Relaxations and approximations of HVdc grid TNEP problem

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***Abstract*—In recent years, the use of HVdc technology has increased and VSC technology has enabled the realization of HVdc grids. This calls for the development of new tools to solve the transmission network expansion planning (TNEP) model. A detailed representation of the dc grid TNEP problem is highly nonlinear and more complex than the traditional ac grid expansion problem due to extra constraints and additional decision variables from the converter station model. The present day industrial solvers have difficulties to tackle the resultant MINLP problem. Therefore, the linearized 'DC' approximation is often used in practice, which may not produce sufficiently accurate answers. In this paper, different relaxations and approximations of the dc grid TNEP problem are presented. The performance of each formulation is evaluated using eight test cases. Although there is no clear formulation that shows the best performance, LPAC approximation and SOC relaxations seem to provide better alternatives to 'DC' approximations. The developed formulations do not guarantee feasibility and a second corrective stage is required to obtain feasible solutions.**

***Index Terms*—Transmission planning, HVdc grid, Convex relaxations, Linear approximations, Nonlinear TNEP problem**

NOMENCLATURE

*E*ntities, indices and sets

$i, j \in \mathcal{I}$	ac nodes
$l \in \mathcal{L}$	ac branches
$lij \in \mathcal{T}^{\text{ac}} \subseteq \mathcal{L} \times \mathcal{I} \times \mathcal{I}$	ac topology
$e, f \in \mathcal{E}$	Candidate dc nodes
$d \in \mathcal{D}$	Candidate dc branches
$def \in \mathcal{T}^{\text{dc}} \subseteq \mathcal{D} \times \mathcal{E} \times \mathcal{E}$	Candidate dc topology
$c \in \mathcal{C}$	Candidate converters
$cie \in \mathcal{T}^{\text{cv}} \subseteq \mathcal{C} \times \mathcal{I} \times \mathcal{E}$	Candidates converter topology
$g \in \mathcal{G}$	Generators
$mi \in \mathcal{T}^{\text{ac}}$	ac load
$me \in \mathcal{T}^{\text{dc}}$	dc load

*P*arameters

C_d	Cost of dc line d
C_c	Cost of converter c
N_d	Number of poles of dc line (link) d
r_d	Resistance of line d
t_c	Transformation ratio in converter station c
a_c, b_c, c_c	Coefficients of polynomial converter power loss
$z_c^{\text{tf}}, r_c^{\text{tf}}, x_c^{\text{tf}}, g_c^{\text{tf}}, b_c^{\text{tf}}$	Impedance, resistance, reactance, conductance and susceptance of converter transformer
$z_c^{\text{pr}}, r_c^{\text{pr}}, x_c^{\text{pr}}$	Impedance, resistance and reactance of phase reactor
b_c^{f}	Filter susceptance
P_{me}	dc load at node e
P_{mi}, Q_{mi}	Active and reactive load at ac node i
$g_i^{\text{shunt}}, b_i^{\text{shunt}}$	Shunt conductance and susceptance elements at ac node i

*V*ariables

ξ_c	Binary decision variable for converter station
ξ_d	Binary decision variable for dc line
U_{de}, U_{df}	dc voltage at node e and f
P_{def}, P_{dfe}	Active power flow from node e to f and f to e
$P_c^{\text{cv,ac}}, P_c^{\text{cv,dc}}$	ac and dc side active power of converter c
$S_c^{\text{cv,ac}}$	Apparent power capacity of converter
i_c^{cv}	ac side current of converter c
$\|U_i\|, \theta_i$	Magnitude and angle of voltage at ac grid node i
$\|U_c^{\text{f}}\|, \theta_c^{\text{f}}$	Magnitude and angle of voltage at filter node
$\|U_c^{\text{cv}}\|, \theta_c^{\text{cv}}$	Magnitude and angle of voltage at ac side node of converter
$P_{cie}^{\text{tf}}, P_{cei}^{\text{tf}}$	Active power flow in from node i to e and node e to i in converter transformer
$Q_{cie}^{\text{tf}}, Q_{cei}^{\text{tf}}$	Reactive power flow from node i to e and node e to i in converter transformer
$P_{cie}^{\text{pr}}, Q_{cie}^{\text{pr}}$	Active and reactive power flow from node i to e in phase reactor
Q_c^{f}	Reactive power absorbed by filter
$P_{lij}^{\text{ac}}, Q_{lij}^{\text{ac}}$	Active and reactive power flow from node i to j in line l
P_{gi}, Q_{gi}	Generator active and reactive power at node i
ϕ	Voltage deviation from 1.0 p.u.

*O*thers

$\overline{x}$	Upper bound of variable x
$\underline{x}$	Lower bound of variable x
$\widehat{x}$	Polyhedral outer approximation of x
$\|x\|$	Magnitude of variable x
x^*	Auxiliary variable for x
W	Lifted variable for square or bilinear voltage terms
i	Lifted variable for current square terms

I. INTRODUCTION

High voltage direct current (HVdc) is the preferred way to transmit power in many applications such as inter-connecting different asynchronous grids, transporting bulk amount of power over a long distance and underwater power transmission using cables. In the context of power systems, the transmission network expansion planning (TNEP) problem solves the 'when' and 'where' questions of adding new transmission capacity, e.g. new dc connections. Traditionally, research on the TNEP problem focused on ac[1] networks. However, with increasing adoption of HVdc systems, it needs to be extended to accommodate this technology.

The fundamental TNEP problem is a red Non-Convex Mixed Integer Nonlinear Program (Non-Convex MINLP), also called 'AC' formulation here. This problem is difficult to solve using the present day solvers. Therefore, many works resort to linearizing techniques and use MILP solvers that are mature and computationally efficient [1]. Out of many variants of linearized formulation, the 'DC' approximation is commonly used. However, it is not the most accurate approach to approximate the original MINLP problem [2]. In recent years, convex relaxations have proven to be a promising alternative for the classic nonlinear optimal power flow (OPF) problem i.e. semi-definite programming (SDP) [3] [4], second order cone programming (SOCP) [5] [6], and quadratic constrained (QC) programming [7]. This has piqued the interest in applying these relaxations for different power system optimization problems. We develop these relaxations for dc grid TNEP problem and evaluate them on various test cases.

We also include a linear programming AC (LPAC) formulation that, unlike 'DC', does not disregard the voltage and reactive power representation. The advantage of such a formulation is that it still can use mature MILP solvers while closely representing 'AC' power flow equations. The LPAC formulation in [1] for ac TNEP problem is taken as a reference and extended for the dc grid.

The formulated dc grid TNEP problem in this paper has the following benefits compared to alternatives found in literature [8] [9] [10]: a) it includes a detailed converter station model that is necessary to capture the full operational behavior of ac/dc grids b) it is not only valid for point-to-point HVdc link expansions, but also for a meshed dc grid expansion, offering wider variety of possible expansions. In this regard, our approach differs from [9] [10] that supply candidate three terminal grids. The proposed formulation does not need to supply such grid candidates separately, but can form multi-terminal grids based on dc line and converter candidates.

This paper extends our previous work in [11] that consisted of three basic dc grid TNEP formulations and implements more formulations for performance analysis. Fig. 1 provides an overview of the implementation. We focus on seven different formulations, that differ from each other on the ac grid side (on the left). Out of them, original MINLP, SOC branch flow model (BFM), 'DC' approximation and linear program AC (LP AC) remain different on the dc grid side, too. However, the SDP and QC models are equivalent to the SOC BIM (bus injection model), hence three separate formulations are not required for dc lines and converter stations. This is due to the radial nature of the converter station that ensures SOC BIM to become the tightest relaxations among the introduced relaxations here and allows its use in conjunction with SDP and QC without affecting their accuracy. Note that two different SOC models (BFM and BIM) exist only for the branch like components (converter station branch, ac branch and dc branch here), but not for ac/dc converter itself. Therefore, both models converge to the same set of equations for the power electronics converter.

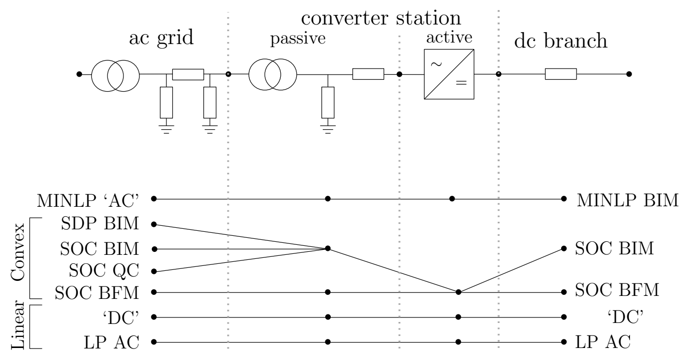

Fig. 1. Mapping of formulations for different parts of the ac/dc grid [12]

All formulations are implemented in Julia/JuMP [13] and conceived as an extension of two existing open-source packages PowerModels.jl [14] and PowerModelsACDC.jl [15]. We take advantage of the modularity offered by these packages and build our problem using existing ac/dc grid power flow models. The implemented code is embedded in PowerModelsACDC.jl and is available for public access along with the test cases introduced in this paper.

II. DC GRID TNEP PROBLEM

The 'AC' TNEP problem formulation is covered in detail in our previous paper [11]. For completeness of this paper, it is briefly described again in this section (see Model 1).

In order to allow a dc grid expansion, it is necessary to decouple the decisions of building new dc lines and new HVdc converters. Therefore, each of them is represented using individual binary decision variables, i.e. ξ_d for candidate dc lines and ξ_c for candidate converters. The objective (M1.1) of the model is to minimize the investment cost of both components. Note that the objective function can also includes the generation cost during the planning horizon, but we restrict it here to expansion related costs to clearly analyze the performance of different formulations for this application.

A. Constraints for grid components

The dc line model is depicted in Fig. 2. U_{de} and U_{df} are the voltage at the nodes e and f for branch d. P_{def} and P_{dfe} are the branch flows at the opposite ends of the branch d. The power flows over dc line are defined using the BIM equation (M1.2). The constraint is also applicable to the reverse node order $\mathcal{T}^{\text{dc,r}}$, i.e. from f to e.

The HVdc converter station model is depicted in Fig. 3. It has four main components: 1) a power-electronic ac/dc converter, 2) a phase reactor as a series impedance, 3) a capacitive filter as a shunt susceptance, and 4) a transformer

[1]we use small letters ac and dc for technology and capital letters 'AC' and 'DC' for formulations

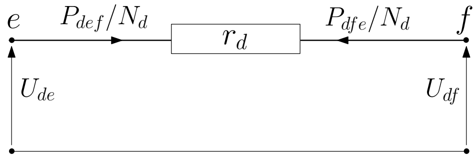

Fig. 2. dc line model (N_d = 1 for monopolar and 2 for bipolar)

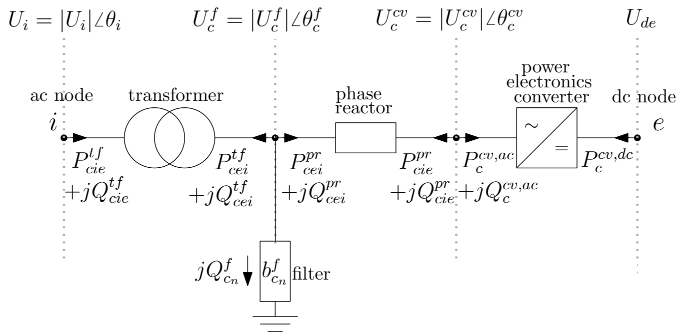

Fig. 3. ac/dc converter station model

with tap and series impedance. Although a VSC (voltage source converter) based HVdc system is assumed in this paper, the LCC (line commutated converter) technology can be easily integrated as described in [12]. The converter constraint (M1.3) links ac and dc side active power injection using the converter losses. The losses are approximated through a quadratic function, dependent on the ac side converter current (i_c^{cv}) [16]. The nonnegative coefficients a_c [MW], b_c [MW/A] and $c_c[\Omega]$ respectively represent the no load, linear and quadratic losses of the converter station. Constraint (M1.4) specifies the converter's apparent power capacity in terms of ac side voltage and current.

The transformer is modeled by its equivalent impedance, $z_c^{\text{tf}} = r_c^{\text{tf}} + jx_c^{\text{tf}}$. Constraints (M1.5) - (M1.8) capture the active and reactive power flows at opposite ends of the transformer [12]. The parameters g_c^{tf} and b_c^{tf} are the series conductance and susceptance respectively. The phase reactor is also modeled in the same manner by impedance $z_c^{\text{pr}} = r_c^{\text{pr}} + jx_c^{\text{pr}}$. Therefore, the same set of constraints can be applied to it when the transformation ratio (t_c) is set to 1.0[2]. Constraints (M1.9) - (M1.10) describe active and reactive power balance at the filter node. The third term in constraint (M1.10) represents the reactive power absorbed by the filter capacitance.

Constraints (M1.2) - (M1.10) should be deactivated when the corresponding decision variable is zero. This is accomplished by integrating the decision variables into the constraints and with additional on/off constraints. The on/off constraints are used for converter current and power flow variables, where the upper and lower limits of variables are defined by the branch and converter specifications. Although the decision variables are already integrated in the most constraints and additional on/off constraints are not strictly required, we add them to keep a uniformity.

[2]As $P_c^{\text{cv,ac}}$ = -P_{cie}^{pr} and $Q_c^{\text{cv,ac}}$ = -Q_{cie}^{pr}, it is not strictly required to define both, but they are retained here to keep the representation clear.

The described model is also subjected to the nodal balance at the dc nodes and the converter connected ac nodes. The balance for dc nodes $e \in \mathcal{E}$ can be given as:

$$\sum_{cie\in\mathcal{T}^{\text{cv}}} P_c^{\text{cv,dc}} + \sum_{def\in\mathcal{T}^{\text{dc}}} P_{def} = \sum_{me\in\mathcal{T}^{\text{dc}}} -P_{me}, \tag{1}$$

The generic ac grid node balance equations are modified to include the dc grid. For every ac node $i \in \mathcal{I}$, the following equality must be respected:

$$\sum_{cie\in\mathcal{T}^{\text{cv}}} P_{cie}^{\text{tf}} + \sum_{lij\in\mathcal{T}^{\text{ac}}} P_{lij}^{\text{ac}} = \sum_{gi\in\mathcal{T}^{\text{ac}}} P_{gi} - \sum_{mi\in\mathcal{T}^{\text{ac}}} P_{mi} - g_i^{\text{shunt}}(|U_i|)^2, \tag{2a}$$

$$\sum_{cie\in\mathcal{T}^{\text{cv}}} Q_{cie}^{\text{tf}} + \sum_{lij\in\mathcal{T}^{\text{ac}}} Q_{lij}^{\text{ac}} = \sum_{gi\in\mathcal{T}^{\text{ac}}} Q_{gi} - \sum_{mi\in\mathcal{T}^{\text{ac}}} Q_{mi} + b_i^{\text{shunt}}(|U_i|)^2. \tag{2b}$$

Model 1: 'AC' formulation

Minimize:

$$\sum_{c\in C} C_c \cdot \xi_c + \sum_{d\in D} C_d \cdot \xi_d \tag{M1.1}$$

dc branch

$$P_{def} = \xi_d \left(\frac{N_d}{r_d}\right) U_{de}(U_{de} - U_{df}) \qquad \forall def \in \mathcal{T}^{\text{dc}} \cup \mathcal{T}^{\text{dc,r}} \tag{M1.2}$$

Converter

$$P_c^{\text{cv,ac}} + P_c^{\text{cv,dc}} = a_c\xi_c + b_c|i_c^{\text{cv}}| + c_c|i_c^{\text{cv}}|^2 \qquad \forall c \in \mathcal{C} \tag{M1.3}$$

$$(P_c^{\text{cv,ac}})^2 + (Q_c^{\text{cv,ac}})^2 = |U_c^{\text{cv}}|^2|i_c^{\text{cv}}|^2 \qquad \forall c \in \mathcal{C} \tag{M1.4}$$

Transformer (or reactor when t_c = 1)

$$P_{cie}^{\text{tf}} = \Big[g_c^{\text{tf}}\left(\frac{|U_i|}{t_c}\right)^2 - g_c^{\text{tf}}\frac{|U_i|}{t_c}|U_c^{\text{f}}|\cos(\theta_i - \theta_c^{\text{f}}) - b_c^{\text{tf}}\frac{|U_i|}{t_c}|U_c^{\text{f}}|\sin(\theta_i - \theta_c^{\text{f}})\Big]\xi_c \qquad \forall cie \in \mathcal{T}^{\text{cv}} \tag{M1.5}$$

$$Q_{cie}^{\text{tf}} = \Big[-b_c^{\text{tf}}\left(\frac{|U_i|}{t_c}\right)^2 + b_c^{\text{tf}}\frac{|U_i|}{t_c}|U_c^{\text{f}}|\cos(\theta_i - \theta_c^{\text{f}}) - g_c^{\text{tf}}\frac{|U_i|}{t_c}|U_c^{\text{f}}|\sin(\theta_i - \theta_c^{\text{f}})\Big]\xi_c \qquad \forall cie \in \mathcal{T}^{\text{cv}} \tag{M1.6}$$

$$P_{cei}^{\text{tf}} = \Big[g_c^{\text{tf}}|U_c^{\text{f}}|^2 - g_c^{\text{tf}}|U_c^{\text{f}}|\frac{|U_i|}{t_c}\cos(\theta_c^{\text{f}} - \theta_i) - b_c^{\text{tf}}|U_c^{\text{f}}|\frac{|U_i|}{t_c}\sin(\theta_c^{\text{f}} - \theta_i)\Big] \cdot \xi_c \qquad \forall cei \in \mathcal{T}^{\text{cv}} \tag{M1.7}$$

$$Q_{cei}^{\text{tf}} = \Big[-b_c^{\text{tf}}|U_c^{\text{f}}|^2 + b_c^{\text{tf}}|U_c^{\text{f}}|\frac{|U_i|}{t_c}\cos(\theta_c^{\text{f}} - \theta_i) - g_c^{\text{tf}}|U_c^{\text{f}}|\frac{|U_i|}{t_c}\sin(\theta_c^{\text{f}} - \theta_i)\Big] \cdot \xi_c \qquad \forall cei \in \mathcal{T}^{\text{cv}} \tag{M1.8}$$

Filter

$$P_{cie}^{\text{pr}} + P_{cei}^{\text{tf}} = 0 \qquad \forall c \in \mathcal{C} \tag{M1.9}$$

$$Q_{cie}^{\text{pr}} + Q_{cei}^{\text{tf}} - \xi_c b_c^{\text{f}}(|U_c^{\text{f}}|)^2 = 0 \qquad \forall c \in \mathcal{C} \tag{M1.10}$$

On/off constraints

$\xi_d \underline{x} \le x \le \overline{x}\xi_d$ where $x = [P_{def}, P_{dfe}]$

$\xi_c \underline{x} \le x \le \overline{x}\xi_c$ where $x = [P_c^{\text{cv,ac}}, P_c^{\text{cv,dc}}, P_{cie}^{\text{tf}}, Q_{cie}^{\text{tf}}, P_{cie}^{\text{pr}}, P_{cei}^{\text{tf}}, Q_{cie}^{\text{pr}}, Q_{cei}^{\text{tf}}, i_c^{\text{cv}}]$

$\xi_d, \xi_c \in \{0, 1\}$

In the following sections, we derive various relaxations and approximations of this MINLP model. Refer to equations in Model 1 wherever needed.

III. Convex relaxations

As shown in Fig. 1, the convex relaxations for the dc grid are implemented through two second order cone models. The BIM represents the grid equations in terms of nodal quantities (e.g. voltages) whereas the BFM represents them using the quantities pertaining to line flows (e.g. branch current). Note that both models are shown to be equivalent in [17].

Model 2: SOC BIM formulation

Minimize:

(M1.1)

dc branch

$$P_{def} = \xi_d \left(\frac{N_d}{r_d}\right)(W^*_{de} - W^*_{def}) \quad \forall def \in \mathcal{T}^{\text{dc}} \cup \mathcal{T}^{\text{dc,r}} \quad \text{(M2.1)}$$

$$(W^*_{def})^2 \leq W^*_{de} W^*_{df} \quad \forall def \in \mathcal{T}^{\text{dc}} \cup \mathcal{T}^{\text{dc,r}} \quad \text{(M2.2)}$$

Converter

$$P^{\text{cv,ac}}_c + P^{\text{cv,dc}}_c = a_c \xi_c + b_c |i^{\text{cv}}_c| + c_c i^{\text{sq,cv,mag}}_c \quad \forall c \in \mathcal{C} \quad \text{(M2.3)}$$

$$(P^{\text{cv,ac}}_c)^2 + (Q^{\text{cv,ac}}_c)^2 \leq (\overline{|U^{\text{cv}}_c|})^2 (|i^{\text{cv}}_c|)^2 \quad \forall c \in \mathcal{C} \quad \text{(M2.4)}$$

$$(P^{\text{cv,ac}}_c)^2 + (Q^{\text{cv,ac}}_c)^2 \leq W^{\text{cv}}_c i^{\text{sq,cv,mag}}_c \quad \forall c \in \mathcal{C} \quad \text{(M2.5)}$$

$$i^{\text{sq,cv,mag}}_c \leq |i^{\text{cv}}_c| I^{\text{cv,rated}}_c \quad \forall c \in \mathcal{C} \quad \text{(M2.6)}$$

Transformer (or reactor when t_c = 1)

$$P^{\text{tf}}_{cie} = g^{\text{tf}}_c \left(\frac{W^*_i}{t^2_c}\right) - g^{\text{tf}}_c \frac{R^{\text{tf}}_{ic}}{t_c} - b^{\text{tf}}_c \frac{T^{\text{tf}}_{ic}}{t_c} \quad \forall cie \in \mathcal{T}^{\text{cv}} \quad \text{(M2.7)}$$

$$Q^{\text{tf}}_{cie} = -b^{\text{tf}}_c \left(\frac{W^*_i}{t^2_c}\right) + b^{\text{tf}}_c \frac{R^{\text{tf}}_{ic}}{t_c} - g^{\text{tf}}_c \frac{T^{\text{tf}}_{ic}}{t_c} \quad \forall cie \in \mathcal{T}^{\text{cv}} \quad \text{(M2.8)}$$

$$P^{\text{tf}}_{cei} = g^{\text{tf}}_c W^{\text{f}}_c - g^{\text{tf}}_c \frac{R^{\text{tf}}_{ic}}{t_c} + b^{\text{tf}}_c \frac{T^{\text{tf}}_{ic}}{t_c} \quad \forall cei \in \mathcal{T}^{\text{cv}} \quad \text{(M2.9)}$$

$$Q^{\text{tf}}_{cei} = -b^{\text{tf}}_c W^{\text{f}}_c + b^{\text{tf}}_c \frac{R^{\text{tf}}_{ic}}{t_c} + g^{\text{tf}}_c \frac{T^{\text{tf}}_{ic}}{t_c} \quad \forall cei \in \mathcal{T}^{\text{cv}} \quad \text{(M2.10)}$$

$$(R^{\text{tf}}_{ic})^2 + (T^{\text{tf}}_{ic})^2 \leq W_i W^{\text{f}}_c \quad \text{(M2.11)}$$

Filter

(M1.9)

$$Q^{\text{pr}}_{cie} + Q^{\text{tf}}_{cei} - \xi_c b^{\text{f}}_c W^{\text{f}}_c = 0 \quad \forall cie \in \mathcal{T}^{\text{cv}} \quad \text{(M2.12)}$$

On/off constraints

$$\xi_d \underline{x} \leq x \leq \overline{x} \xi_d \quad \text{where} \quad x = [P_{def}, P_{dfe}]$$

$$\xi_c \underline{x} \leq x \leq \overline{x} \xi_c \quad \text{where} \quad x = [P^{\text{cv,ac}}_c, P^{\text{cv,dc}}_c, Q^{\text{cv,ac}}_c, P^{\text{tf}}_{cie}, Q^{\text{tf}}_{cie}, P^{\text{pr}}_{cie}, P^{\text{tf}}_{cei}, W^{\text{f}}_c, i^{\text{cv}}_c, i^{\text{sq,cv,mag}}_c, R^{\text{tf}}_{ic}, T^{\text{tf}}_{ic}]$$

Auxiliary variable constraints

$$\xi_d \underline{x} \leq x^* \leq \overline{x} \xi_d \quad \text{where} \quad x = \left[W^*_{de}, W^*_{def}\right]$$

$$x - (1-\xi_d)\overline{x} \leq x^* \leq x - (1-\xi_d)\underline{x} \quad \text{where} \quad x = \left[W^*_{de}, W^*_{def}\right]$$

$$\xi_c \underline{x} \leq x^* \leq \overline{x} \xi_c \quad \text{where} \quad x = W^*_i$$

$$x - (1-\xi_c)\overline{x} \leq x^* \leq x - (1-\xi_c)\underline{x} \quad \text{where} \quad x = W^*_i$$

$$\xi_d, \xi_c \in \{0, 1\}$$

A. *Second order cone BIM*

The SOC-BIM formulation is provided in Model 2. The dc branch equation (M1.2) can be linearized by assigning a new lifted variable W for voltage square and bilinear terms such that,

$$(U_{de})^2 \rightarrow W_{de}, (U_{df})^2 \rightarrow W_{df}, (U_{def})^2 \rightarrow W_{def}.$$

The new variables are coupled through a non-convex equality, $(W^*_{def})^2 = W^*_{de} W^*_{df}$ that can be replaced by a valid inequality constraint (M2.2) in the form of a rotated SOC. Similarly, the current square in the converter losses (M1.3) and voltage square in the capacity constraint (M1.4) are replaced by lifted variables $i^{\text{sq,cv,mag}}_c$ and W^{cv}_c. Hence, the converter loss constraint becomes linear (M2.3). The converter capacity constraint (M1.4) is relaxed through two valid inequalities, rotated SOCs (M2.4) and (M2.5). Note that the upper-bounds of all lifted variables must correspond to the upper-bounds of the original variables, e.g. $i^{\text{sq,cv,mag}}_c \in [0, |i^{\text{cv}}_c|^2]$. In addition, to provide a tighter bound on $i^{\text{sq,cv,mag}}_c$, we adopt constraint (M2.6).

Since the transformer (and reactor) models are similar to ac branch model (i.e. $z = r + jx$), the classic approach from [5] is used to prepare an SOC model for them. Along with replacing the voltage square terms by respective W variables, the approach relies on the following substitutions for nonlinear terms,

$$|U^{\text{f}}_c||U_i| \cos(\theta^{\text{f}}_c - \theta_i) \rightarrow R^{\text{tf}}_{ic}, \quad |U^{\text{f}}_c||U_i| \sin(\theta^{\text{f}}_c - \theta_i) \rightarrow T^{\text{tf}}_{ic}$$

leading to a linearized power flow constraints (M2.7) - (M2.10). The coupling among lifted variables is defined through the rotated SOC (M2.11). The filter constraint (M2.12) formulation is straightforward using the lifted variable. The binary decision variables for dc branches and converters (ξ_d, ξ_c) are integrated using the on-off constraints for shown variables. The product of bus voltages (both ac or dc) with the binary variables are treated separately using the auxiliary variables (marked with *). The shown disjunctive constraints ensures that the auxiliary variables equals the original variables, i.e. $W^* = W$ when corresponding binary variable is 1, else they are zero. This way the dc and ac node voltages (W_{de}, W_{df} and W_i) are not forced to be zero when a candidate converter is not constructed between them.

B. *Second order cone BFM*

Model 3 represents the SOC-BFM formulation. We explain the converter transformer constraints first for this formulation and then exclude the reactive component to obtain the dc line constraints. Note that the transformer model is similar to π model of ac branch, except the shunt element. Hence, the SOC-BFM relaxation (or distflow) for ac branch from [18] is adapted here. The relaxation can be defined using a lifted variable for transformer current square such that,

$$(i^{\text{tf}}_c)^2 = i^{\text{sq,tf}}_c = \frac{(P^{\text{tf}}_{cie})^2 + (Q^{\text{tf}}_{cie})^2}{U^2_i}. \quad (3)$$

The power loss along the converter transformer can be defined using constraints (M3.4) and (M3.5). In BFM, ohm's law over a line is defined as constraint (M3.6). The non-convexity in (3) is resolved by converting it to a rotated SOC constraint for the converter transformer (M3.7). Based on the transformer model, the dc branch model (M3.1 - M3.3) can be easily derived by removing the reactive power terms and making necessary adjustments for t_c and N_d. The converter and filter constraints remain the same as in the SOC-BIM formulation. The logic behind on/off and auxiliary variable constraints remains unchanged.

IV. Linear formulations

The linear formulations are particularly attractive since they can employ MILP solvers that are more mature than MINLP and Mixed Integer Convex Programming (MICP) solvers. We select two linear formulations for our study: 1) linear AC approximation that represents the reactive power and the dc voltage in an approximate manner and 2) classic 'DC'

Model 3: SOC BFM formulation

Minimize:

(M1.1)

dc branch

$$P_{def} + P_{dfe} = r_d N_d i^{\text{sq}}_{def} \quad \forall def \in \mathcal{T}^{\text{dc}} \quad \text{(M3.1)}$$

$$(P_{def})^2 \leq (N_d)^2 W^*_{de} i^{\text{sq}}_{def} \quad \forall def \in \mathcal{T}^{\text{dc}} \quad \text{(M3.2)}$$

$$W^*_{df} = W^*_{de} - 2\frac{r_d}{N_d} P_{def} + (r_d)^2 i^{\text{sq}}_{def} \quad \forall def \in \mathcal{T}^{\text{dc}} \quad \text{(M3.3)}$$

Converter

same as SOC-BIM

Transformer (or reactor when t_c = 1)

$$P^{\text{tf}}_{cie} + P^{\text{tf}}_{cei} = r^{\text{tf}}_c i^{\text{sq,tf}}_c, \quad \forall cie \in \mathcal{T}^{\text{cv}} \quad \text{(M3.4)}$$

$$Q^{\text{tf}}_{cie} + Q^{\text{tf}}_{cei} = x^{\text{tf}}_c i^{\text{sq,tf}}_c, \quad \forall cei \in \mathcal{T}^{\text{cv}} \quad \text{(M3.5)}$$

$$W^{\text{f}}_c = \frac{W^*_i}{(t_c)^2} - 2\left(r^{\text{tf}}_c P^{\text{tf}}_{cie} + x^{\text{tf}}_c Q^{\text{tf}}_{cie}\right) + \left(\left(r^{\text{tf}}_c\right)^2 + \left(x^{\text{tf}}_c\right)^2\right) i^{\text{sq,tf}}_c \quad \forall cei \in \mathcal{T}^{\text{cv}} \quad \text{(M3.6)}$$

$$(P^{\text{tf}}_{cie})^2 + (Q^{\text{tf}}_{cie})^2 \leq \frac{W^*_i}{(t_c)^2} i^{\text{sq,tf}}_c \quad \forall cie \in \mathcal{T}^{\text{cv}} \quad \text{(M3.7)}$$

Filter

M1.7, M2.12

On/off constraints

$\xi_d \underline{x} \leq x \leq \overline{x}\xi_d$ where $x = [P_{def}, P_{dfe}]$

$\xi_c \underline{x} \leq x \leq \overline{x}\xi_c$ where $x = [P^{\text{cv,ac}}_c, P^{\text{cv,dc}}_c, Q^{\text{cv,ac}}_c, P^{\text{tf}}_{cie}, Q^{\text{tf}}_{cie}, P^{\text{pr}}_{cie}, P^{\text{tf}}_{cei}, W^{\text{f}}_c, i^{\text{cv}}_c, i^{\text{sq,cv,mag}}_c, i^{\text{sq,tf}}_c]$

Auxiliary variable constraints

$\xi_d \underline{x} \leq x^* \leq \overline{x}\xi_d$ where $x = [W^*_{de}, W^*_{df}]$

$x - (1-\xi_d)\overline{x} \leq x^* \leq x - (1-\xi_d)\underline{x}$ where $x = [W^*_{de}, W^*_{df}]$

$\xi_c \underline{x} \leq x^* \leq \overline{x}\xi_c$ where $x = W^*_i$

$x - (1-\xi_c)\overline{x} \leq x^* \leq x - (1-\xi_c)\underline{x}$ where $x = W^*_i$

$\xi_d, \xi_c \in \{0, 1\}$

approximation that discards the reactive power and assumes the dc voltage magnitudes as a constant.

A. Linear Programming AC

The linear AC approximations are developed using the LPAC formulation from [1] and [19] which approximate nonlinear power flow equations based on the following modifications:

(a) Approximation of cosine by its polyhedral relaxation ($\widehat{cos}$) and approximation of $sin(x)$ by x

(b) Voltage magnitudes are assumed to be near to the nominal voltages with tolerance ϕ such that $|V| = 1.0 + \phi$

(c) Approximation of the branch capacity constraint by its piecewise linear (PWL) approximation

(d) Taylor series approximation for any remaining nonlinear terms

While using the already implemented LPAC model in [14] for ac grid, we develop and extend the model for the dc grid that is shown in Model 4. The nonlinear dc branch flow (M1.2) with new voltage representation can be reformulated as:

$$\begin{aligned} P_{def} =& N_d/r_d \cdot (1+\phi_{de})[(1+\phi_{de}) - (1+\phi_{df})] \\ =& N_d/r_d \cdot (1 + \phi^2_{de} - \phi_{df} - \phi_{de}\phi_{df}) \end{aligned} \quad (4)$$

Taking the taylor series approximation of the function near $\phi_{de} = 0$ and $\phi_{df} = 0$ and only considering the linear terms results in constraint (M4.1). The converter losses are approximated by accounting for the linear terms as shown in (M4.2). The nonlinear converter capacity constraint (M1.4) is approximated by replacing voltage and current variables on the right with the converter apparent power rating as shown in (M4.3). Then, the constraint is linearized using the PWL approximation using Algorithm 1 [19]. The transformer and reactor models are created using the ac branch model derived in [20]. The polyhedral approximations of cosine ($\widehat{cos}$) are applied between $[-\pi/6, \pi/6]$ using 20 piecewise segments [1]. Finally, the voltage square term in the filter reactive power is approximated through taylor series expansion as characterized by constraint (M4.8).

Algorithm 1 PWL: converter capacity constraint

Input: $Ns, l, S^{\text{cv,ac}}_c, P^{\text{cv,ac}}_c, Q^{\text{cv,ac}}_c$

Output: Ns no. of linear constraints (= 20 in this paper)

```
l ← 0
inc ← 2 * pi/Ns
for k ← 1 to Ns do
    a ← S̄_c^{cv,ac} · sin(l), b ← S̄_c^{cv,ac} · cos(l)
    Constraint(k) ← a · P_c^{cv,ac} + b · Q_c^{cv,ac} ≤ (S̄_c^{cv,ac})^2
    l ← l + inc
end for
```

B. 'DC' approximation

The typical assumptions made to obtain 'DC' approximation model are:

(a) Phase angle difference of nodal voltages along an ac branch is small enough to ensure $cos(\theta) \approx 1.0$ and $sin(\theta) \approx \theta$

(b) Magnitudes of nodal voltages are fixed at constant value, i.e. 1.0 p.u.

(c) all lines are lossless

The dc branch constraint for a lossless line can defined by (M5.1). Similar to LPAC, the converter losses are accounted through only the linear terms as shown in constraint (M5.2), where $|i^{\text{cv}}_c|$ is replaced by the ratio of $P^{\text{cv,ac}}_c$ and the voltage magnitude. Under the mentioned assumptions, the power flow equations for the transformer reduce to (M5.3) - (M5.4). In absence of the reactive power representation, only the active power balance equation is present at the filter node.

Note that the nodal balance (1) - (2b) remain the same for different relaxations and approximations except the voltage square term $(|U_i|)^2$ that is replaced by lifted variable (W_i) for SOC, $(1 + 2\phi)$ for LPAC and 1.0 for 'DC', respectively.

V. Numerical Experiments

A. Test case preparation

At first, we analyze the performance of the developed formulations using two variants of Garver 6 bus system [21]. The basic 6 bus Garver system (case 6) has 5 interconnected nodes and one isolated generator node [22]. The interconnected part

Model 4: LPAC formulation

Minimize:

(M1.1)

dc branch

$$P_{def} = \left(\frac{N_d}{r_d}\right)(\phi^*_{de} - \phi^*_{df}) \qquad \forall def \in \mathcal{T}^{\text{dc}} \cup \mathcal{T}^{\text{dc,r}} \qquad \text{(M4.1)}$$

Converter

$$P_c^{\text{cv,ac}} + P_c^{\text{cv,dc}} = a_c\xi_c + b_c|i_c^{\text{cv}}| \qquad \forall c \in \mathcal{C} \qquad \text{(M4.2)}$$

$$(P_c^{\text{cv,ac}})^2 + (Q_c^{\text{cv,ac}})^2 \leq (\overline{S_c^{\text{cv,ac}}})^2 \qquad \forall c \in \mathcal{C} \qquad \text{(M4.3)}$$

Transformer (or reactor when t_c = 1)

$\forall cie \in \mathcal{T}^{\text{cv}}$,

$$P^{\text{tf}}_{cie} = g^{\text{tf}}_c\left(\frac{\xi_c + 2\phi^*_i}{t_c^2}\right) - g^{\text{tf}}_c\left(\frac{\widehat{cos}(\theta_i - \theta^{\text{f}}_c) + \phi^*_i + \phi^{\text{f}}_c}{t_c}\right) - b^{\text{tf}}_c\left(\frac{\theta^*_i - \theta^{\text{f}}_c}{t_c}\right) \quad \text{(M4.4)}$$

$$Q^{\text{tf}}_{cie} = -b^{\text{tf}}_c\left(\frac{\xi_c + 2\phi^*_i}{t_c^2}\right) + b^{\text{tf}}_c\left(\frac{\widehat{cos}(\theta_i - \theta^{\text{f}}_c) + \phi^*_i + \phi^{\text{f}}_c}{t_c}\right) - g^{\text{tf}}_c\left(\frac{\theta^*_i - \theta^{\text{f}}_c}{t_c}\right) \quad \text{(M4.5)}$$

$$P^{\text{tf}}_{cei} = g^{\text{tf}}_c\left(\xi_c + 2\phi^*_i\right) - g^{\text{tf}}_c\left(\frac{\widehat{cos}(\theta_i - \theta^{\text{f}}_c) + \phi^*_i + \phi^{\text{f}}_c}{t_c}\right) - b^{\text{tf}}_c\left(\frac{\theta^{\text{f}}_c - \theta^*_i}{t_c}\right) \quad \text{(M4.6)}$$

$$Q^{\text{tf}}_{cie} = -b^{\text{tf}}_c\left(\xi_c + 2\phi^*_i\right) + b^{\text{tf}}_c\left(\frac{\widehat{cos}(\theta_i - \theta^{\text{f}}_c) + \phi^*_i + \phi^{\text{f}}_c}{t_c}\right) - g^{\text{tf}}_c\left(\frac{\theta^{\text{f}}_c - \theta^*_i}{t_c}\right) \text{(M4.7)}$$

Filter

(M1.7)

$$Q^{\text{pr}}_{cie} + Q^{\text{tf}}_{cei} - b^{\text{f}}_c(\xi_c + 2\phi^{\text{f}}_c) = 0 \qquad \forall cie \in \mathcal{T}^{\text{cv}} \qquad \text{(M4.8)}$$

On/off constraints

$\xi_d\underline{x} \leq x \leq \overline{x}\xi_d$ where $x = [P_{def}, P_{dfe}]$

$\xi_c\underline{x} \leq x \leq \overline{x}\xi_c$ where $x = [P_c^{\text{cv,ac}}, P_c^{\text{cv,dc}}, Q_c^{\text{cv,ac}}, P^{\text{tf}}_{cie}, Q^{\text{tf}}_{cie}, P^{\text{pr}}_{cie}, P^{\text{tf}}_{cei}, \phi^{\text{f}}_c, \theta^{\text{f}}_c, i^{\text{cv}}_c, \widehat{cos}(\theta_i - \theta^{\text{f}}_c)]$

Auxiliary variable constraints

$\xi_d\underline{x} \leq x^* \leq \overline{x}\xi_d$ where $x = [\phi^*_{de}, \phi^*_{df}]$

$x - (1-\xi_d)\overline{x} \leq x^* \leq x - (1-\xi_d)\underline{x}$ where $x = [\phi^*_{de}, \phi^*_{df}]$

$\xi_c\underline{x} \leq x^* \leq \overline{x}\xi_c$ where $x = [\phi^*_i, \theta^*_i]$

$x - (1-\xi_c)\overline{x} \leq x^* \leq x - (1-\xi_c)\underline{x}$ where $x = [\phi^*_i, \theta^*_i]$

$\xi_d, \xi_c \in \{0, 1\}$

Model 5: 'DC' formulation

Minimize:

(M1.1)

dc branch

$$P_{def} + P_{dfe} = 0 \qquad \forall def \in \mathcal{T}^{\text{dc}} \qquad \text{(M5.1)}$$

Converter

$$P_c^{\text{cv,ac}} + P_c^{\text{cv,dc}} = a_c\xi_c + b_c\frac{P_c^{\text{cv,ac}}}{1.0} \qquad \forall c \in \mathcal{C} \qquad \text{(M5.2)}$$

Transformer (or reactor when t_c = 1)

$$P^{\text{tf}}_{cie} = -b^{\text{tf}}_c\left(\frac{\theta^*_i - \theta^{\text{f}}_c}{t_c}\right) \qquad \forall cie \in \mathcal{T}^{\text{cv}} \qquad \text{(M5.3)}$$

$$P^{\text{tf}}_{cei} = -b^{\text{tf}}_c\left(\frac{\theta^{\text{f}}_c - \theta^*_i}{t_c}\right) \qquad \forall cie \in \mathcal{T}^{\text{cv}} \qquad \text{(M5.4)}$$

Filter

(M1.7)

On/off constraints

$\xi_d\underline{x} \leq x \leq \overline{x}\xi_d$ where $x = [P_{def}, P_{dfe}]$

$\xi_c\underline{x} \leq x \leq \overline{x}\xi_c$ where $x = [P_c^{\text{cv,ac}}, P_c^{\text{cv,dc}}, P^{\text{tf}}_{cie}, Q^{\text{tf}}_{cie}, P^{\text{pr}}_{cie}, P^{\text{tf}}_{cei}]$

Auxiliary variable constraints

$\xi_c\underline{x} \leq x^* \leq \overline{x}\xi_c$ where $x = \theta^*_i$

$x - (1-\xi_c)\overline{x} \leq x^* \leq x - (1-\xi_c)\underline{x}$ $x = \theta^*_i$

$\xi_d, \xi_c \in \{0, 1\}$

has maximum generation capacity of 530 MW against the total demand of 760 MW. The expansion must take place to connect the generator node 6 to the rest of the network in order to compensate for the lack of generation there. The second variant of Garver system (case 6-fs) does not include any pre-existing network. The 6 isolated nodes are equipped with the same generation and demand as the first case, and expansion must take place in order to form a network. We convert ac line candidates for this bus system (provided in [21]) to the dc line candidates. Even though ac and dc lines have different characteristics, for simplicity, the data for dc line candidate (i.e. resistance, cost, capacity, no. of candidates, connection nodes) are taken from the ac line candidate. One converter candidate is considered per ac node. Table I summarizes the modifications made for this and all other test systems in this paper to motivate a dc grid expansion and Table II lists the total number of candidates for all test cases. The specifications of converter station are prepared as follows. The active and reactive power capability are set to 1000 MW and 500 MVar. The reactance of transformer and reactor are set to 10 % and 7 % of short circuit impedance respectively, and the resistance of both components are assumed to be 0.01 times their reactance values. The filter reactance is assumed to be 8 % of the short circuit admittance value [23]. The converter cost is calculated using an approximate relation [24]:

$$C_c[M€] = 28 + 0.083|\overline{P_c^{\text{cv,ac}}}|[MW] \qquad (5)$$

TABLE I
TEST CASE PREPARATION

Case	ac grid changes	Converter candidate Nos./bus	Converter candidate Cost	Branch candidate Nos./corridor	Branch candidate Cost
case 6, 24	None	1	eq. 5	available ac → dc	
case 6-fs	None	1	eq. 5	available ac → dc	
case 9, 14, 30	3xgen,3xload	1	3 pu	3	1 pu
case 73 (node 1-23)	3xgen,3xload	1	3 pu	3	1 pu
case 118 (node 1-33)	3xgen,3xload	1	3 pu	3	1 pu

TABLE II
TOTAL NUMBER OF CANDIDATES

Case	Total candidates: converter	Total candidates: branch
case 6	6	75
case 6-fs	6	75
case 9	9	27
case 14	14	60
case 24	24	123
case 30	30	123
case 73	28	126
case 118	37	150

B. Evaluation criteria

The indexes for evaluating the quality of solutions are the objective value, the number of built candidates and the feasibility of solutions. The solution obtained using local MINLP solver is used to compare the objective value. The feasibility of the solutions is analyzed by running 'AC' optimal power flow problem of the reinforced grid. The rationale behind doing this instead of an 'AC' power flow check as done in optimal fuel cost studies is as follows. The TNEP problem is usually solved in more than one step and generator and converter set-points are expected to change over the time by underlying operational problems. An 'AC' OPF run should be sufficient to ensure if the obtained reinforcements are physically adequate.

Approximations and relaxations do not always provide a feasible solution. A common approach to deal with this is to have a second corrective step to attain feasibility for TNEP problem. If the solution is infeasible, we apply constraint tightening approach used in [1] to retrieve a feasible solution.

C. Case analysis

'AC' formulation is solved using Juniper 0.5.2 [25], supplied with Ipopt 3.12.10 [26] as a nonlinear solver and Cbc 2.10.3 [27] as a mixed integer solver. The SDP is solved using Pajarito 0.6.0 [28], supplied with Mosek 9.0 [29] as a convex solver and CPLEX 12.7 [30] as a mixed integer solver. The rest of the formulations are solved using Gurobi 8.11 [31]. The tests are run on a server with Intel Xenon, 3.30 GHz processor and 128 GB RAM. The 'AC' formulation is allowed to run for 10 hours but others are set to a time limit of 2 hours. Table III shows the objective function values for both 6 bus cases. The convex relaxations (SOCs, SDP, QC) obtain the same value as 'AC' local solution. LPAC and 'DC' perform well for case 6-fs, but are infeasible for case 6. Refer to table IV to review the candidates built for case 6. All convex relaxations build the same converter stations as the 'AC' solution, but not the same line candidates. It indicates that many local solutions with similar objective function values exist. This is expected since the cost of individual line candidates or combinations of line candidates are equal among themselves, leading to multiple solutions with equal objective values. Since such instances are common among all the test cases, we focus on the number of built candidates rather than particular candidate itself for test cases above 6 bus system. The candidates built for case 6-fs are shown in table V. All formulations build the same candidates and are feasible.

We further include the IEEE 9, 14, 30, 73 and 118 bus systems from library in [32] and 24 bus system from [21] to provide a larger test cases set. The following changes are made to motivate the expansion. For cases 9, 14 and 30, the generation and load values in the whole grid and for 73 and 118 bus systems, the generation and load values in a section of the grid, are increased by a factor of 3 to motivate the expansion. The sections are selected as the first 23 buses for 73 bus system and the first 33 buses for 118 bus system. The buses directly connected to these sections are also included. To limit the node numbers for candidate dc line connections, it is assumed that they can only be installed using the existing ac line corridors. A corridor here refers to the existing ac branch installation and small area available around it along its route. Three branch candidates per corridor and one converter candidate per node are considered for which the cost are taken as 1 pu and 3 pu respectively. For 73 and 118 bus systems, the candidates are only provided in the selected section of the grid. The 24 bus system (case 24) is already a TNEP test case, so it does not require the increase in the generation and load to motivate the expansion. The candidate preparation for this test case is similar to 6 bus system described before. Also, the converter specifications for all test cases are prepared based on the assumption mentioned in section V-A. Refer to Table I and II for a summary on the grid modification and expansion candidates.

For case 9, all formulations provide a feasible solution (see table III) with the same objective value. For case 14 onwards, all formulations generate infeasible reinforcements except for LPAC in two instances. The SDP formulation does not converge within the time limit for any test case above 9 bus system. The MINLP does not converge within the time limit for any test case beyond and including 30 bus system. The number of built line and built converter candidates are shown in table VI. Among the feasible instances, all non-'AC' formulations build same number of candidates as 'AC' for case 9 and LPAC builds same number of candidates as 'AC' for case 14.

We use the constraint tightening approach for all infeasible instances. The thermal limits of ac lines are reduced progressively, in four steps of 5 % until a feasible solution is found. If feasible solution is not found in four steps, the solution at the last step (i.e. 20 % line rating reduction) is reported here. A relaxation or approximation may overestimate the power flows in the ac branches. The constraint tightening would remove such instances by setting a lower thermal limits so that even an overestimation remains within the original thermal limit. The constraint tightening is not done for SDP because of its scalability issues.

For case 6, LPAC successfully gains the feasibility with the same objective value as 'AC', but 'DC' solution remains infeasible. The built candidates are indicated in table IV with 'ct'. After constraint tightening, LPAC builds an additional converter at node 2 and becomes feasible whereas 'DC' approximation builds more lines instead and still miss one converter in obtaining feasibility. The case 6-fs and case 9 are already feasible, hence constraint tightening is not required for them.

After constraint tightening, all convex formulations for case 14 achieve the objective value same as 'AC' solution (see table III) and are feasible. They also build the same number of candidates (see table VI). For case 24, all convex relaxations and LPAC reach to feasibility at a higher objective value than 'AC' solution. It is interesting to note that even though the SOC solution proposes one dc line less than 'AC', SOC formulation builds more expensive lines. This can be explained as follows: the constraint tightening may cut through the original feasible space with the reduced thermal rating of lines. For case 30 and above, no benchmark to compare the results

TABLE III
OBJECTIVE VALUES AND COMPUTATIONAL TIMES (NUMERICAL ISSUES INDICATED*)

	objective value							computational time (sec)						
case	'DC'	LPAC	SOCBF	SOCWR	QC	SDP	'AC'	'DC'	LPAC	SOCBF	SOCWR	QC	SDP	'AC'
case 6	483	484	**595**	**595**	**595**	**595**	**595**	0.34	0.31	1.69	13.47	31.3	279	3442
case 6-fs	**755**	**755**	**755**	**755**	**755**	**755**	**755**	0.18	0.16	0.45	3.42	2.71	72.96	5590
case 9	**8.0**	**8.0**	**8.0**	**8.0**	**8.0**	**8.0**	**8.0**	0.069	0.23	0.19	0.27	0.31	26.62	59
case 14	13.0	**14**	12.0	12.0	12.0	*	**14.0**	0.18	2.05	0.43	2.74	5.65	limit	4729
case 24	320	527	494	494	510	*	**638**	0.42	8.93	7.52	18.39	65.74	limit	31183
case 30	23	27	24	24	24	*	*	1.08	7.91	11.96	35.07	53.98	limit	limit
case 73	16	25	22	22	23	*	*	0.31	14.25	16.61	26.23	1050	limit	limit
case118	25	**27**	22	22	23	*	*	1.71	45.69	73.35	162	2765	limit	limit
constraint tightening														
case 6	515	**595**	NA	NA	NA	NA	NA	0.2	0.3	NA	NA	NA	NA	NA
case 14	14.0	NA	**14.0**	**14.0**	**14.0**	NA	NA	0.12	NA	4.18	7.28	7.28	NA	NA
case 24	547	**662**	**694**	**694**	**703**	NA	NA	0.8	8	33	86	362	NA	NA
case 30	27	**28**	**28**	**27**	**28**	NA	NA	0.76	9.57	116	45.29	68.16	NA	NA
case 73	26	31	**27**	**27**	**27**	NA	NA	1.01	17.11	13.19	109	302	NA	NA
case 118	**31**	NA	**29**	**29**	32*(23%)	NA	NA	4.57	NA	194	213	limit	NA	NA

(Objective values are in M$ for original TNEP test cases and in pu for 9, 14, 30, 73 and 118 bus systems. Feasible solutions are marked in bold)

TABLE IV
CANDIDATES BUILT FOR CASE 6 (C-CONVERTER AND L-LINE)

	built converters (✓)				No. of built lines			
	C2	C4	C5	C6	L26	L46	L25	L56
'AC'	**✓**	**✓**	**✓**	**✓**	**1**	**2**	-	**1**
SOC BFM	**✓**	**✓**	**✓**	**✓**	-	**3**	-	**1**
SOC BIM	**✓**	**✓**	**✓**	**✓**	**2**	**2**	**1**	-
SDP	**✓**	**✓**	**✓**	**✓**	**3**	**1**	**1**	-
QC	**✓**	**✓**	**✓**	**✓**	**1**	**2**	-	**1**
LPAC	-	✓	✓	✓	1	3	1	-
LPAC-ct	**✓**	**✓**	**✓**	**✓**	**1**	**3**	**1**	-
'DC'	✓	✓	-	✓	1	4	-	-
'DC'-ct	✓	-	✓	✓	4	-	2	-

feasible solutions are in bold. ct = solution after constraint tightening

TABLE V
CANDIDATES BUILT FOR CASE 6-FS (C-CONVERTER AND L-LINE)

	built converters (✓)					No. of built lines			
	C2	C3	C4	C5	C6	L26	L46	L35	L23
All formulations	**✓**	**✓**	**✓**	**✓**	**✓**	**2**	**2**	**3**	**1**

is available. Instead, we comment the quality of solutions based on the feasibility. For case 30 and case 73, all feasible solutions are in the vicinity to each other, with maximum difference of 1 pu among them and building almost the same number of candidates. For case 118, the objective values of feasible formulations after constraint tightening are higher than the feasible LPAC solution before constraint tightening, hence it is not the best possible solution. This can be due to the reduction in feasibility space as mentioned above. The objective value of QC does not reach to an optimality within time limit and the value shown here is with 23 % gap from incumbent. The number of candidates proposed for this case are considerably different for among different formulations. It is worth to mention that a lower resolution of constraint tightening steps ($<$ 5 % here) may able to provide better solutions.

The computational time is of an importance since it can indicate the scalability of formulations as problem size gets bigger e.g. multi-period TNEP [11]. The SDP takes the most time among all non-'AC' formulations. LPAC and SOC models seem to be the computationally more efficient than QC while resulting in feasibility in most cases.

VI. CONCLUSION

The nonlinear, non-convex TNEP problem is difficult and computationally demanding to solve. We formulate different linear approximations and convex relaxations for the problem in order to find a trade-off between speed and accuracy. Additionally, the LPAC approximations are developed for dc grid. A number of dc grid TNEP test cases are introduced based on the existing ac grid TNEP or OPF test cases.

Although none of the non-'AC' formulations are feasible for all test cases, most convex relaxations and LPAC attain the feasibility after the constraint tightening procedure, except LPAC for case 73 and QC for case 118. Particularly, both SOC models - BIM and BFM are feasible for all test cases. 'DC' approximation remains infeasible even after the constraint tightening except for 118 bus system. We observe that the constraint tightening does not always result in the best solution but performs well to achieve the feasibility of a difficult mixed integer problem.

From a computational point of view, SOC and LPAC formulations are more attractive compared to QC. The MISDP solvers are not matured yet, therefore, this formulation is not interesting for the TNEP problem at this point of time.

TABLE VI
NO. OF CANDIDATES BUILT FOR 9 - 118 BUS SYSTEMS (C-CONVERTER AND L-LINE)

	Case 9		Case 14		Case 24		Case 30		Case 73		Case 118	
	C	L	C	L	C	L	C	L	C	L	C	L
'AC'	**2**	**2**	**3**	**5**	**4**	**5**	*	*	*	*	*	*
SOC BIM	**2**	**2**	3 (**3**)	3 (**5**)	4 (**4**)	1 (**4**)	5 (**5**)	9 (**12**)	5 (**6**)	7 (**9**)	4 (**6**)	10 (**11**)
SOC BFM	**2**	**2**	3 (**3**)	3 (**5**)	4 (**4**)	1 (**4**)	5 (**5**)	9 (**13**)	5 (**6**)	7 (**9**)	4 (**6**)	10 (**11**)
SDP	**2**	**2**	*	*	*	*	*	*	*	*	*	*
QC	**2**	**2**	3 (**3**)	3 (**5**)	4 (**4**)	2 (**5**)	5 (**5**)	9 (**13**)	5 (**6**)	8 (**9**)	4	11
LPAC	**2**	**2**	**3**	**5**	3 (**4**)	5 (**6**)	5 (**5**)	12 (**13**)	6 (6)	7 (13)	**5**	**12**
'DC'	**2**	**2**	3 (3)	4 (5)	2 (3)	4 (5)	4 (4)	11 (15)	3 (5)	7 (11)	4 (**5**)	13 (**16**)

results after constraint tightening are shown in bracket.

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