WHY MOTIVATION NECESSARILY PRECEDES HIGH MATHEMATICAL PERFORMANCE – BUT NOT VICE VERSA

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Motivation provides a learner's impetus to initiate learning and later the driving force to sustain the learning. Through the analysis of the motivation concept, empirical studies and related research methodology, we conclude that positive motivations precede high mathematical performance. More experimental evidence seems desirable to understand how affective factors influence students' achievement.

INTRODUCTION

Outsiders often consider mathematics to be a rational activity in which logical deduction determines the flow of thinking and problem solving, and in which affective factors play no role (Törner, 2014). However, learners experience a wide range of feelings and moods in relation to learning mathematics, to problem solving, to performing on a test, and so on. Various terms are used to denote the diversity in affective variables. One major dimension that is used to distinguish affective variables is the temporal one (McLeod, 1992). Certain affects can be described as emotions: rapidly changing states of feelings that are directly related to specific activities and experiences. Other affective variables are considered to be longer lasting and moderately stable, including attitudes that learners have regarding mathematics as a school subject, and beliefs about mathematics as a scientific discipline (Phillip, 2007).

As Zan, Brown, Evans, and Hannula (2006) argue, there are two major argumentations to conduct research on affective variables regarding mathematics. First of all, positive affect regarding mathematics is considered to be related to a better mathematics achievement. This will be the main focus of the current paper, and we will provide theoretical and empirical arguments to substantiate this idea. Second, some consider positive affect regarding mathematics as significant *per se*. Indeed, in various mathematics curricula around the world, it is a common desirable goal that learners develop positive attitudes and beliefs about mathematics. While this goal seems to be an intrinsic one, it serves a purpose in the longer run: If during their education learners have acquired a positive affect regarding mathematics, they will also in the future (be it in a school setting, in daily life, or in professional circumstances) be inclined to practice more, undertake more challenging tasks, be more persistent when solving problems,

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etcetera. This is already a first argument to make the point that was made in our title: high mathematical performance is preceded by positive affect.

FOCUS ON MOTIVATION

The term “positive affect” is very general. To make it more specific, we have decided to focus the panel discussion on the construct of motivation. In line with Ames (1992), we would define motivations as reasons individuals have for behaving in a given manner in a given situation. They exist as part of one’s goal structures, one’s beliefs about what is important, and they determine whether or not one will engage in a given pursuit. As Hannula (2006) argues, motivation can be conceptualised as a potential to direct behaviour through the mechanisms that control emotion. In that sense, it is related to other affective variables as it regulates them.

Motivation is not a unidimensional construct in the sense that it only varies in intensity (learners can just be more or less motivated). The self-determination theory (Ryan & Deci, 2000) focuses on major qualitative differences in the way in which learners can be motivated. While nowadays these differences are formulated in a subtler way, the major distinction is that between intrinsic and extrinsic motivation. Intrinsic motivation is the drive or desire of the student to engage in learning “for its own sake”. Students who are intrinsically motivated engage in academic tasks because they enjoy them. They feel that learning is important with respect to their self-image, and they seek out learning activities for the sheer joy of learning (Middleton, 1995). Their motivation tends to focus on learning goals such as understanding and mastery of mathematical concepts. Students who are extrinsically motivated engage in academic tasks to obtain rewards (e.g., good grades, approval) or to avoid punishment (e.g., bad grades, disapproval). These students’ motivation tends to centre on such performance goals as obtaining favourable judgments of their competence from teachers, parents, and peers or avoiding negative judgments of their competence (Ames, 1992).

This distinction between types of motivation is an important one, as different types of motivation lead learners to do different things. Lepper (1998), for instance, has shown that learners who are motivated intrinsically exhibit a behaviour that can be considered as pedagogically desirable, such as showing an increased time on task, a persistence in face of failure, a more elaborative processing and monitoring of comprehension, a selection of more difficult tasks, greater creativity and risk taking, etcetera. The link with mathematical performance seems obvious and it may also have different effects on mathematics learning in different cultures (Zhu & Leung, 2011). We would argue that mathematics educators would generally agree that when learners exhibit activities associated with intrinsic motivation, they will learn more and in a deeper way, and as such show higher mathematical performance.

CONCEPTUAL ARGUMENTATION

So far, we have argued that intrinsic motivation would lead to a higher mathematical performance than extrinsic motivation. However, one can argue further that extrinsic motivation is still better for performance than no motivation at all, and so, more generally, that motivation–of any kind–precedes high performance. Our claim is that a high motivation *necessarily* precedes a high mathematical performance, while the opposite (motivation following high performance) is not necessarily the case.

Our first argument is a theoretical one, based on simple logic. While there are various definitions of motivation available in the literature, they all let motivation precede in time before any mathematical performance. We just take some excerpts from these definitions to make our point: "the reason we engage in any pursuit, mathematical or otherwise" (Middleton, Jansen, & Goldin, 2016, p. 18), "determine whether or not one will engage in a given pursuit" (Ames, 1992), "reasons individuals have for behaving in a manner in a given situation" (Middleton & Spanias, 1999, p. 66).

An essential point in each of these definitions is that motivation precedes performance in time, and that it is causally related to that performance. Learners will not show any behaviour in the total absence of motivation, so motivation *necessarily* precedes performance. Importantly, the opposite is *not* necessarily the case: A high motivation for mathematics may also occur after a learner did not perform well at all. For instance, the learner may, for some reason, not be aware of the actual quality of his or her performance and believe it was excellent. This may motivate him for the future.

A second aspect in the definitions of motivation deserves attention too: Even if we would accept the idea that motivation may follow after a good mathematical performance, this motivation is still oriented on a *future* mathematical performance, and therefore is necessarily preceding it. We will use an analogy to make our point: Most members of PME engage in a similar activity in the period of end December – beginning January: the writing of a Research Report (RR) to submit for the forthcoming conference. While some experience a pleasure in writing up their RR as such (this pleasure-oriented behaviour is intrinsically motivated), many see the writing of the RR at least also as a means towards a further end (this productivity-oriented behaviour is extrinsic motivated): The RR has to be written in order to communicate one's research results to the research community, and/or an RR (or any other contribution) needs to be written in order to be allowed to attend the forthcoming conference as such. The motivation that is experienced in these cases by necessity is always preceding the actual PME conference. If in the course of December, the announcement would be made that the forthcoming PME conference is cancelled, the (extrinsic) motivation to write a contribution will almost certainly disappear. In a similar way, motivation for mathematics is necessarily preceding a mathematical performance, in the sense that it originates with that forthcoming mathematical performance (be it in the near or in the far future) in mind.

STATISTICAL EVIDENCE

We have just shown how a conceptual analysis of the notion of motivation already shows how motivation necessarily has to precede mathematical performance. A next question could be whether such claims would also be supported by empirical evidence.

A lot of criticism can be given on research that relates attitudes towards mathematics to mathematical performance, particularly from a methodological point of view (for an overview, see Zan et al., 2006), and we will come back to this in the next section. Still, it seems worthwhile to look at the general trend in this empirical research.

An older meta-analysis by Ma and Kishor (1997) looks exactly at the issue under consideration here: the relationship between attitude toward mathematics and achievement in mathematics. Specific questions asked were what the strength of this relationship is (in correlational terms), what *causal* evidence there is, and what the magnitude of the causal relationship is. A total of 113 primary studies were included in the meta-analysis. Regarding the overall strength of the relationship, the conclusion based on 108 effect sizes was that the relation was significantly positive and reliable, but it was not a strong one. More importantly, the meta-analysis also investigated specifically the causal relationship between attitudes and performance. Among the 113 studies, only 5 studies applied a causal modelling of the data; all others looked merely in a correlational way. The 5 studies that applied causal modelling reported 10 effect sizes derived from testing 20 227 students. The finding was that the causal relation in the direction achievement→attitudes was not significantly different from zero, while the causal relation in the direction attitudes→achievement was statistically significant, with an effect size of 0.08. Even though, as the authors commented, the magnitude of this effect size was small and therefore cannot be described as practically meaningful, we still think this is a very important finding from a theoretical point of view. It provides clear evidence for the point we make in our paper that affective variables causally precede high performance and not vice versa. We explicitly want to contest the strong focus on (standardized or other) effect sizes and the practical conclusions that can be drawn from them. Silberzahn et al. (2017) have clearly shown this by involving 29 research teams working on the same data set to answer the same research question. Each team came up with its own analysis strategy. The effect sizes that the teams obtained from empirical studies varied greatly. The conclusion they made was that the effect sizes highly depend on subjective analytical choices, so one can argue that "the" effect size does not exist. Still, the vast majority of teams arrived at the same conclusion on the existence and direction of an effect. So, while effect size claims can be discussed, a *theoretical* claim about the existence of an effect can be made reliably.

So while Ma and Kishor (1997) found a small effect, the fact that it was significant and in the causal direction that we expected is a very important finding for our central claim. In this respect, we also want to refer to Mook's (1983) argument that psychological investigations are too often criticized for lacking direct practical relevance: Often, such psychological studies are intended to test specific predictions that derive from a theory. The theory is assumed to be true for various kinds of settings, including laboratory and real-world settings. The prediction, however, is tested in a controlled lab setting. This does not imply that the instruments, manipulations, etc. of that lab study would *directly* generalize to the real-world setting. Most often, the study was not intended as such at all. With Mook (1983, p. 379), we therefore wish to warn for "A misplaced preoccupation with external validity (…) to dismiss good research for which generalization to real life is not intended nor meaningful".

THE PROBLEM WITH SELF-REPORTS

In the previous section, we have explained how there is a significant statistical relation between motivation and high performance, and that it is specifically in the causal direction that motivation precedes performance. We further explained that the small effect size generally obtained is not necessarily problematic. However, there are also other criticisms that can be made about the statistical evidence that we have provided above. A lot of research on motivation (and attitudes in general) is based on questionnaires. While a lot of questionnaires nowadays may have good psychometric qualities, one can always pose questions about the validity of such questionnaires when it comes to measuring motivation or attitudes. The central problem is that such questionnaires are based on self-reports by the learners, and such self-reports can be questioned on various levels. First (and maybe ironically), there may be motivation issues: Questionnaire data are only valid if the participants wish to put effort in reporting how they really feel and think about a certain problem. Data are no longer valid if (some) participants are not motivated to take part in the study, and give random responses. Second, there may be desirability issues: Participants may take into consideration what they think the researcher wants to find, and adapt their answers to comply with this expectation. Third, questionnaires assume that participants are aware of their affect/motivation and are able to report about it. Murphy and Alexander (2000, p. 8) note that in motivation research nowadays "one assumption seemingly (…) is that individual's motives, needs, or goals are explicit knowledge that can be reflected upon and communicated to others". We agree with Hannula (2006, p. 166) that "The present view emphasises the importance of the unconscious in human mind. Motivation, like much of our mind, is only partially accessible to introspection." Fourth, questionnaires are retrospective. One can argue that participants in questionnaire research may not be able to report reliably and validly about their affect when learning mathematics or when solving a mathematical problem while responding to a questionnaire, simply because they are not experiencing this at the moment of reporting about it.

Research nowadays often moves away from quantitative approaches, and turns towards more qualitative approaches, for instance by means of interviews, diaries, etcetera. However, just like the quantitative questionnaire-based approaches, such qualitative approaches rely on self-reports and therefore bring with them the very same problems that were just discussed.

EXPERIMENTAL EVIDENCE AS AN ALTERNATIVE

Regardless of whether one uses a quantitative or qualitative approach to map learners' affect, there are various methodological problems regarding validity of the data. Another substantial concern with a lot of research on the relation between motivation and performance is that it is correlational in nature (see e.g. the meta-analysis by Ma &

Kishnor, 1997). This makes it difficult to deduce a causal influence, and to exclude that the effects are explained partially or in full by a third variable.

In order to address all these issues, it may be desirable to seek experimental evidence. In the limited space that is available to us in this paper, we restrict ourselves to examples. One major problem when pupils solve word problems is that they tend to exclude real-world considerations from the solution process, and tend to give unrealistic answers (see e.g., Greer, Verschaffel, Van Dooren, & Mukhopadyay, 2009). Some studies have experimentally manipulated the setting in which pupils solved such problems, in order to stimulate them to include more realistic considerations, and as such come to a better mathematical performance. For example, DeFranco and Curcio (1997) have offered the following word problem in a typical school test format: *328 Senior citizens are going on a trip. A bus can seat 40 people. How many buses are needed so that all the senior citizens can go on a trip?* Nearly all students answered in an unrealistic way, for instance "8.2 buses". The day after, the same pupils received a real-life problem that – from a mathematical point of view – was completely parallel: The pupils received a facts sheet containing information on a party that needed to be organised for a group of classmates in a specific restaurant. Pupils had to make a phone call to order minivans to transport all children to that restaurant. In this case, nearly all children ordered a whole number of buses.

Such a study shows clearly that the pupils had all relevant knowledge to come to a good mathematical performance, but in the typical school setting they were not inclined to engage in making realistic considerations; they felt it sufficient to just report the results of arithmetical operations. However, when the same students were involved in a more authentic setting, they engaged not only in doing the correct arithmetical operations, but also considered whether that outcome was realistic, and adapted their answer accordingly. In that sense, a realistic mathematical problem embedded in an authentic setting should motivate students realistically at first, and then their mathematical performance and understanding can be really good.

This is a simple study that illustrates how one can experimentally manipulate the motivation of students in relation to the mathematical problem they are solving, and show the causal impact on the quality of the solution they obtain. In this case, the causal impact of a specific kind of motivation on mathematical performance has been shown. Several other studies have been conducted that show how making mathematical tasks more authentic has an impact on students' performance (see, e.g., Palm, 2002). If one wants convincing evidence for the opposite relation (of students' performance on motivation) too, experimental studies should experimentally manipulate mathematical performance, and measure the motivation that follows from it. To the best of our knowledge, such evidence is not (yet) available.

CONCLUSIONS AND DISCUSSION

Our arguments have shown how positive affect or motivation necessarily precedes high mathematical performance. First, even though motivation is complex, conceptually there is a very clear link between motivation and a subsequent high performance. It is believed that motivation provides the primary impetus to initiate learning and later the driving force to sustain the learning. Simple logic further makes us conclude motivation must precede (and cause) high performance. Secondly, from the statistical evidence, even from a meta-analysis, the causal direction that motivation precedes performance is clearly shown. One can argue about the effect size but we have explained why this is not really an issue. The crucial point is that the theoretically assumed relation has been empirically verified. Thirdly, we propose that, to investigate the relation between motivation and performance, sole reliance on self-report measures entails a danger, and experimental evidence seems to be the way to go, while the experimental evidence for the opposite direction (high performance leading to motivation) seems totally lacking. When individuals are doing mathematics, the affective system is not merely auxiliary to cognition, it is central (Goldin, 2002). Thus, more direct experimental evidence would be desirable to show how students' motivation or other affective factors influence achievement.

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