The parametrized Lanczos method for multiple right-hand sides

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Abstract

The solution of the parametrized system

$$Ax = f \quad \text{with} \quad A = K - \omega^2 M \tag{1}$$

with K real symmetric, and M symmetric positive definite arises in applications, including structural engineering and acoustics. The parameter ω is often the frequency (or the wave number) and lies in $[\omega_{\min}, \omega_{\max}]$, which is the frequency interval where the numerical model is valid. Usually, $\omega_{\max}$ is determined by the level of mesh refinement. In many cases, $\omega_{\min} = 0$. The solution x is called the frequency response function.

The traditional method in engineering is modal superposition where (1) is projected on the eigenvectors associated with the eigenvalues of

$$Ku = \lambda M u \tag{2}$$

in $\Lambda = [\lambda_{\min}, \lambda_{\max}]$, where $\lambda_{\min} \ll \omega_{\min}^2$ and $\lambda_{\max} \gg \omega_{\max}^2$. This method is usually experienced as very efficient when the eigenvectors and eigenvalues are available, since (1) is transformed to a diagonal linear system. The practical problem is that it is not always clear how $\lambda_{\min}$ and $\lambda_{\max}$ need to be chosen. For example, when $\omega_{\min} = 0$, we use $\lambda_{\min} = 0$ and $\lambda_{\max} = \eta\omega_{\max}^2$ with $\eta \in [2, 10]$.

Efficient methods for solving (1) have been developed over the last decade, in the context of iterative linear system solvers for parametrized problems [5] [4], and the Padé via Lanczos method in the context of modelreduction [3] [1] [2].

In this paper, we study the use of eigenvectors to precondition the Lanczos method from [4] for solving (1). Modal superposition is used as a preconditioner to the parameterized Lanczos method. This work is an extension of recycling Ritz vectors for the solution of linear systems with multiple right-hand sides to parameterized linear systems with multiple right-hand sides. We show a numerical example arising from structural analysis.

References

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