (2011) and Di Comite et al. (2014) pointed out the granularity aspects of firm size distributions, suggesting these determine the aggregate impact of idiosyncratic shocks.

In terms of previous related literature on the determinants of sectoral differences in firm size heterogeneity, Melitz (2003) was one of the first to introduce the concept of heterogeneous firms in the international trade literature. In this seminal model, firms use a homogeneous input to satisfy a symmetric output demand, where the output of firms only differs in the horizontal dimension.[4] While Melitz (2003) did not make specific distributional assumptions, Helpman et al. (2004) used a combination of a power law distribution of firm productivity on the producer side with CES preferences on the consumer side. Theoretically, this combination results in a firm size distribution that is characterised in an autarky scenario by a power law distribution.[5] In this framework, the power law coefficient is explained by two parameters, i.e. the elasticity of substitution and the dispersion of the productivity of firms.[6] The idea that firm size distributions follow a power law probability distribution, characterised by many small firms and fewer large firms, is also based on previous empirical evidence such as Axtell (2001), Fujiwara et al. (2004), Helpman et al. (2004), Zhang et al. (2009) and Gabaix (2016) that have shown empirically that firm size distribution can be fitted by power law functions, which holds for different countries, sectors and measures of firm size.

In follow-up literature, several papers have then identified additional determinants that can affect the power law parameter of firm size distributions. Di Giovanni et al. (2011) and Di Giovanni and Levchenko (2012) showed that as we allow foreign trade, the dispersion of firm size increases. This effect is driven by the right-hand side tail of the distribution by making bigger firms even bigger. Bonfiglioli et al. (2017) also found that openness to trade increases the dispersion of firm size distributions but for a different reason, i.e. trade openness raises the incentive for firms to invest, which positively affects the productivity levels of firms and the firm size distributions. This mechanism may be hampered by firms facing financial constraints to invest, as suggested by Bonfiglioli et al. (2016). Other factors

[4]Although some researchers have interpreted the Melitz (2003) in terms of quality-differentiated outputs, the standard interpretation takes output demand as symmetric where the only differentiating factor is the horizontal dimension. This complements other recent studies such as Di Comite et al. (2014) who found evidence that horizontal attributes should be taken into consideration when evaluating international trade performance. Complementing the previous study, Aw-Roberts et al. (2018) suggested that the horizontal dimension explains around 50% of the variation in export revenue.

[5]See Mrázová et al. (2017) for a detailed discussion on the outcomes of using different functional forms in the demand and supply side.

[6]Helpman et al. (2004) was one of the first to use empirically power law coefficients on the size of firms as a measure of within-industry firm heterogeneity.

that affect the firm size distributions include the average age or the portfolio of products of the firm population within a given sector, as shown by Cabral and Mata (2003) and Hutchinson et al. (2010), respectively. Recent studies have also pointed out the role of institutions and regulations as a determinant of firm size distributions. Using French data, Garicano et al. (2016) showed that France has a large number of firms where the number of employees is below fifty. He argues that the reason for this peculiar distortion in firm size distributions was the increase in the labour costs originating from higher regulatory requirements for firms with more than 50 employees.

This paper contributes to this previous literature by identifying and testing the mechanism that the sectoral scope for quality differentiation contributes to sectoral differences in firm size heterogeneity, which to our knowledge has not been shown explicitly before. Sutton (2001) has long argued that firms invest in quality in sectors where there is a higher consumer willingness to pay for quality products. Our framework builds on this idea as we show that in sectors where consumers have higher taste for quality and are willing to reward firms producing higher quality goods, firms will have an incentive to use higher quality inputs and sell higher quality products, at least those firms that can afford to invest in quality. Consequently, the higher the reward of firms for increasing quality, the higher the firm size heterogeneity.

The remainder of this paper is organised as follows. In Section 2, we present the theoretical model of endogenous quality choice. In Section 3, we discuss the empirical methodology and discuss data sources. Section 4 presents the empirical findings and section 5 concludes.

2. Theoretical Framework

In this section, we develop a model that builds on Kugler and Verhoogen (2011), which we augment with the insights of Di Giovanni et al. (2011) and Di Giovanni and Levchenko (2012). Our theoretical framework explicitly results in a new expression, identifying the power law coefficient of firm size distributions, featuring an additional parameter, i.e. the sectoral scope for vertical differentiation.[7] We first show that under an autarky scenario, firm size heterogeneity varies when firms operate in sectors with different scopes to upgrade the quality of their products. Next, we show how the opening to trade interacts with the sectoral scope for quality differentiation increasing firm size heterogeneity. It is important to remember throughout the paper that an increase in firm size heterogeneity corresponds to a decrease in the absolute power law coefficient.

2.1. Power Laws in autarky allowing for vertical differentiation

Consider a world economy that consists of N countries and two types of sectors in each country. Countries have final goods sectors,(FS), that operate under monopolistic competition and intermediate-input sectors (IS) operating under a perfectly competitive environment.

In each country, the utility of a representative consumer is derived from the consumption of a final homogeneous good, $s = 0$, and from various differentiated final goods varieties in sectors $s \geq 1$. The utility of the representative consumer follows a Cobb-Douglas function:

$$U = \prod_{s=0}^{S} \left[X_s^{\beta_s} \right], \; \sum_{s=0}^{S} \beta_s = 1, \; \beta_s \geq 0 \tag{1}$$

where $\beta_s \geq 0$ indicates the sectoral expenditure shares.

In a differentiated sector, preferences follow an asymmetric constant elasticity of substitution (CES) form:

$$U_s = \left[\int_{i \in s} \left[q_{is} \alpha_{is} \right]^{\frac{\sigma_s - 1}{\sigma_s}} di \right]^{\frac{\sigma_s}{\sigma_s - 1}} \tag{2}$$

[7] Throughout this paper, vertical differentiation in terms of product qualities occurs when all consumers agree on the quality ranking of varieties and thus, quality positively affects prices in all destination countries.

where q_{is} is the quantity of the final good of variety i within a differentiated sector s, σ_s is the elasticity of substitution between varieties within a differentiated sector s and α_{is} is the quality of variety i in sector s, which acts as a demand shifter. This means that high quality products, everything else constant, increases the utility of the representative consumer. Given that utility follows a Cobb-Douglas utility function, consumers exogenously spend $I_s = \beta_s I$ on goods produced by sector s, where I is the total income of a country. Our theoretical framework can be interpreted as describing an individual differentiated sector s. Hence, given that all sectoral variables refer to sector s, we omit the subscript s from this point onward.

The representative consumer within a final differentiated sector s maximises its utility following:

$$U = \left[\int_{i\in s} [q_i\alpha_i]^{\frac{\sigma-1}{\sigma}}\, di\right]^{\frac{\sigma}{\sigma-1}} s.t. \int_{i\in s} p_i q_i di = I \tag{3}$$

where q_i is consumption of a variety i in a final differentiated sector, $\sigma > 1$ is the sectoral elasticity of substitution, p_i is the price of this variety i within a differentiated final sector and I is the total expenditure in sector s.

As is well-known in the trade literature, the final demand for an individual variety i is given by:

$$q_i = \frac{I\ \alpha_i^{\sigma\, -\, 1}\ p_i^{-\sigma}}{P^{1-\sigma}} \tag{4}$$

where $P = \left[\int_{i\in s} \left[\frac{p_i}{\alpha_i}\right]^{1-\sigma} di\right]^{\frac{1}{1-\sigma}}$ is the price index adjusted by quality.

From Equation 4, we see that demand for a particular variety i is an increasing function of its quality and the total expenditure, and a decreasing function of its price and the market concentration as captured by the price index.

In the intermediate sector, each producer operates in a perfectly competitive environment where it transforms homogeneous inputs into heterogeneous inputs in terms of quality. Producers are subject to an input bundle price ρ, which for simplicity is normalised to one. For example, if labour is one factor of production, then ρ is equal to wages w.

Therefore, the production function in the intermediate sector (IS) is given by:

$$Y^{IS} = \frac{a}{\mu} \tag{5}$$

where a is the amount of homogeneous labour inputs, i.e. the number of labour-hours used, and μ is the quality of the intermediate sector unit. More precisely, quality can be defined as $\mu = (z\ \rho)$ where z is a subset of a. This means that producing one unit of an intermediate input Y^{IS} of quality μ requires z number of inputs of labour at a given price of ρ. Therefore, μ will be the cost of producing one unit in the intermediate sector.

Given the assumption that producers in the intermediate sector operate in a perfectly competitive market under free entry condition, we can obtain the following equilibrium price:

$$p^{IS} = \mu \tag{6}$$

where p^{IS} is the price of the intermediate sector input and μ is the cost of producing one unit. The importance of this equation is that there is a linear relationship between the quality of the intermediate input and the price of the input.

Final producers operate under a monopolistic competition market structure. The production function in the final good sector is given by:

$$Y_i^{FS} = n\varphi_i \tag{7}$$

where n is the number of intermediate inputs used and φ_i reflects the capability draw obtained from a Pareto distribution, $Pr\left(\varphi_i \geq \varphi\right) = \left[\frac{\varphi_{min}}{\varphi}\right]^{\theta}$, where θ measures the exogenous dispersion of firm capabilities. The production function of the final sector indicates that $\frac{1}{\varphi_i}$ captures productivity, as it is the inputs required to produce one unit of output. Therefore, given that the variable cost is expressed as $\frac{p_{i,}^{IS}}{\varphi_i}$, we observe that higher firm capability φ_i lowers variable cost.[8]

[8]Note that firms and their respective varieties are both indexed with the subscript i. This comes from the assumption that each of the n firms, within a sector s, offers a single product variety.

As in Kugler and Verhoogen (2011), the quality of the final good α_i depends on different combination of firms' capability draw φ_i and the input quality μ.

$$\alpha_i = \left[\frac{1}{2} \left(\varphi_i^{\psi} \right)^{\delta} + \frac{1}{2} \left(\mu^2 \right)^{\delta} \right]^{\frac{1}{\delta}} \tag{8}$$

where capability and input quality of firms are assumed to be complements in generating output quality, given by $\delta < 0$, suggesting that we cannot substitute either factor for the other. The complementarity between capability and input quality ensures that quality for a given increase in input quality is greater for more capable firms. The variable of interest of this paper is the scope for quality/vertical differentiation, represented by ψ. In other words, it is the return that firms obtain by improving the demand-side element α_i. A high ψ will be observed in a highly vertical differentiated sector as firms have an incentive to invest in improving quality. This intuition is based on the idea of Sutton (2001) who argues that firms will only invest in quality development in sectors where there is consumer willingness to consume quality products.

Similarly to the Melitz (2003) model, firms in the final differentiated sector have to decide whether to serve the domestic market. The outcome of this decision is given by the assumption originating from using a CES world, where each potential entrant can produce a variety with some degree of market power, and the idea that producers incur in both fixed and variable costs of production. Given that firms learn about their capability type at the beginning of the period, they decide whether to serve the domestic market if their profit maximisation results in a positive outcome taking into account the common fixed cost f_d that firms must pay to serve the domestic market.

Therefore, the producer maximises its profits in terms of its output price p_i and the input price p_i^{IS}. The input price of the intermediate sector is equal to the cost of producing one unit, as seen in Equation 6, and it refers to the quality of the intermediate sector unit.

On the one hand, the profit maximising price in the final good sector is equal to:

$$p_i^* = \frac{\sigma}{\sigma - 1} \frac{p_i^{IS}}{\varphi_i} \tag{9}$$

where higher firm capability φ_i affects the price negatively in the final good sector p_i.[9]

On the other hand, the profit maximizing input price or equivalently quality of the intermediate sector unit is given by:

$$p_i^{IS*} = \mu_i^* = \varphi_i^{\frac{\psi}{2}} \tag{10}$$

where higher capability firms φ_i incur in higher quality inputs μ_i and consequently, higher priced intermediate inputs p_i^{IS}. The degree to which capability affects the quality choice of inputs is given by the scope for vertical differentiation in a given sector ψ. This means that we expect to see more sophisticated and higher quality inputs the higher the scope of vertical differentiation observed. Hence, higher firm capability has a negative effect on the price of the final good sector and a positive effect on the price of the intermediate input.We follow Khandelwal (2010) and assume that technology developments do not change the range of potential qualities within a product market in the short and medium perspective. [10]

The firm's domestic revenues are given by:

$$r^*(\varphi_i) = \frac{I}{P^{1-\sigma}} \left[\frac{\sigma}{\sigma-1}\right]^{1-\sigma} \varphi_i^{(\sigma-1)\left(\frac{1}{2}\psi+1\right)} = D_m A_i \tag{11}$$

where total domestic revenues are affected by two components. The first element, $D_m = \frac{I}{P^{1-\sigma}}\left[\frac{\sigma}{\sigma-1}\right]^{1-\sigma}$, is common to all firms and it can be interpreted as the size of the domestic market m. It implies that domestic sales will increase with higher aggregate consumption $\frac{I}{P}$ and a higher price index adjusted by quality P^{σ}. The second element is firm's specific and it is a function of the capability of firms, $A_i = \varphi_i^{(\sigma-1)\left(\frac{1}{2}\psi+1\right)}$. Therefore, Equation 11

[9] This is a simplification of reality given that it excludes the idea that mark-ups vary across firms. Kugler and Verhoogen (2008) and Hallak and Sivadasan (2009) suggested that firms of high quality goods, while using high quality inputs, can charge higher mark-ups. De Loecker and Warzynski (2012) found evidence of higher mark-ups in exporters compared to non-exporters.

[10] This model follows most of the literature on firm size distributions where market conditions are given, i.e. the scope of vertical differentiation in a sector is given for any sector. We set aside the notion that the scope of vertical differentiation can be affected by an increase in sector-level competition as assumed by Grossman and Helpman (1991), Sutton (1996) or Sutton (2007) .

implies that highly capable firms have higher revenues, and this effect is enhanced by a higher sectoral scope for vertical differentiation ψ.[11]

In line with the trade literature, we also assume that a firm will only operate in the domestic market if variable profits for a given firm's capability high φ_i exceeds the domestic fixed cost $\frac{r*(\varphi_i)}{\sigma} > f_d$, which defines the minimum size of domestic firms. Therefore, given that firm size is equivalent to total revenues $s_i = r^*(\varphi_i)$ and the assumption that the capabilities of firms are drawn from a Pareto distribution, $Pr(\varphi_i \geq \varphi) = \left[\frac{\varphi_{min}}{\varphi}\right]^{\theta}$ where $\varphi > \varphi_{min}$ and θ is exogenously determined, we get an expression indicating that firm size follows a power law distribution as seen below:

$$Pr[s_i \geq s] = Pr[D_m A_i \geq s] = Pr\left[\varphi_i \geq \left[\frac{s}{D_m}\right]^{\frac{1}{(\sigma-1)\left(\frac{1}{2}\psi+1\right)}}\right]$$

Assuming that the capabilities of firms are drawn from a Pareto distribution $Pr(\varphi_i \geq \varphi) = \left[\frac{\varphi_{min}}{\varphi}\right]^{\theta}$:

$$Pr[s_i \geq s] = \left[\varphi_{min}^{(\sigma-1)\left(\frac{1}{2}\psi+1\right)} D_m\right]^{\frac{\theta}{(\sigma-1)\left(\frac{1}{2}\psi+1\right)}} s^{-\frac{\theta}{(\sigma-1)\left(\frac{1}{2}\psi+1\right)}} \tag{12}$$

where $Pr[s_i \geq s] = C\ s^{-\zeta}$ follows a power law distribution.

More specifically, $C = \left[\varphi_{min}^{(\sigma-1)\left(\frac{1}{2}\psi+1\right)} D_m\right]^{\frac{\theta}{(\sigma-1)\left(\frac{1}{2}\psi+1\right)}}$ is a constant and the power law coefficient of a cumulative distribution function is given by:

$$\zeta = \left(\frac{\theta}{\sigma-1}\right)\left(\frac{1}{\boldsymbol{\frac{1}{2}\psi+1}}\right) \tag{13}$$

where a lower absolute power law coefficient ζ indicates a higher firm heterogeneity in terms of the dispersion in the firm size distribution.

As in Melitz (2003), Helpman et al. (2004) or Di Giovanni et al. (2011) the power law coefficient is affected by the dispersion of the capability of firms captured θ and the elasticity of substitution σ, as observed by the first expression of Equation 13. A high elasticity of

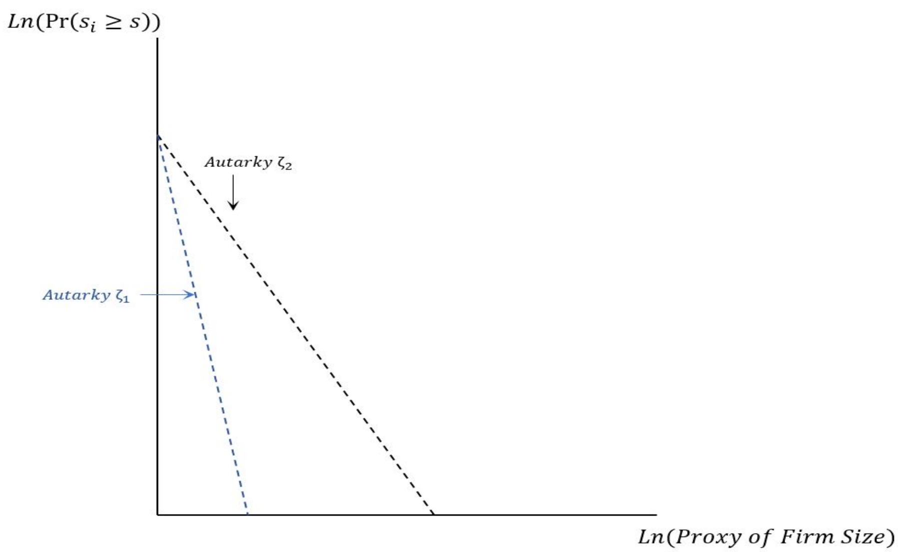
[11] The first derivative of Equation 11 is positive for $\varphi_i > 1$ and it increases with higher levels of firm capabilities φ_i. $\left\{\frac{dr^*(\varphi_i)}{d\psi} = \frac{Y}{P^{1-\sigma}}\left[\frac{\sigma}{\sigma-1}\right]^{1-\sigma}\left[\frac{\sigma-1}{2}\right] ln(\varphi_i)\,\varphi_i^{(\sigma-1)\left(\frac{1}{2}\psi+1\right)}\right\}$

substitution σ indicates that taste is not particularly important as consumers switch easily among products, indicating a low horizontal differentiation. In other words, horizontal differentiation is defined as the inverse of the elasticity of substitution.

Complementing the previous papers, we explicitly show that a higher scope of vertical differentiation ψ also increases the firm size heterogeneity in a particular sector, as observed by the second expression of Equation 13. For example, assuming that the scope of vertical differentiation ψ varies across sectors, $\psi_2 > \psi_1$ where $s = (1, 2)$, we expect to see, *ceteris paribus,* the absolute power law coefficient of a higher differentiated sector to be lower than a sector with a lower scope for vertical differentiation, $\zeta_2 = \frac{\theta}{(\sigma_s - 1)\left(\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{\psi_2}+\mathbf{1}\right)} < \zeta_1 = \frac{\theta}{(\sigma_s - 1)\left(\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{\psi_1}+\mathbf{1}\right)}$.

Figure 1 shows that different scopes of vertical differentiation ψ results in different power law coefficients in a closed economy.

Figure 1: Power Laws in Autarky



Note: This illustration represents the firm size distributions under an autarky scenario. In this illustration, we see that higher levels of scopes of vertical differentiation ψ result in lower estimates of power law coefficients (lower slopes). In this example, we assume that sectors are characterised by two levels of vertical differentiation $\psi_2 > \psi_1$, which results in the following power law coefficients $\zeta_2 = \frac{\theta}{(\sigma_s - 1)\left(\frac{1}{2}\psi_2+1\right)} < \zeta_1 = \frac{\theta}{(\sigma_s - 1)\left(\frac{1}{2}\psi_1+1\right)}$.

2.2. Power laws and vertical differentiation when trade opens

In this section, we show that similarly to Di Giovanni et al. (2011), we find that the power law coefficient, ζ , tilts to the right when we allow for foreign trade. This is due to the idea that the entry of firms into foreign markets increases the revenues of firms progressively. However, what is new here is the additional effect on firm size distributions coming from the scope of vertical differentiation, ψ. In line with the trade literature, we assume that firm i must pay a fixed cost f_z^m to start exporting from the domestic market m to a foreign market z, and we introduce an additional variable trade cost, also known as an iceberg cost $\tau_z^m > 1$.

The final producer in market m maximises its foreign profit in terms of its final price in the foreign market z, $p_{i,z}$, and in terms of the quality of its intermediate inputs given by $p_{i,z}^{IS} = \mu_i$.

The profit maximizing price in market z is given by:

$$p_{i,z}^{*} = \frac{\sigma}{\sigma - 1}\tau_z^m \left(\frac{p_{i,z}^{IS}}{\varphi_i}\right) \tag{14}$$

where the profit maximizing price in market z depends on the constant mark-up $\frac{\sigma}{\sigma-1}$ and the marginal cost, including the so-called iceberg cost, $\tau_z^m \frac{p_{i,z}^{(IS)}}{\varphi_i}$.[12]

As in the autarky scenario, the profit maximizing input price or quality of the intermediate sector unit is given by:

$$p_{i,z}^{IS*} = \mu_i^* = \varphi_i^{\frac{\psi}{2}} \tag{15}$$

where higher capability firms φ_i also incur in higher quality inputs μ_i and consequently, a higher price of intermediate inputs $p_{i,z}^{IS}$. Once again, the degree to which capability affects the quality choice of inputs is given by the scope for vertical differentiation ψ and the

[12] The constant mark-ups in the foreign price excludes the idea that mark-ups could vary by destination. Some papers such as Aw-Roberts et al. (2018) relaxed this assumption and use data with destination specific information to allow mark-ups to vary by destination.

choice of input price/quality is not affected by trade openness.[13]

The revenues obtained in the foreign market z are given by:

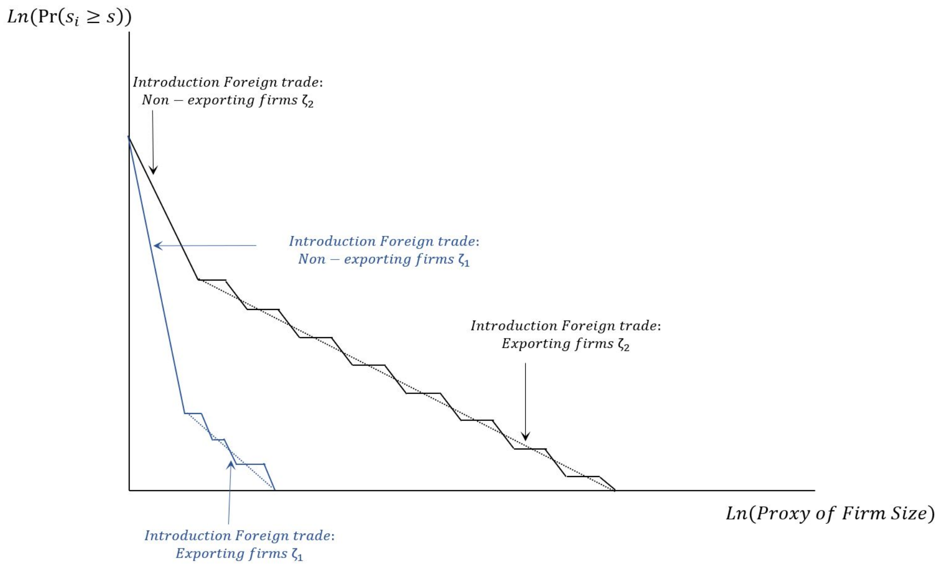
$$r_z^*(\varphi_i) = \frac{I_z}{P_z^{1-\sigma}} \left[\frac{\sigma}{\sigma-1}\right]^{1-\sigma} \tau_z^{m(1-\sigma)} \varphi_i^{(\sigma-1)\left(\frac{1}{2}\psi+1\right)} = D_z A_i \tag{16}$$

where total foreign revenues are affected by two components. The first component, $D_z = \frac{I_z}{P_z^{1-\sigma}} \left[\frac{\sigma}{\sigma-1}\right]^{1-\sigma} \tau_z^{m(1-\sigma)}$, is common to all firms in market m and can be interpreted as the size of the foreign market z and a second component which is firm's specific $A_i = \varphi_i^{(\sigma-1)\left(\frac{1}{2}\psi+1\right)}$. Once again, Equation 16 implies that high capable firms affect foreign revenues positively, and this effect is enhanced by a higher sectoral scope for vertical differentiation ψ.

Figure 2 summarises the results of our theoretical framework, which we document for two sectors with different scope of vertical differentiation, $\psi_2 > \psi_1$, as in the previous example. Once we allow for international trade, the power law in firm size distribution tilts for each sector. The reason is that participation in export markets results in multiple parallel shifts in the power law cumulative distribution function, one for each foreign market that a certain number of firms export to. Figure 2 clearly illustrates that with a higher sectoral scope of vertical differentiation in a sector, the difference between the two lines is exacerbated, i.e. the power law coefficients become much more different across the two industries. Thus, Figure 2 shows the scope of vertical differentiation plays an important role in explaining firm size heterogeneity, especially in the right-hand tail of the firm size distribution.

[13] We assume that opening to trade does not affect the import behaviour of firms. However, we acknowledge that a separate strand of trade literature has shown that offshoring, i.e. importing intermediate inputs, could lead to firms adjusting their behaviour, increasing in some cases the productivity of firms (see for instance, Amiti and Konings (2007), Goldberg et al. (2010) or Halpern et al. (2015)). That being said, the effect of opening to trade on the dispersion of the firms' size dispersion would only be enhanced if the effect of offshoring is taken into account. Consequently, our framework can be seen as an underestimation of the argument that opening to trade increases the dispersion of the firms' size distribution.

Figure 2: Power Law with multiple export markets



Note: This illustration represents the firm size distributions once we allow for foreign trade. This illustration shows once we allow for foreign trade, the dispersion of the firms' size increases even further for sectors observing higher levels of vertical differentiation. This mechanism can be seen by looking at the distributions of sector 2 (see the black line) and sector 1 (see the blue line).

In the next section, we set out to see whether the scope for vertical differentiation is a determinant of firm size heterogeneity as predicted by this theoretical framework. In this framework, both trade openness and quality differentiation operate in the same direction, i.e. they both increase firm size heterogeneity within an industry. Empirically, we can disentangle the importance of the two by using regression analysis, where we include measures of trade openness alongside with sectoral measures of vertical differentiation. Our findings clearly indicate that after controlling for the elasticity of substitution, trade openness and country fixed characteristics, the scope for vertical differentiation significantly and additionally reinforces firm size dispersion within an industry.

3. Implementation Framework and Data Sources

3.1. Power Law Coefficients

We use data from the reported unconsolidated company accounts of manufacturing firms, which is commercialised under the ORBIS database, to estimate sectoral power law coefficients within 2-digit manufacturing sub-sectors.[14] ORBIS is one of the most comprehensive and comparable firm level databases containing firm characteristics measuring firm size.[15] In this analysis, we define each individual firm by their unique VAT national number,[16] and we use information on the ten largest countries in the European Union (EU) in terms of GDP, i.e. Germany, France, Great Britain, Italy, Spain, Netherlands, Sweden, Poland, Belgium and Austria.[17] The limited number of firms observed in sectors of relatively smaller countries result in an insufficient number of observations to obtain unbiased estimates of the power law coefficient at country-sector level. For the countries included in our analysis, we evaluate the coverage of the ORBIS database with the Eurostat Structural Business

[14]We classify a firm's sector with their primary reported 2-digit Classification of Products by Activity (CPA). See Table 2 in the Appendix for more details on the 2-digit CPA (2008 version) aggregates considered in this analysis.

[15]Following the theoretically framework, we would only consider information of firms producing final goods. However, the ORBIS database does not allow us to restrict our sample to firms producing only final goods. In this analysis, we assume that all firms produce goods to the final consumer.

[16]Using the unique VAT national number to identify a firm is closer to our single product firm framework and it is consistent with the majority of the empirical firm level literature in international trade. However, while this approach allows us to test the main predictions of our theoretical framework, we acknowledge that an alternative approach could rely on consolidated accounts. This alternative approach will deal with the limitation that many firms identified by a unique VAT number could belong to the same group, potentially indicating that the true firm size distributions will be even more skewed than the estimates used throughout this paper. Moreover, we use national unique VAT numbers without considering foreign affiliates. Limitations regarding missing firm level data across countries prevents us from using the total distribution of firms without considering national boarders in a complete manner.

[17]The selection of these countries is done in terms of their size given that power law estimates require a minimum number of observations within a sector to be unbiased. The choice to focus on European Member States is given by the higher regulatory requirements faced by European firms to report most of the balance sheet variables, which makes firm coverage and comparability superior to many other regions.

Statistics (SBS) aggregate statistics across different firm size groups in order to evaluate the ORBIS representation in the sub-sample being analysed. As an illustration, Table 1 compares the coverage in the manufacturing sector of the ORBIS and Eurostat database in 2013.[18] Column 1 lists the total number of firms in ORBIS that report one of the three measures of firm size, i.e. "Total Assets", "Total Employment" or "Total Turnover", expressed as a percentage of the total number of firms reported in Eurostat. Column 2 and 3 restrict the comparison between ORBIS and Eurostat to the total number of firms reporting "Total Employment" and "Total Assets", respectively.[19] Column 4 restricts the sample to medium and large enterprises and compares the two data sources in terms of the number of firms reporting "Total Employment". Column 5 makes the comparison of the two data sources in terms of employed people reported by medium and large enterprises.

Using "Total Employment" as a measure of firm size, we observe that firm coverage in ORBIS is more representative of the true sample for medium and large enterprises. We cannot compare the coverage using "Total Assets" in ORBIS as Eurostat does not provide information distinguishing between firms size categories. For this reason, we rely mostly on "Total Employment" as our preferred measure of firm size, and we restrict our sample to medium and large size enterprises using the European Commission definition of medium-sized and large companies.[20] Alternatively, "Total Assets" as a measure of firm size will be used as a robustness check. Given that our theoretical model predicts that differences in firm size heterogeneity will be stronger in the right-hand side tail of the firm size distribution due to the export activity of some firms, we will focus on the medium and large sized firms.

[18]In this analysis, we use the 2016 ORBIS historic disk. Researchers should not take the last two years reported as there is a reporting lag of financial data when using historical data from ORBIS. Given that we consider the period between 2009 and 2014, firm coverage does not change significantly if we were to change the reference year.

[19]In accordance with accounting rules, firms are not obliged to report all financial and operational items. Given these accounting protocols, "Total Employment" and "Total Asset" (including intangible, tangible, financial, and current assets) have a particularly high coverage.

[20]The European Commission defines medium-sized and large companies as those reporting higher or equal than 50 employees or higher than 10 million euros in terms of total assets. (Small and medium-sized enterprises (SMEs) are defined in the EU recommendation 2003/361.)

This is supported by the empirical finding by Mayer and Ottaviano (2008) who observed that exporting is more likely to occur among this medium and large sized firms.[21]

Our theoretical framework indicates that firm size distributions are characterised by power law distributions. The properties of this type of distribution are particular relevant in our analysis, as these distributions are used when small values are extremely common and large values are extremely rare. In the case of firm's size measures, the majority of the distribution occurs for small size firms, however, there are a small number of firms with sizes much higher than the typical value. This produces the long tail to the right of a histogram. A power law distribution can be represented by a power law function $p(x) = Cx^{-\alpha}$, where x is the positive variable of interest, such as a proxy of firm size, α is the power law exponent that determines the shape of the distribution, C is a constant and $p(x)$ is the probability of observing the value x.[22] Consequently, the probability of measuring a particular value of some quantities varies inversely as a power of that value. While in our theoretical framework, firms size distributions follow a power law as a result of combining a Pareto distribution of firm capabilities with a CES preferences on the consumer side,[23] this functional form is also supported by previous empirical evidence.[24] Axtell (2001) was the first to approximate firm size distributions with a power law function. Using data on the entire population of taxable US firms, he provides evidence that sales of firms follow a power law distribution. Thereafter, many other authors have used this form of distribution when measuring the size of firms. For instance, Helpman et al. (2004),

[21] The ORBIS database provides several measures of firm size but it does not report exports at firm-level.

[22] Newman (2005) shows that a power law distribution has the particularity to be the only distribution to be scale-free.

[23] See Mrázová et al. (2017) for a detailed discussion on the outcomes of using different functional forms in the demand and supply side.

[24] Although some recent researches such as Head et al. (2014) have observed that the log-normal distribution provides a better description of the distribution of firms size, they also find an important overlap with the power law distribution (see Mrázová et al. (2017)). Considering this overlap between distributions, it is important to note that the law of proportionate growth proposed by Gibrat (1931) is shown to give rise to a distribution characterised by a log-normal (see Sutton (1997) for more details on the Gibrat's legacy).

Fujiwara et al. (2004), Zhang et al. (2009), Di Giovanni et al. (2011), Bonfiglioli et al. (2017) have found evidence of power law distributions in various countries and sectors using various firm size proxies such as the employees, revenues, sales, assets or net worth.[25] Consequently, empirical evidence supports the validity of the outcome of our theoretical framework, which suggests that firm size distributions are observed to follow a power law function.

Previous empirical analysis has used various statistical techniques to obtain estimates of the power law coefficient. To our knowledge, most of the previous studies in economics dealing with firm size distributions have based their analysis on graphical methods. This refers to the idea that power law coefficients can be obtained making use of the definition of the power law. This method uses the idea that if the distribution follows a power law function, once plotted on logarithmic scales, it will produce a characteristic straight-line. Ordinary Least Squares (OLS) is then used to calculate the power law exponent. Among others, Axtell (2001) used the cumulative distribution function of firms in the US to obtain the power law coefficient. On the other hand, Helpman et al. (2004) used the ranked distribution to obtain the power law estimator, whereas Di Giovanni et al. (2011) obtained the power law exponents using the cumulative, density and ranked distribution. Appendix 5 shows the formal derivation and the correspondence between the exponents obtained using these three different distribution functions.

Several studies have started to question the validity of the so-called graphical methods using OLS fitting. Goldstein et al. (2004), Bauke (2007), Clauset et al. (2009) stated that the graphical methods can lead to bias estimates of parameters for power-law distributions. To justify their argument, they use a numerical experiment using synthetic power-law data in order to compare the fitted coefficient with the estimates obtained using OLS. In these papers, the density and the cumulative distribution functions are estimated using constant

[25] Gabaix (2016) shows that type of distribution function is not unique to economics and he details topics in economics and elsewhere where power law distributions have been observed. These include the size of firms, size of cities, stock market returns, income and wealth, the popularity of websites or the usage of words in a text among others.

and exponentially increasing size bins. This alternative estimation method, referred to as "logarithmic binning", normalises the sample counts by the width of the bin they fall into, in order to get a count per unit interval. The bins in the tail of the distribution get more observations than they would if bins sizes were fixed, which consequently reduces the statistical errors in the tails despite not respecting one of the assumptions for using OLS. Bauke (2007) shows that the standard error is increasing with higher values of our variable of interest, violating the OLS assumption that error terms should be independent identically distributed (iid) and these should not be correlated for all the data points. In addition, methods using the cumulative distribution violate the assumption that statistical errors of the dependent variable should be independent for higher values of our variable of interest. Since the cumulative distribution function is adding up to 1 (towards 0 in the case of the Counter Cumulative Distribution (CCDF)) error terms are not independent across all the observations.

Alternatively, Goldstein et al. (2004), Bauke (2007), Clauset et al. (2009) proposed the Maximum Likelihood Estimator (MLE) as a way to obtain the power law exponents. Using an equivalent numerical experiment with synthetic power-law data, they argue that the MLE estimates were unbiased and asymptotically efficient considering that the sample size is large enough.[26] Appendix 5 shows the formal derivation of the maximum likelihood estimation in the context of power law coefficient estimations.

To test the implications of the theoretical framework, we use two methods to estimate power law coefficients, namely the MLE and the CCDF with logarithmic binning. We observe a correlation of coefficients between these two estimating techniques of 0.73 when "Total Employment" is used as a proxy of firm size and 0.90 when "Total Assets" is used. An important observation is the significant variation in the power law coefficients across sectors, which is a fundamental empirical observation to test the main predictions of our theoretical framework. Table 3 in the Appendix shows the average sectoral power law coefficient across the countries considered in this analysis in 2012 using the two estimating

[26] Clauset et al. (2009) argue that biased estimates can be present in small databases of sample sizes lower than 50 observations.

techniques.

To test the validity of the assumption that firm size distributions are characterised by a power law distribution, we use the Kolmogorov-Smirnov test to assess the equality of two distributions. This assesses the hypothesis that the coefficient obtained from assuming a power law function does not differ significantly from a theoretical fitted distribution. In other words, we evaluate the maximum distance between the CDF's of the actual data and the fitted model. Given that the null-hypothesis of the Kolmogorov-Smirnov test is the equality of the distributions, if the null-hypothesis is not rejected at a 5% significance level, it implies that we cannot reject the hypothesis that the estimated and the fitted power law distributions are equal. Using the predicted power law coefficients using "Total Employment" ("Total Assets") as a proxy of firm size, we observe that we cannot reject the hypothesis of equality between firm size distributions in 75% (69%) of the cases in terms of the coefficients estimated by Maximum Likelihood (MLE) and 67% (67%) when the coefficients are estimated using CCDF with logarithmic binning. This empirical evidence justifies the validity of the assumption that the majority of the distributions of firms' size follow a power law distribution in our sample of firms.

3.2. Measuring the Scope of Vertical and Horizontal Differentiation

To capture the different levels of scope for vertical differentiation across countries and sectors, we are going to rely on various indicators.

The first indicator uses the world dispersion of export unit prices (FOB). We use our theoretical mechanism to justify this measure of vertical differentiation. From Equation 9 and Equation 10, we see that the extent to which firm capability affects the quality choice of inputs is given by the scope of vertical differentiation in each country-sector pair, given that we assume a constant mark-up across firms within each sector. Hence, we expect to see that, *ceteris paribus*, a higher predetermined scope for vertical differentiation leads to higher quality of inputs, affecting positively the dispersion of final prices in the final sector.

However, using price information at a firm level will be affected both by the scope of vertical differentiation and the underlying distribution of firm productivity in each sector. In other to avoid that the dispersion of prices is affected by the underlying firm productivity, we are going to use the dispersion in export unit prices (FOB) at a 6-digit Harmonized System (HS) product level considering each origin-destination country pair in the world.

This indicator of vertical differentiation is based on the assumption that different countries produce unique varieties within the 6-digit HS product level. Moreover, it is also based on three empirical observations, i.e. the average quality of these varieties changes across destination markets, high income countries produce higher quality goods than low-income countries, and quality of exports vary across destinations.[27] Relying on this empirical evidence, we expect to see a lower (higher) variation in unit prices when the scope for vertical differentiation of a given product is low (high). Using unit values as a measurement of quality differences has been widely used in international trade literature. Among others, Schott (2004) uses unit values to show that the per capital income and relative factor endowments of a country affect the quality of exports. Hallak (2010) uses unit values to predict the role that quality plays as a determinant of bilateral trade, and Harding and Javorcik (2012) show evidence that foreign direct investment (FDI) upgrades the quality of exports, measured in unit values.

We rely on the BACI database to obtain our measure of vertical differentiation. This database harmonises annual bilateral trade in values (FOB) and volumes for more than 200 countries with a level of product disaggregation up to 6-digit HS. Consequently, BACI contains information to compute unit values of exports, which enables us to obtain the world dispersion of unit prices (FOB) within a given 6-digit HS product level across origin-destination pairs in the world.[28] In this paper, the dispersion of unit prices is measured by the coefficient of variation. This unit-free measure of volatility is defined as the ratio of the standard deviation and the mean of the different product unit prices. In this context, the coefficient of variation is used to capture the variability in unit prices relative to their average worldwide unit price. The unit value of a product k in country c towards a specific

[27] See Schott (2004), Hummels and Klenow (2005), Fajgelbaum et al. (2011) and Manova and Zhang (2012) for more details.

[28] See Gaulier and Zignago (2010) for more details on the database.

destination is obtained using the ratio of export value and quantity to a specific destination. Measuring unit price at an aggregate level could capture the composition structure of this product instead of the average price. To minimise the error originating from the composition of a given aggregated product, we use the lowest level of product disaggregation available in the BACI database. Consequently, we compute the vertical differentiation for 5,046 6-digit HS products, which are common across countries. Table 4 shows examples of products observed as being the three most and least vertical differentiated products according to their coefficient of variation in their worldwide unit prices. The left-hand side column shows the three products with the highest degree of vertical differentiation and as expected, these products refer to elaborated manufacturing products such as refrigerators or televisions. On the other hand, the column on the right-hand side shows less elaborated manufacturing products such as daily products or basic metals.

While the previous measure is consistent with our theoretical framework, using unit values as a proxy for quality differentiation excludes the possibility that price may capture other factors other than quality, as proposed by some studies in the international trade literature. For instance, Khandelwal (2010) suggested that unit values contain both vertical and horizontal attributes.[29] For example, he suggested that within the clothes sector, some consumers might give more value to a horizontal attribute such as the style than to a vertical attribute such as the comfort. Using US import data, he measured the quality of a variety using the assumption that an increase in the quality of the variety allows its price to increase without losing market share. From these quality measures, he estimated the scope for vertical differentiation using the range of the estimated qualities within a product in what he labelled as "quality ladders". To test the hypothesis that firm heterogeneity is positively affected by the scope for vertical differentiation, we are going to use these "quality ladders" at a product level common to all countries as a second measure of vertical differentiation.[30]

[29]Additional factors identified as capturing unit prices are production costs or undervalued exchange rates as proposed by Hallak and Schott (2011).

[30]The RAMON correspondence tables are used to find the equivalence between HS92 and HS07.

A third measure of sectoral scope for quality differentiation is proposed by Sutton (2001) and Kugler and Verhoogen (2011). They use sectoral R&D intensity as a measure of vertical differentiation, as they argue that firms will only invest in R&D in sectors where there is consumer willingness to consume quality products and it is feasible for firms to affect quality. For this measure, we use the ratio of sectoral R&D expenditure relative to total turnover as obtained by Eurostat.[31] The main limitation of this proxy is that we cannot distinguish between product and process innovation. Intuitively, product innovation would capture the part aimed at improving the quality of the product, whereas process innovation aims at reducing the costs of producing. Moreover, although this measure of scope for vertical differentiation is not consistent with our theoretical framework, given that we assume that increasing quality is obtained by higher variable costs in the form of higher quality input, this proxy is used in the empirical analysis for completeness.

As Tang and Zhang (2012), we control for horizontal differentiation using the elasticity of substitution between varieties σ as provided by Broda and Weinstein (2006). A high elasticity of substitution indicates a low horizontal differentiation as varieties are easily substituted. Consequently, sectoral horizontal differentiation is measured as the inverse of the elasticity of substitution between varieties σ. Using varieties imported into the US, Broda and Weinstein (2006) estimate product-specific elasticities of substitutions between varieties at the 6-digit HS level.[32]

Since the power law coefficient is at a sectoral level, the sectoral vertical differentiation and the elasticity substitution across country-sector pairs are obtained using the weighted average of the 6-digit HS product level of the scope for vertical differentiation and the elasticity of substitution.

$$\psi_{cs} = \sum_{k \in cs}^{K} w_{csk} \psi_k \tag{17}$$

[31] Information on total R&D by economic activity (NACE Rev. 2) is collected from "Business enterprise R&D expenditure (BERD)".

[32] Broda and Weinstein (2006) report estimates of sigma using various aggregation levels. In this analysis we use the SITC Rev.3 5-digit. The RAMON correspondence tables are used to find the equivalence between SITC Rev.3 5-digit and 6-digit HS07.

$$\sigma_{cs} = \sum_{k \in cs}^{K} w_{csk} \sigma_k \tag{18}$$

where the weights w_{csk} are country-sector-product specific, computed as the export share (in values) of product k within the total world exports of a given country-sector pair cs. We use weights based on the initial year 2009 as the measures of vertical and horizontal differentiation are assumed to be time invariant. As previously mentioned, the measurement of vertical and horizontal differentiation are at a product level k. As previously mentioned, in this paper a sector s is described as a 2-digit CPA aggregate.[33]

3.3. Trade openness

The fourth important element to test our theoretical framework is a measure of sectoral trade openness. This measure should be exogenous to the distribution of the country-sector underlying productivity. Taking into account this condition, we use a proxy of the sectoral transport simplicity to export, measured by the value shipped per dollars of transport cost. This measure is obtained by dividing the "Value of Exports" by the "import charges", which captures the sum of all freight and insurance costs, excluding US import duties. Using this proxy of sectoral trade cost, we capture cost determinants including the sector's average product weight or transport complexity, which affects exogenously the sectoral trade openness. The underlying idea is that the higher the value shipped per dollar of cost the simpler it is for firms within a sector to engage in exports.

A public database that enables the calculation of sectoral export cost intensities is the US International Trade Commission.[34] To obtain this measure of trade cost, we use the aggregate exports from the EU-28 to the US. Using these trading partners enables us to obtain information on a wide range of products with enough variation in transport cost across products, given the size and the geographical distance between the EU-28 and

[33] RAMON correspondence tables are used to find the equivalence between the HS 2007 level and the 2-digit CPA (version 2008).

[34] An additional public database that can be used to obtain trade costs at a product level is the "International Transport and Insurance Cost of Merchandise Trade (ITIC)" by the The Organisation for Economic Co-operation and Development (OECD).

the US. Using the above information, we estimate a yearly sectoral trade cost using the 2-digit CPA 2008 aggregate.[35] We keep the year dimension given that trade costs can vary exogenously in a short period of time. For instance, fluctuations of world oil price can affect transport costs across sectors.[36] Table 5 shows the summary statistics of the different elements described above.

4. Empirical Implementation

To test our theoretical framework, we rely on the following specification.

$$\zeta_{cst} = \beta_0 + \beta_1 \psi_{cs} + \beta_2 \sigma_{cs} + \beta_3 T_{st} + \lambda_{ct} + \epsilon_{cst} \tag{19}$$

where ζ_{cst} is the absolute power law coefficient of the firm size distribution estimated at a country c, sector s at a given time t, ψ_{cs} refers to the time invariant measurements of vertical differentiation of the country-sector pairs, σ_{cs} refers to the time invariant sectoral elasticity of substitution of the country-sector pairs, where a high elasticity of substitution indicates a low horizontal differentiation as varieties are easily substituted. In addition, T_{st} refers to the sectoral simplicity to export in year t and λ_{ct} controls for country-year fixed effects in order to control for particular country characteristics that affect the firm size distribution in all sectors in a given country-year pair. This captures country elements that affects the trade openness of all sectors within a country, i.e. the level of GDP per capita, national population, country size, geographical location, infrastructure or strength of institutions. Moreover, these country-year fixed characteristics control for other determinants such as the access to credit or quality of the education that can impact equally the capability dispersion of sectors in a given country. Following the theoretical framework, the empirical implementation assumes that the dispersion of the capabilities of firms is exogenous and

[35] We use the RAMON correspondence tables to find the equivalence between 6-digit NAICS classification and the 2-digit CPA 2008.

[36] In the time frame of this analysis (2009-2014), the average annual OPEC crude oil price went from 60.86 US dollars per barrel to 109.45 dollars in 2012, decreasing to 96.29 dollars in 2014.

consequently, it can be treated as an error term. Given that this omitted variable is uncorrelated with the exogenous explanatory variables described above, we argue that our estimates are unbiased. The standard errors ϵ_{cst} are clustered at a country-sector level in order to avoid that errors for a given country-sector pair could be correlated in different years.

Table 6 and Table 7 present the results of the different econometric specifications using "Total Employment" and "Total Assets" as measures of firm size, respectively. While Columns (1), (2) and (3) in both tables use the Maximum Likelihood Estimator (MLE) to obtain the the power law coefficients, Columns (4), (5) and (6) use the Counter Cumulative Distribution (CCDF) with logarithmic binning. In each column, we control for the different elements identified in Specification 19 but altering the proxies of the sectoral scope for vertical differentiation. Hereof, Column (1) uses the sectoral variation of world export unit prices, Column (2) uses the sectoral "Quality Ladders" taken from Khandelwal (2010), and Column (3) uses the sectoral R&D intensity. As predicted by our theoretical framework, we see that the different proxies of the scope for vertical differentiation and the elasticity of substitution are negatively related to the power law coefficient. Moreover, we see a negative relation between the power law coefficient and the proxy of sectoral simplicity to export, suggesting that sectors where export transport cost is less complex, the size of firms are more dispersed. Column (4), (5) and (6) repeat the same exercise but this time using power law coefficients estimated using CCDF with logarithmic binning. Consequently, we observe that the conclusions are robust to different statistical methods for estimating the power law coefficients. Unsurprisingly, comparing Tables 6 and 7, we find that these results are also robust to different proxies of firm size, i.e. "Total Employment" and "Total Assets".

Given that our dependent variable is estimated, i.e. estimated power law coefficients, we need to correct for a potential heteroscedasticity problem. In other words, we need to deal with the observation that the disturbance variance is not constant across observations, which could yield inefficient estimates of the different estimated parameters. As suggested by Saxonhouse (1976), to correct for this potential problem, we weigh all the variables in the Specification 19 by the inverse of the estimated standard error of the dependent variable for each observation. Tables 8 and 9 show that our results are robust to the potential problem of heteroscedasticity.[37]

In order to evaluate the impact of each element on the measure of firm size heterogeneity, we multiply the coefficients by the standard deviation of the independent variables. As an illustration, we use the coefficients of Columns (1) to (3) as reported in Table 6. Table 10 shows the result of the multiplication of the estimated coefficients by one standard deviation of the respective independent variable, and how the increase of the independent variable impacts the power law coefficient. The negative impact on the independent variable is then expressed as a share of the standard deviation of the power law coefficient. For example, Column (4) of Table 10 shows that an increase of one standard deviation on the scope for vertical differentiation, measured by the unit price coefficient of variation, decreases the absolute power law coefficient by 0.05. This drop represents a decrease of 18% of the standard deviation of the estimated power law coefficients. The same interpretation can be used to understand the magnitude of the different variables reported in the Specification 19.

5. Conclusion

In this paper, we have studied the role of sectoral characteristics in explaining differences in firm size heterogeneity, measured by power law coefficients. Until now, sectoral variations in firm size heterogeneity were attributed to differences in the exogenous dispersion of the productivity of firms, differences in horizontal differentiation and trade openness. We build on this result by additionally examining the role of the sectoral scope for quality differentiation. To illustrate this mechanism, we used a standard firm level model of endogenous input quality choice in order to evaluate the aggregate effect on the firm size heterogeneity under two scenarios of trade openness. Under a closed economy scenario, we showed that there is a positive relation between the scope of vertical differentiation and the firm size heterogeneity. This positive relation is justified by the idea that revenues of

[37] Except R&D of Table 9 where, even though it follows the predicted sign, the coefficient is not statistical significant.

high productive firms are positively affected by the degree of quality differentiation. In a second scenario, we allowed for international trade in order to illustrate how opening to trade affects the dispersion of firm size distributions. In this setting, some of the high productive firms are able to generate foreign revenues by exporting to foreign markets, increasing their firm revenues, and consequently their firm size. We build on this result by showing that the effect of trade openness on the degree of the firm size heterogeneity is intensified by a higher sectoral scope for vertical differentiation. This aggregate outcome is complementary to previous firm level empirical evidence that identified, within a given sector, firms producing quality products as larger, more productive, using more expensive inputs, charging higher prices, earning greater revenues, exporting more and entering a higher number of foreign markets when compared to firms offering low quality products.

To show empirical evidence of the mechanism described above, we began by estimating power law coefficients of firm size distributions using firm level data on a large number of European country-sectors for the time period between 2009 and 2014. Using these estimates, we tested the theoretical framework using different elements theoretically identified as affecting the sectoral level of firm size heterogeneity. For this purpose, we used various proxies measuring the sectoral scope for vertical differentiation, i.e. sectoral coefficient of variation of export unit prices (FOB), sectoral "quality ladders" borrowed from previous economic literature, and sectoral R&D intensities. In addition, estimates of the elasticities of substitution between varieties are taken from previous economic literature so in order to control for sectoral differences in the horizontal differentiation. Moreover, we use a measure of the sectoral simplicity (transport cost) to export as a proxy of the sectoral trade openness. Using a cross-sectional econometric specification, the coefficients from the different specifications were found to be statistically significant and with the expected sign. As predicted, sectors where we observed a higher scope of vertical differentiation, a lower level of horizontal differentiation and lower transport costs tend to experience stronger inequalities in firm size. Moreover, in line with our theoretical framework, the effect of the scope for quality differentiation on the firm size distribution is observed to be reinforced by sector characteristics that facilitate access to international trade. An important implication of this last finding is that, *ceteris paribus*, firms operating in sectors where quality of the product is more appreciated will benefit more from opening up to trade.

Tables

Table 1: Number of Firms: Coverage Relative to Eurostat in the Manufacturing Sector (2013)

Variable	**N-Firms**	**N-Firms**	**N-Firms**	**N-Firms**	**Employment**
Proxy of Firm Size	**Output, Employment, Assets**	**Assets**	**Employment**	**Employment**	**Employment**
Sample	**Total Sample**	**Total Sample**	**Total Sample**	**Medium and Large Firms**	**Medium and Large Firms**
N. of Column	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**
AT	53%	38%	38%	80%	108%
BE	60%	59%	37%	86%	143%
DE	50%	35%	44%	88%	144%
ES	56%	43%	51%	85%	104%
FR	30%	30%	11%	48%	162%
GB	76%	73%	11%	87%	222%
IT	59%	27%	52%	88%	112%
NL	42%	32%	33%	66%	411%
PL	79%	10%	71%	39%	54%
SE	45%	4%	43%	82%	236%

Note: This table compares the coverage of the ORBIS and the Eurostat database. The first column shows the number of total firms captured in our database either reporting "Total Assets", "Employment" or "Turnover" relative to the number of firms reported in Eurostat. The second and third column limits to the share of total firms reporting information on "Total Assets" and "Employment" respectively. The fourth column restricts the comparison to medium and large firms reporting employment data. The fifth column compares the total employment reported by medium and large firms in the two databases.

Table 2: Description of 2-digit CPA Sectors

Code	**Legend: 2-digit CPA 2008**
C10	Manufacture of food products
C11	Manufacture of beverages
C13	Manufacture of textiles
C14	Manufacture of wearing apparel
C15	Manufacture of leather and related products
C16	Manufacture of wood and of products of wood and cork, ... except furniture; manufacture of articles of straw and plaiting materials
C17	Manufacture of paper and paper products
C18	Printing and reproduction of recorded media
C20	Manufacture of chemicals and chemical products
C21	Manufacture of basic pharmaceutical products and pharmaceutical preparations
C22	Manufacture of rubber and plastic products
C23	Manufacture of other non-metallic mineral products
C24	Manufacture of basic metals
C25	Manufacture of fabricated metal products, except machinery and equipment
C26	Manufacture of computer, electronic and optical products
C27	Manufacture of electrical equipment
C28	Manufacture of machinery and equipment n.e.c.,
C29	Manufacture of motor vehicles, trailers and semi-trailers
C30	Manufacture of other transport equipment
C31	Manufacture of furniture
C32	Other manufacturing

Note: This table contains information on the 2-digit Classification of Products by Activity (CPA) version 2008 considered in this analysis

Table 3: Power Law Coefficients (2-digit CPA Sectors)

Legend: 2-digit CPA 2008	**Total Employment**		**Total Assets**	
	MLE	**CCDF-bins**	**MLE**	**CCDF-bins**
Manufacture of food products	1.96	2.09	1.91	1.94
Manufacture of beverages	1.90	1.82	1.82	1.78
Manufacture of textiles	2.15	2.27	2.29	2.32
Manufacture of wearing apparel	1.94	2.13	1.97	1.86
Manufacture of leather and related products	2.17	2.25	2.38	2.17
Manufacture of wood and of products of wood and cork...	2.27	2.27	2.22	2.06
Manufacture of paper and paper products	1.90	2.05	1.79	1.82
Printing and reproduction of recorded media	2.23	2.24	2.29	2.02
Manufacture of chemicals and chemical products	1.91	2.05	1.74	1.80
Manufacture of basic pharmaceutical products ... **... and pharmaceutical preparations**	1.72	1.79	1.58	1.62
Manufacture of rubber and plastic products	2.09	2.19	2.10	2.13
Manufacture of other non-metallic mineral products	1.98	2.07	1.88	1.90
Manufacture of basic metals	1.82	1.89	1.70	1.73
Manufacture of fabricated metal products, except machinery and equipment	2.26	2.37	2.19	2.11
Manufacture of computer, electronic and optical products	1.84	1.86	1.76	1.77
Manufacture of electrical equipment	1.93	1.94	1.86	1.82
Manufacture of machinery and equipment n.e.c.,	1.98	2.08	1.93	1.94
Manufacture of motor vehicles, trailers and semi-trailers	1.69	1.76	1.69	1.69
Manufacture of other transport equipment	1.71	1.69	1.64	1.62
Manufacture of furniture	2.22	2.34	2.45	2.40
Other manufacturing	2.07	2.09	2.05	1.96

Note: This table presents the average sectoral power law coefficient across across our sample for the year 2012 using "Total Employment" and "Total Assets" as proxies of firm size. Maximum Likelihood Estimator (MLE) and a Counter Cumulative Distribution Function (CCDF) with logarithmic bins are used to obtain the different sectoral power law coefficients ($\alpha = \zeta + 1$).

Table 4: Examples of Scope for Vertical Differentiation: 6-digit HS Product Level

Highest Vertical Differentiation	**Lowest Vertical Differentiated**
1. New pneumatic tyres, of kind used on buses or lorries	1. Alkaloids, vegetable
2. Refrigerators and freezers	2. Ferro-alloys; ferro-niobium
3. Transmission apparatus for radio-broadcasting or television	3. Metals; gold, non-monetary
4. Valves; pressure reducing, for pipes, boilers shells, tanks	4. Dairy produce; cheese, blue-veined
5. Shirts; men's or boys', of textile materials	5. Meat and edible meat offal

Note: According to their coefficient of variation in unit prices, this table lists the five most and the least vertical differentiated products of the 5,046 6-digit HS reported in the BACI database. On the left-hand side column, we see that products refer mainly to elaborated manufacturing products such as refrigerators or televisions, whereas the right-hand side column refers to less elaborated products such as daily products or basic metals. The full list of products can be provided upon request.

Table 5: Summary Statistics

	Observations	**Mean**	**Std. Dev**	**Minimum**	**Maximum**
Power Law Coefficient (MLE), Total Employment	1,150	1.99	0.26	1.47	3.22
Power Law Coefficient (MLE), Total Assets	1,135	1.95	0.29	1.42	3.55
Power Law Coefficient (CCDF-bins), Total Employment	1,150	2.05	0.30	1.35	4.75
Power Law Coefficient (CCDF-bins), Total Assets	1,135	1.92	0.29	1.36	3.62
Scope for Vertical Differentiation, Unit price Coef. Var	1,260	8.75	2.99	2.84	19.68
Scope for Vertical Differentiation, Quality Ladders	1,140	1.75	0.36	0.76	2.83
Scope for Vertical Differentiation, R&D Intensity	831	7.7	19	0.0003	170.19
Elasticity of Substitution	1,260	10.14	46.76	1.19	635.98
Transport Simplicity	1,260	0.036	0.017	0.006	2.82

Note: The table provides summary statistics for the main elements theoretically identified as important when evaluating firm size heterogeneity.

Table 6: Firm Size Dispersion, Vertical and Horizontal Differentiation and Trade Costs(Total Employment)

Firm Size measured in Total Employment	ζ_{cst} **obtained using MLE**			ζ_{cst} **obtained using CCDF binning**		
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
ψ_{sc} **: (Unit price Coef. Variation)**	-0.0158***			-0.0171***		
	(0.005)			(0.007)		
ψ_{sc} **: (Quality Ladders)**		-0.190***			-0.156**	
		(0.047)			(0.060)	
ψ_{sc} **: (R&D intensity)**			-0.0010**			-0.0017***
			(0.0005)			(0.0005)
σ_{sc} **: (Elasticity of substitution)**	-0.0018***	-0.0030***	-0.0017***	-0.0025***	-0.0036***	-0.0025***
	(0.0004)	(0.0005)	(0.0003)	(0.0004)	(0.0006)	(0.0004)
T_{st} **: (Transport Simplicity)**	-0.0033***	-0.0024***	-0.0036***	-0.0036***	-0.0029***	-0.0037***
	(0.0004)	(0.0003)	(0.0005)	(0.0004)	(0.0004)	(0.0004)
Constant	2.415***	2.572***	2.243***	2.199***	2.294***	2.089***
	(0.159)	(0.171)	(0.224)	(0.166)	(0.181)	(0.230)
Observations	1,150	1,046	779	1,150	1,046	779
R-squared	0.472	0.515	0.483	0.467	0.514	0.488
λ_{ct}**: Country-Year FE**	YES	YES	YES	YES	YES	YES

Robust standard errors in parentheses . Standard errors are clustered at the country-sector level. All regression include *** p<0.01, ** p<0.05, * p<0.1

Note: "Total employment" is used as a measurement of firm size. Column (1), Column (2) and Column (3) test whether higher scope for vertical differentiation results in a higher dispersion of firm size heterogeneity using different proxies of vertical differentiation, i.e. the sectoral dispersion of export unit prices, sectoral quality ladders and the sectoral R&D intensity, respectively. Column (4), (5) and (6) repeat the same exercise but this time using power law coefficients estimated using CCDF with logarithmic binning.

Table 7: Firm Size Dispersion, Vertical and Horizontal Differentiation and Trade Openness(Total Assets)

Firm Size measured in Total Assets	ζ_{cst} **obtained using MLE**			ζ_{cst} **obtained using CCDF binning**		
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
ψ_{sc} **: (Unit price Coef. Variation)**	-0.0201***			-0.0158***		
	(0.005)			(0.006)		
ψ_{sc} **: (Quality Ladders)**		-0.243***			-0.130**	
		(0.059)			(0.064)	
ψ_{sc} **: (R&D intensity)**			-0.0011*			-0.0011**
			(0.0006)			(0.0005)
σ_{sc} **: (Elasticity of substitution)**	-0.0020***	-0.0035***	-0.0020***	-0.0022***	-0.0032***	-0.0022***
	(0.0006)	(0.0007)	(0.0006)	(0.0004)	(0.0005)	(0.0003)
T_{st} **: (Transport Simplicity)**	-0.0040***	-0.0029***	-0.0040***	-0.0032***	-0.0028***	-0.0031***
	(0.0004)	(0.0003)	(0.0004)	(0.0003)	(0.0003)	(0.0004)
Constant	2.208***	2.430***	2.055***	2.140***	2.239***	2.006***
	(0.069)	(0.120)	(0.056)	(0.076)	(0.130)	(0.050)
Observations	1,135	1,032	771	1,135	1,032	771
R-squared	0.395	0.456	0.408	0.396	0.438	0.417
λ_{ct}**: Country-Year FE**	YES	YES	YES	YES	YES	YES

Robust standard errors in parentheses. Standard errors are clustered at the country-sector level. All regression include *** p<0.01, ** p<0.05, * p<0.1

Note: "Total Assets" is used as a measurement of firm size. Column (1), Column (2) and Column (3) test whether higher scope for vertical differentiation results in a higher dispersion of firm size heterogeneity using different proxies of vertical differentiation, i.e. the sectoral dispersion of export unit prices, sectoral quality ladders and the sectoral R&D intensity, respectively. Column (4), (5) and (6) repeat the same exercise but this time using power law coefficients estimated using CCDF with logarithmic binning.

Table 8: Firm Size Dispersion, Vertical and Horizontal Differentiation and Trade Costs: Correction for potential heteroscedasticity(Total Employment)

Firm Size measured in Total Employment	ζ_{cst} **obtained using MLE**			ζ_{cst} **obtained using CCDF binning**		
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
ψ_{sc} **: (Unit price Coef. Variation)**	-0.0193***			-0.0178***		
	(0.0041)			(0.0057)		
ψ_{sc} **: (Quality Ladders)**		-0.163***			-0.141**	
		(0.0477)			(0.0584)	
ψ_{sc} **: (R&D intensity)**			-0.0005			-0.0008**
			(0.0005)			(0.0004)
σ_{sc} **: (Elasticity of substitution)**	-0.0017***	-0.0028***	-0.0015***	-0.0022***	-0.0030***	-0.0021***
	(0.0004)	(0.0005)	(0.0003)	(0.0003)	(0.0004)	(0.0003)
T_{st} **: (Transport Simplicity)**	-0.0031***	-0.0024***	-0.0037***	-0.0035***	-0.0029***	-0.0039***
	(0.0003)	(0.0003)	(0.0004)	(0.0004)	(0.0004)	(0.0005)
Constant	2.386***	2.474***	2.154***	2.134***	2.186***	1.986***
	(0.165)	(0.188)	(0.213)	(0.195)	(0.207)	(0.219)
Observations	1,150	1,046	779	1,149	1,045	778
R-squared	0.519	0.527	0.511	0.542	0.561	0.569
λ_{ct}**: Country-Year FE**	YES	YES	YES	YES	YES	YES

Robust standard errors in parentheses. Standard errors are clustered at the country-sector level . All regression include *** p<0.01, ** p<0.05, * p<0.1

Note: "Total employment" is used as a measurement of firm size. In this table, we correct for a potential heteroscedasticity problem by weighting all variables by the inverse of the estimated standard error of the dependent variable for each observation. Column (1), Column (2) and Column (3) test whether higher scope for vertical differentiation results in a higher dispersion of firm size heterogeneity using different proxies of vertical differentiation, i.e. the sectoral dispersion of export unit prices, sectoral quality ladders and the sectoral R&D intensity, respectively. Column (4), (5) and (6) repeat the same exercise but this time using power law coefficients estimated using CCDF with logarithmic binning.

Table 9: Firm Size Dispersion, Vertical and Horizontal Differentiation and Trade Openness: Correction for potential heteroscedasticity(Total Assets)

Firm Size measured in	ζ_{cst} **obtained using MLE**			ζ_{cst} **obtained using CCDF binning**		
Total Assets	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
ψ_{sc} **: (Unit price Coef. Variation)**	-0.0199***			-0.0142***		
	(0.0046)			(0.0043)		
ψ_{sc} **: (Quality Ladders)**		-0.189***			-0.137***	
		(0.0549)			(0.0505)	
ψ_{sc} **: (R&D intensity)**			-0.0004			-0.0006
			(0.0005)			(0.0004)
σ_{sc} **: (Elasticity of substitution)**	-0.0016***	-0.0028***	-0.0017***	-0.0023***	-0.0033***	-0.0023***
	(0.0005)	(0.0006)	(0.0004)	(0.0004)	(0.0005)	(0.0003)
T_{st} **: (Transport Simplicity)**	-0.0034***	-0.0027***	-0.0037***	-0.0027***	-0.0022***	-0.0029***
	(0.0003)	(0.0003)	(0.0003)	(0.0003)	(0.0003)	(0.0004)
Constant	2.171***	2.310***	2.004***	2.091***	2.220***	2.003***
	(0.0628)	(0.114)	(0.0524)	(0.0561)	(0.103)	(0.0454)
Observations	1,135	1,032	771	1,135	1,032	771
R-squared	0.462	0.487	0.444	0.463	0.499	0.473
λ_{ct}**: Country-Year FE**	YES	YES	YES	YES	YES	YES

Robust standard errors in parentheses.Standard errors are clustered at the country-sector level. All regression include *** p<0.01, ** p<0.05, * p<0.1

Note: "Total Assets" is used as a measurement of firm size. We correct for a potential heteroscedasticity problem by weighting all variables by the inverse of the estimated standard error of the dependent variable for each observation. Column (1), Column (2) and Column (3) test whether higher scope for vertical differentiation results in a higher dispersion of firm size heterogeneity using different proxies of vertical differentiation, i.e. the sectoral dispersion of export unit prices, sectoral quality ladders and the sectoral R&D intensity, respectively. Column (4), (5) and (6) repeat the same exercise but this time using power law coefficients estimated using CCDF with logarithmic binning.

Table 10: Interpretation of the coefficients independently of the scale

			ψ_{sc}	ψ_{sc}	ψ_{sc}	σ_{sc}	T_{st}
			(Unit price Coef. Variation)	**(Quality Ladders)**	**(R&D intensity)**	**(Elasticity of substitution)**	**(Transport Simplicity)**
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Column 1	**Table 6**	**(beta)*(std.dev of the independent variable)**	-0.05			-0.08	-0.00006
		Impact (as % of the std. dev of the dependent variable)	-18%			-32%	0.022%
Column 2	**Table 6**	**(beta)*(std.dev of the independent variable)**		-0.07		-0.14	0.00004
		Impact (as % of the std. dev of the dependent variable)		-26%		54%	0.016%
Column 2	**Table 6**	**(beta)*(std.dev of the independent variable)**			-0.02	-0.07	0.00006
		Impact (as % of the std. dev of the dependent variable)			7%	31%	0.024%

Note: Using the coefficients reported in Columns (1), (2) and (3) of Table 6, this table shows two elements: (1) The result of multiplying the coefficients and one standard deviation of the each of the independent variables ("(beta)*(std.dev of the independent variable)") and how this impact translates in terms of the standard deviation of the dependent variable ("Impact (as % of the std. dev of the dependent variable)").

Appendix: Estimation of Power Law Coefficients

Cumulative Distribution, Density Distribution and Ranked Distribution

A power law Probability Density Function (pdf) is defined as: $p(x) = K\ x^{-\alpha}$. The constant is given by the normalisation requirement $Pr\left[X_i \geq x_{min}\right] = 1$ where:

$$P\left[X_i \geq x_{min}\right] = \int_{x_{min}}^{\infty} p\left(x\right) dx = K \int_{x_{min}}^{\infty} x^{-\alpha} dx = \frac{K}{\alpha - 1}\ x_{min}^{1-\alpha} = 1$$

$$K = x_{min}^{\alpha-1}(\alpha - 1)$$

Therefore, the power law pdf is given by:

$$p(x) = K\ x^{-\alpha} = \left[\frac{\alpha - 1}{x_{min}}\right]\left[\frac{x}{x_{min}}\right]^{-\alpha} = \zeta\ x_{min}^{\zeta}\ x^{-\alpha}$$

where, $K = \zeta\ x_{min}^{\zeta}$ is constant, x is the variable of interest, $\zeta = \alpha - 1$ is the power law coefficient of the pdf.

The power law coefficient α of a pdf can be obtained using Ordinary Least Squares (OLS) by taking logs on both sides:

$$Lnp\left(x\right) = Ln\left(K\right) - \alpha Ln\left(x\right)$$

In addition, we are able to obtain the “Cumulative Distribution Function (CDF)” using the “Counter Cumulative Distribution Function” (CCDF).

Remember that:

$$Pr\left[X_i \geq x\right] = CCDF$$

$$Pr\left[X_i < x\right] = CDF$$

From,

$$P\left[X_i \geq x'\right] = \int_{x'}^{\infty} p(x)dx = K \int_{x'}^{\infty} x^{-\alpha} dx = \frac{K}{\alpha - 1} x'^{1-\alpha}$$

Once again, the constant K is given by the normalisation requirement that $P\left[X \geq x_{min}\right] = 1$.

If $\alpha > 1$ and $K = (\alpha - 1)\, x_{min}^{\alpha-1}$

$$CCDF = P[X_i \geq x] = \frac{(\alpha-1)\ x_{min}^{\alpha-1}}{\alpha-1} x^{1-\alpha} = \left[\frac{x_{min}}{x}\right]^{\alpha-1} = C\, x^{-\zeta}$$

$$CDF = 1 - P\left[X_i \geq x\right] = 1 - \frac{(\alpha-1)\ x_{min}^{\alpha-1}}{\alpha-1} x^{1-\alpha} = 1 - \left[\frac{x_{min}}{x}\right]^{\alpha-1} = 1 - C\, x^{-\zeta}$$

where $C = x_{min}^{\alpha-1}$ is constant, x is the variable of interest, ζ is the power law coefficient of the CCDF , $P\left[X_i \geq x\right]$ is the sample of firms in the sample greater or equal than "x" divided by the total number of firms.

Cumulative distributions following a power-law function are sometimes referred to as distributions following a "Zipf's Law" or "Pareto distribution".

The power law coefficient α of a CCDF can be obtained using Ordinary Least Squares (OLS) by taking logs on both sides:

$$LnP\left[X_i \geq x\right] = Ln\left(K\right) - \zeta\, Ln\left(x\right)$$

From the previous expression of CCDF following a power law, $P\left[X_i \geq x\right] = K\, x^{-\zeta}$, we are able to obtain the same expression using the ranked distribution. This is done using the fact that $P\left[X_i \geq x\right]$ is the sample of firms in the sample greater or equal than "x" divided by the total number of firms. If our sample has η firms in total, we see that the expected number of firms above x, in other words its rank, is given by $P\left[X_i \geq x\right] = r = \eta K\, x^{-\zeta}$.

Therefore, if we express x in terms of r we obtain what referred as the Zipf rank exponent:

$$x = \left[\frac{r}{\eta K}\right]^{-\frac{1}{\zeta}} = Dr^{-\rho}$$

where $D = \left[\frac{1}{\eta K}\right]^{-\frac{1}{\zeta}}$ is constant, x is the variable of interest, r is the rank of the firm and $\rho = \frac{1}{\zeta} = \frac{1}{\alpha-1}$ is the Zipf rank exponent.

Appendix: Maximum likelihood estimators

Newman (2005) and Clauset et al. (2009) proposed the following methodology to obtain the power law coefficients.

Given a set of n firms with a firm size proxy of x_i, the probability that those values are generated from a power law distribution is proportional to:

$$Pr\left[x|\alpha\right] = \prod_{i=1}^{n} p\left(x_i\right) = \prod_{i=1}^{n} \left[\frac{\alpha-1}{x_{min}}\right]\left[\frac{x_i}{x_{min}}\right]^{-\alpha}$$

In order to find the value of α that best fits the data on firm size distribution, we need to obtain the probability $P\left[\alpha|x\right]$ of a particular value of α given the observed data x_i. This relates to $P\left[x|\alpha\right]$ by the Bayes' Law: $P\left[\alpha|x\right] = P\left[x|\alpha\right]\,\frac{P(\alpha)}{P(x)}$. We see that $P\left[\alpha|x\right] \propto\ P\left[x|\alpha\right]$ given that $P\left(x\right)$ is fixed since x is observed in the data and the prior probability of the exponent $P\left(\alpha\right)$ states that it is independent of α.

Setting the likelihood of the data set in logarithms:

$$\mathscr{L} = lnP\left[\alpha|x\right] = ln\left[\prod_{i=1}^{n} p\left(x_i\right)\right] = \sum_{i=1}^{n}\left[ln\left(\alpha-1\right) - lnx_{min} - \alpha ln\left(\frac{x_i}{x_{min}}\right)\right]$$

$$\mathscr{L} = \left[n\ ln\left(\alpha-1\right) -\ n\ lnx_{min} - \alpha\sum_{i=1}^{n} ln\left(\frac{x_i}{x_{min}}\right)\right]$$

Maximising with respect to α, we obtain the most likely α:

$$\frac{d\mathscr{L}}{d\alpha} = 0 = \frac{n}{\alpha-1} - \sum_{i=1}^{n} ln\left(\frac{x_i}{x_{min}}\right)$$

$$\{\hat{\alpha}\} = 1 + n\left[\sum_{i=1}^{n} ln\frac{x_i}{x_{min}}\right]^{-1}$$

where x_i, for $i = 1...n$ are the observed values of firm size x_i such that $x_i \geq x_{min}$ and $\hat{\alpha}$ denotes the estimates power law coefficient derived from empirical data.

References

Mary Amiti and Jozef Konings. Trade Liberalization, Intermediate Inputs, and Productivity: Evidence from Indonesia. *American Economic Review*, 97(5):1611–1638, 2007.

Bee Yan Aw-Roberts, Yi Lee, and Hylke Vandenbussche. Decomposing Firm-Product Appeal: How important is Consumer Taste? *CEPR Working Paper, DP12707*, 2018.

Robert L. Axtell. Zipf distribution of u.s. firm sizes. *Science*, 293(5536):1818–1820, 2001.

Heiko Bauke. Parameter estimation for power-law distributions by maximum likelihood methods. *The European Physical Journal B*, 58(2):167–173, 2007.

Andrew B Bernard, Jonathan Eaton, J Bradford Jensen, and Samuel Kortum. Plants and Productivity in International Trade. *American economic review*, 93(4):1268–1290, 2003.

Alessandra Bonfiglioli, Rosario Crinò, and Gino Gancia. Trade, Finance and Endogenous Firm Heterogeneity. *Journal of the European Economic Association, forthcoming*, 2016.

Alessandra Bonfiglioli, Rosario Crinò, and Gino Gancia. Betting on exports: Trade and endogenous heterogeneity. *The Economic Journal*, 2017.

Christian Broda and David E Weinstein. Globalization and the Gains from Variety. *The Quarterly journal of economics*, 121(2):541–585, 2006.

Luis Cabral and Jose Mata. On the Evolution of the Firm Size Distribution: Facts and Theory. *The American Economic Review*, 93(4):1075–1090, 2003.

Aaron Clauset, Cosma Rohilla Shalizi, and Mark EJ Newman. Power-law distributions in empirical data. *SIAM review*, 51(4):661–703, 2009.

Arnaud Costinot, Dave Donaldson, and Ivana Komunjer. What Goods Do Countries Trade? A Quantitative Exploration of Ricardo's Ideas. *The Review of economic studies*, 79(2): 581–608, 2011.

Matthieu Crozet, Keith Head, and Thierry Mayer. Quality Sorting and Trade: Firm-level Evidence for French Wine. *The Review of Economic Studies*, 79(2):609–644, 2011.

Jan De Loecker and Frederic Warzynski. Markups and Firm-Level Export Status. *The American Economic Review*, 102(6):2437–2471, 2012.

Francesco Di Comite, Jacques-François Thisse, and Hylke Vandenbussche. Verti-zontal differentiation in export markets. *Journal of International Economics*, 93(1):50–66, 2014.

Julian Di Giovanni and Andrei A Levchenko. Country size, international trade, and aggregate fluctuations in granular economies. *Journal of Political Economy*, 120(6): 1083–1132, 2012.

Julian Di Giovanni, Andrei A Levchenko, and Romain Ranciere. Power laws in firm size and openness to trade: Measurement and implications. *Journal of International Economics*, 85(1):42–52, 2011.

Pablo Fajgelbaum, Gene M Grossman, and Elhanan Helpman. Income Distribution, Product Quality, and International Trade. *Journal of Political Economy*, 119(4):721–765, 2011.

Yoshi Fujiwara, Corrado Di Guilmi, Hideaki Aoyama, Mauro Gallegati, and Wataru Souma. Do pareto–zipf and gibrat laws hold true? an analysis with european firms. *Physica A: Statistical Mechanics and its Applications*, 335(1):197–216, 2004.

Xavier Gabaix. The Granular Origins of Aggregate Fluctuations. *Econometrica*, 79(3): 733–772, 2011.

Xavier Gabaix. Power laws in economics: An introduction. *The Journal of Economic Perspectives*, 30(1):185–205, 2016.

Luis Garicano, Claire Lelarge, and John Van Reenen. Firm Size Distortions and the Productivity Distribution: Evidence from France. *The American Economic Review*, 106 (11):3439–3479, 2016.

Guillaume Gaulier and Soledad Zignago. Baci: International Trade Database at the Product-Level (the 1994-2007 version). 2010.

Robert Gibrat. Les inégalits économiques. *Sirey*, 1931.

Pinelopi Koujianou Goldberg, Amit Kumar Khandelwal, Nina Pavcnik, and Petia Topalova. Imported Intermediate Inputs and Domestic Product Growth: Evidence from India. *The Quarterly journal of economics*, 125(4):1727–1767, 2010.

Michel L Goldstein, Steven A Morris, and Gary G Yen. Problems with fitting to the power-law distribution. *The European Physical Journal B-Condensed Matter and Complex Systems*, 41(2):255–258, 2004.

Gene M Grossman and Elhanan Helpman. Quality ladders and Product Cycles. *The Quarterly Journal of Economics*, 106(2):557–586, 1991.

Juan Carlos Hallak. A Product-Quality View of the Linder Hypothesis. *The Review of Economics and Statistics*, 92(3):453–466, 2010.

Juan Carlos Hallak and Peter K Schott. Estimating Cross-Country Differences in Product Quality. *The Quarterly Journal of Economics*, 126(1):417–474, 2011.

Juan Carlos Hallak and Jagadeesh Sivadasan. Firms' Exporting Behavior under Quality Constraints. Technical report, National Bureau of Economic Research Working Paper 14928, 2009.

László Halpern, Miklós Koren, and Adam Szeidl. Imported Inputs and Productivity. *American Economic Review*, 105(12):3660–3703, 2015.

Torfinn Harding and Beata S Javorcik. Foreign Direct Investment and Export Upgrading. *Review of Economics and Statistics*, 94(4):964–980, 2012.

Keith Head, Thierry Mayer, and Mathias Thoenig. Welfare and trade without pareto. *American Economic Review*, 104(5):310–16, 2014.

Elhanan Helpman, Marc J. Melitz, and Stephen R. Yeaple. Export versus fdi with heterogeneous firms. *American Economic Review*, 94(1):300–316, March 2004. doi: 10.1257/000282804322970814. URL http://www.aeaweb.org/articles?id=10.1257/000282804322970814.

David Hummels and Peter J Klenow. The Variety and Quality of a Nation's Exports. *The American Economic Review*, 95(3):704–723, 2005.

John Hutchinson, Jozef Konings, and Patrick Paul Walsh. The firm size distribution and inter-industry diversification. *Review of Industrial Organization*, 37(2):65–82, 2010.

Amit Khandelwal. The Long and Short (of) Quality Ladders. *The Review of Economic Studies*, 77(4):1450–1476, 2010.

Maurice Kugler and Eric Verhoogen. The Quality-Complementarity Hypothesis: Theory and Evidence from Colombia. Technical report, National Bureau of Economic Research Working Paper 14418, 2008.

Maurice Kugler and Eric Verhoogen. Prices, plant size, and product quality. *The Review of Economic Studies*, 79(1):307–339, 2011.

Kalina Manova and Zhiwei Zhang. Export Prices Across Firms and Destinations. *The Quarterly Journal of Economics*, 127(1):379–436, 2012.

Thierry Mayer and Gianmarco IP Ottaviano. The Happy Few: The Internationalisation of European Firms. *Intereconomics*, 43(3):135–148, 2008.

Marc J Melitz. The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6):1695–1725, 2003.

Monika Mrázová, J Peter Neary, and Mathieu Parenti. Sales and Markup Dispersion: Theory and Empirics. 2017.

Mark EJ Newman. Power laws, pareto distributions and zipf's law. *Contemporary physics*, 46(5):323–351, 2005.

Gary R Saxonhouse. Estimated parameters as dependent variables. *The American Economic Review*, 66(1):178–183, 1976.

Peter K Schott. Across-Product versus Within-Product Specialization in International Trade. *The Quarterly Journal of Economics*, 119(2):647–678, 2004.

John Sutton. Technology and Market Structure. *European Economic Review*, 40(3):511–530, 1996.

John Sutton. Gibrat's legacy. *Journal of Economic Literature*, 35(1):40–59, 1997.

John Sutton. *Technology and Market Structure: Theory and History*. MIT Press, 2001.

John Sutton. Quality, trade and the moving window: The globalisation process. *The Economic Journal*, 117(524):F469–F498, 2007.

Heiwai Tang and Yifan Zhang. Quality Differentiation and Trade Intermediation. 2012.

Jianhua Zhang, Qinghua Chen, and Yougui Wang. Zipf distribution in top chinese firms and an economic explanation. *Physica A- statistical Mechanics and its applications*, 388 (10):2020–2024, 2009.