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Leobacher, Gunther; **Pillichshammer, Friedrich**

Introduction to quasi-Monte Carlo integration and applications. 2nd edition. (English)

Zbl 08107850

Compact Textbooks in Mathematics. Cham: Birkhäuser (ISBN 978-3-032-05445-6/pbk; 978-3-032-05446-3/ebook). xviii, 244 p. (2026).

In this revised edition, of the original textbook [*G. Leobacher* and *F. Pillichshammer*, Introduction to quasi-Monte Carlo integration and applications. Cham: Birkhäuser/Springer (2014; Zbl 1309.65006)] the basics of Quasi-Monte Carlo integration are reviewed together with some applications. Monte Carlo considers the integral to be an expected value which can be approximated by averages of N randomly sampled values. Convergence is slow (essentially $N^{-1/2}$), but without the curse of dimensionality it is useful to evaluate high dimensional integrals. The 'quasi' refers to the fact that the samples are not purely random, but taken at deterministically selected point sets that are uniformly distributed.

In the book the standard region is the hypercube $[0,1]^s$, so the problem is to generate uniformly distributed numbers modulo 1. This branch of number theory is introduced in Chapter 2. Discrepancy is a measure for how much a distribution deviates from being uniform, and hence it is important to know its lower bounds. Theorems for these bounds are proved and examples of classical point sets are given (van der Corput, and its generalizations: Halton and Hammersley points, all based on the digits in the expansion of integers in a certain basis b).

Studying their properties requires the introduction of continuous and discrete L_p spaces. In particular reproducing kernel Hilbert spaces, are useful since the kernels are an essential tool to define the worst case error in the QMC method. This relies on the Koksma-Hlawka inequality obtained in Chapter 3 using more functional analysis.

The previous ingredients are needed to come to the construction of lattice point sets with low discrepancy in Chapter 4 (using e.g. the (fast) component-by-component or CBC algorithm which is closely related to FFT). This allows to study QMC integration in Korobov spaces (i.e. when the Fourier coefficients decrease fast enough). Also tractability is discussed, i.e., how many evaluations are needed to reduce the worst case error by a factor ε?

Chapter 5 deals with the further construction of low discrepancy point sets, in particular (t,m,s)-sets (sets of b^m points in dimension s with t some quality parameter for the discrepancy) and (t,s)-sequences (an infinite sequence that will generate a (t,m,s)-set in basis b).

Chapter 6 is really brief and adds some notes on discrepancy bounds and tractability.

While the treatment has been theoretical so far, a glance at applications is given in the remaining two chapters. Chapter 7 introduces some elements from financial mathematics, especially the pricing of financial derivatives, which are further worked out in Chapter 8 where an introduction to the use of (Q)MC methods in a wider context such as price sensitivity or Brownian motion generation, etc. is given.

This is just an introduction and there is much more to be explored, but all the basic ingredients are introduced, based on a good mathematical basis with precise definitions given and properties carefully proved. Some previous knowledge of functional analysis is helpful. All chapters are extended with a set of exercises.

As compared to the first edition, there is some refinement and extension about weighted discrepancies and tractability, nonperiodic functions, and polynomial lattice point sets, and more details for the CBC algorithm, but the basics have been maintained.

Reviewer: Adhemar Bultheel (Leuven)

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MSC:

65-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to numerical analysis	Cited in **5** Documents
65C05	Monte Carlo methods	
91G60	Numerical methods (including Monte Carlo methods)	
65C10	Random number generation in numerical analysis	
65C30	Numerical solutions to stochastic differential and integral equations	
11K38	Irregularities of distribution, discrepancy	

Keywords:

Halton point set; Koksma-Hlawka inequality; Korobov space; textbook; quasi-Monte Carlo methods; multidimensional numerical integration; Monte Carlo integration; component-by-component algorithm; financial mathematics; options valuating; stochastic differential equation; pseudo-random numbers generation

Full Text: DOI

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