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RUNNING HEAD: Development of multiplication representation

The representation of multiplication facts:

Developmental changes in the problem size, five, and tie effects

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Abstract

In this study we investigated the development of basic effects that have been found in single-digit multiplication arithmetic: the problem size, five and tie effects. Participants (9-, 10- and 11-year-old children and adults) performed a production task on simple multiplication. The procedure replicated Campbell and Graham's (1985) study, but the results show that the gradual decrease of the problem size effect ends in sixth grade. We report analyses on raw latencies and state trace analyses, which take into account reaction time scaling as a function of age (Verhaeghen & Cerella, 2002). The results show that 11-year-old children do not differ significantly from adults, on either of the three effects. Before the age of 11, interesting developmental changes occur.

Key words: mental arithmetic, multiplication, development, memory retrieval

Introduction

In the literature on simple arithmetic, there is a consensus that solving multiplication facts relies to a large extent on memory retrieval (e.g., Campbell & Xue, 2001; Kirk & Ashcraft, 2001; Roussel, Fayol & Barrouillet, 2002). However, the question about how the representation of multiplication facts is organized in memory is still heavily debated. One of the open questions is how these representations evolve with age. This article focuses on the evolution of three effects that are robustly found in adult multiplication arithmetic, namely the problem size effect (small problems are easier than large problems, both in RTs and in error rates), the tie effect (problems with two equal operands are easier compared to the other problems) and the five effect (problems including the operand five are easier than problems without the operand five). The problem size effect and the five effect have also been observed in children (Campbell & Graham, 1985; Koshmider & Ashcraft, 1991) and the development of the problem size effect has been described (Campbell & Graham, 1985), but, to our knowledge, the three effects have never been studied simultaneously. However, understanding the developmental interdependencies is an important source of information to understand the nature and organization of the multiplication network in memory.

General trends

In multiplication retrieval three robust effects are observed, both in RTs and error rates. The problem size effect is the first: problems with small operands are solved faster and with fewer errors than problems with large operands (Stazyk, Ashcraft & Hamann, 1982). Another robust effect is the tie effect: problems with two equal operands (e.g., '6×6') are solved faster and with fewer errors than problems with two unequal operands. In addition to these two effects, an interaction between problem size and tie has been reported (Campbell & Gunter, 2002), in the sense that the size effect is weaker for ties than for non-ties. A third and last effect that has been widely documented is the five effect, meaning that problems with

operand 5 are solved faster and with fewer errors than would be predicted based on their size (Siegler, 1988; Lemaire & Reder, 1999; Masse & Lemaire, 2001).

An important challenge for theories on the representation of multiplication facts (e.g. Campbell, 1995; Siegler, 1988; Verguts & Fias, 2005) in memory lies in explaining these general effects. However, an adequate model should also be able to account for the following three developmental trends that have been observed for multiplication.

Developmental trends

As mentioned previously, researchers agree that memory retrieval is a frequently used strategy in single digit multiplication (e.g., Campbell & Xue, 2001; Roussel et al., 2002). In fact, teaching programs are explicitly oriented towards establishing these long-term memory representations. However, there is also a consensus about the existence of multiple procedures or strategies (such as counting or use of rules) in addition to retrieval (e.g., Siegler, 1988; Campbell & Xue, 2001). Although multiple strategies have been observed even in adults, previous research has shown that the use of retrieval from memory as a strategy increases with age (Cooney, Swanson & Ladd, 1988; Imbo & Vandierendonck, unpublished results), which is the first developmental trend. Nevertheless, retrieval quickly becomes the dominant strategy. Fourth graders (with a mean age of 9 years) already solve most simple multiplication problems by means of direct retrieval from memory. Cooney et al. (1988) found in a simple multiplication production task that fourth graders report the use of a retrieval strategy in 74 % of all trials. In a recent study by Imbo and Vandierendonck (unpublished results), fourth graders reported the use of retrieval in 81% of all trials. Even by the end of second grade, children already reported using retrieval in 60% of all trials (Imbo & Vandierendonck, unpublished results). These data are in accordance with Lemaire and Siegler (1995), who found that by the end of second grade over 90% of all multiplication problems were solved using retrieval. Lemaire, Barrett, Fayol and Abdi (1994) found clear evidence for the

existence of sufficiently strong associations between a number pair and its sum or product to produce retrieval-induced interference effects in different tasks, even as early as in third grade. Similarly, Lemaire, Fayol and Abdi (1991) showed that, from fourth grade on, children exhibit an associative confusion effect. When verifying simple additions (e.g. 8 + 4), they reject incorrect answers more slowly when these answers are the correct solution for the corresponding multiplication problem (e.g., 32).

Another developmental trend concerns RT and error rates: with increasing age, performance in mental multiplication becomes faster and less error prone. Campbell and Graham (1985) reported RTs of 3830 ms, 3630 ms, 1870 ms and 830 ms for third graders, fourth graders, fifth graders and adults respectively. Error rates also decreased, from 23% for third graders to 17% for fifth graders and 8% for adults. The RTs and error rates reported by Koshmider and Aschraft (1991) showed the same pattern. Moreover, this pattern has been observed in a wide range of tasks that measured RTs in childhood and adolescence (e.g., Kail, 1991).

A last developmental trend, observed by Campbell and Graham (1985) and Koshmider and Ashcraft (1991), is that the problem size effect decreases gradually with age (although it never disappears as it is still robustly observed in adulthood). This finding can be considered as another important developmental trend. However, no attention has been given to the fact that the smaller problem size effect for the older children could be an effect of RT scaling, as older children show faster RTs and this in itself may lead to the observed smaller effect size. It is often observed that effect size increases as general processing time elapses (e.g., Hale & Jansen, 1994; Salthouse & Hedden, 2002; Verhaeghen & Cerella, 2002); consequently, slower RTs implicate larger effect sizes (see Verhaeghen & Cerella, 2002, for a simple demonstration of this fact). Furthermore, age differences in individual performance sometimes completely disappear when differences in speed are controlled for statistically

(Schaie, 1989; Salthouse & Coon, 1994; Verhaeghen & Cerella, 2002). Because Campbell and Graham (1985) and Koshmider and Ashcraft (1991) did not correct for between-group differences in general processing speed, their reported conclusions could be an artifact of these differences.

The present study

Previous developmental studies on simple multiplication arithmetic indicate that the problem size effect decreases gradually with increasing age (Cooney et al., 1988; Koshmider & Ashcraft, 1991). But, as already mentioned, differences in mean RTs between the studied age groups were not taken into account. Moreover, no study has yet investigated the evolution of the five and tie effects and of the interaction between the size and tie effect (i.e., the weaker size effect for tie problems). Therefore, this study investigated the sizes of the problem size, five and tie effects simultaneously while controlling for RT scaling effects. Because the focus of this article is on the retrieval process in simple multiplication arithmetic, we investigated the performance of children from fourth grade (mean age of 9 years) on, assuming that these children have mastered the multiplication tables and consequently that they have a complete representation of the multiplication network in memory. Moreover, fourth graders use retrieval as a strategy in the majority of the trials in a simple multiplication production task (Cooney et al., 1988; Lemaire & Siegler, 1995; Imbo & Vandierendonck, unpublished results).

To accomplish these goals we measured the performance of fourth, fifth and sixth grade children in a simple multiplication production task and compared their performance with each other and with a group of adults.

Method

Participants

All children in grades 4, 5, and 6 of an elementary school participated in this study. Due to technical problems[1], the data of only 13 out of 26 fourth graders were available for analysis (7 girls and 6 boys). Their mean age was 9.43 years (*SD* = .33 years). The fifth and sixth grade classes consisted of 22 (13 girls and 9 boys) and 15 (8 girls and 7 boys) children, respectively. Mean ages were 10.48 years (*SD* = .33 years) and 11.46 years (*SD* = .33 years) respectively. As a reward for participation, children received some fruit. Parents were informed through a letter in which they were asked to contact us if they objected to participation of their child, but none did. Data of ten adults (two researchers and eight first-year psychology students) on the same task were also included in the analyses.

Material

The stimuli consisted of all simple multiplication problems from 2 × 2 to 9 × 9. The software was developed with E-Prime V1.1 (Psychology Software Tools, 2002). All testing was performed on a Pentium PC (Windows 98) with a 17'' screen. RTs were measured by means of a voice-activated relay connected to the parallel port of the PC.

Procedure

All testing took place in November 2003 and December 2003 during school hours. All children were tested individually and participated in two identical sessions. Each session consisted of 72 simple multiplication problems. Commutative non-ties (e.g., 3 × 4 and 4 × 3) were considered as two different problems and were administered once in each session. Each tie problem was tested twice (cf. Campbell & Gunter, 2002). A trial consisted of a fixation point (hash mark) in the center of the screen for 1000 ms, after which the problem (a × b) appeared in the same position. The problem remained on screen until a response was given, with a maximum RT of 10 sec. RTs were measured by means of a voice-activated relay. All incorrect trials and all trials that were corrupted due to failure of the voice-activated relay were repeated at the end of each session to decrease the amount of data loss.

Stimuli appeared in a font size of 18 points. Ten practice trials[2] were administered to make the children familiar with the procedure and the voice-activated relay. These practice trials were the same for each participant and in each session. They were shown in the same order to each participant.

Children were told to respond as quickly and accurately as possible. After each trial they received visual feedback. The feedback consisted of a white screen with a 'correct' or 'wrong' message on top and a consistent picture (e.g., a smiley face if the answer was correct) in the middle of the screen.

Results

Error percentages were 7.50%, 3.85%, 3.22% and 5.56% for fourth-, fifth-, and sixth-grade children, and adults respectively. The data did not show any evidence of a speed-accuracy trade off, as indicated by the positive correlation between RTs and error rates, computed across the 64 cells of the design and for each grade separately, $r = 0.59$; 0.65; 0.69 and 0.51 for fourth, fifth, and sixth graders and adults respectively, $n = 64$, $p<.01$. Because errors were infrequent, they are not described further.

Due to occasional failures of the voice activated relay, 10% of correct trials were lost (respectively 15.0%, 8.90%, 17.78% and 5.48%). These high percentages of voice-key failure (in the children's data) are due to the fact that all testing took place at school, where there was continuous background noise from an adjacent classroom. For this reason, all invalid trials were repeated once at the end of each session. Hence, the proportion of data loss from the 144 experimental trials per participant was reduced to on average 7.05%, 4.39% and 8.33% for fourth, fifth, and sixth graders, respectively.

Uncorrected RTs

All reported analyses were performed on median RTs. To evaluate the presence or absence of a problem size effect, we divided the set of multiplication problems in a small

and large set, following the criterion used by Campbell (1994). Small problems were defined as those with both operands smaller than 5, large problems as those with both operands larger than five. Hence five problems and problems with one operand smaller than 5 and one larger than 5 are excluded from analysis. An ANOVA with age group (fourth grade, fifth grade, sixth grade, adults) as between-subjects factor and problem size (small, large), and tie (tie, non-tie) as within-subject variables revealed a main effect of problem size: Small problems were answered faster than large problems, $F(1, 56) = 140.28$; $MSe = 167625$; $p<.01$. Also, tie problems were answered faster than non-tie problems, $F(1, 56) = 90.89$; $MSe = 104943$; $p<.01$. The main effect of age group was significant, $F(3, 56) = 7.72$; $MSe = 484094$; $p<.01$. Fourth graders responded in 1748 ms, compared to 1539, 1266, and 1096 ms, respectively, for fifth graders, sixth graders and adults. Planned comparisons demonstrated that median RTs differed significantly between all groups, except between sixth graders and adults, $F<1$.

Problem size interacted significantly with age group, $F(3, 56) = 2.77$; $MSe = 167625$; $p<.05$. Planned comparisons revealed that the problem size effect was significantly larger for fourth graders than adults, $F(1, 56) = 7.88$, $MSe = 167625$; $p<.01$. Also, the planned comparison between the problem size effect of fourth and sixth graders was marginally significant, $F(1, 56) = 3.75$, $MSe = 167625$; $p=.057$, and the planned comparison between the problem size effect of fourth and fifth graders showed a trend towards significance, $F(1, 56) = 2.85$, $MSe = 167625$; $p=.097$. No other planned comparisons between age groups reached significance for the problem size effect.

The interaction of tie and age group was not significant, $F(3, 56) = 1.56$. Planned comparisons revealed that the tie effect of fourth graders was significantly larger than the tie effect of sixth graders, $F(1, 56) = 4.13$, $MSe = 104943$; $p<.05$. The other planned comparisons did not reach significance.

A significant interaction between problem size and tie status was observed, $F(1, 56) = 93.37$; $MSe = 119032$; $p<.01$: the size effect was larger for non-tie problems than for tie problems. The interaction between problem size and tie status is embedded in the significant three-way interaction between problem size, tie and age group, $F(3, 56) = 4.14$; $MSe = 119032$; $p=.01$. Figure 1 illustrates that the interaction between problem size and tie is larger for fourth graders than fifth graders, $F(1, 56) = 9.02$, $MSe = 119032$; $p<.01$, sixth graders, $F(1, 56) = 7.65$, $MSe = 119032$; $p<.01$, and adults, $F(1, 56) =8.74$, $MSe = 119032$; $p<.01$, while the interaction is equally large for fifth graders, sixth graders and adults, Fs<1.

To evaluate the five effect, tie problems were excluded from the subsequent analysis with age group as a between-subject variable and five status (five, non-five) as a within-subject variable. The main effect of five status reached significance, $F(1, 56) = 115.48$; $MSe = 50039$; $p<.01$; five problems were answered faster ($Md = 1119$ ms) than non-five problems ($Md = 1328$ ms). The interaction with age group, shown in Figure 2, was highly significant, $F(3, 56) = 5.73$; $MSe = 50039$; $p<.01$. Planned comparisons revealed that the five effect was larger for fourth graders than sixth graders, $F(1,56) = 8.08$; $MSe = 50038$; $p<.01$, and adults, $F(1,56) = 15.06$; $MSe = 50038$; $p<.01$. The comparison of fourth graders and fifth graders showed a trend towards significance, $F(1,56) = 2.96$; $MSe = 50038$; $p=.09$. Also, the five effect appeared to be larger for fifth graders compared to adults, $F(1,56) = 7.31$; $MSe = 50038$; $p<.01$. No other planned comparisons reached significance.

From this pattern of results, we would conclude that fourth grade children indeed show evidence for a larger problem size, five and tie effect compared to adults. Fifth grade children showed evidence for a larger five effect compared to adults. However, as we already mentioned, it is essential to disentangle specific age-related changes in the three effects from the generic change in mean reaction time. Consequently, we used state trace analysis to

distinguish between an actual larger effect size in younger (and consequently slower) children and the generic effect of slower mean reaction times.

RTs controlled for general speeding

Whenever one studies effect sizes in groups that differ in overall performance, the problem of interpreting a group by condition interaction is encountered (Loftus, 1978). Most researchers rely on tests of interactions in an analysis of variance framework and interpret interactions with age group as age-related changes in the specific cognitive components accessed by their experimental manipulations. However, analysis of variance assumes *additive* effects both of age and the experimental manipulations. If general speeding is operating and age differences are not additive but rather multiplicative, the age × experimental factor interaction test will yield false positives. Some methods have been proposed in the literature to solve this problem, like transforming raw RTs into logarithms or ratios (see Salthouse & Hedden, 2002). However, these transformations are based on the assumption of *multiplicative* age and experimental effects, namely, that each group is slowed by the same relative amount *for all processes* (including input and output processes) by the experimental manipulation, and that all differences between groups are multiplicative and not additive. As these assumptions are not always satisfied (e.g., Verhaeghen & Cerella, 2002), we believe that transformations are not the optimal method to interpret age by condition interactions. Madden, Pierce and Allen (1992) proposed analyses of proportional slowing to control for group differences in processing speed. This method also assumes multiplicative effects. Consequently, we argue that analyses of proportional slowing/speeding are not general enough, because in many cases one does not know a priori whether age and experimental effects are multiplicatively related. One solution (proposed by Faust, Balota, Spieler and Ferraro, 1999) is to use standardized solution times. This method is potentially valid but only works when both the age and experimental effect are linear (i.e., additive or

multiplicative). Although this is certainly a step in the right direction, there is an even more general framework that does not assume linearity of either age or experimental effects, which is the state trace method (Bamber, 1979; Verhaeghen & Cerella, 2002; Loftus, Oberg & Dillon, 2004). Our particular implementation of state trace methodology also assumes linearity, but due to its graphical nature, deviations from linearity can be detected, as linear versus non-linear effects lead to clearly distinct signatures, as do additive versus multiplicative experimental effects. We therefore believe that the state trace method is the more informative one. Following Verhaeghen and Cerella (2002) and Verhaeghen, Steitz, Sliwinski and Cerella (2003), we constructed state traces for each of the three effects at test (the problem size, five and tie effect). A state trace displays the performance (in mean RT) of one age group in one condition (e.g., small problems) as a function of the performance of the same age group in the other condition (e.g., large problems). Each point of the resulting scatter plot corresponds to the mean RT of one participant in one condition plotted against the mean RT of the same participant in the other condition. Regression analyses with performance in one condition as the predictor variable and performance in the other as the dependent variable are performed to quantitatively describe the relation between the two conditions. Verhaeghen and Cerella (2002) constructed a framework in which the configuration of these state traces is open to straightforward interpretation. In the case of additive effects of the experimental manipulation, the resulting state trace with performance on the simple task depicted on the abscissa and performance on the complex task on the ordinate will be a line parallel to the diagonal. The intercept of the state trace indicates the cost of the complex condition. In contrast, state traces with a slope larger than one indicate a multiplicative experimental effect and the inflation factor is given directly by the slope of the state trace.

If RTs are only influenced by the general scaling effect of increased processing speed, a single line suffices to explain the relation between both conditions for the different age groups. If different regression lines are obtained, this signals a specific age-related effect in the process that is associated with the complexity cost, irrespective of whether the type of complexity is additive or multiplicative. In any case, the emergence of different lines rather than one line implies a true difference between the age groups concerning the effect at test (e.g., the problem size effect). In particular, we were able to test statistically whether a single line suffices to explain the data or whether different lines (one for each age group) are needed to describe the relationship between both conditions (e.g., small and large problems).

Median RTs for each participant and for each problem type (ties and non-ties, small and large problems, five and non-five problems) were calculated on correct and unspoiled (by voice-key failure) answers. We performed a series of regression analyses with RTs in one (the 'difficult') condition (large problems, non-tie problems, or non-five problems) as the dependent variable. Predictor variables were (a) RTs in the other (the 'easy') condition (small problems, tie problems, or five problems), (b) a dummy variable that coded for a contrast between one age group and the other (older) age groups (e.g., the contrast 'age = 3' if the participant is in fourth grade and 'age = -1' if in fifth grade, sixth grade or in the adult group so that sum of contrast weights = 0 was used to compare fourth graders with the older age groups) and (c) a variable that is the product of the 'easy condition' variable and the corresponding contrast dummy variable. The resulting regression equation is shown below.

$$RT_{large\ /\ non\text{-}tie\ /\ non\text{-}five} = \beta_1 + (\alpha_1 \cdot RT_{small\ /\ tie\ /\ five}) + (\beta_2 \cdot age) + (\alpha_2 \cdot age \cdot RT_{small\ /\ tie\ /\ five}) \qquad (1)$$

The parameters β_1 and α_1 convey the main effect of condition (e.g., the problem size, tie or five effect). The parameter β_2 conveys the effect of age group on the intercept of the state trace. If this parameter differs significantly from zero, it points to an additive interaction

effect of age with the size, five or tie effect. The parameter α_2 conveys the effect of age group on the slopes of the state traces. If these parameters differ significantly from zero, it implicates a multiplicative interaction of age with the size, five and tie effect.

Because the predictors age and 'age × RT' are strongly correlated, their contribution was not evaluated separately. Instead, we opted for a hierarchical approach where the restricted model with RTs in the easy condition as the only predictor was fitted first, followed by the full model in which the predictors age and age × RT were added to the regression equation. F-change values (full model versus a restricted model) are reported.

Concerning the problem size effect, when comparing fourth graders on the one hand with fifth graders, sixth graders and adults on the other hand, ΔF of the model with all three predictors (RTs on small problems, age and 'age × RT') included was significant, $R^2 = .29$; $F(2, 56) = 4.91$; $p<.05$. When comparing fifth graders on the one hand with sixth graders and adults on the other, ΔF for the full model also reached significance, $R^2 = .31$; $F(2, 56) = 4.5$; $p<.05$. For the comparison of sixth graders with adults, ΔF of the full model was not significant, $F<1$. Hence, a single line fits the problem size effect for sixth graders and adults, but for the problem size effect of the fourth graders another fit is needed, as well as for the problem size effect of the fifth graders (see Figure 3). Interestingly, when comparing the problem size effect of fourth graders and fifth graders, ΔF of the full model reached significance, $R^2 = .31$; $F(2, 56) = 4.27$; $p<.05$. This indicates that two lines are needed to fit the problem size effect of these age groups.

Regarding the tie effect (Figure 4), ΔF of the full model showed a trend towards significance, $R^2 = .57$; $F(2, 56) = 2.55$; $p=.09$, when comparing fourth graders with the older age groups. When comparing the tie effect of the fifth graders with the tie effect of sixth graders and adults, ΔF of the full model did not reach significance, $F(2, 56) = 1.92$; $p=.16$. When comparing sixth graders with adults, ΔF of the full model did not reach significance,

$F(2, 56) = 1.48$; $p=.24$. Consequently, a single line fits the tie effect for fifth graders, sixth graders and adults, although a different line is needed for fourth graders (see Figure 4).

Regarding the five effect, similar results emerged from the analysis for all three comparisons (fourth graders versus older; fifth graders versus older and sixth graders versus adults). ΔF of the full model never reached significance, all Fs<1. Consequently, the five effect can be fitted by one single line for all age groups (see Figure 5).

The adult sample in the present study was relatively slow (*Md* = 1095.5 ms in this study compared to 830 ms in Campbell and Graham's study, 1985), but the children were relatively fast (*Md* = 1748 ms and 1539 ms for fourth and fifth graders respectively; compared to 3630 ms and 1870 ms in Campbell and Graham's study, 1985). To investigate whether the relative slowness of the adult sample and the relative speed of the children were responsible for the lack of age-related changes[3], we performed additional analyses using data from the 50% slowest children of each grade and the 50% fastest adults. The analyses yielded the same general pattern as those on the full sample.

Discussion

This study investigated developmental change in the problem size, five and tie effect from fourth grade on. Previous studies that investigated the evolution of the problem size effect found that the effect size declines gradually with increasing age (Campbell & Graham, 1985; Koshmider & Ashcraft, 1991). However, these studies also reported major differences in mean RTs between age groups. As effect sizes scale up or down with increasing or decreasing RT, their conclusions can be an artifact of the observed differences in mean RT. Therefore, the objective of this study was to take into account these group differences in investigating the evolution of the problem size, five and tie effect.

As children in fourth grade already retrieve the majority of the solutions to simple multiplication problems from long-term memory (Cooney, Swanson & Ladd, 1988; Lemaire

& Siegler, 1995; Imbo & Vandierendonck, unpublished results), we compared the RTs of fourth, fifth and sixth grade children on simple multiplication with each other and with the performance of adults on the same task.

Because of huge differences in median RTs between groups, no conclusions can be drawn from the present analyses on uncorrected RTs. Therefore, we used the state trace method, first proposed by Bamber (1979) and applied by Verhaeghen and Cerella (2002) and Loftus, Oberg and Dillon (2004) to disentangle the specific age-related changes in the problem size, five and tie effect from the generic change in mean RT.

Taken together, the results of the present study indicate that the underlying nature of the size and tie effect is different for fourth graders compared to fifth graders, sixth graders and adults. Also, the size effect of fifth graders appeared to differ from sixth graders and adults. State trace analysis showed that different regression lines are needed to fit both the size and tie effect for fourth graders on the one hand and fifth graders, sixth graders and adults on the other. Moreover, a different fit was needed for the size effect of fifth graders on the one hand and sixth graders and adults on the other.

The present results indicate that the problem size effect indeed decreases gradually up to fifth grade. Once in sixth grade, age-related changes are no longer observed. In contrast, the tie effect decreases up to fourth grade and does not seem to evolve further (although this decrease was only a trend in the data).

The present results also indicate that the five effect is already fully fixed from fourth grade on, indicating that five problems are already fully represented at that age. An explanation for this observation can be found in the curriculum of Belgian elementary schools. Children start to learn the multiplication tables by learning problems involving 1, 2, 5, and 10. This implies that these numbers are the first ones integrated into the memory

network. The gradual decrease in effect size of the five effect should then be observed in younger children.

The observation that the size, five and tie effect remains equally large from sixth grade on (when scaling is taken into account) clearly implies that from that age on, children already have developed a complete memory network similar to an adult network: All effects that are observed in adults are present. Furthermore, all three effects remain equally large from that moment on.

These conclusions contrast with those of both Campbell and Graham (1985) and Koshmider and Ashcraft (1991), who maintained that the problem size effect declines dramatically *and* gradually with age. Our data indeed suggest that the problem size effect decreases gradually with age but that from sixth grade on, nothing really changes, apart from a generic increase in processing speed. However, we do not claim that this finding implies that there are no changes whatsoever in general arithmetic knowledge. We acknowledge that fact knowledge (like the arithmetic tables) is part of a richer representational structure in which both conceptual and factual knowledge are integrated (Baroody, 1994). Nevertheless, the present results indicate that the factual part of this richer representation is fully formed in sixth grade.

The finding that sixth graders show evidence for a fully formed multiplication network in memory that is similar to an adult network is noteworthy. To find an explanation, consider the process of learning the multiplication tables. In the curriculum of Belgian elementary schools the multiplication tables are introduced in second grade; in third and fourth grade the tables are often repeated to ensure their integration in memory. Moreover, children learn the multiplication tables in a very structured way. Each problem is introduced explicitly and is repeated frequently during and after the learning process. The structured learning process may facilitate the formation of a memory representation in a relatively short period of time.

Whether the same conclusion of fast network formation also holds for other (e.g., American) curricula remains to be tested. Such comparative studies would require the application of an appropriate method, like the one we adopted here, to correct for overall group differences. Our results indeed demonstrate the importance of cautiously interpreting age group by condition interactions in developmental research when confronted with between-group differences in mean RTs. We are convinced that the generality of the state trace method, and its combination of graphical and statistical procedures, is useful for developmental researchers.

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Footnotes

1. These 13 participants were excluded because of proven anomalies in reaction time registration through the game port of the computer. Therefore we used the parallel port for RT registration in subsequent testing, but unfortunately 13 fourth graders had already participated in both sessions of the experiment.
2. The practice trials consisted of the following problems: 2×3, 3×4, 4×5, 5×6, 6×7, 7×8, 8×9, 9×3, 2×4 and 3×5.
3. We are thankful to P. Lemaire for this suggestion.

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Figure Captions

Figure 1. Interaction problem size by tie by age group on uncorrected RTs. Black lines correspond to tie problems, dotted lines to non-tie problems. Error bars denote within standard errors.

Figure 2. Interaction five by age group on uncorrected RTs. Black lines correspond to five problems, dotted lines to non-five problems. Error bars denote within standard errors.

Figure 3. State trace of the problem size effect. The thin black line is the fit for the fourth grade children. The dotted line is the fit for the fifth graders. The thick black line is the fit for the 6th graders and the adults. The thin grey line is the fit for Y=X, the diagonal. The text labels (4, 5, 6 or A) point to the age group to which the corresponding observation belongs (fourth, fifth, or sixth grade or adult).

Figure 4. State trace of the tie effect. The thin black line is the fit for the fourth graders. The dashed black line is the fit for the fifth graders, sixth graders and the adults. The thin grey line is the fit for Y=X, the diagonal. The text labels (4, 5, 6 or A) point to the age group to which the corresponding observation belongs (fourth, fifth, or sixth grade or adult).

Figure 5. State trace of the five effect. The dotted grey line is the fit for all age groups. The thin grey line is the fit for Y=X, the diagonal. The text labels (4, 5, 6 or A) point to the age group to which the corresponding observation belongs (fourth, fifth, or sixth grade or adult).

Figure 1

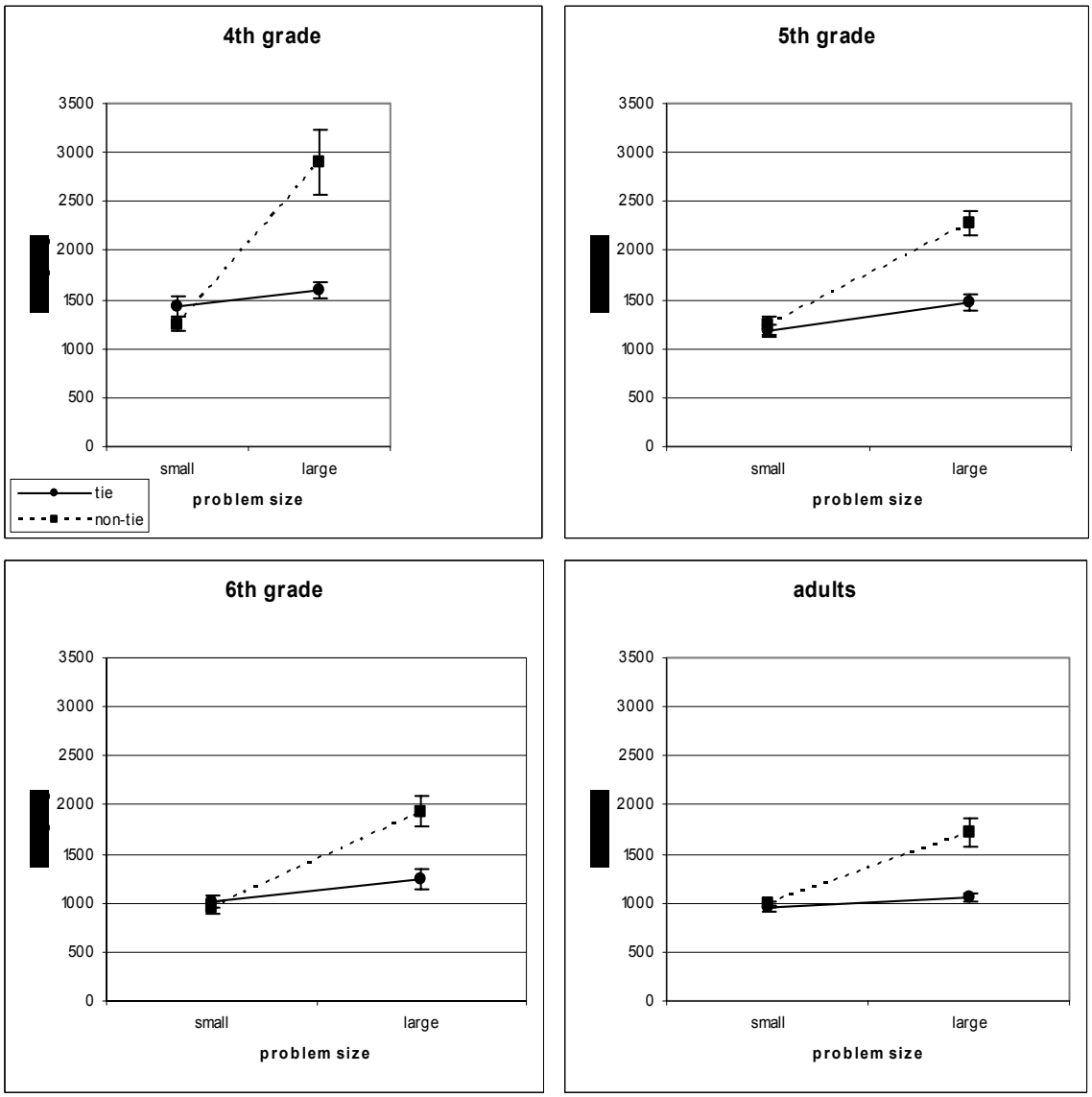

4th grade
5th grade
6th grade
adults
3500
3000
2500
2000
1500
1000
500
0
small
large
problem size
tie
non-tie

Figure 2

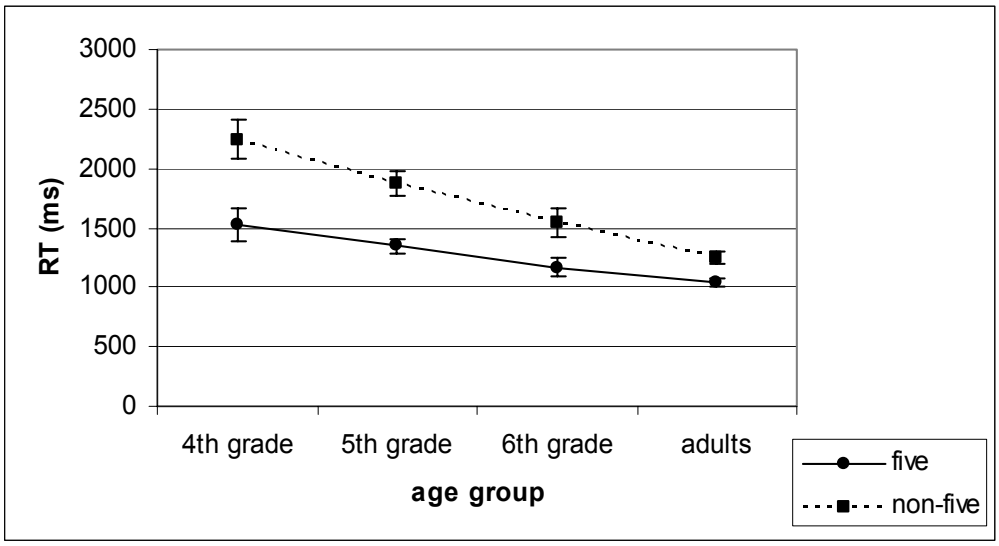

3000
2500
2000
1500
1000
500
0
RT (ms)
4th grade
5th grade
6th grade
adults
age group
five
non-five

Figure 3

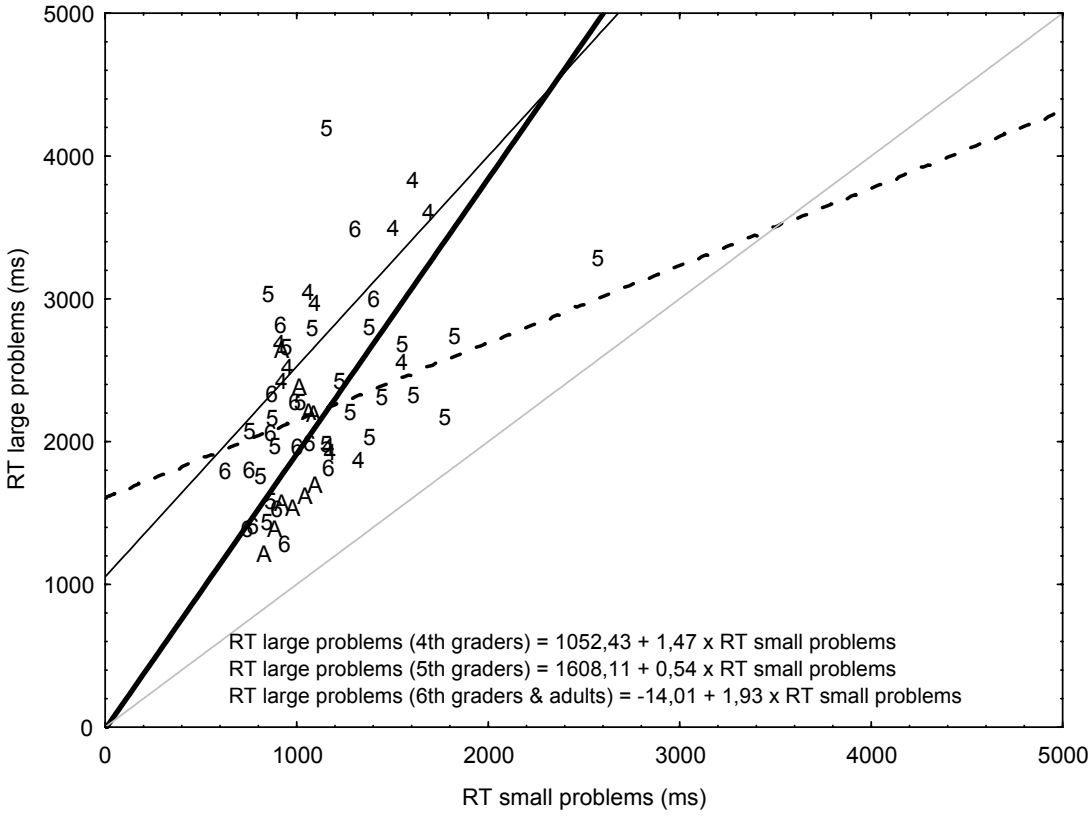

RT large problems (ms)
RT small problems (ms)
0
1000
2000
3000
4000
5000
RT large problems (4th graders) = 1052,43 + 1,47 x RT small problems
RT large problems (5th graders) = 1608,11 + 0,54 x RT small problems
RT large problems (6th graders & adults) = -14,01 + 1,93 x RT small problems

Figure 4

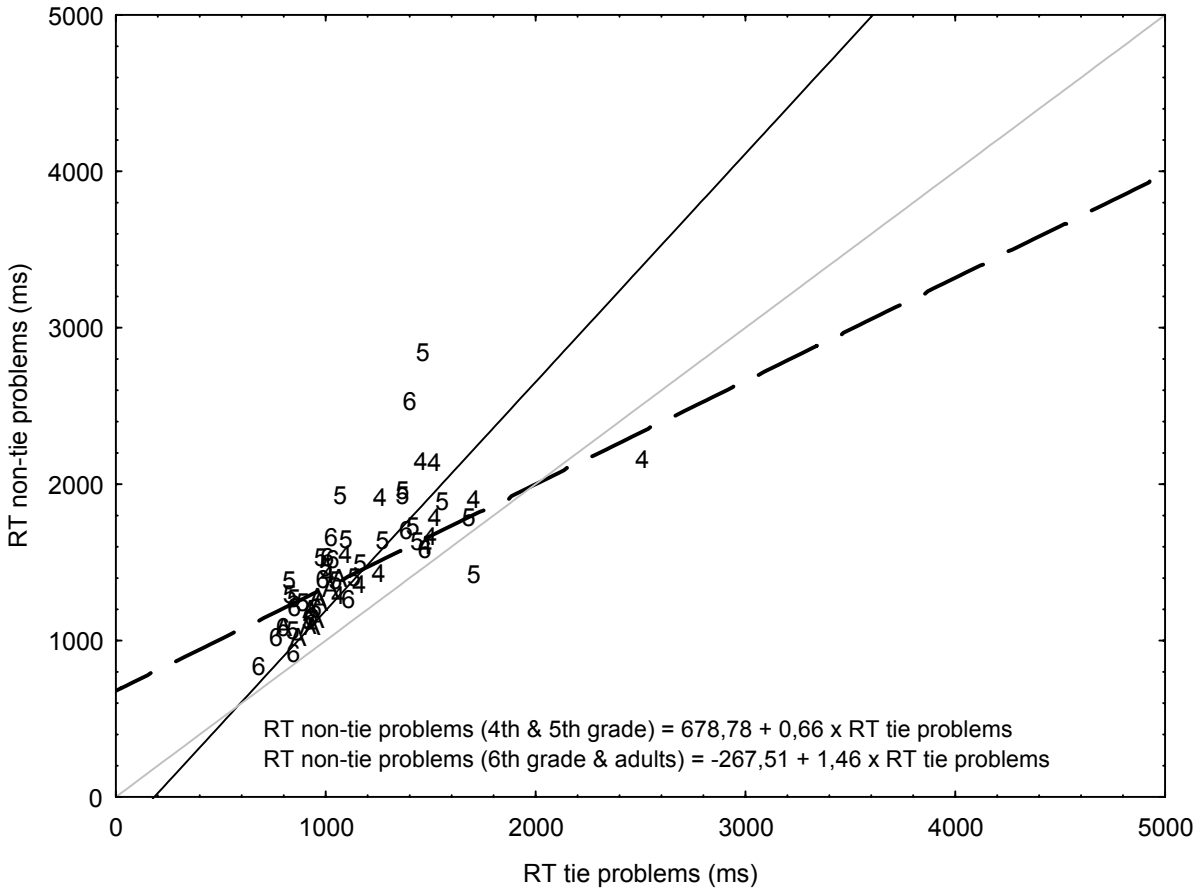
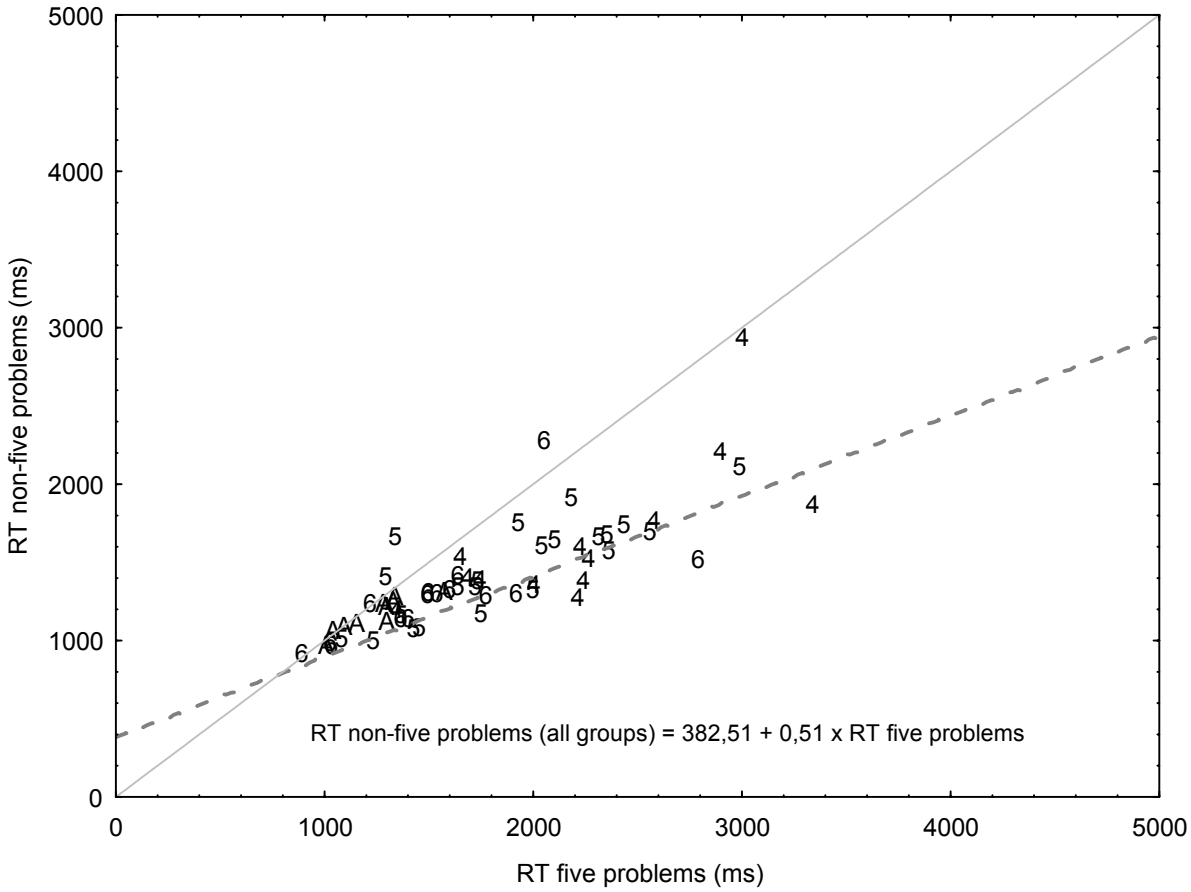

RT non-tie problems (ms)
RT tie problems (ms)
0
1000
2000
3000
4000
5000
RT non-tie problems (4th & 5th grade) = 678,78 + 0,66 x RT tie problems
RT non-tie problems (6th grade & adults) = -267,51 + 1,46 x RT tie problems

Figure 5


RT non-five problems (ms)
RT five problems (ms)
RT non-five problems (all groups) = 382,51 + 0,51 x RT five problems
0
1000
2000
3000
4000
5000