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Compressibility and temperature effects on turbulent spot growth

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Abstract

The production, growth and propagation of turbulent spots is a driving mechanism behind the process of bypass transition. An increased understanding of the behaviour of turbulent spots will therefore help the understanding and the prediction of transition onset. Recently an intermittency-based transition model by the same authors used the turbulent spot growth mechanism in their calculation of the intermittency production. A mathematical model for the effect of compressibility and wall temperature effects on the turbulent spot is thus required to accurately predict and replicate the transition behaviour in hypersonic flows. The current work synthesises experimental and numerical data for spot growth and aims to present an empirical model for the spot propagation parameter that includes compressibility and wall temperature effects.

Keywords: *turbulent spot, transition, modelling, compressibility, wall temperature*

Nomenclature

Latin
M – Mach number
T – Temperature
U – Velocity
x – Streamwise coordinate
z – Spanwise coordinate
Greek
β – Turbulent spot spreading angle
γ – Intermittency
σ – Spot propagation parameter
Subscripts
∞ – Freestream value
ad – Adiabatic
e – Edge value
le – Leading edge
te – Trailing edge
w – Value at the wall
wt – Wing tip

1. Introduction

During the super- and hypersonic flight in the atmosphere, both for ascent or re-entry, the boundary layer will experience a transition from a laminar towards a turbulent flow behaviour. As a turbulent flow results in a stronger mixing, the skin friction and heat transfer increase easily with a factor 3 to 4. Hence, the transition point and length are of great importance to evaluate drag and heat load. Moreover, close to the end of transition, the skin friction and heat transfer rates overshoot the pure turbulent values with at least 15%-30%. Finally, the presence of a SWBLI impinging near the start of transition can even enlarge the effect up to a factor 6 [1].

Compressibility of the flow has the tendency to postpone the transition point and to extend the transitional zone (e.g. at only Mach=1.2 already with a factor 2). So far, correlations to account for compressibility provide large variations and need to be assessed. According to the NASP Task Force report [2], estimates of transition onset range from 20% to 80% along the body. This estimate range made for the point of transition can affect the design vehicle gross take-off weight by a factor of two or more. Hence, the importance to dedicate both experimental and numerical efforts to evaluate the

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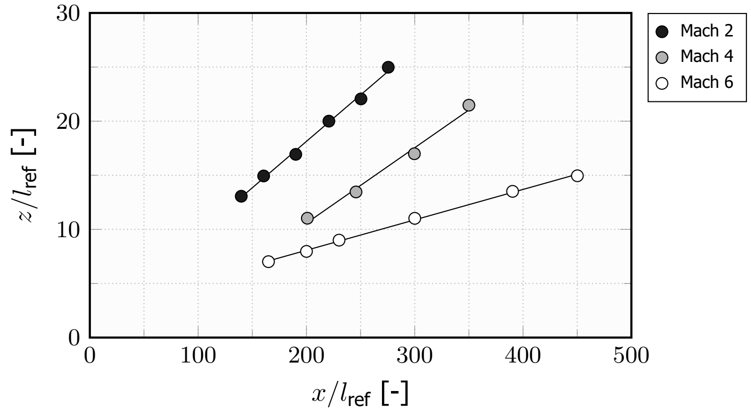

Fig 1. Lateral wingtip locations of spot with axial distance. (Reproduced from Krishnan & Sandham [3])

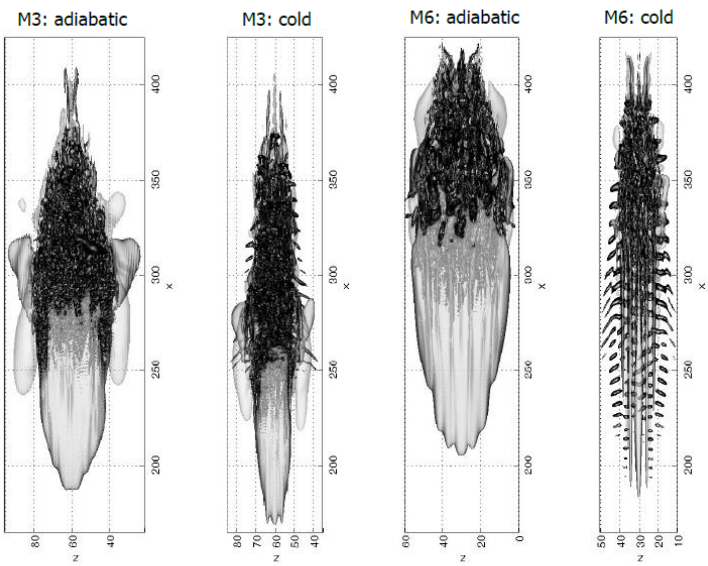

Fig 2. Effect of wall temperature on turbulent spot planform. (Redford et al. [4])

effects of compressibility and wall temperatures on the overall transition onset and extent. Measuring the compressibility effects by performing experiments is tedious and very costly. The recent progress obtained in LES- and DNS-techniques for transitional flow simulation helps in understanding supersonic features in boundary layers and turbulent spots. In particular detailed investigation of the turbulent spot growth is necessary to assess the effect of compressibility and wall temperature on transition length (Figs. 1 and 2).

A mathematical model accounting for the effect of compressibility and wall temperature effects on the turbulent spot is thus required to accurately predict and replicate the transition behaviour in hypersonic flows. The current work synthesises experimental and numerical data for spot growth and aims to present an empirical model for the spot propagation parameter that includes compressibility and wall temperature effects.

2. Background

Several researchers have been investigating turbulent spots over the years, and a large number of experiments and direct numerical simulations have been performed on this topic.

Narasimha [5] presented an extensive review on the role of turbulent spots in the transition process. They concluded that the concept of concentrated breakdown can describe the intermittency distribution observed in transitional flows for a large range of flow conditions. They also determined that the non-dimensional spot breakdown parameter would have to be dependent on freestream disturbances, compressibility and pressure gradient. In addition they concluded that *"spot propagation characteristics need to be investigated in a variety of environments, and may reveal unsuspected surprises yet"*.

Clark et al. [6] performed experiments of natural transition and tracked the evolution of turbulent spots. They extracted the convection speeds of leading and trailing edges of a turbulent spot and found that both the leading and trailing edge convection speeds are fairly constant for Mach numbers up to 1.86. Both the convection speeds and the spot spreading angle were found to be dependent on the pressure gradient however.

Zanchetta & Hillier [7] did experiments on 5° cones at Mach 9 with various degrees of nose bluntness. They used high-speed heat transfer measurements and extracted the streamwise intermittency profile. From the time history of the different thin-film gauges given in the article, the convection speed of a turbulent spot can be estimated. Also Mee and Goyne [8] used thin-film gauges to identify turbulent spot convection speeds. Their experiments were performed on flat plates at Mach 5.6.

Mee [9] performed experiments on natural and forced transition, by means of roughness elements, in low and high enthalpy conditions at Mach 6. They did not observe a large difference between the high-enthalpy and lower-enthalpy condition. They did see a strong compressibility effect on the spot spreading angle, with higher Mach numbers yielding smaller spreading angles. Fiala et al. [10] performed experiments at Mach 8.9 on a blunted cylindrical body instrumented with thin-film gauges and investigated in detail the characteristics of individual turbulent spots.

Krishnan & Sandham [3] were one of the first to run Direct Numerical Simulations (DNS) focussing on turbulent spots characteristics at hypersonic speeds. Spot spreading angle, convection speeds and propagation parameter σ were computed and listed for Mach 2, 4 and 6. Because of the detailed spatial and temporal nature of data generated by DNS, extracted spot properties like convection speed or spreading angle are potentially more precise than those obtained from experimental measurements. This does not necessarily mean however that simulations are always more accurate.

Sivasubramanian & Fasel [11] also performed DNS of wave packets and turbulent spots on a sharp cone at Mach 6. They looked in detail at the three-dimensional structure of the turbulent spot and associated structures.

Redford & Sandham [4] looked at the effect of compressibility and wall temperature on the spreading angle of a turbulent spot at Mach 3 and 6. Using DNS they found that turbulent structures advecting outwards form the spot core and an instability of lateral jets coming from the spot are responsible for the lateral growth of a turbulent spot.

Jewel et al. [12] performed a large experimental test campaign on turbulent spot propagation on a cone. They used a high-enthalpy shock tunnel and different gas mixtures to analyse spot convection speeds and intermittency distributions. They also made a survey on spot propagation studies and summarised the reported convection speeds in literature. Raghunath et al. [13] presented a method to estimate turbulent spot initiation rates and intermittency from experimental data. They also showed that it could be used as a prediction tool for the intermittency distribution.

3. Empirical model of turbulent spot growth

Results for the spot convection speed, spreading angle and/or spot propagation parameter σ were extracted and synthesised from the literature described in Section 2. This data is given in Table 1, and was taken directly from the original article or from secondary articles (e.g. [12, 13]).

3.1. Turbulent Spot Structure

A turbulent spot consists of essentially an arrowhead-shaped turbulent region pointing downstream, followed by a region of laminar calmed flow. The lateral spot growth is measured by the spot spreading

Table 1. Sources and data from experiments and numerical simulations used in the current analysis.

Reference	M_e [-]	T_w/T_e [-]	U_{front} [-]	U_{tail} [-]	β [deg]	σ [-]
Narasimha (1985)	0.00	1.00	0.88	0.50	11.3	0.27
Clark et al. (1994)	0.24	0.71	0.86	0.56	10.9	0.15
Clark et al. (1994)	0.55	0.67	0.81	0.56	9.0	0.12
Clark et al. (1994)	1.32	0.53	0.85	0.54	7.0	0.10
Clark et al. (1994)	1.86	0.42	0.83	0.53	—	—
Zanchetta & Hillier (1995)	9.00	4.38	0.98	0.68	—	—
Mee and Goyne (1996)	5.60	—	—	—	2.8	—
Mee (2002)	6.20	0.37	0.90	0.50	3.5	0.05
Krishnan & Sandham (2006)	2.00	1.67	0.87	0.53	5.0	0.08
Krishnan & Sandham (2006)	4.00	3.69	0.87	0.59	4.0	0.05
Krishnan & Sandham (2006)	6.00	7.00	0.87	0.68	1.7	0.02
Fiala et al. (2006)	3.50	0.97	0.81	0.40	6.8	0.15
Sivasubramanian & Fasel (2010)	5.35	7.48	0.92	0.79	2.3	0.01
Redford & Sandham (2012)	3.00	2.50	0.90	0.60	7.5	0.07
Redford & Sandham (2012)	3.00	1.00	0.85	0.60	4.5	0.04
Redford & Sandham (2012)	6.00	7.00	0.90	0.75	2.5	0.01
Redford & Sandham (2012)	6.00	1.00	0.90	0.75	1.5	0.01
Jewel et al. (2017)	4.75	0.18	0.91	0.63	3.3	0.03
Jewel et al. (2017)	6.14	0.38	0.91	0.63	2.7	0.02

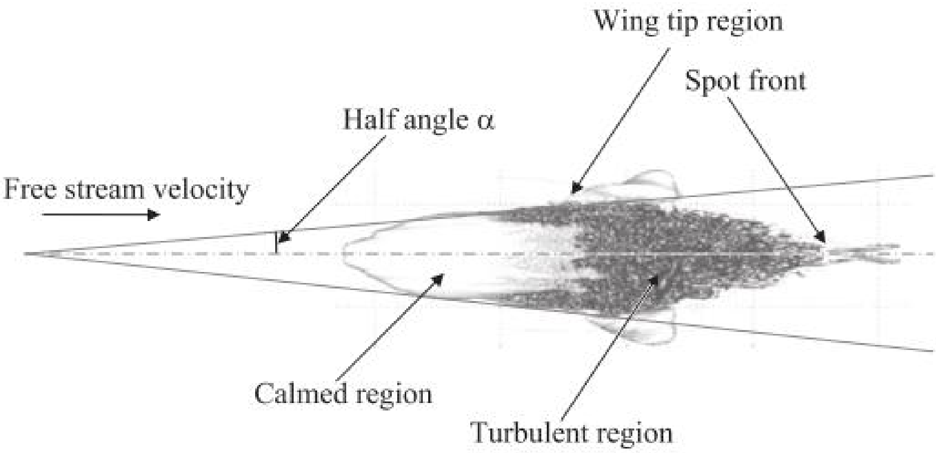

Fig 3. Planform shape of a turbulent spot. (Redford et al. [4])

(half-)angle β formed by the *wing tips* of the turbulent region. A visualisation of the planform shape of a turbulent spot, as presented by Redford et al. [4], is shown in Fig. 3.

The formation, propagation and merging of individual turbulent spots will eventually yield a fully turbulent boundary layer. The transitional state of the boundary layer can then be described by the intermittency γ, governed by this turbulent spot behaviour. Dhawan & Narasimha [14] used the concept of concentrated breakdown to formulate a function for the intermittency distribution in the transitional region,

$$\gamma(x) = 1 - \exp\left[-(x-x_t)^2 \frac{n\sigma}{u_\infty}\right] \qquad (\text{for } x > x_t) \tag{1}$$

where x_t is the transition onset location, n is the breakdown rate and σ the spot propagation parameter. A model for the spot breakdown rate and propagation parameter could therefore yield a predictive model for the intermittency during the transition process. The current work aims to present an empirical model for the compressibility and wall temperature effect on $n\sigma$.

3.2. Spot Convection Speed

The arrowhead-shaped turbulent spot, as shown in Fig. 3, grows in streamwise extent due to the different speeds of the leading edge and trailing edge. The leading edge speed is widely report to be in the range of 81-89% of the freestream velocity, while the trailing edge speed lies between 40-68% [13]. The data on convection speeds, summarized in Table 1, is plotted against the edge Mach number in Figs. 4 and 5. From these figures it can be seen that the scatter in the resulting convection speeds is very high.

The leading edge speed is sometimes described as being constant with Mach number [3], however this does not seem to hold when looking at the data in Fig. 4. Similarly to the trailing edge speed, as shown in Fig. 5, a weakly positive correlation seems to exist between the convection speeds and the edge Mach number M_e.

In the current model the following convection speeds of the front and tail are used and indicated in Figs. 4 and 5:

$$\frac{U_{\text{front}}}{U_\infty} = (0.82 + 0.017\, M_e) \tag{2a}$$

$$\frac{U_{\text{tail}}}{U_\infty} = (0.50 + 0.025\, M_e) \quad . \tag{2b}$$

The wall temperature could have an effect on the convection speed by altering the local Mach number close to the wall. This could potentially explain some of the scatter in Figs. 4 and 5, since it does not

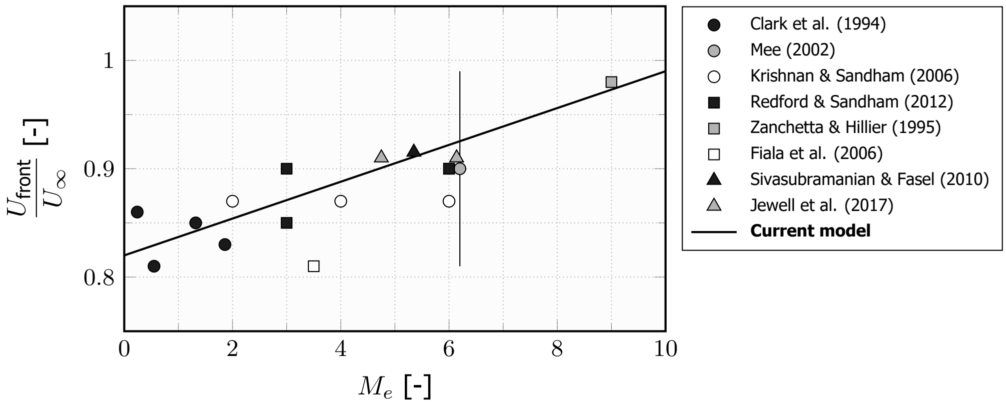

Fig 4. Convection speed of the front part of a turbulent spot.

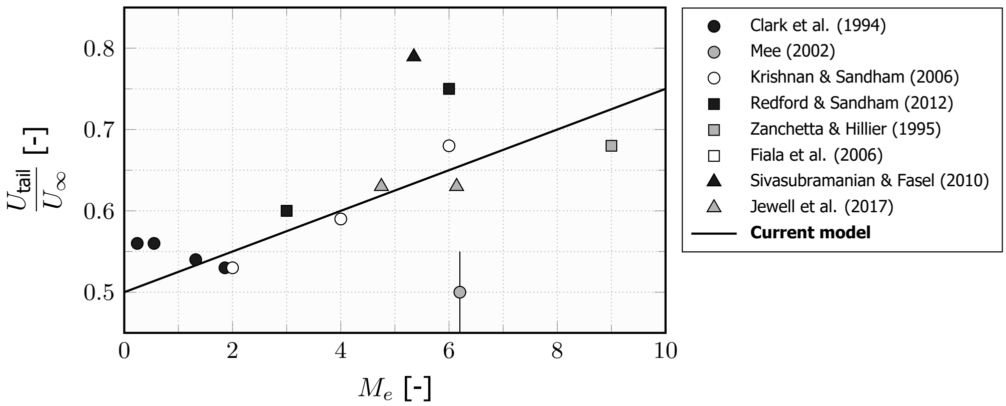

Fig 5. Convection speed of the tail of a turbulent spot.

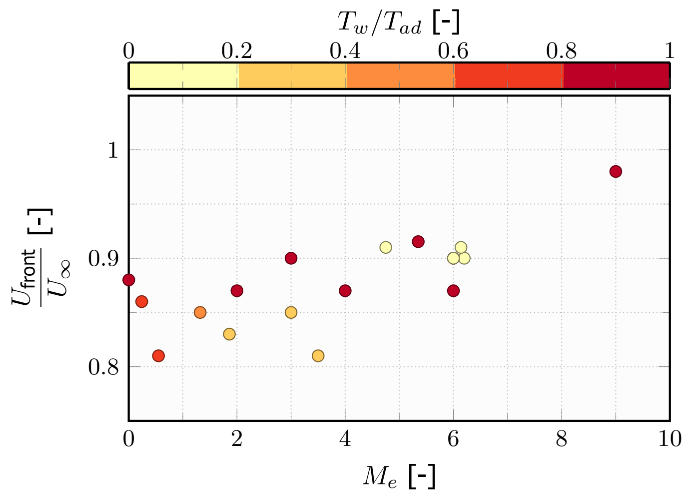

Fig 6. Temperature dependency of the convection speed of the front part of a turbulent spot.

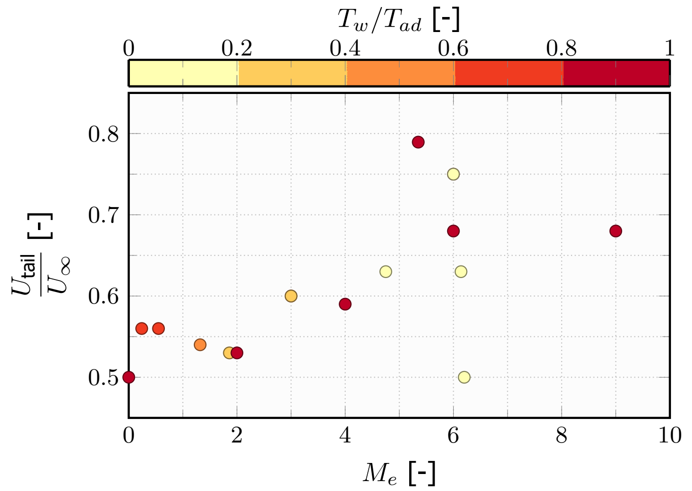

Fig 7. Temperature dependency of the convection speed of the tail of a turbulent spot.

take into account the wall temperature. However, when looking at Figs. 6 and 7, which incorporates the effect by including the ratio of wall temperature over adiabatic wall temperature, no clear trend is immediately visible.

3.3. Spot Spreading Angle

Due to the lateral spreading of the turbulent spot, it forms a spreading angle when convected downstream, as shown in Fig. 1. This lateral spreading is due to turbulent structures advecting outwards from the spot core and an instability of lateral jets coming from the spot. [4]

The spreading angle is known to be strongly reduced with Mach number [15], which is also clear when looking at the data plotted in Figs. 8 and 9. Still a significant amount of scatter is present in the data. This could be partly explained by differences in measurement techniques. For example, measurements of the spreading angle taken at the wall (e.g. using liquid crystals or infra-red thermography) will yield a smaller turbulent spot width than measurements taken at some distance from the wall (e.g. using hot-wire anemometry) because of the three-dimensional shape of the turbulent spot. This was observed by Chong and Zhong [16] and later also reported by Krishnan and Sandham [3].

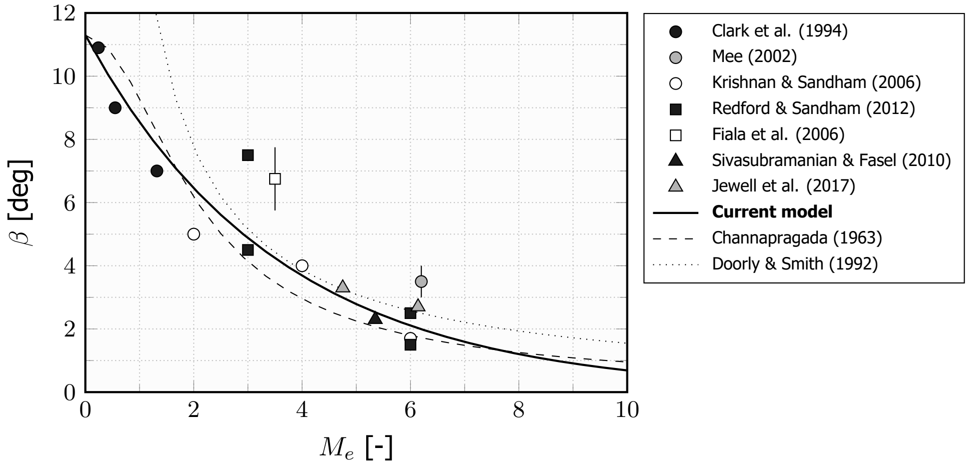

Fig 8. Spot spreading angle β of a turbulent spot.

Doorly & Smith [17] performed a theoretical analysis of spot development in incompressible and compressible boundary layers. They formulated a theory in the hypersonic limit for the half-angle of convecting waves, essentially being inversely proportional to the Mach number. Channapragada [18] derived a model for the spreading of a compressible jet based in terms of Mach number and stagnation temperature ratio. This model can be applied to spreading angle of a turbulent spot by multiplying the Mach-dependent correction with the spreading angle in the incompressible limit. Both models are plotted in Fig. 8, for which can be seen that the latter has a fairly good agreement with the data points of Table 1.

In the current work, a simple exponential decay model is used for the spreading angle of the turbulent angle:

$$\frac{\beta}{\beta_0} = \exp\left(-0.28\, M_e\right) \tag{3}$$

in which the incompressible angle β_0 is taken from Narasimha [5] to be $\beta_0 = 11.3$ deg. This function is shown in Fig. 8.

The effect of wall temperature on the spot spreading angle is found to be significant and cold wall conditions seem to result in a smaller spreading [3, 4]. However, this does not seem to be very clearly represented as a general trend in Fig. 9, as the scatter in the published results is very large. For example, at Mach 6 some spreading angles with cold wall conditions ([9, 12]) are actually greater than the hot wall cases of [3, 4].

3.4. Spot Propagation Parameter

A model for the spot propagation parameter σ is important when developing a predictive model for the intermittency. Understanding the effect of compressibility and wall temperature is thus crucial to have accurate predictive capabilities of such a model. The spot propagation parameter can be modelled using the convection speed and spreading angle of the spot. Vinod & Rama [19] defined the spot propagation parameter as

$$\sigma = \left[\frac{U_\infty}{U_{te}} - \frac{U_\infty}{U_{le}}\right] \tan\beta \tag{4}$$

while Krishnan & Sandham [3] defined it as

$$\sigma = \frac{1}{2}\frac{U_\infty}{U_{wt}} \tan\beta \quad . \tag{5}$$

In Fig. 10 the spot propagation parameter σ as given in Table 1 is plotted. Note that this data is either taken from values declared directly by the corresponding authors, or otherwise computed using the

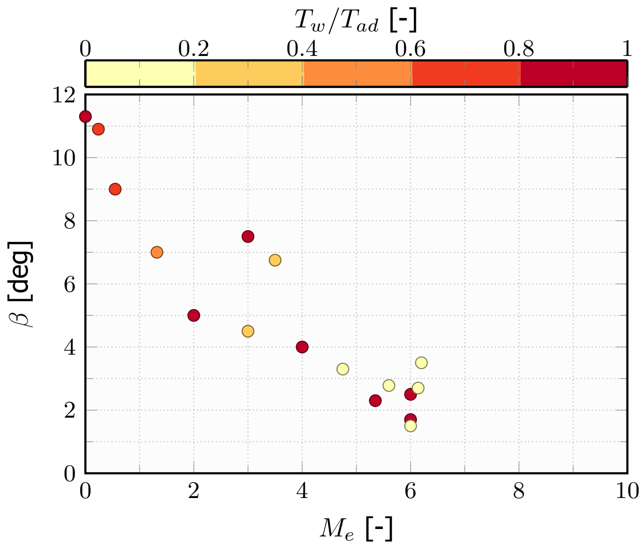

Fig 9. Spot spreading angle β of a turbulent spot.

definition of Vinod & Rama. Similarly to the data of convection speeds and spreading angle, significant scatter exists for this parameter. The correlation by Steelant & Dick [20, 21], given as

$$\frac{\sigma_{\text{S\&D}}}{\sigma_0} = \left(1 + 0.58\, M_e^{0.6}\right)^{-2} \tag{6}$$

with the incompressible value $\sigma_0 = 0.27$, is also shown in this figure.

By combining the model equations for the convection speeds and the spot spreading angle, i.e. Eqs. (2) and (3), within the spot propagation parameter equation of Vinod & Rama [19], Eq. (4), a model for the spot propagation parameter is obtained. Because it is found to underpredict the incompressible value of Klebanoff & Schubauer [22], an additional premultiplication term for low Mach numbers is added to the model. The overall model equation for the spot propagation parameter, shown in Fig. 10, then becomes:

$$\sigma_{\text{model}} = F_0 \left[\frac{1}{u_{te}} - \frac{1}{u_{le}}\right] \tan\beta \tag{7}$$

with

$$\begin{aligned} F_0 &= 1 + 0.8\left[1 - \tanh\left(8\, M_e\right)\right] \\ u_{\text{te}} &= 0.82 + 0.017\, M_e \\ u_{\text{le}} &= 0.50 + 0.025\, M_e \\ \beta &= 11.3\, \exp\left(-0.28\, M_e\right) \quad . \end{aligned}$$

4. Conclusion

A short review of turbulent spot characteristics was performed and data on spot convection speed, spreading angle and propagation parameter was collected. Very large scatter exists in the data, partially arising from difficulties of the measurements and definition of the spot geometry.

In this work, some basic empirical correlations have been proposed for these turbulent spot characteristics and spot propagation parameter. A model for the spot propagation parameter can then be used in transition models that are based on empirical correlations. Large uncertainties and scatter exists in the data however, so the currently proposed correlations are not capturing all data points well.

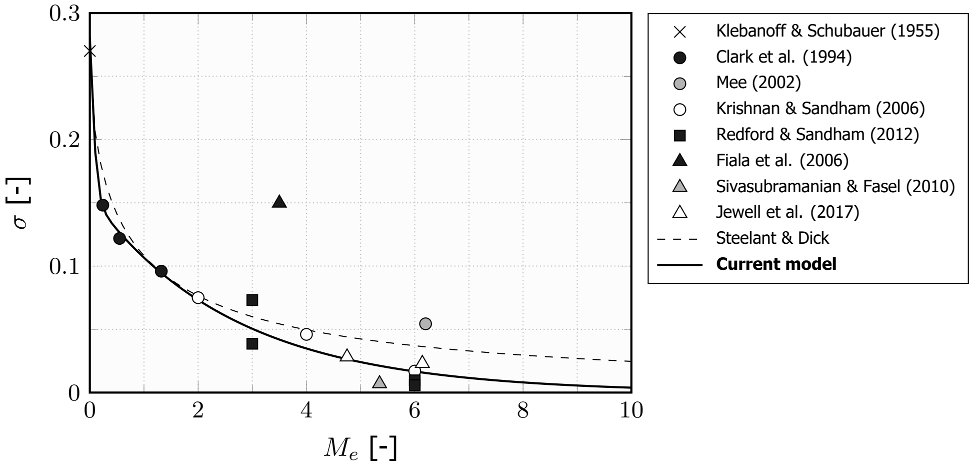

Fig 10. Spot propagation parameter σ of a turbulent spot.

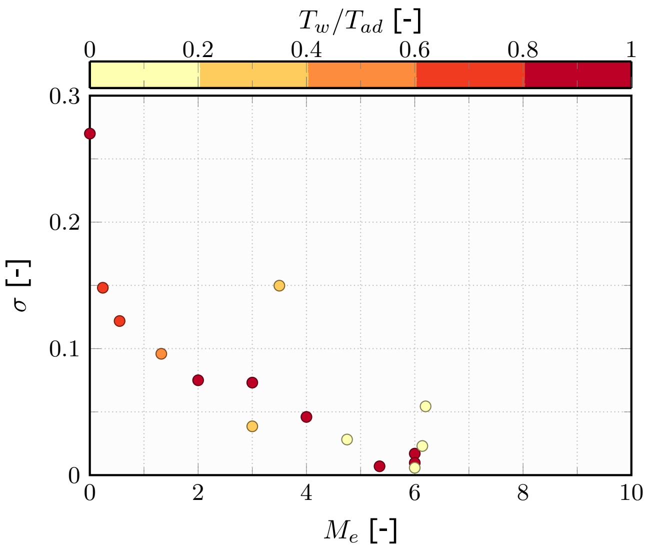

Fig 11. Spot propagation parameter σ of a turbulent spot.

A more in-depth review of the data, including measurement techniques and definitions of the spot geometry, is needed. In addition, the effect of wall temperature is currently not yet addressed in the proposed correlations and will need to be included in the future work.

Acknowledgements

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