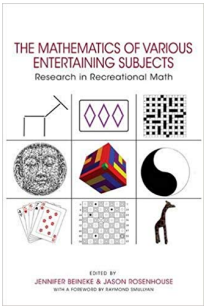
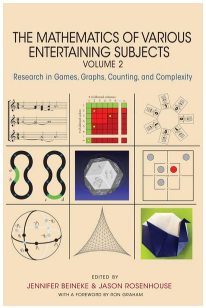
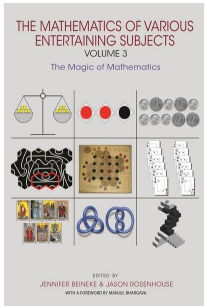
The mathematics of various entertaining subjects,
Jennifer Beineke and Jason Rosenhouse (eds.), Princeton University Press,
vol. 1 Research in recreational mathematics: 2015 (288 p.) isbn: 9780691164038;
vol. 2 Research in games, graphs, counting, and complexity: 2017 (408 p.) isbn: 9780691171920;
vol. 3 The magic of mathematics: 2019 (352 p.) isbn: 9780691182582.





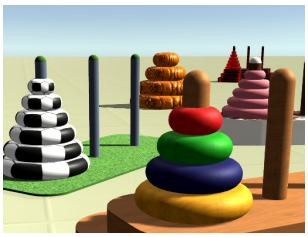
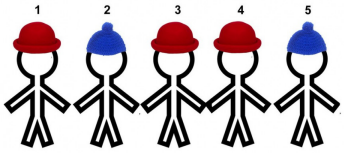



J. Beineke

J. Rosenhouse

Since 2013, every two years there is a MOVES (Mathematics Of Various Entertaining Subjects) conference organized in the MoMath museum in New York. These are conferences that bring together mathematicians, magicians, puzzlers, and mathematical gamers. The topics of the conference is not only the fun aspect, but most of all the mathematics that is behind the fun. It is the study of recreational mathematics as a mathematical subject. This topic has grown to maturity and there are some serious problems that can be formulated and solving them requires the use of many different mathematical tools: number theory, graphs, combinatorics, geometry, topology, probability, stochastic dynamical systems, complexity etc. It's recreational mathematics for mathematicians or at least math enthusiasts.

Jennifer Beineke and Jason Rosenhouse have collected texts as proceedings of these conferences that are published with a delay of about two years. It is obvious that the possible readership for the topics of the conference will be quite diverse and not all of them are professional mathematicians. So the texts are maintained at an accessible level for a broad audience.

The subjects of the papers are very diverse in every volume but similar topics are recurrent over the successive volumes. For example *puzzles and brainteasers* are to be found in every volume. Take for example the classical problem *Towers of Hanoi*. Do you know the different strategies to solve the problem? How do you recognize a regular but random distribution of the disks over the pegs as a starting position that will or will not allow a solution? Suppose you are interrupted half way your moves, how do you know how to continue? How to solve an inverse problem: give disks perfectly arranged on one peg, can you rearrange them to obtain a prescribed (regular) distribution over the pegs?
Other classics are magic squares and 'prisoner's hat colour riddles' for which many variants exist. Volume 3 has a version of the latter in which the prisoners have to guess simultaneously the colour of their own hat (red or blue), which they have to deduce from the colours of the hats of all the other prisoners. They can pass, but not all of them are allowed to pass. This seems to be an impossible problem, but there is a solution that will allow to make a correct decision with high probability.

There is a huge number of different *games* that can be played with two or more players. Also this topic is recurrent in the proceedings. Suppose you are playing checkers with your daughter, and you want her to win, but you do not want to violate the rules of the game. What strategy should you follow to optimize the chance that you will loose? Classical dice have the convention that opposite sides of the cube sum to 7, but it is still possible to have different dice. You have to look at the vertex sums: the sum of the value of the three faces that have a common vertex. But for a d20 die (an icosahedron with 20 trangular faces), how should one number the faces, and how many ways are there to produce a "fair" die (supposing there is a slight deviation problem with the center of gravity of the die)?

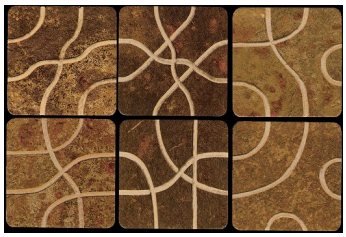
Graphs is another recurrent subject. Here is an interesting problem. Suppose you have a regular triangular planar grid. On some of the grid points we have cities, and the problem is to design a railroad network that makes it possible to connect every city with every other city. What is the shortest possible set of railroad tracks that solves this problem when the tracks can only be along the grid lines? The problem originates from a game called TransAmerica, but it is a good approximation of a practical problem. Suppose one wants to fix lights on some of the grid points of the metal framework that shapes the geodesic dome of the Dalí museum in St. Petersburg, USA, and the electrical wiring is only allowed along the frame since it is filled with 1100 triangular glass windows that are all different but nearly of the same size.

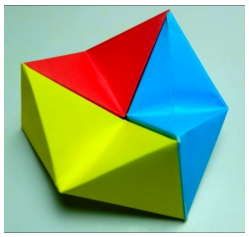


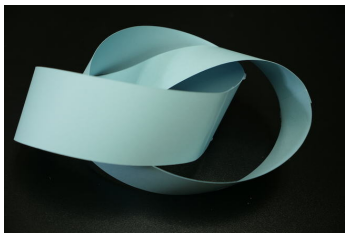


Magic cart tricks are often based on number theory. In volume 3, there is a long paper by P. Diaconis and R. Graham who explain step by step the number theoretical issues that are the backbone of what is sometimes called the most complicated cart trick ever. It was invented by Charles Sanders Peirce (1839-1914), an American logician and philosopher who is known as the father of pragmatism. He had however a soft spot for card tricks and the trick discussed consists in repeatedly laying out and dealing the cards and collecting them in different ways, seemingly reshuffling them, but it ends with two stacks where the value of the card at position j in the first stack indicates to position of the card with value j in the other stack. Peirce describes it in a publication, but like many of his publications it is nearly unreadable. The paper explains exactly the complicated modulo calculus and prime numbers that is making the trick work.

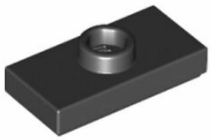








Geometry and topology, is a recurrent topic as well. The Tsuro game is definitely of this class. The players have square cards that have two entry points on each of its 4 sides and they are connected in pairs. There are many ways to do the latter pairing. How could you encode the topology of the square tiles and how many different tiles are there, taking symmetry into consideration?
Knots, origami, flexagons, Möbius bands, and their generalizations are obviously part of this section.



In many of the topics mentioned there is some complexity involved. The number of possible chess games is finite but quasi infinite for any realistic computation. So many of the strategies or the possibilities in any of the above mentioned problems quickly rise to an impractical number of possibilities which requires an analysis of the complexity, or defining the complexity class of the problem so that *computational complexity* is also one of the returning issues that are addressed in these proceedings. Take for example one of the simplest Lego blocks: a jumper plate which has only one notch at the top and 3 slots at the bottom. Only 8 of these can be connected in 393314 ways. One might ask what is the entropy, that is $\frac{1}{n}\log N(n)$ where $N(n)$ is the number of possibilities with n blocks as n becomes very large and if exact values are impossible. Ccan upper or lower bounds be given?

As can be seen from the few examples that I gave above, there are quite some problems that are worth investigating and some of these are quite challenging from a mathematical point of view involving many different areas of mathematics, from logic to computational complexity. Clearly recreational mathematics has become a proper research area.

Adhemar Bultheel