Identifying linear systems from binary output data

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1 Motivation

This work considers the identification of linear systems based on binary output measurements. This can be useful for applications where sensors measuring the output are too costly or impractical to install. If it is possible to determine whether the output is above or below a given threshold, this information can then be used instead. An example is a moving object whose position is not measured, but where an optical sensor can detect when it passes a specific point.

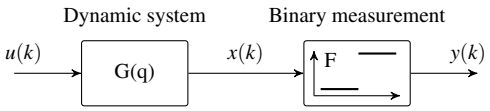


Figure 1: Schematic representation of considered systems.

Fig. 1 gives a schematic representation of the considered systems, with the binary measurement represented by a non-linear function F. It relates the output of the linear system $x(k)$ to the binary value $y(k)$ through

$$y(k) = F(x(k)) = \begin{cases} 0 & \text{if} \quad x(k) < x_{th}, \\ 1 & \text{if} \quad x(k) \geq x_{th}, \end{cases}$$

where x_{th} is the threshold at which the binary output changes in value between 0 and 1. The goal is now to identify a good model $\hat{G}(q)$ without measuring $x(k)$, using only $u(k)$ and $y(k)$, as well as the known behavior of F.

2 Methodology

In contrast to existing techniques with strict requirements on the excitation signals, the available data is assumed to be obtained from short and independent experiments, during which only a single transition of the binary output is observed. This situation often arises in practical cases, such that data measured during normal system operation can be used. However, a consistent identification can then only be performed if data from multiple experiments is combined.

The identification relies on making predictions $\hat{x}(k)$ for the different experiments. These are determined using the known input signals and an estimated Finite Impulse Response (FIR) model $\hat{G}(q)$. The best model is then found by minimzing a cost function that is composed by comparing the values of $\hat{x}(k)$ with the measured binary values $y(k)$, taking into account the known behaviour of the binary measurement. Two different methods are developed to do so:

- The first method directly uses the binary information and applies penalties for predicted values of $\hat{x}(k)$ that are not in the range corresponding to the binary value. This achieves a high prediction accuracy but yields constrained optimization problems.
- The second method is developed as an alternative to reduce the required computational load of the first. By assuming the output equals the threshold near the detected transition, a small error is introduced but the estimation reduces to an unconstrained linear least squares optimization problem. Solutions can therefore be found much faster, and it also becomes possible to find recursive solutions.

3 Experimental validation

For the experimental validation the test setup shown in Fig. 2 is available. It contains a sliding mass which is driven by an electromotor. The mass's position is measured with a linear encoder, while an optical sensor also provides binary measurements. The models estimated using the binary data can thus be evaluated by comparing their predictions to the true measurements, or by comparing the models to those estimated with the data from the linear encoder.

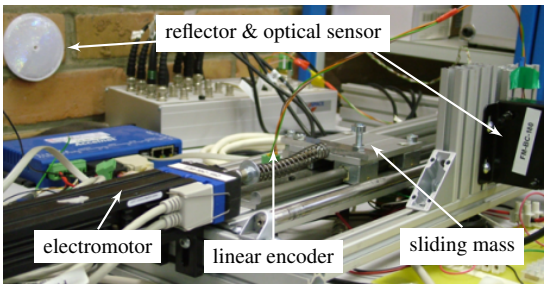


Figure 2: Test setup with binary output sensor.

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