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# The Role of Aristotelian Diagrams in Scientific Communication

## 1. Aristotelian Diagrams and Logical Geometry

An *Aristotelian diagram* is a visual representation of a number of formulas or expressions, and certain logical relations holding between them (in particular, contradiction, (sub)contrariety and subalternation). Without a doubt, the oldest and most widely used Aristotelian diagram is the so-called *square of opposition* for the categorical statements from syllogistics. The relations of contradiction and contrariety were systematically studied for the first time in the logical works of Aristotle, while the first actual square diagrams are due to the late ancient authors Apuleius and Boethius. Throughout history, distinguished philosophers and logicians such as William of Ockham, John Buridan, Gottlob Frege and Roderick Chisholm have used squares of opposition (as well as larger, more complex Aristotelian diagrams) in order to explain and illustrate their theorizing. Because of the ubiquity of the relations that they visualize, Aristotelian diagrams are nowadays also frequently used in other disciplines that are concerned with logical reasoning, such as linguistics, cognitive science and artificial intelligence.[1]

This widespread usage of Aristotelian diagrams has recently led to the development of the framework of *logical geometry*. Rather than using Aristotelian diagrams as a mere tool to explain some given idea or theory, logical geometry shows that these diagrams can be fruitfully studied as objects of independent interest, with respect to both their abstract-logical properties and their visual-geometric fea-

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[1] Cf. the papers mentioned in notes 2 and 3 below for a plethora of concrete examples of the widespread (historical and contemporary) usage of Aristotelian diagrams.

tures.[2] This theoretical work has also led to the discovery of new applications of Aristotelian diagrams (often in logic, but also in more unexpected fields, such as philosophy of religion), and to interesting new case studies on historical authors (e.g. John Buridan).[3]

The aim of this short paper is not to present any new technical results or to develop a new application or case study, but rather to ask a more fundamental question: *why are Aristotelian diagrams used so frequently to begin with?* Or more generally: what is the role of Aristotelian diagrams in the scientific communication of logicians, philosophers, linguists, computer scientists, etc.? Despite their apparent simplicity, these questions go directly to the philosophical foundations of logical geometry. After all, the widespread usage of Aristotelian diagrams constitutes one of the main motivations for systematically developing this framework. A potential danger is that an un-

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[2] For the abstract-logical investigations, cf. Lorenz Demey and Hans Smessaert, "Combinatorial Bitstring Semantics for Arbitrary Logical Fragments", *Journal of Philosophical Logic* 47 (2018), pp. 325–363; Lorenz Demey, "Computing the Maximal Boolean Complexity of Families of Aristotelian Diagrams", *Journal of Logic and Computation* 28 (2018), pp. 1323–1339; Hans Smessaert and Lorenz Demey, "Logical Geometries and Information in the Square of Oppositions", *Journal of Logic*, *Language and Information* 23 (2014), pp. 527–565. For the visual-geometrical topics, cf. Lorenz Demey and Hans Smessaert, "Geometric and Cognitive Differences between Aristotelian Diagrams for the Boolean Algebra $B_4$", *Annals of Mathematics and Artificial Intelligence* 83 (2018), pp. 185–208; Demey–Smessaert, "Logical and Geometrical Distance in Polyhedral Aristotelian Diagrams in Knowledge Representation", *Symmetry* vol. 9, issue 10 (2017), article no. 204; Demey–Smessaert, "The Interaction between Logic and Geometry in Aristotelian Diagrams", in Mateja Jamnik, Yuri Uesaka and Stephanie Elzer Schwartz (eds.), *Diagrammatic Representation and Reasoning*, LNAI 9781, 2016, Berlin: Springer, pp. 67–82. For further discussion in the context of logic diagrams in general, cf. Jens Lemanski, "Means or End? On the Valuation of Logic Diagrams", *Logic-Philosophical Studies* 14 (2016), pp. 98–122.

[3] Lorenz Demey, "Aristotelian Diagrams for Semantic and Syntactic Consequence", *Synthese*, forthcoming; Demey, "Using Syllogistics to Teach Metalogic", *Metaphilosophy* 48 (2017), pp. 575–590; Demey, "A Hexagon of Opposition for the Theism/Atheism Debate", *Philosophia*, forthcoming; Demey, "Boolean Considerations on John Buridan's Octagons of Opposition", *History and Philosophy of Logic*, forthcoming.

bridgeable abyss might arise between the purely mathematical investigation of (the logical and geometrical properties of) Aristotelian diagrams on the one hand, and their concrete applications on the other.[4] To some extent, these worries can be alleviated by pointing to the variety of new applications and historical case studies that have been made possible by the recent theoretical advances, which illustrate the tight connection between the theoretical and application-oriented sides of logical geometry. Nevertheless, the widespread usage of Aristotelian diagrams is not simply to be taken for granted, but rather calls for a substantial philosophical explanation. The issue becomes all the more pressing if one realizes that the most common (albeit often implicit) conception of Aristotelian diagrams cannot adequately account for all aspects of their widespread usage.

## 2. The Received View

Although Aristotelian diagrams are used very frequently, authors rarely pause to comment explicitly on their decision to include such a diagram in their writings, or to explain why/how they expect the diagram to be useful. Nevertheless, it is quite clear that there is an underlying common view on the role of Aristotelian diagrams. This view holds that Aristotelian diagrams mainly function as *pedagogical devices*. Because of their visual nature, they have a high mnemonic value, which can be helpful for introducing novice students to the abstract discipline of logic. This view is probably based on the fact that many Aristotelian diagrams can be found in logic textbooks (especially in the scholastic tradition, but also in contemporary times). Furthermore, examination of historical student notebooks shows that

[4] A similar danger exists for the mathematical study of Euler and Venn diagrams; cf. Amirouche Moktefi and A. W. F. Edwards, “One More Class: Martin Gardner and Logic Diagrams”, in Mark Burstein (ed.), *A Bouquet for the Gardener: Martin Gardner Remembered*, New York: The Lewis Carroll Society of North America, 2011, pp. 160–174.

students actively drew their own Aristotelian diagrams in order to master the logical subject matter.[5]

This view on the role of Aristotelian diagrams is usually left implicit, especially by authors who actively put these diagrams to use in their logical work. However, in more historical or methodological contexts, we sometimes do find explicit expressions of this received view. For example, a historical overview of graph drawing states that "[s]quares of opposition were pedagogical tools used in the teaching of logic… They were designed to facilitate the recall of knowledge that students already had." Similarly, in his methodological reflection on the square of opposition, Dale Jacquette writes that "[t]he square provides a memory device … and as such can be thought of largely as a crutch for students to lean on when they are first learning syllogistic logic".[6]

The view that Aristotelian diagrams are primarily pedagogical devices has two important consequences. First of all, it entails that the use(fulness) of these diagrams is mainly due to the practical circumstance that novice students are typically less-than-ideally prepared for abstract logical thinking, and will thus benefit significantly from the mnemonic aids provided by the diagrams.[7] Under "ideal" circumstances, however, the use(fulness) of Aristotelian diagrams

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[5] Christophe Geudens and Jan Papy, "The Teaching of Logic at Leuven University (1425–1797): Perpetually Peripatetic? A First Survey of a Research Project on Student Notebooks and their European Context", *Neulateinisches Jahrbuch: Journal of Neo-Latin Language and Literature* 17 (2015), pp. 360–378.

[6] Eriola Kruja, Joe Marks, Ann Blair and Richard Waters, "A Short Note on the History of Graph Drawing", in P. Mutzel, M. Jünger and S. Leipert (eds.), *Graph Drawing 2001*, LNCS 2265, Berlin: Springer, 2002, pp. 272–286, the quoted passage on pp. 274–276; Dale Jacquette, "Thinking Outside the Square of Opposition Box", in Jean-Yves Béziau and Dale Jacquette (eds.), *Around and Beyond the Square of Opposition*, Berlin: Springer, 2012, pp. 73–92, the quoted passage on p. 80.

[7] In this context, we should also point out the pejorative term used for another famous logic diagram, viz. the *pons asinorum* or "bridge of asses", which was called like that because it primarily "helped dull-witted students"; cf. Martin Gardner, *Logic Machines and Diagrams*, New York: McGraw Hill, 1958, the quoted passage on p. 30.

would vanish almost completely: if one were teaching logic to an audience that consists exclusively of highly gifted students, then the mnemonic aids would be superfluous, and the diagrams could be dispensed with altogether. Secondly, this view holds that Aristotelian diagrams can help students to reproduce knowledge that was previously acquired, but they cannot produce any genuinely *new* knowledge.

The received view is correct in emphasizing the pedagogical value of Aristotelian diagrams. However, it cannot adequately account for the heterogeneous usage of these diagrams. After all, in the contemporary scientific landscape, Aristotelian diagrams are not only found in textbooks, but also, and most frequently, in *research-level* monographs and journal papers. This observation is hard to square with some of the basic tenets of the received view. In particular, the appearance of Aristotelian diagrams in research-level publications does not fit well with the idea that these diagrams are primarily targeted at novice (or even “dull-witted”) students, nor with the idea that these diagrams do not produce any new knowledge. Unlike textbooks, research-level publications are written by scientists for their fellow scientists, and with the explicit aim of contributing to the production of new knowledge.

## 3. Two Alternative Views

A first alternative to the received view emphasizes the potential *cognitive advantages* offered by the multimodal (visual and symbolic/textual) nature of Aristotelian diagrams. An Aristotelian diagram is taken to provide a “visual summary” of the logical system or lexical field under investigation. This can be compared to the way in which a 2D graph functions as a “visual summary” (e.g. in a regression analysis) of the underlying raw numerical data and calculations. The graph contains exactly the same information as the corresponding spreadsheet with raw numerical data; however, because of its visual nature, the graph is much easier to work with, and thus facilitates producing new insights (e.g. discovering general trends in the data).

Just like with the received view, we find a strong emphasis on the cognitive advantages of Aristotelian diagrams. The crucial difference, however, is that these cognitive advantages are no longer restricted to pedagogical contexts, but can also occur in research contexts. For example, some researchers explicitly indicate that Aristotelian diagrams are "very useful to understand in a direct, quick and synthetic way basic notions of modern logic, corresponding to the notion of *Übersichtlichkeit* [surveyability] that Wittgenstein was fond of", or that they constitute "a powerful tool to express all properties of rough sets and fuzzy rough sets with respect to negation in a synthetic way".[8] The main problem, however, is that some Aristotelian diagrams (e.g. 3D polyhedra) have a high degree of visual complexity, and thus do not seem to offer many cognitive advantages. Applications of such visually complex diagrams (which effectively occur in the literature) cannot be accounted for in cognitive terms.

The second alternative view emphasizes the rich and respectable tradition of Aristotelian diagrams within the broader history of logic. The tradition of using these diagrams thus gets endowed with a certain degree of (implicit) *normativity*: because logicians have been using Aristotelian diagrams for a very long time, it is "normal" or "expected" that we continue to use them today. For example, in a paper on artificial intelligence (i.e. *not* on the history of logic), we find side-remarks such as: "The study on oppositions starts in ancient Greece and has its main result is the Square of Opposition by Aristotle."[9] Once the tradition of using Aristotelian diagrams is in place, this view can explain why it continues; however, the main problem is

[8] Jean-Yves Beziau, "The Metalogical Hexagon of Opposition", *Argumentos* 5 (2013), pp. 111–122, the quoted passage on p. 118; Davide Ciucci, Didier Dubois and Henri Prade, "Structures of Opposition in Fuzzy Rough Sets", *Fundamenta Informaticae* 142 (2015), 1–19, the quoted passage on p. 17.

[9] Davide Ciucci, "Orthopairs and Granular Computing", *Granular Computing* 1 (2016), pp. 159–170, the quoted passage on p. 167.

that this view cannot explain how the tradition came into existence in the first place.

## 4. The Heuristic Role of Aristotelian Diagrams

Both the received view and the two alternatives outlined above face serious problems. I would therefore like to put forward a new explanation for the widespread usage of Aristotelian diagrams. Note that what follows is merely a sketch of this new account; many details still need to be filled in, which will be done in future work. The key idea is that Aristotelian diagrams owe their popularity to the fact that they function as powerful *heuristic tools*. They enable researchers to draw unexpected analogies between seemingly unrelated logical, philosophical and scientific frameworks. Once such a "bridge" between two disciplines has been established, it can also be used to introduce new concepts, by transferring them from one discipline into the other. Aristotelian diagrams are situated at precisely the right level of abstraction to play this heuristic role. On the one hand, they are not too specific, which would otherwise impede the search for genuinely surprising cross-disciplinary analogies. On the other hand, they are not too general either, which would otherwise render the resulting analogies devoid of any substance: if the level of abstraction gets too high, everything will become analogous to everything else. Using ideas from logical geometry (in particular, the notion of *Aristotelian isomorphism*), the idea that Aristotelian diagrams occupy a heuristic "sweet spot" can be made mathematically precise.

Many authors who make use of Aristotelian diagrams, explicitly point to their heuristic potential. For example, the medieval logician John Buridan was able "to exhibit a strong analogy between modal, oblique and nonnormal propositions in his three octagons".[10] Furthermore, in artificial intelligence research, Aristotelian diagrams

[10] Stephen Read, "John Buridan's Theory of Consequence and his Octagons of Opposition", in Jean-Yves Béziau and Dale Jacquette (eds.), *Around and Beyond the Square of Opposition* (cf. note 6 above), pp. 93–110, the quoted passage on p. 109.

have explicitly been called "a new bridge" between different knowledge representation formalisms,[11] and they have also led to the introduction of new concepts: "With respect to the four logic expressions of the square of opposition, we can identify four subsets of attributes. … While [one subset] is well studied, the other [three subsets have] received much less attention".[12]

By enabling us to discover unexpected analogies, Aristotelian diagrams clearly contribute to the production of *new* knowledge. In particular, these diagrams are not mere mnemonic devices that facilitate the recall of pre-existing knowledge. This squares well with the observation that most contemporary applications of Aristotelian diagrams are found in research-level writings from a wide variety of disciplines. Finally, it should be noted that the heuristic view on Aristotelian diagrams is by no means incompatible with the other views described in this paper. For example, historically speaking, it seems plausible that (i) Aristotelian diagrams were initially used almost exclusively as mnemonic devices in teaching contexts, but (ii) over time, they gradually shifted toward the role of heuristic devices in research contexts.

## 5. Concluding Remarks

In this paper I have sketched several explanations for the widespread usage of Aristotelian diagrams, with a particular focus on the view that these diagrams primarily perform a heuristic role. It bears emphasizing that I have *not* argued for the absolute indispensability of Aristotelian diagrams. One might observe that ultimately, everything that can be done by means of an Aristotelian diagram, can also be done *without* such a diagram. I agree with this observation, but I think

[11] Davide Ciucci, Henri Prade and Didier Dubois, "The Structure of Oppositions in Rough Set Theory and Formal Concept Analysis – Toward a New Bridge between the Two Settings", in C. Beierle and C. Meghini (eds.), *FoIKS 2014*, LNCS 8637, Berlin: Springer, 2014, pp. 154–173.

[12] Yiyu Yao, "Duality in Rough Set Theory Based on the Square of Opposition", *Fundamenta Informaticae* 127 (2013), pp. 49–64, the quoted passage on p. 59.

it is not problematic for any of the views discussed above. In order to explain this, I will finish this paper by drawing an analogy with Feynman diagrams from physics. Already in 1948, Freeman Dyson proved the formalism of Feynman diagrams to be mathematically equivalent to a more conventional, non-diagrammatic formalism.[13] Hence, everything that can be done by means of a Feynman diagram, can also be done *without* such a diagram, at least in principle. However, in many cases, the non-diagrammatic equivalent will be hugely more complex than the original Feynman diagram, to the extent that it becomes utterly infeasible for humans to work with those non-diagrammatic equivalents. For example, the Nobel Prize winner Frank Wilczek recently declared that "[t]he calculations that eventually got me a Nobel Prize in 2004 would have been literally unthinkable without Feynman diagrams".[14] Even though Feynman diagrams are not indispensable *in principle*, they certainly seem to be so *in practice*. In future work, I will argue that something similar can also be said about Aristotelian diagrams.

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[13] David Kaiser, *Drawing Theories Apart: The Dispersion of Feynman Diagrams in Postwar Physics*, Chicago, IL: University of Chicago Press, 2005, especially pp. 74–75.

[14] Frank Wilczek, "How Feynman Diagrams Almost Saved Space", *Quanta Magazine*, 2016, https://www.quantamagazine.org/20160705-feynman-diagrams-nature-of-empty-space.