Bases for C^1 quadratic splines on Powell-Sabin triangle splits

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In this paper we look at different bases for the space $S_2^1(\Delta_{PS})$ of C^1 continuous quadratic splines on Powell–Sabin triangle splits. First, we describe a general framework that leads to the notion of control points and control triangles. This is a useful property that gives insight in the shape of the surface. Then we consider two bases in particular: a normalised B-spline basis, and one based on minimal determining sets. In the normalised B-spline representation the basis functions are all positive, have local support and form a partition of unity. Geometrically these requirements can be interpreted as finding a triangle containing a specific set of Bézier points. We show why the triangle with minimal area is a suitable choice. The approach with minimal determining sets leads to bases that are stable as a function of the smallest angle in the triangulation. The basis functions are not only linearly independent but also local linearly independent. Compared to the normalised B-spline representation, we have given up positivity for other properties. In between these two are other possibilities that depend on the requested properties of the basis functions.