Published as:
García-Barragán, J.F., Eyckmans, J. and Rousseau, S. (2019),
Defining and Measuring the Circular Economy: A Mathematical Approach,
Ecological Economics 157, 369-372

Defining and Measuring the Circular Economy: A Mathematical Approach

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Abstract

The circular economy literature lacks unambiguous definitions. We argue that a convenient solution to this problem consists of defining the circular economy as a function of a metric, departing from a well-defined material flow and value system. In particular, we propose a metric that is derived from maximizing the value to society of materials used in the production of commodities that provide services to consumers. Our metric can accommodate for recycling but also for alternative strategies like lifetime extension and new business models that intensify the productivity of commodities. Following our methodology, we provide unambiguous definitions for linear economy, circular economy, and circular economic growth.

Keywords: recycling, circular economy, linear economy, circular economic growth, circular economy metric
JEL classification: Q53

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†This document belongs to the Integrated economic modeling of material flows (IECOMAT) project, supported by Belgian Science Policy (BELSPO). The authors would like to thank Balazs Zelity, Irina Bakalova, and IECOMAT researchers of VITO and UCLouvain CORE for their valuable comments and suggestions.

Introduction

According to the *Handbook of Recycling* (Worrell and Reuter (2014)), recycling is defined as the "reprocessing of recovered materials at the end of product life, returning them into the supply chain". Statistics designed to account for recycling activity reflect the same intuition, with the recycling rate, the old scrap ratio, the recycled content, and the recycling input rate being the most well-known ones (Graedel et al. (2011)). These estimates require minimal information to be computed and thus serve as convenient tools for monitoring the re-flow of material into the economy's system. On the other hand, the *Ellen MacArthur foundation (EMF)* conceives the circular economy as

> [R]estorative and regenerative by design, (...) [that] aims to keep products, components, and materials at their highest utility and value at all times. The concept (....) is a continuous positive development cycle that preserves and enhances natural capital, optimizes resource yields, and minimizes system risks by managing finite stocks and renewable flows. It works effectively at every scale.[1]

Despite the fact that recycling and the circular economy are closely related concepts, the latter lacks a generally accepted metric. Research on circular metrics has been advanced in EMF (2015), Haupt et al. (2016), Linder et al. (2017), Tisserant et al. (2017), Lèbre et al. (2017), and Lonca et al. (2018).

The complexity of determining a credible and usable indicator for the circular economy that also reflects the role of recycling is highlighted by the following two observations. First, as the definitions of the two concepts suggest, circular economies take the maximization of a material's value as an explicit benchmark of efficiency, while recycling activity on its own just describes an industrial activity among many others. From this discrepancy, it follows that a direct implementation of recycling indicators as metrics of circular economy activity is methodologically unsatisfactory. Second, the circular economy does not have a rigorous and unambiguous definition (Reike et al. (2018)), nor is there a consensus on previously suggested definitions. In a recent study, Kirchherr et al. (2017) show that there are at least 114 definitions of the circular economy. As a matter of fact, all definitions appear to have some degree of ambiguity.

Aside from the problem of consensus and subsequent limitations, the absence of unambiguous definitions compromises the development of widely accepted metrics and thus also the policy development needed for the transition towards a circular economy. In this reflection, we argue that a convenient solution to this problem consists of defining the circular economy as a function of a metric, departing from an equally well-defined material flow

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[1] This definition appears in Moreau et al. (2017).

and value system. Moreover, we show that when a metric is designed to operate according to mathematical logic, definitions are unambiguous. To the best of our knowledge, this approach is novel in the circular economy literature.

Our methodology works in three steps. First, we depart from a *given and mathematically well-defined circular system.* As a second step, we characterize optimal material flows and carefully build well-defined metrics that measure the linear and recycling activity of the economy. Finally, we use the metrics to unambiguously define *linear economy*, *circular economy*, *more circular economy* and *circular economic growth.*

1 The model

We consider a dynamic economy consisting of a representative consumer with preferences over consumption on n different types of functionalities $\hat{c}_{i,t} \geq 0$, with $i = 1, 2, ..., n$ and t, denoting time, running from 1 to T. Preferences are represented by a strictly concave instantaneous utility function $u_t(\hat{c}_{1,t}, \hat{c}_{2,t}, ..., \hat{c}_{n,t})$ satisfying $\partial u_t / \partial \hat{c}_{i,t} > 0$.[2] In line with an increasing focus on functionalities and services rather than physical products, we assume that consumers want to satisfy different needs such as mobility, lighting or communication and thus focus on the functionality of the produced goods and services rather than the specific material that is used to produce them. We assume that functionality $\hat{c}_{i,t}$ is by itself a sub-utility function composed of goods disposed of after h_i periods of having been produced. Sub-utility functions are expressed as $\hat{c}_{i,t} = z^i_t(c_{i,t}, c_{i,t-1}, ..., c_{i,t-h_i})$, and are assumed strictly concave and increasing in all arguments. In other words, we assume that $\partial z^i_t / \partial c_{i,t-s} > 0$, with $s = 0, 1, ..., h_i$.

There are N different stocks of virgin material and N different stocks of recycled material. We denote the stock of virgin material as $M_{j,t}$ and the stock of recycled material as $M^r_{j,t}$, with $j = 1, 2, ..., N$. The production of $c_{i,t}$ requires a certain non-negative amount of virgin and recycled material. We denote the amount of virgin material of type j used in the production of $c_{i,t}$ as $m^j_{i,t}$, and the amount of recycled material of type j used in the production of $c_{i,t}$ as $r^j_{i,t}$. The degree of substitution between $m^j_{i,t}$ and $r^j_{i,t}$ is not necessarily perfect, reflecting for example quality loss in subsequent recycling rounds. Capital is also required, and it is denoted as $k_{i,t}$. The technology that transforms virgin and recycled material into physical goods is

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[2] Throughout this article, time subscripts imply that functions can vary over time and are properly discounted. Dynamic preferences and changes in the productivity of final and recycling sectors are typical cases.

$$c_{i,t} = f_{i,t}\left(m_{i,t}^{1}, m_{i,t}^{2}...., m_{i,t}^{N}, r_{i,t}^{1}, r_{i,t}^{2}, ..., r_{i,t}^{N}, k_{i,t}\right),$$

where $f_{i,t}$ is assumed to be a concave function satisfying $\partial f_{i,t}/\partial m_{i,t}^{j} \geq 0$ and $\partial f_{i,t}/\partial r_{i,t}^{j} \geq 0$. We assume there is at least one $m_{i,t}^{j'}$ satisfying $\partial f_{i,t}/\partial m_{i,t}^{j'} > 0 \; \forall i, t$. In the case in which the condition $\partial f_{i,t}/\partial m_{i,t}^{j'} = 0$ holds for some j', virgin material of type j' cannot be used to produce good i at time t. In particular, we assume that $\partial f_{i,t}/\partial k_{i,t} > 0$.[3]

Externalities associated with consumption of virgin and recycled material have a negative impact on environmental quality, which is represented by a concave instantaneous quality function

$$\sum_{j}\left(q_{j,t}^{v}\left(m_{1,t}^{j}, m_{2,t}^{j}, ..., m_{n,t}^{j}\right) + q_{j,t}^{r}\left(r_{1,t}^{j}, r_{2,t}^{j}, ..., r_{n,t}^{j}\right)\right),$$

with $\partial q_{j,t}^{v}/\partial m_{i,t}^{j} < 0$ and $\partial q_{j,t}^{r}/\partial r_{i,t}^{j} < 0$. This is to say, we allow for different environmental impacts depending on the specific path that each type of material has followed through the production process.

The equation of motion that characterizes the evolution of the stock of virgin material is given by $M_{j,t} - M_{j,t-1} = -\sum_{i} m_{i,t}^{j} + \phi_{j,t}$, for some $\phi_{j,t} \geq 0$. When $\phi_{j,t} > 0$, the virgin material of type j is renewable at time t, otherwise it is non-renewable.

We proceed to describe the recycling process. We assume that each good $c_{i,t}$ is transferred to a specialized recycler facility after h_i periods of having been produced, this is to say when the good faces its end-of-life stage. Recycler i recovers a non-negative fraction of each type of material embedded in good $c_{i,t}$, including virgin and recycled components. We assume that the recycling possibilities are given by the multivariate rule

$$\begin{aligned} r_{i,t}^{v,1} &= g_{i,t}^{v,1}\left(m_{i,t-h_i}^{1}, k_{i,t}^{v,1}\right) \\ r_{i,t}^{v,2} &= g_{i,t}^{v,2}\left(m_{i,t-h_i}^{2}, k_{i,t}^{v,2}\right) \\ &... \end{aligned}$$

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[3] More realistic conditions on production functions could be imposed without necessarily affecting our main results. We refer the reader to Anderson (1987), Krysiak and Krysiak (2003), Baumgärtner (2004), and Pethig (2006) for an extensive discussion on production functions and technical assumptions.

$$
\begin{aligned}
r^{v,N}_{i,t} &= g^{v,N}_{i,t}\left(m^{N}_{i,t-h_i}, k^{v,N}_{i,t}\right) \\
r^{r,1}_{i,t} &= g^{r,1}_{i,t}\left(r^{1}_{i,t-h_i}, k^{r,1}_{i,t}\right) \\
r^{r,2}_{i,t} &= g^{r,2}_{i,t}\left(r^{2}_{i,t-h_i}, k^{r,2}_{i,t}\right) \\
&\dots \\
r^{r,N}_{i,t} &= g^{r,N}_{i,t}\left(r^{N}_{i,t-h_i}, k^{r,N}_{i,t}\right),
\end{aligned}
$$

where $k^{v,j}_{i,t}$ and $k^{r,j}_{i,t}$ denote capital used for recovering virgin and recycled material embedded in good $c_{i,t}$. We denote recycled virgin material of type j recovered by recycler i as $r^{v,j}_{i,t}$ and re-recycled material of type j recovered by recycler i as $r^{r,j}_{i,t}$. In order to ensure a correct material balance and to rule out the possibility of 100% recycling efficiency, we follow Eichner and Pethig (2001) and assume that recycling functions are concave and satisfy $r^{v,j}_{i,t} < m^{j}_{i,t-h_i}$, $r^{r,j}_{i,t} < r^{j}_{i,t-h_i}$, $g^{v,j}_{i,t}(0,.) = g^{r,j}_{i,t}(.,0) = 0$, $\lim_{k^{v,j}_{i,t} \to +\infty} g^{v,j}_{i,t} < m^{j}_{i,t-h_i}$, and $\lim_{k^{r,j}_{i,t} \to +\infty} g^{r,j}_{i,t} < r^{j}_{i,t-h_i}$. Waste is disposed of in a landfill with capacity W_t, according to the rule $W_t - W_{t-1} = -\left(\sum_i \sum_j \left(m^{j}_{i,t-h_i} - r^{v,j}_{i,t} + r^{j}_{i,t-h_i} - r^{r,j}_{i,j}\right)\right)$.

Recovered materials $r^{v,j}_{i,t}$ and $r^{r,j}_{i,t}$ are transferred to be stockpiled and re-introduced into the economy according to the motion $M^{r}_{j,t} - M^{r}_{j,t-1} = \sum_i \left(r^{v,j}_{i,t} + r^{r,j}_{i,t} - r^{j}_{i,t}\right)$. Since there is a unique stock of recycled material for each type of material j, we consider a general situation in which loops can be either *open* (recycled material is used by different sectors) or *closed* (recycling is a sector-specific activity). We assume that a fixed, positive and finite amount of capital $K_t > 0$ is available for all sectors, this is to say $K_t = \sum_i k_{i,t} + \sum_i \sum_j \left(k^{v,j}_{i,t} + k^{r,j}_{i,t}\right)$.

2 Optimal behavior

We proceed to characterize the optimal behavior of the material flows of the system. The maximization problem of the economy subject to technological constraints can be written in Lagrangian notation as

$$
\begin{aligned}
\mathcal{L} = &\sum_t u_t(z_t^1(c_{1,t}, ..., c_{1,t-h_1}), ..., z_t^n(c_{n,t}, ..., c_{n,t-h_n})) \\
&+ \sum_t \sum_j \left(q^v_{j,t}\left(m^j_{1,t}, m^j_{2,t}, ..., m^j_{n,t}; t\right) + q^r_{j,t}\left(r^j_{1,t}, r^j_{2,t}, ..., r^j_{n,t}\right)\right) \\
&+ \sum_t \lambda^c_t \left(c_{i,t} - f_{i,t}\left(m^1_{i,t}, m^2_{i,t} ...., m^N_{i,t}, r^1_{i,t}, r^2_{i,t}, ..., r^N_{i,t}, k_{i,t}\right)\right) \\
&+ \sum_t \sum_i \sum_j \lambda^{r^v}_{i,j,t} \left(r^{v,j}_{i,t} - g^{v,j}_{i,t}\left(m^j_{i,t-h_i}, k^{v,j}_{i,t}\right)\right) \\
&+ \sum_t \sum_i \sum_j \lambda^{r^r}_{i,j,t} \left(r^{r,j}_{i,t} - g^{r,j}_{i,t}\left(r^j_{i,t-h_i}, k^{r,j}_{i,t}\right)\right) \\
&+ \sum_t \lambda^k_t \left(K_t - \sum_i k_{i,t} - \sum_i \sum_j \left(k^{v,j}_{i,t} + k^{r,j}_{i,t}\right)\right) \\
&+ \sum_t \lambda^W_t \left(W_t - W_{t-1} + \left(\sum_i \sum_j \left(m^j_{i,t-h_i} - r^{v,j}_{i,t} + r^j_{i,t-h_i} - r^{r,j}_{i,t}\right)\right)\right) \\
&+ \sum_t \sum_j \lambda^M_{j,t} \left(M_{j,t} - M_{j,t-1} + \sum_i m^j_{i,t} - \phi_{j,t}\right) \\
&+ \sum_t \sum_j \lambda^{M^r}_{j,t} \left(M^r_{j,t} - M^r_{j,t-1} - \sum_i \left(r^{v,j}_{i,t} + r^{r,j}_{i,t} - r^j_{i,t}\right)\right),
\end{aligned}
$$

where λ^c_i is the the shadow price of physical good $c_{i,t}$, $\lambda^{r^v}_{i,j,t}$ is the the shadow price of recycled virgin material $r^{v,j}_{i,t}$, $\lambda^{r^r}_{i,j,t}$ is the the shadow price of re-recycled material $r^{r,j}_{i,t}$, λ^k_t is the shadow price of capital, λ^W_t is the shadow price of landfill capacity, $\lambda^M_{j,t}$ is the shadow price of the stock of virgin material j, and $\lambda^{M^r}_{j,t}$ is the shadow price of the stockpiled recycled material j. Initial and terminal conditions are standard and therefore omitted.

For an interior solution, first-order conditions with respect to $m^j_{i,t}$ and $r^j_{i,t}$ imply that

$$
\sum_{s=0}^{h_i} \frac{\partial u^*_{t+s}}{\partial z^*_{t+s}} \frac{\partial z^*_{t+s}}{\partial f^*_{i,t}} \frac{\partial f^*_{i,t}}{\partial m^j_{i,t}} = -\frac{\partial q^v_{j,t}}{\partial m^j_{i,t}} + \lambda^{r^v}_{i,j,t+h_i} \frac{\partial g^{v,j}_{i,t+h_i}}{\partial m^j_{i,t}} - \lambda^W_{t+h_i} - \lambda^M_t \;\; \forall \; i, j, t
$$

$$
\sum_{s=0}^{h_i} \frac{\partial u^*_{t+s}}{\partial z^*_{t+s}} \frac{\partial z^*_{t+s}}{\partial f^*_{i,t}} \frac{\partial f^*_{i,t}}{\partial r^j_{i,t}} = -\frac{\partial q^r_{j,t}}{\partial r^j_{i,t}} + \lambda^{r^r}_{i,j,t+h_i} \frac{\partial g^{r,j}_{i,t+h}}{\partial r^j_{i,t}} - \lambda^W_{t+h_i} - \lambda^{M^r}_{j,t} \;\; \forall \; i, j, t, \tag{1}
$$

with the economic interpretation that the streams of materials maximize social welfare when present and future marginal benefits, accounting for durability, are equal to the marginal social costs of recycling, including the impacts on scarcity of material resources and landfilling space. The marginal benefit of an additional unit of a material consists of the marginal value attached by the consumer to the functionality times the marginal productivity of the good to provide functionality (product intensity of functionality) times the marginal productivity of materials to produce goods (material intensity of products). Hence, if more functionality can be produced with the goods, and/or more goods can be produced with a unit of material, it will be reflected in the left-hand side of the expression. For example, increased use of car sharing could lead to more intensive use of vehicles, which in turn improves the productivity of the car stock to provide mobility as a functionality.

3 Proposed metric

Having characterized the optimal flow of virgin and recycled materials, we proceed to build some auxiliary metrics. We define the *endogenous* recycling rate of material type j in sector i at time t ($\alpha^i_{j,t}$) as

$$\alpha^i_{j,t} = \frac{r^j_{i,t}}{m^j_{i,t} + r^j_{i,t}} \in [0,1],$$

and the size of optimal recycling activity of material type j in sector i at time t ($R^*_{i,j,t}$) as

$$R^*_{i,j,t} = \left(\sum_{s=0}^{h_i} \frac{\partial u^*_{t+s}}{\partial z^*_{t+s}} \frac{\partial z^*_{t+s}}{\partial f^*_{i,t}} \frac{\partial f^*_{i,t}}{\partial r^j_{i,t}} \right) {\alpha^i_{j,t}}^* \quad , \tag{2}$$

where asterisks denote optimal levels. Thus metric (2) measures the marginal benefit of recycled material $r^j_{i,t}$ multiplied by the recycling rate $\alpha^i_{j,t}$, taking into account that consumers value $c_{i,t}$ which requires $r^j_{i,t}$ as input.

Metric (2) has the following partial sensitivities: $\partial R^*_{i,j,t}/\partial \alpha^i_{j,t} > 0$ and $\partial R^*_{i,j,t}/\partial \left[\partial u^*_{t+s}/\partial r^j_{i,t}\right] > 0$. This shows that the recycling metric responds positively to increases in the level of recycling and the marginal utility induced by each additional unit of material $r^j_{i,t}$ used in the economy. Moreover, it is apparent from (1) that our metric gives more importance to recycled material streams that are relatively more valuable to the consumer when environmental externalities are fully internalized. From our perspective, these properties are attractive since they allow one to measure the size of the recycling activity when the flows are meant to maximize the whole system's value, including its inter-temporal dimension.

Similarly, we define the size of optimal linear activity of sector i of material type j at time t ($L^*_{i,j,t}$) as the metric

$$L^*_{i,j,t} = \left(\sum_{s=0}^{h_i} \frac{\partial u^*_{t+s}}{\partial z^*_{t+s}} \frac{\partial z^*_{t+s}}{\partial f^*_{i,t}} \frac{\partial f^*_{i,t}}{\partial m^j_{i,t}} \right) \left(1 - {\alpha^i_{j,t}}^*\right). \tag{3}$$

Metric (3) displays a similar sensitivity with respect to the marginal utility of virgin material, i.e. $\partial L^*_i / \partial \left[\partial u^*_{t+s} / \partial m^j_{i,t}\right] > 0$. However, for a fixed level of marginal utility $\partial u^*_{t+s}/\partial m^j_{i,t}$, the linear activity is expected to increase as the recycling rate $\alpha^i_{j,t}$ shrinks. We consider that this property makes $L^*_{i,j,t}$ appealing as to measure sectoral linear activity.

We combine our two auxiliary metrics to build a metric for the circular activity of the system. We define the aggregate level of optimal recycling activity at time t as $R^*_t = \sum_i \sum_j R^*_{i,j,t}$ and the aggregate level of optimal linear activity at time t as $L^*_t = \sum_i \sum_j \mu_{i,j} L^*_{i,j,t}$. We define the optimal size of the circular activity of the economy at time t (C^*_t) as

$$C^*_t = R^*_t - L^*_t. \tag{4}$$

This is to say, the circular activity of the economy is defined as the difference between the optimal recycling activity and the optimal linear activity penalized by intolerance factors $\mu_{i,j}$. In principle, the values of the intolerance parameters are exogenous and can be set equal to one. However, when the degree of substitution between virgin and recycled materials tends to infinity, the metrics that measure circular and linear activity can converge. In this extreme case, the presence of $\mu_{i,j}$ serves to ensure that the circular activity is distinguishable from the linear.

Note that the behavior of a circular economy cannot be satisfactorily characterized by a metric of recycling activity of the R^*_t type only. The reasoning is simple. In the plausible case in which *at least one* of the recycling rates is zero, metric R^*_t is neutral to partial increments in the marginal utility of the zero recycling sector. This is to say, if R^*_t is taken as the metric of the circular economy, a relatively more linear economy could appear equally circular. This issue is solved using metric L^*_t as a penalty.

We can decompose metric (4) in a more familiar way. Letting $p_{i,t}$ be the competitive market price of good $c_{i,t}$, then

$$\sum_{s=0}^{h} \frac{\partial u^*_{t+s}}{\partial z^*_{t+s}} \frac{\partial z^*_{t+s}}{\partial f^*_{i,t}} = \sum_{s=0}^{h} \frac{\partial u^*_{t+s}}{\partial z^*_{t+s}} \frac{\partial z^*_{t+s}}{\partial c^*_{i,t}} = \lambda'_t p_{i,t}$$

should hold. We let $\tau^v_{i,j,t}$ be the corresponding Pigouvian tax levied on the price of virgin material of type j faced by producer i and $\tau^r_{i,j,t}$ the corresponding Pigouvian tax levied on the price of recycled material faced by producer i. Then, letting $p^v_{j,t}$ be the competitive price of virgin material j, $\partial f_{i,t}/\partial m^j_{i,t} = (p^v_{j,t} + \tau^v_{i,j,t})/p_{i,t}$ should hold. Similarly, letting $p^r_{j,t}$ be the competitive price of recycled material j, we have that $\partial f_{i,t}/\partial r^j_{i,t} = (p^r_{j,t} + \tau^r_{i,j,t})/p_{i,t}$. Thus linear and recycling metrics can be re-expressed in monetary terms as ${L'_{i,j,t}}^* = (p^v_{j,t} + \tau^v_{i,j,t})(1 - {\alpha^i_{j,t}}^*)$ and ${R'_{i,j,t}}^* = (p^r_{j,t} + \tau^r_{i,j,t}){\alpha^i_{j,t}}^*$.

4 Definitions of circular economy, linear economy, more circular economy, and circular economic growth

Finally, we use metric (4) to unambiguously define the circular economy. We say that the economy is *circular* at time t if $C^*_t > 0$, that is, if the metric measuring the optimal circular activity is strictly positive. In the case that $C^*_t \leq 0$, we say that economy is *linear*. Moreover, we say that economy is *more circular* with respect to a metric ${C^*_{t+s}}''$ if $C^*_t > {C^*_{t+s}}''$ for some $s \in \mathbb{N}$.[4] We define the *circular growth rate* between periods τ and τ' as $\rho(\tau, \tau') = (C^*_{\tau'} - C^*_{\tau})/C^*_{\tau}$. We say that the economy is experiencing *circular economic growth* between periods τ and τ' if $\rho(\tau, \tau') > 0$, that is, if the metric measuring the circular growth rate is strictly positive.

5 Conclusion

We have illustrated that, departing from a well-defined circular system, the construction of optimal linear and recycling metrics that fully incorporate the social value of economic activity serves as a convenient base on which currently vague terminology as *circular economy*, *linear economy* or *more circular economy* can be built. Given the mathematical structure imposed upon our system, our definitions, while not perfect, are certainly unambiguous. Naturally, circular modeling features and metrics as those presented in this paper can and

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[4] Note that inter-temporal comparisons are possible given that utility is discounted.

are expected to be enriched. However, despite of potential expansions and refinements, unambiguous definitions of the circular economy can be derived following our methodology, i.e. by building recycling and linear activity metrics based on maximizing social welfare. The development of unambiguous definitions and meaningful indicators capable of measuring circular economy activity may help authorities to set and coordinate circular policy targets and advance environmental legislation. An interesting direction for future research is to operationalize the proposed metric using real-world data on an existing system of functionalities, for example mobility, housing or nutrition.

References

Anderson, C. (1987). The production process: Inputs and wastes. *Journal of Environmental Economics and Managment*, 14(1):1–12.

Baumgärtner, S. (2004). The inada conditions for material resource inputs reconsidered. *Environmental and Resource Economics*, 29(3):307–322.

Eichner, T. and Pethig, R. (2001). Product design and efficient management of recycling and waste treatment. *Journal of Environmental Economics and Management*, 41(1):109–134.

EMF (2015). Circularity indicators: An approach to measuring circularity. Methodology.

Graedel, T. E., Allwood, J., Birat, J., Buchert, M., Hagelüken, C., Reck, B. K., Sibley, S. F., and Sonnemann, G. (2011). What do we know about metal recycling rates? *Journal of Industrial Ecology*, 15(3):355–366.

Haupt, M., Vadenbo, C., and Hellweg, S. (2016). Do we have the right performance indicators for the circular economy?: Insight into the swiss waste management system. *Journal of Industrial Ecology*, 21(3):615—-627.

Kirchherr, J., Reike, D., and Hekkert, M. (2017). Conceptualizing the circular economy: An analysis of 114 definitions. *Resources, Conservation and Recycling*, 127:221–232.

Krysiak, F. and Krysiak, D. (2003). Production, consumption, and general equilibrium with physical constraints. *Journal of Environmental Economics and Management*, 46(3):513–538.

Linder, M., Sarasini, S., and van Loon, P. (2017). A metric for quantifying product-level circularity. *Journal of Industrial Ecology*, 21(3):545—-558.

Lonca, G., Muggéo, R., Imbeault-Tétreault, H., Bernard, S., and Margni, M. (2018). Does material circularity rhyme with environmental efficiency? case studies on used tires. *Journal of Cleaner Production*, 183:424—-435.

Lèbre, , Corder, G., and Golev, A. (2017). The role of the mining industry in a circular economy: A framework for resource management at the mine site level. *Journal of Industrial Ecology*, 21(3):662—-672.

Moreau, V., Sahakian, M., Griethuysen, P., and François, V. (2017). Coming full circle: Why social and institutional dimensions matter for the circular economy. *Journal of Industrial Ecology*, 21(3):497–506.

Pethig, R. (2006). Non-linear production, abatement, pollution and materials balance reconsidered. *Journal of Environmental Economics and Management*, 51(2):185–204.

Reike, D., Vermeulen, W., and Witjes, S. (2018). The circular economy: New or refurbished as ce 3.0? — exploring controversies in the conceptualization of the circular economy through a focus on history and resource value retention options. *Resources, Conservation and Recycling*, 135:246–264.

Tisserant, A., Pauliuk, S., Stefano, M., Jannick, S., Fry, J., Wood, R., and Tukker, A. (2017). Solid waste and the circular economy: A global analysis of waste treatment and waste footprints. *Journal of Industrial Ecology*, 21(3):628—640.

Worrell, E. and Reuter, M. (2014). *Handbook of Recycling: State-of-the-art for Practitioners, Analysts, and Scientists.* Elsevier, United Kingdom.