Qualitative Comparison of Techniques for Evaluating Performance of Short Term Power System Reliability Management

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***Abstract*—Adequate performance evaluation of different short term reliability management approaches and criteria is crucial in order to convince transmission system operators to substitute or complement the currently used N-1 criterion with more efficient, alternative approaches. This paper qualitatively compares existing evaluation techniques. These techniques are also applied in other contexts, such as reliability assessment, which have similarities with performance evaluation, but differ in some crucial aspects. Evaluation techniques typically make a trade-off between accuracy, complexity and computational burden. Easy-to-use, non-sequential, analytical state enumeration techniques can be useful to obtain indicative results. More computationally intensive, sequential simulation techniques are on the other hand better suited for in depth analysis, as they allow to take into account the dynamic character of the decision making process under evaluation. Emulation seems promising from a theoretical perspective, but handling the complex, multi-dimensional nature of short term reliability management is a challenge.**

***Index Terms*—Power system operation, Power system reliability management, Performance evaluation techniques**

I. Introduction

Evolutions in power systems, such as increased uncertainty due to an increase of renewable energy sources, challenge currently used deterministic reliability criteria. More advanced reliability management approaches and criteria (RMACs) to overcome those challenges are under development [1]–[3]. These alternative RMACs are typically probabilistic in nature and can have different requirements compared to traditional deterministic approaches, for instance in terms of data and flexibility. Smart grids will provide a framework to exploit the full potential of alternative reliability management strategies. An example of this is given by Ovaere et al., who have indicated that for a basic short term probabilistic reliability management strategy, using a more detailed representation

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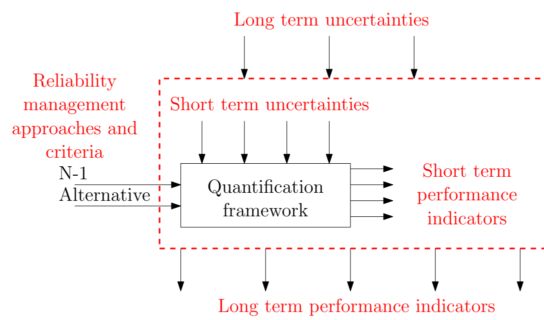


Fig. 1. Overview of the procedure for selecting an RMAC

of value of lost load (VOLL) data has a high potential for efficiency improvements [4]. In existing power systems, it is practically impossible to differentiate between consumers in load curtailment, but this becomes an option when smart grids will have been deployed in the future.

In order to convince power system stakeholders to move towards an alternative RMAC, it is important to have an accurate and reliable performance evaluation. The overall decision making process of selecting an appropriate RMAC is influenced by long term and short term uncertainties, as shown in Fig. 1. In order to assess the long term impact of using alternative RMACs, similar uncertainties need to be considered as in transmission expansion planning [5]. The performance of short term reliability management is also influenced by several short term uncertainties, such as contingencies, load and renewable energy sources, behaviour of corrective control, behaviour of exogenous actors and unforeseeable events. The focus of this paper is on the performance evaluation of short term reliability management impacted by short term uncertainties, as included in the dashed box in Fig. 1.

Techniques for evaluating performance of short term reliability management have similarities with reliability assessment techniques. Reliability assessment can be considered as part of the performance evaluation. A major and important difference between reliability assessment and performance evaluation is that reliability assessment mainly focuses on the final system state, while in order to obtain a complete and reliable performance evaluation, both the final system state resulting from reliability management and the trajectory followed while executing reliability management should be evaluated [6], [7]. Another important difference is that especially failure states are of interest for reliability assessment, while performance evaluation also has to evaluate the performance of reliability management in normal states. Complete performance evaluation of RMACs, considering both the decision making trajectory and the final system states, has not been specifically covered in literature so far. Nevertheless, a decent and complete performance evaluation of different RMACs is crucial, given the importance for society of an adequate reliability level.

Performance evaluation of different short term reliability management approaches and criteria is an offline process and consists of four main steps:

1) Selection of a performance evaluation technique and appropriate sampling technique
2) Simulation of decision making behaviour for different short term RMACs
3) Selection and calculation of performance indicators
4) Post processing of results and comparison of performance of different RMACs

The focus of this paper will be on the first point, while points 2 to 4 are introduced for sake of completeness. The paper gives an overview of different techniques that are used in other contexts, but are also applicable to evaluate the performance of short term power system reliability management. Advantages and shortcomings of the different techniques are discussed in order to guide the selection or development of an appropriate performance evaluation technique. Furthermore, the authors bring together the pieces required in order to analyse and evaluate different reliability management approaches and criteria and combine them in an analytical formulation, which facilitates the comparison between evaluation techniques.

Section II introduces the simulation of the dynamic process of short term reliability management, the performance evaluation based on performance indicators and the comparison of the performance. Section III describes different performance evaluation techniques. Section IV discusses the advantages and shortcomings of the techniques and the characteristics of an ideal performance evaluation technique. Section V concludes the paper.

II. Simulation, Evaluation and Comparison of Short Term Reliability Management

In order to frame the performance evaluation techniques, the simulation of short term reliability management and its characteristics are introduced together with the evaluation procedure.

A. Simulation of short term reliability management

The decision making process of short term reliability management based on a particular reliability criterion j is a dynamic process:

$$X_j(t) = f_j(X_j(t-1), Y(t)) \tag{1}$$

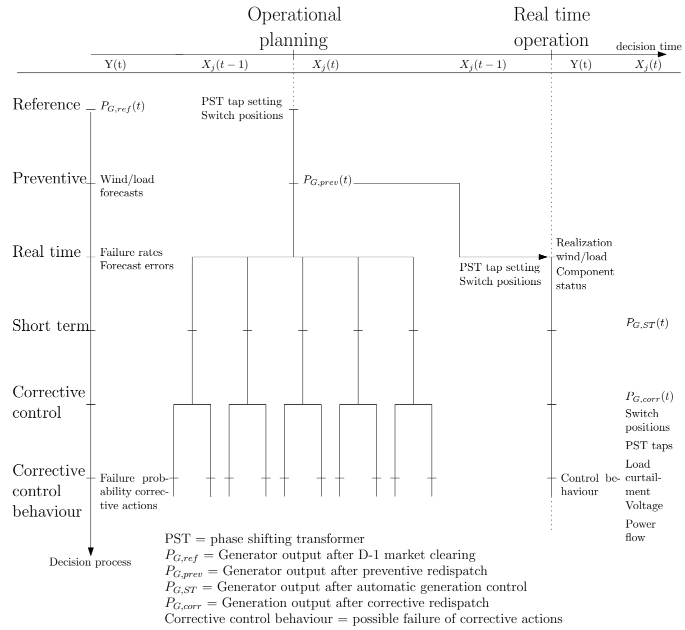


Fig. 2. Multi-stage procedure of the decision making process of short term reliability management

where $X_j(t)$ is the state vector at time t according to criterion j, consisting of generator active power set points, phase shifting transformer tap settings, switch positions, voltage/reactive power set points, etc., $X_j(0)$ is the initial state of the system and $Y(t)$ is the vector of external forcing inputs at time t, containing forecast values of net demand and wind generation (aggregated or per node), net demand and wind generation realizations per node, status of system components, failure data, etc. Some input parameters are non-stationary, e.g. due to daily cycles, which should be handled appropriately in order to obtain a decent performance evaluation. The function f_j is a deterministic function representing the decision making behaviour of a system operator in short term reliability management based on a particular reliability criterion j. Short term reliability management typically ranges from a few days ahead of real time up to real time and consists of multiple stages, as illustrated for a two stage decision making process, consisting of operational planning and real time operation, in Fig. 2. This multi-stage decision making process represented by f_j is typically simulated using consecutive multi-stage optimizations, also denoted as security constrained optimal power flows (SCOPF). The exact, non-linear AC-SCOPF typically has convergence issues, bringing alternative, approximate, linear and convex implementations in the picture, such as DC-SCOPF or LPAC [8], [9]. The computational burden of these optimizations is a challenge due to the many binary variables that is introduced in the formulation, especially in real systems with thousands of nodes.

Different reliability criteria imply different security constraints in the optimization formulations, leading to different functions f_j. Differences exist in terms of which and how system states are considered in the decision stages ahead of real time. Moreover, the way costs at different stages are considered can differ and additional thresholds can be imposed on reliability indicators, such as energy not supplied or power not supplied, either aggregated or separated per contingency, node or consumer.

B. Performance evaluation and comparison

Evaluation of short term reliability management based on criterion j requires the calculation of quantitative performance indicators Q_{ij} based on the state vectors $X_j(t)$:

$$Q_{ij,traj.}(t) = g_i(X_j(t)) \tag{2}$$

$$Q_{ij} = \sum_{t=1}^{T} Q_{ij,traj.}(t) \cdot \Delta t \tag{3}$$

where g_i is a deterministic function translating the state space vectors $X_j(t)$ into a performance indicator $Q_{ij,traj.}(t)$. Performance indicators $Q_{ij,traj.}(t)$ can be system related, such as over- or undervoltage or line overloading, consumer related, such as energy not supplied or outage cost, or can consider aspects of both, such as total system cost.[1] The value of the performance indicator at time t implicitly depends on the previous state and the external forcing inputs at time t. For this reason, the performance indicators follow a trajectory over time that depends on the applied RMAC. This trajectory, taking into account the dependence on the previous system state, should be evaluated in an ideal performance evaluation.

The objective of the performance evaluation is to verify whether one criterion performs significantly better than another one in terms of particular quantitative performance indicators Q_i. In order to obtain reliable conclusions about the relative performance of different reliability criteria, stationary distributions of the performance indicators Q_{ij} should be used.

III. Performance evaluation techniques

Performance evaluation techniques can be classified in simulation techniques and analytical approaches. A distinction can be made between sequential and non-sequential techniques. Sequential techniques allow evaluation of the complete dynamic behaviour of decision making, taking into account temporal correlation in the input and state variables. Non-sequential techniques on the other hand ignore the time dependency of the external forcing inputs Y and the time correlations.

Simulation techniques, such as Monte Carlo simulation, simulate the actual process and random behaviour of the system. The uncertainty in terms of external forcing inputs Y is included in the sampling process, as values with a higher probability occur more frequently in the sample [10]. Simulation techniques allow to make conclusions about the distribution of the output variables and propagate the uncertainty from the input to the output variables.

[1]Next to the quantitative indicators, qualitative aspects need to be considered in a complete performance evaluation, such as data issues, ease of use and social acceptance, resulting in a multifaceted analysis.

Analytical techniques are typically based on a mathematical model resulting in a specific solution for a given input. Uncertainties can be included using stochastic models.

A. Sequential simulation

Sequential simulations allow to make conclusions about the distribution of performance indicators for time periods of length T, e.g. a year. A sample of $Y(0), \ldots, Y(T)$ with N realizations is generated, which is denoted by $y_n(t)$. For each realization, $x_{jn}(t) = f_j(x_{jn}(t-1), y_n(t))$ is recursively calculated with $x_{jn}(0) = x_0$. Performance indicators q_{ijn} can be evaluated per time period n of length T in the sample following equations 2 and 3. In order to obtain stationary distributions of the performance indicators, the first few iterations of the simulations including the transient behaviour of the dynamic decision making process should be dropped.

A sample should represent the variation between different time periods of length T in terms of uncertainties regarding load and wind forecasts and realizations and availabilities of system components. A sample can be generated based on historical time series of forecasts and realizations of load and wind and system component statuses or based on statistical models of load, wind power and failure and repair of system components [11]. However, the former is challenging due to non stationarities in the time series, while the latter is challenging due to correlations between the parameters in the multi-dimensional input parameter space.

The mean of the performance indicator and its confidence interval can be approximated as:

$$E[Q_{ij}] \approx \frac{1}{N} \sum_{n=1}^{N} q_{ijn} \pm t_\alpha \cdot \frac{s_D}{\sqrt{N}} \tag{4}$$

where $s_D = \sqrt{\frac{1}{N-1} \sum_{n=1}^{N} (q_{ijn} - \bar{q}_{ij})^2}$ is the sample standard deviation, $\bar{q}_{ij} = \frac{1}{N} \sum_{n=1}^{N} q_{ijn}$, and t_α is the α-percentile of the t-distribution.[2] Moreover, the (joint) marginal distribution of performance indicators $Q_{ij,traj.}(t)$ can be determined based on the simulations, which allows to verify whether a particular criterion performs better than another one at each time instant t in the period T.

B. Non-sequential simulation

Non-sequential simulation techniques allow to make conclusions about the (relative) performance of different reliability criteria at an average point in time and the uncertainty on this value. Evaluations are made for random snapshots. For each snapshot, the different decision stages in short term reliability management are simulated. Non-sequential simulations of short term reliability management can be represented as:

$$x_j = f_j(x_0, y_n) \quad \forall n \in N \tag{5}$$

[2]Alternatively, bootstrapping can be used in order to obtain asymmetric confidence intervals taking into account the asymmetry of the distribution of the performance indicators. However, uncertainty is typically underestimated with bootstrapping techniques.

where y_n is a realization of external forcing inputs in the sample and x_0 the initial conditions. Time dependence is omitted in these simulations. Mean and variance of the performance indicators Q_{ij} can be calculated similarly to Eq. 4.

The sample of N system states should represent the correlation between the input parameters and the distributions of the external forcing inputs. The latter is graphically illustrated in Fig. 3(a) and 3(d) for an example with a one dimensional external forcing input Y and constant initial conditions. The sample of external forcing inputs can be generated based on stochastic models, but this is challenging due to correlations between the parameters of the input space. Alternatively, samples can be randomly drawn from (historical) time series of the different parameters in the external forcing input space, however, non-stationarities in the time series make this challenging.

Each realization y_n in the sample has an equal probability $\frac{1}{N}$, if random sampling is applied. However, the performance of short term reliability management strategies might be strongly affected by a set of high impact contingencies that only occur with a low probability. The effect of these contingencies only becomes visible in the result after a large number of simulations. In order to reduce the number of simulations, importance sampling can be applied. By sampling based on a different distribution than the distribution of interest, highly impacting states of the external forcing input space appear more often in the sample. However, finding such an alternative distribution is typically challenging [12].

C. Emulation

An emulator is a statistical representation of a simulator and is typically developed using a Gaussian process or analogous Bayes linear theory based on a reduced number of simulations [13], [14]. It allows to determine uncertainties in model outputs arising from numerous sources of uncertainty, e.g. parametric uncertainty, condition uncertainty, functional uncertainty, stochastic uncertainty, etc. [15] Emulation is applied in other application contexts requiring highly complex models that are computationally intensive to simulate, such as transmission expansion planning [16], system generation planning [17], climate models or to predict the behaviour of nuclear power reactors [14].

The single step function f_j, representing the simulator of short term reliability management according to criterion j, is a deterministic function, which can be approximated by a function $\tilde{f}_j$:

$$X_j(t) = \tilde{f}_j(X_j(t-1), Y(t)) \tag{6}$$

The function $\tilde{f}_j$ is determined based on simulations for a training sample of external forcing inputs and system states (x_j, y). The training sample is a subspace of the input region of interest of the single step function f_j. Moreover, prior beliefs about the simulator, i.e. before the training data are considered, are taken into account. These prior beliefs are represented by the mean and covariance structures of the Gaussian process [14]. The emulation of a one-dimensional function based on a reduced number of simulations is graphically illustrated in Fig. 3(c).

Instead of simulating the exact function f_j for different external forcing inputs y and different previous system states x_j, the function $\tilde{f}_j$ can be evaluated directly in terms of y and x_j. This results in an approximate value of the non-sequential performance indicator $\tilde{Q}_{ij}$. The expected value of the approximate indicator can be calculated directly if the multivariate distribution $\Pi(x_j, y)$ is known, however this is rarely the case in practice as the multivariate distribution of the system states is hard to determine:

$$E[\tilde{Q}_{ij}] = \int_{X_j} \int_Y \Pi(x_j, y) \cdot g_i(\tilde{f}_j(x_j, y)) dX_j dY \tag{7}$$

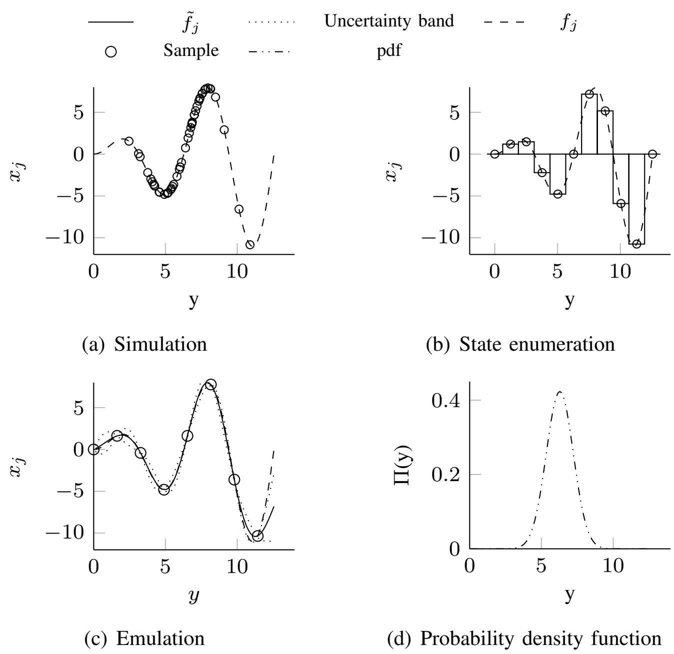
Alternatively, direct evaluations of the approximate function $\tilde{f}_j$ for a sample of external forcing inputs and system states that represents the multivariate distribution $\Pi(x_j, y)$ can be used.

Eq. 6 corresponds to the emulation of the single step function $x_j(t) = f_j(x_j(t-1), y(t))$. The emulation of this single step function can be used to construct an emulator for the dynamic simulator of the decision making process $(x_j(1), \ldots, x_j(T)) = f_j(x_0, y(1), \ldots, y(T))$. In this case, the full simulator output $(x_j(1), \ldots, x_j(T))$ is approximated by iteratively applying $x_j(t) = \tilde{f}_j(x_j(t-1), y(t))$, with $\tilde{f}_j$ the single step function in Eq. 6 for different time series of external forcing inputs and initial system states $(x_0, y(1), \ldots, y(T))$ within the input region of interest of the full simulator. The distribution of the sampled trajectories $(x_j(1), \ldots, x_j(T))$ needs to be verified in order to determine whether the applied training data for the single step emulator are adequate. If not, further runs of the single step function are required and the procedure needs to be repeated [14].

If an emulator of the dynamic simulator can be obtained, the calculation time can significantly be reduced compared to the sequential simulation approach, as time consuming simulations are replaced by analytical function evaluations. However, a challenge of emulation is the sampling of an appropriate training set. Short term reliability management is subject to a complex, highly dimensional parameter space of external forcing inputs, which might be hard to process in an emulation technique. A sufficiently high number of simulations is required in order to obtain a satisfactory approximation $\tilde{f}_j$, if the function f_j is highly variable. This aspect is difficult to verify without knowing the exact behaviour of the function. Moreover, high impact low probability events might not be well represented in the emulator if only a small number of system states is simulated. Therefore, it is important that the emulator is also trained to extreme events.

D. Analytical state enumeration

Analytical state enumeration (ASE) considers a prescribed set of combinations of external forcing inputs and initial conditions with probabilities assigned to them. The fact that the probability distribution of the initial state x_0 is not known analytically and is hard to determine in practice leads to a similar challenge as in non-sequential simulation. Therefore, initial conditions are typically assumed to be constant. The



(a) Simulation (b) State enumeration (c) Emulation (d) Probability density function

Fig. 3. Differences between non-sequential performance evaluation techniques and the applied probability density function for a one dimensional space of external forcing inputs Y and constant initial conditions x_0.

state space of external forcing inputs Y is divided in intervals Δy_m for which the function f_j is simulated at one point y_m in the interval. The function f_j is approximated by assigning the same function value $f_j(x_0, y_m)$ to all y within the interval Δy_m. The approach is graphically illustrated for a one-dimensional set Y and fixed initial conditions x_0 in Fig. 3(b). The accuracy of the results strongly depends on the number of intervals M and the sizing of the intervals Δy_m, which can be improved using appropriate snapshot selection techniques.

The probability of occurrence of a state in the interval Δy_m is calculated as:

$$\Pi(x_0, \Delta y_m) = \int_{\Delta y_m} \Pi(x_0, y) dy \tag{8}$$

The expected value of the performance indicator can be approximated as:

$$E[Q_{ij}] \approx \sum_{m=1}^{M} \Pi(x_0, \Delta y_m) \cdot f_j(x_0, y_m) \tag{9}$$

Applying state enumeration in a sequential context is challenging and would require the simulation of a prescribed set of time series of external forcing inputs. However, the set of all possible time series is hard to approximate with a reduced set of time series due to the many possible combinations of external forcing inputs at different time instants. Also the probability of occurrence of a certain time series is hard to obtain.

IV. Comparison and Discussion

An ideal performance evaluation technique should exploit the advantages of the existing techniques and overcome their drawbacks. A qualitative comparison of the different techniques for different aspects is summarized in table I.

TABLE I
QUALITATIVE COMPARISON OF DIFFERENT PERFORMANCE EVALUATION TECHNIQUES

	(1)	(2)	(3)	(4)	(5)	(6)
Seq. simulation	- -	-	++	+	++	- -
Non-seq. simulation	-	-	-	++	-	-
Seq. emulation	+	+	+	- -	+	+
Non-seq. emulation	++	++	-	-	- -	+
Analytical state enumeration	-	+	- -	++	- -	+

1: Time per simulation 4: Simplicity
2: Number of simulations 5: Suitability short term evaluation
3: Accuracy 6: Suitability long term evaluation
- -: very bad -: bad +: good ++: very good

Simulation techniques require a sufficiently large sample size in order to obtain a representative solution. Especially sequential techniques are challenged by this aspect, because parallellization is possible in sequential simulations in terms of the sample size N, but within the simulation of a time period of length T possibilities for parallel simulations are limited. Non-sequential simulation techniques allow for parallel simulations of single time instances rather than time periods, which reduces the computation time significantly, especially if a lot of computational power is available. Studies in the context of power system reliability assessment have already shown that similar results can be obtained in terms of energy not supplied and load curtailment if non-sequential simulations are used compared to sequential simulations [18]–[20]. However, the major drawback of non-sequential simulation techniques is that the dynamic characteristics of short term reliability management and time correlations are ignored. Pseudo-sequential approaches are developed that allow to determine interruption duration and interruption frequency indicators in a traditional reliability assessment [21]–[23], but the main problem with these techniques in the context of performance evaluation of short term reliability management is that they mainly focus on the final system states, while omitting the decision making trajectory.

Emulation and analytical state enumeration can give an indication of the change in performance based on a limited number of simulations. This makes them more applicable to assess the long term impact of using alternative reliability criteria, because detailed simulations of short term reliability management for multiple years are not suitable from a computational perspective. Emulation has an advantage compared to analytical state enumeration, namely that it allows to quantify uncertainty for all points which have not been evaluated [16], while state enumeration typically focuses on expected values of performance indicators [10]. ASE is easy to use on the other hand, but results are sensitive to the set of selected system states. A major drawback of analytical techniques is that simplifying assumptions and approximations need to be made due to the complex nature of short term reliability management [10]. This makes them hard to apply in highly-dimensional systems with a lot of uncertain parameters. Principal component analysis in a preprocessing step, which analyses the importance of different parameters in the parameter space, can (partly) overcome this issue. It allows to reduce the dimensions of the parameter space and to focus on the most influential parameters. Furthermore, it is challenging to consider the trajectory of reliability management in analytical approaches. Therefore, analytical techniques are less suitable than sequential simulation techniques to provide accurate conclusions in short term performance evaluation of RMACs.

A major challenge in the performance evaluation techniques is the appropriate choice of the initial state x_0. The initial state x_0 impacts the state vector x_j that is obtained and therefore the performance of the reliability management strategy. The multi-variate stationary distribution of the initial system conditions x_0 is unknown analytically and is hard to estimate in practice. Moreover, it might differ across reliability criteria. In practice, the initial state x_0 is typically assumed to be constant, however, an additional sensitivity analysis might be useful in order to verify the impact of the state x_0 on the performance evaluation.

V. Conclusion

Two types of techniques to evaluate performance of reliability management approaches and criteria can typically be distinguished: analytical techniques and simulation techniques, which can be sequential or non-sequential in nature. In order to accurately quantify the short term impact of using alternative short term reliability management approaches and criteria, the dynamic characteristics of the decision making behaviour of short term reliability management should be considered in the evaluation. Sequential simulations allow to evaluate both the final system states and the decision making trajectory, but purely sequential simulations are impractical from a computational perspective, especially in real systems with thousands of nodes. The reduced computation time of analytical techniques, such as emulation or state enumeration, comes with an increased complexity or a reduced accuracy.

Future work should focus on obtaining an accurate and computationally tractable performance evaluation technique. This is important, because an adequate performance evaluation is crucial to convince transmission system operators to move towards alternative approaches. Emulation looks promising from a theoretical point of view, but more in depth research is required in order to fully grasp the possibilities of this technique in the context of assessing the short term impact of alternative reliability criteria given the complex and dynamic simulation and its multi-dimensional input parameter space. Alternatively, looking for (pseudo)-sequential evaluation techniques that combine the accuracy of sequential simulations and the parallellization options of non-sequential techniques might be promising in order to move a step forward.

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