WAVES AND OSCILLATIONS IN MAGNETIC FIELDS

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ABSTRACT

This paper gives an overview of the theory of MHD waves in magnetic plasma configurations in the solar atmosphere. The emphasis is on basic properties that are independent of specific equilibrium models but are rather related to the intrinsic structuring and non-uniformity of the plasma. The discussion is confined to MHD waves in uniform and 1-d cylindrical equilibrium models of magnetic flux tubes with a straight magnetic field. These models contain sufficient physics for understanding basic properties of MHD waves and still allow for a relatively straightforward and transparent mathematical analysis.

1. INTRODUCTION

The magnetic field in the solar atmosphere is not uniformly distributed but organized in typical configurations: intense flux tubes and sunspots in the photosphere, magnetic loops and prominences in the corona, plumes in the solar wind. Theory predicts that each of these magnetic plasma configurations must support MHD waves and that MHD waves have important consequences for the solar atmosphere. Observations now show beyond any doubt that MHD waves are indeed ubiquitous in the solar atmosphere.

- Oscillations and waves have been observed in, around, above sunspots. In addition sunspots interact with global acoustic oscillations of sun, and sunspots have been identified as strong absorbers of acoustic flux (see e.g. Bogdan 2000, Muglach & O'Shea 2001).
- All solar prominences are oscillating with periods in the range from less than 1 minute to more than 1 hour (see e.g. Engvold 2001). Tentative identifications point to Alfvén waves and hybrid fast MHD waves. In addition to periods, there are also reports on damping times.
- Compressive propagating waves have been observed in polar plumes (DeForest & Gurman 1998) and interpreted as slow magnetoacoustic waves (Ofman et al. 1999).
- TRACE registered decaying oscillating displacements of hot coronal loops on 14th July 1998 (Aschwanden et al. 1999, Nakariakov et al. 1999, see also Nakariakov 2000, 2001). These oscillations were very probably generated by a flare which went off shortly before the loops started oscillating. New examples of coronal loop oscillations have been discovered in TRACE data since then. The oscillations undergo strong damping. They are interpreted as fast magneto-acoustic kink mode oscillations.
- Compressive propagating disturbances have been observed in coronal loops with SOHO/EIT (Berghmans & Clette 1999) and with TRACE (Berghmans et al. 1999, De Moortel et al. 2000ab) and interpreted as slow magnetoacoustic waves (Nakariakov et al. 2000).
- MHD waves and oscillations have been observed from the low chromosphere to the transition region (see e.g. Banerjee et al. 2001).

The interested reader should consult recent reviews on the same subject by Roberts (2000, 2001).

2. MHD EQUATIONS

MHD waves and oscillations are studied by use of the equations of Magnetohydrodynamics (MHD). The original full set of non-linear time dependent partial differential equations of MHD are:

$$\begin{aligned}
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \vec{v}), \\
\frac{dp}{dt} &= \frac{\gamma p}{\rho}\frac{d\rho}{dt} + \text{gain and loss terms}, \\
\rho \frac{d\vec{v}}{dt} &= -\nabla p + \frac{1}{\mu}(\nabla \times \vec{B}) \times \vec{B} + \text{viscous forces}, \\
\frac{\partial \vec{B}}{\partial t} &= \nabla \times (\vec{v} \times \vec{B}) + \eta \nabla^2 \vec{B}.
\end{aligned} \tag{1}$$

In these equations ρ is density, p is plasma pressure, $\vec{v}$ is velocity and $\vec{B}$ is the magnetic field; η is the coefficient of magnetic diffusivity and γ is the ratio of specific heats.

3. LINEAR THEORY OF WAVES AND OSCILLATIONS

3.1. Basic Equations

MHD waves are often studied by using linear theory of wave motions superimposed on a static or stationary background. The various physical quantities are written as a sum of a time independent background value f_0 and a time varying part f_1 as

$$f(\vec{r},t) = f_0(\vec{r}) + f_1(\vec{r},t), \qquad \frac{\mid f_1 \mid}{\mid f_0 \mid} \ll 1. \qquad (2)$$

f_1 is the Eulerian perturbation of f. Linear theory is applicable if $|f_1| \ll |f_0|$. When the background is static, the condition for velocity is $|\vec{v}| \ll |\max(v_A, v_S)|$. The full set of non-linear partial differential equations of MHD is then reduced to a set of linear partial differential equations. This set of equations can be studied as an initial value problem, a boundary value problem, or an eigenvalue problem. Here, we concentrate on the eigenvalue problem of linear waves and oscillations and put the perturbed quantities proportional to

$$\exp(-i\omega t). \qquad (3)$$

3.2. Uniform plasma of infinite extent

The fundamental frequencies for MHD waves already show up in the study of MHD waves in a uniform plasma of infinite extent so that we do not have to worry about boundary conditions. Follow Goedbloed (1983) and consider MHD waves in a uniform plasma with wave vectors $\vec{k} = (k_x, k_y, k_z)^t$ where the z-direction is along the constant equilibrium magnetic field B_z. Anticipate that for a more general basic state the x-direction will be the direction normal to the magnetic surfaces and that the y-direction is the direction in the magnetic surfaces perpendicular to the magnetic field lines. Since the background state is uniform, the equations that describe the linear motions have constant coefficients and the perturbed quantities can be put proportional to

$$\exp(ik_x x + ik_y y + ik_z z).$$

For a uniform plasma the MHD waves can be nicely separated in Alfvén waves and magnetosonic waves. Their dispersion relations are

$$\omega_A^2 = k_z^2 v_A^2, \qquad (4)$$

$$\omega_{sl,f}^2 = \frac{k^2(v_S^2+v_A^2)}{2}\left(1 \pm \sqrt{1 - \frac{4\omega_C^2}{k^2(v_S^2+v_A^2)}}\right), \qquad (5)$$

where

$$\omega_C^2 = \frac{v_S^2}{v_S^2+v_A^2}\omega_A^2, \qquad (6)$$

$$v_A^2 = \frac{B^2}{\mu\rho}, \qquad v_S^2 = \frac{\gamma p}{\rho}, \qquad k^2 = k_x^2 + k_y^2 + k_z^2.$$

v_A and v_S are the Alfvén velocity and the velocity of sound respectively. ω_A and ω_C are the Alfvén frequency and the cusp frequency. $\omega_{sl,f}$ denote the frequencies of the slow and fast magnetosonic waves respectively.

The spectrum as a function of k_x is then characterized by the following facts (see Fig. 1):

- The point eigenvalue $\omega_A^2 = k_z^2 v_A^2$ of the Alfvén point spectrum is infinitely degenerate.
- The slow point eigenvalues have an accumulation frequency ω_C and a cut-off frequency ω_I:
$$\lim_{k_x\to\infty} \omega_{sl}^2 = \omega_C^2 \le \omega_{sl}^2 \le \omega_{sl}^2(k_x=0) = \omega_I^2.$$
- The point eigenvalues of the fast waves have a cut-off frequency ω_{II} and an accumulation frequency at ∞:
$$\omega_f^2(k_x=0) = \omega_{II}^2 \le \omega_f^2 \le \lim_{k_x\to\infty} \omega_f^2 = \infty.$$
- Hence, there are four characteristic frequencies and three well separated types of MHD waves:
$$\omega_C^2 \le \omega_{sl}^2 \le \omega_I^2 \le \omega_A^2 \le \omega_{II}^2 \le \omega_f^2 \le \infty.$$
- In a pressureless plasma with $\beta = 0,\ v_S^2 = 0$, the slow waves are absent and the fast waves are driven by magnetic pressure:
$$\omega_C = 0, \qquad \omega_I = 0, \qquad \omega_{sl} = 0,$$
$$\omega_f^2 = k^2 v_A^2, \qquad \omega_{II} = (k_y^2+k_z^2)v_A^2 \ge \omega_A^2.$$

3.3. Non-uniform plasmas

In a non-uniform plasma the equilibrium values of density $\rho(x)$, pressure $p(x)$, magnetic field $B_z(x)$, the Alfvén velocity $v_A(x)$ and speed of sound $v_S(x)$ are functions of position. Consequently the four characteristic frequencies found for a uniform plasma of infinite extent

$$\omega_C(x),\ \omega_I(x),\ \omega_A(x),\ \omega_{II}(x)$$

are functions of position and map out four intervals of frequencies. The dispersion relations for Alfvén waves and slow waves (with $k_x \to \infty$) are locally satisfied on each magnetic surface. In ideal MHD each individual magnetic surface can oscillate at its own local Alfvén frequency or local slow frequency without interaction with neighboring magnetic surfaces. Non-uniformity creates a range of local Alfvén frequencies and a range of local cusp frequencies known as the Alfvén continuum and the cusp continuum respectively. In ideal MHD these local

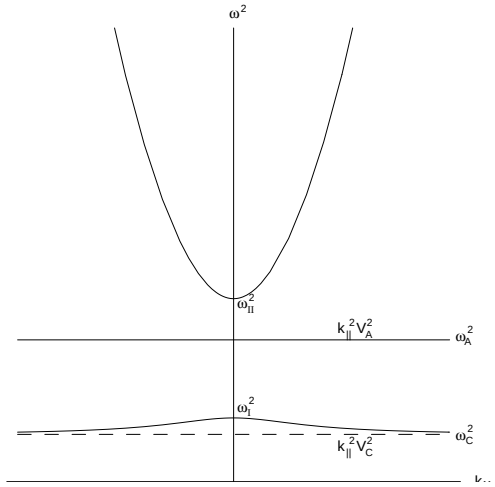

Fig. 1. Variation of ω^2 as function of k_x for fixed k_z and fixed $k_y \neq 0$.

Alfvén waves and local cusp waves are confined to the magnetic surfaces on which their dispersion relations are satisfied locally. Dissipative effects produce coupling to the neighboring surfaces, but the local Alfvén/cusp waves still have steep gradients across the magnetic surfaces. Because of these steep gradients, excitation of these local Alfvén/cusp continuum waves provides a means for dissipating wave energy which is far more efficient in weakly dissipative plasmas than classic resistive or viscous MHD wave damping in a uniform plasma (see e.g. Poedts 2002).

In addition to the Alfvén continuum waves and the cusp continuum waves, a non-uniform plasma can support discrete slow and fast magnetosonic modes and discrete Alfvén modes. The inequalities for the four characteristic frequencies found for a uniform plasma, still hold locally. However the intervals defined by these four characteristic frequencies can (partially) overlap leading to MHD waves with mixed character and wave transformation.

4. MHD WAVES IN 1-D MODELS OF FLUX TUBES

We now turn to the MHD waves in magnetic flux tubes. The flux tube is considered to be in static equilibrium and on this static background we superimpose linear compressible motions. The flux tube is idealized as a 1-dimensional cylindrically symmetric column of plasma. This is an obvious simplification of reality. However, it is necessary to have a clear understanding of MHD waves in these relatively simple equilibrium models before we can hope to understand what is happening in more complicated equilibrium models. Moreover, it turns out that results for these simple 1-dimensional models can actually be used for understanding the observations. A system of cylindrical coordinates r, φ, z with the z-axis coinciding with the axis of symmetry is used. The equilibrium quantities, magnetic field $\vec{B} = (0, B_\varphi(r), B_z(r))$, pressure $p(r)$ and density $\rho(r)$ are functions of the radial coordinate only and satisfy the radial force balance equation

$$\frac{d}{dr}(p + \frac{B^2}{2\mu}) = -\frac{B_\varphi^2}{\mu r} \ .$$

For what follows it is important to note that the magnetic surfaces are concentric cylinders : $r =$ constant.

Since the equilibrium quantities depend on r only, the perturbed quantities can be Fourier-analyzed with respect to the ignorable coordinates φ, z and put proportional to

$$\exp(i(m\varphi + k_z z)).$$

Here m (an integer) and k_z are the azimuthal and axial wave numbers. Special names have been given to the waves that have their azimuthal wave numbers equal to 0 or 1. Waves with $m = 0$ are called slow/fast sausage modes and torsional Alfvén waves respectively. For $m = 0$ the axis of the tube remains undisturbed. For $m = 1$ the waves are called kink modes. These waves involve lateral displacements of the tube; they maintain a circular cross section with the axis of the tube resembling a wriggling snake. Only the kink waves displace the central axis of the vibrating tube.

In ideal MHD the equations that govern the linear motions superimposed on a static 1-dimensional cylindrical equilibrium model can be reduced to two first order ordinary differential equations for the radial component of the Lagrangian displacement ξ_r and the Eulerian perturbation of total pressure P' (see Appert et al. 1974). The remaining perturbed quantities can be computed once ξ_r and P' are known. For a straight magnetic field,

$$\vec{B} = B(r)\vec{1}_z,$$

the differential equations for ξ_r and P', originally derived by Appert et al., take the following form:

$$\begin{aligned}
D\frac{d(r\xi_r)}{dr} &= -C_2 r P', \\
\frac{dP'}{dr} &= \rho(\omega^2 - \omega_A^2)\xi_r, \\
\rho(\omega^2 - \omega_A^2)\xi_\varphi &= \frac{im}{r}P', \\
\rho(\omega^2 - \omega_C^2)\xi_z &= ik_z\frac{v_S^2}{v_S^2 + v_A^2}P'. \qquad (7)
\end{aligned}$$

The equations relating ξ_φ and ξ_z to ξ_r and P' are also given, as they play an important role in the coupling of different wave motions. The original formulation of Appert et al. involved additional coefficient functions C_1 and C_3. For a straight equilibrium magnetic field with $B_\varphi = 0$ the coefficient C_1 equals zero and $C_3 = \rho(\omega^2 - \omega_A^2)D$. For a straight field the coefficient functions D and C_2 take the form:

$$\begin{aligned}
D &= \rho(c^2 + v_A^2)(\omega^2 - \omega_A^2)(\omega^2 - \omega_C^2), \\
C_2 &= \omega^4 - (c^2 + v_A^2)(\omega^2 - \omega_C^2)(m^2/r^2 + k_z^2) \\
&= (\omega^2 - k_z^2 v_S^2)(\omega^2 - \omega_A^2) \\
&\quad -\frac{m^2}{r^2}(v_S^2 + v_A^2)(\omega^2 - \omega_C^2) \\
&= (\omega^2 - \omega_I^2)(\omega^2 - \omega_{II}^2) \qquad (8)
\end{aligned}$$

Since the equilibrium magnetic field is straight $\vec{B} = B(r)\vec{1}_z$, it follows that the φ-direction and the z-direction are two perpendicular directions in the magnetic surfaces, respectively perpendicular and parallel to the magnetic field lines. The r-direction is normal to the magnetic surfaces. It then follows that ξ_φ (A), ξ_r (F) and ξ_z (S) is the relevant component for respectively Alfvén waves, fast waves and slow waves.

Eqs. (7ab) have regular singular points at the positions where $D = 0$ or consequently at the resonant positions r_A and r_C where $\omega = \omega_A(r_A)$ and $\omega = \omega_C(r_C)$. These singularities and the fact that $\omega_A(r)$ and $\omega_C(r)$ are functions of position, give rise to two continuous ranges in the spectrum which are associated with resonant waves with singular spatial solutions in ideal MHD. The two continuous ranges in the spectrum are classically referred to as the Alfvén continuum and the slow (or cusp) continuum. In ideal MHD the solutions for $P', \xi_r, \xi_\varphi, \xi_z$ behave close to the resonant position ($s = r - r_{A,C}$) as (see e.g. Goedbloed 1975, 1983, 1998; Sakurai et al. 1991):

$$\begin{aligned} P' \ \& \ \xi_z &: \text{constant}, \\ \xi_r &: \text{logarithmic singularity and a jump}, \\ \xi_\varphi &: 1/s - \text{singularity and } \delta(s) \text{ contribution} \end{aligned}$$

for the resonant Alfvén waves, and

$$\begin{aligned} P' \ \& \ \xi_\varphi &: \text{constant}, \\ \xi_r &: \text{logarithmic singularity and a jump}, \\ \xi_z &: 1/s - \text{singularity and } \delta(s) \text{ contribution} \end{aligned}$$

for the resonant slow waves. Hence, the dominant dynamics resides in the components in the magnetic surfaces, respectively perpendicular to the magnetic field lines for resonant Alfvén waves and parallel to them for the resonant slow/cusp waves. These continuum waves imply that in ideal MHD each magnetic surface can oscillates at its own Alfvén (slow) continuum frequency. This is the physical mechanism behind phase mixing and resonant absorption (Ionson 1978; Heyvaerts & Priest 1983; Poedts et al. 1989,1990; Goossens 1991, 1994; Goossens & De Groof 2001a,b). In dissipative MHD the singular solutions are replaced with large but finite solutions (see Goossens et al. 1995, Tirry & Goossens 1996)

Let us now go back to Eqs. (7) and see what happens when the azimuthal wave number m is equal to zero. In planar geometry this choice corresponds to $k_y = 0$. This case $m = 0$ is popular because it simplifies the mathematics. However, it should be remembered that it is one of the infinitely many choices for m and a choice which leads to very particular results. From Eqs. (7) it follows that the equation for ξ_φ is decoupled from the other equations. This means that for $m = 0$ the eigenmodes are decoupled into

• Torsional Alfvén continuum eigenmodes:

$$\xi_r = 0, \ P' = 0, \ \xi_z = 0, \ \xi_\varphi \neq 0. \qquad (9)$$

• Discrete / continuum eigenmodes:

$$\xi_r \neq 0, \ P' \neq 0, \ \xi_z \neq 0, \ \xi_\varphi = 0. \qquad (10)$$

The Alfvén waves do not interact with the sausage magnetosonic waves. The (real) eigenvalues of the fast magnetosonic waves can lie in the Alfvén continuum, but there is no interaction and no coupling between the discrete fast wave and the local continuum Alfvén wave. However, for $m \neq 0$ the equations for $\xi_r, \ P', \ \xi_z, \ \xi_\varphi$ are coupled and pure Alfvén waves and pure magnetosonic waves do not exist. The eigenmodes now have all perturbation quantities different from zero:

$$\xi_r \neq 0, \ P' \neq 0, \ \xi_z \neq 0, \ \xi_\varphi \neq 0. \qquad (11)$$

An important consequence of this behaviour is that fast discrete eigenmodes with an eigenfrequency in the Alfvén continuum couple to a local Alfvén continuum eigenmode and produce the famous quasi-modes. These quasi-modes are the natural oscillation modes of system (Balet et al. 1982, Steinolfson & Davila 1993). They combine the properties of a localized resonant Alfvén wave and of a global fast eigenoscillation. Quasi-modes are damped due to resonant coupling for a static equilibrium and the damping is independent of dissipation for small dissipation (Poedts & Kerner 1992; Tirry & Goossens 1996). Asymptotic analysis in dissipative MHD allows to compute jump relations of the quasi-modes over the resonant position. For a resonance with an Alfvén wave these jump relations are

$$\begin{aligned} [P] &= 0, \qquad [\xi_r] = -i\pi \ \mathrm{sgn}(\Omega) \frac{m^2 P'}{\rho |\Delta|}, \\ [F] &= -\frac{\pi |\Omega| m^2}{2\rho |\Delta|} |P'|^2. \end{aligned} \qquad (12)$$

where $[F]$ is the jump in the energy flux. This jump clearly shows that in a static equilibrium the global eigenmode is damped because of a transfer of energy to local continuum modes. Quasi-modes play an important role in MHD wave heating scenarios (Poedts et al. 1990; Wright & Rickard 1995; Ofman et al. 1995; De Groof & Goossens 2000, 2002; Goossens & De Groof 2001). In the presence of an equilibrium flow these quasi-modes can become overstable (Andries et al. 2000; Andries & Goossens 2001a,b).

The first point that this paper aims to make is that MHD waves with $m = 0$ are very particular in the sense that only for $m = 0$ the Alfvén waves and the magnetosonic waves are decoupled. For all other values of m there is a natural and unavoidable interaction between Alfvén waves and the magnetosonic waves. The second point is that, because of this interaction between Alfvén waves and the magnetosonic waves, a non-uniform magnetic plasma supports quasi-modes. These quasi-modes are the natural oscillation modes of the system and very hard to avoid.

5. DISCRETE MODES IN CYLINDRICAL FLUX TUBES

Let us now look at the discrete eigenmodes of the 1-dimensional cylindrical configuration. More than four

decades ago the linear motions of a 1-dimensional cylindrical plasma were described by a second order differential equation for ξ_r. This is the famous Hain-Lust equation (Hain & Lust 1958). In solar physics applications the set of two differential Eqs. (7ab) is rewritten as a second order differential equation for P' and an equation that relates ξ_r to P':

$$D\frac{d}{dr}\left[\frac{r}{\rho(\omega^2-\omega_A^2)}\frac{dP'}{dr}\right]+rC_2P'=0, \quad (13)$$

$$\xi_r=\frac{1}{\rho(\omega^2-\omega_A^2)}\frac{dP'}{dr}. \quad (14)$$

The reason for using the second order differential equation for P' rather than that for ξ_r is that for a uniform plasma column the solutions for P' can be obtained in terms of Bessel or Hankel functions while the solutions for ξ_r involve derivatives of Bessel or Hankel functions. For a uniform plasma Eq. (13) can be written as

$$\frac{d^2P'}{dr^2}+\frac{1}{r}\frac{dP'}{dr}+\left[\frac{(\omega^2-k_z^2v_S^2)(\omega^2-\omega_A^2)}{(v_S^2+v_A^2)(\omega^2-\omega_C^2)}-\frac{m^2}{r^2}\right]P'=0.$$

where

$$k_\perp^2=\pm\frac{(\omega^2-k_z^2v_S^2)(\omega^2-\omega_A^2)}{(v_S^2+v_A^2)(\omega^2-\omega_C^2)}.$$

The solutions to this equation can be obtained in terms of Bessel / Hankel functions:

$$I_m(k_\perp r), J_m(k_\perp r), K_m(k_\perp r), H_m^1(k_\perp r), H_m^2(k_\perp r).$$

The MHD waves can have a variety of spatial behavior. The radius of the loop is denoted as R. In the interior of the loop $0 \le r \le R$, the MHD waves can be either surface waves or body waves (see Fig. 2). A surface wave is a wave which decreases in amplitude with distance from the tube's surface, a body wave is a wave which decreases with distance from the surface externally but not internally. It is supported by the tube as a whole and not just by the discontinuity at the boundary (Cally 1986). In addition, for $m \neq 0$ the discrete magnetosonic waves can couple to continuum waves and produce resonantly damped quasi-modes. In the latter case ω and $k_\perp$ are complex. Exterior to the loop ($r \ge R$) the MHD waves can be non-leaky waves, or leaky waves which are damped due to outgoing MHD radiation. A leaky wave is characterized by an external solution which carries energy away from the tube. For leaky waves ω and $k_\perp$ are complex. This lead Stenuit et al. (1998, 1999) to distinguish four types of wave modes: non-leaky & non-resonant (NL/NR), leaky & non-resonant (L/NR), non-leaky & resonant (NL/R), leaky & resonant (L/R). In order to have resonant waves, non-uniformity in the equilibrium state is required.

The dispersion relation is obtained by using the conditions that the normal component of the displacement and the Lagrangian perturbation of total pressure have to be continuous at the boundary of the tube. Usually, these conditions can be combined into:

$$\frac{P_i'(R)}{\xi_{r,i}(R)}=\frac{P_e'(R)}{\xi_{r,e}(R)} \quad (15)$$

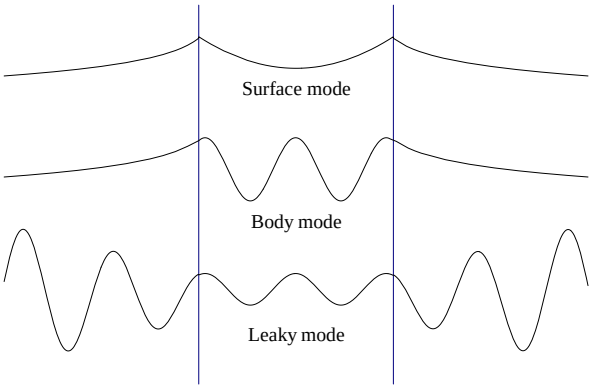

Fig. 2. Different types of waves.

(quantities related to the interior have a subscript i, those related to the exterior have subscript e).

Before we proceed with a discussion of specific results, it is instructive to recall that two distinct types of tubes arise frequently in solar atmosphere: the isolated photospheric tube and the embedded coronal tube. In the isolated tube, the magnetic field is confined by an external plasma pressure. The plasma around the tube is field-free or has possible a weak field. The isolated photospheric tube is partially evacuated so that its plasma density is reduced below that of its surroundings. On the other hand, the embedded coronal flux tube is not distinct because of the magnetic field which may be uniform. Instead the embedded tube is distinguished by density, with a region of high density defining the embedded tube. Hence

$$B_i >> B_e,\ \rho_i < \rho_e,\ v_{A,i} > v_{A,e} \quad (16)$$

for the isolated photospheric tube, and

$$B_i \approx B_e,\ \rho_i > \rho_e,\ v_{A,i} < v_{A,e} \quad (17)$$

for the embedded coronal flux tube.

6. NON-LEAKY DISCRETE MODES OF UNIFORM CYLINDRICAL FLUX TUBES

Let us now turn to the non-leaky discrete modes of uniform cylindrical flux tubes. A complete discussion of these modes can be found in Edwin & Roberts (1983). Since there is no damping for these modes, the frequencies ω and wave numbers $k_\perp$ are real. Edwin & Roberts report dispersion relations for surface waves and body waves for isolated photospheric tubes and for embedded coronal tubes. The dispersion relations are given in terms of $I_m,\ K_m,\ J_m$ and their derivatives and involve real expressions only. They read:

surface waves:

$$\frac{I_m(k_{\perp,i}R)}{k_{\perp,i}I_m'(k_{\perp,i}R)\rho_i(\omega^2-\omega_{A,i}^2)}=\frac{K_m(k_{\perp,e}R)}{k_{\perp,e}K_m'(k_{\perp,e}R)\rho_i(\omega^2-\omega_{A,e}^2)}, \quad (18)$$

body waves:

$$\frac{J_m(k_{\perp,i}R)}{k_{\perp,i}J'_m(k_{\perp,i}R)\rho_i(\omega^2-\omega^2_{A,i})} = \frac{K_m(k_{\perp,e}R)}{k_{\perp,e}K'_m(k_{\perp,e}R)\rho_i(\omega^2-\omega^2_{A,e})}. \quad (19)$$

Solutions to these dispersion relations are given by Edwin & Roberts for uniform photospheric flux tubes in their Fig. 3 and for uniform coronal flux tubes in their Fig. 4 both for sausage ($m = 0$) waves and kink ($m = 1$) waves. The solutions are given as ω/k_z in function of k_zR in our notation. An important point that this paper aims to make is that quasi-modes are ubiquitous when the flux tube is allowed to be non-uniform. In that respect it is important to observe that for the uniform photospheric flux tube, the frequencies ω of all modes (without any exception) given on their Fig. 3 satisfy

$$v_{A,e} < \omega/k < v_{A,i}$$

and similarly, that for a uniform coronal flux tube (their Fig. 4), the frequencies of all the fast body waves satisfy

$$v_{A,i} < \omega/k < v_{A,e}.$$

This means that, when the discontinuous transition from the constant value $v_{A,i}$ to the constant value $v_{A,e}$ is replaced with a continuous variation, these discrete eigenmodes will have their frequency in the Alfvén continuum. All modes with $m \neq 0$ with a frequency the Alfvén continuum are damped quasi-modes. In particular, let us have a look at the fundamental kink ($m = 1$) mode. Take the limit $x_i = k_{\perp,i}R << 1$ & $x_e = k_{\perp,e}R << 1$ and find the classic result

$$\omega^2_{kink} \approx \frac{\rho_i\,\omega^2_{A,i} + \rho_e\,\omega^2_{A,e}}{\rho_i + \rho_e}. \quad (20)$$

Hence, when the discontinuous transition from $v_{A,i}$ to $v_{A,e}$ is replaced with a continuous variation, the fundamental kink mode has its frequency in the Alfvén continuum. The obvious conclusion is that the classic kink mode is always a resonantly damped quasi-mode!

7. DISCRETE MODES OF NON-UNIFORM CYLINDRICAL FLUX TUBES

Let us now further elaborate on the findings at the end of the previous section for non-uniform coronal flux tubes. The focus is on non-leaky waves which might be non-resonant or resonant. Our interest goes to the resonant waves. Leaky waves are not very popular in studies of coronal flux tubes. They are discarded on the basis of the argument that MHD waves in a very low β-plasma, cannot propagate in the external medium as $v_{A,i} < v_{A,e}$. However, Cally (1986) notes that the "fast" body waves, which are found in the coronal loop models do not have a low wave-number cut-off, they merely transform to leaky waves at large wavelengths, when their phase speeds exceed the external Alfvén speed. Apart, nobody has looked into leaky waves for coronal tubes. In order to illustrate the ubiquitous presence, we follow Tirry & Goossens (1996). They studied MHD waves in a non-uniform pressureless ($\beta = 0$) tube with

$$\rho(r) = \rho_0 \exp(-(r/R)^4), \;\; B_z = \text{constant}.$$

The variation in density leads to a variation in Alfvén speed and to an Alfvén continuum for every value of k_z. Inspired by Wright and Rickard (1995), Tirry and Goossens first computed the spectrum of fast discrete modes for $m = 0$. The results are shown on Fig. 3. The eigenfrequencies of the first three fast body modes (dotted lines) are plotted as a function of k_z. The full lines are the lower and upper bound of the Alfvén continuum. There are several fast modes with frequencies in the Alfvén continuum. However, since $m = 0$ these modes do not couple to the Alfvén continuum modes. In order to see what happens to these fast modes in the Alfvén continuum when $m \neq 0$, Tirry and Goossens computed the eigenmodes in resistive MHD using the analytical solutions in terms of the $\tilde{F}$ and $\tilde{G}$ functions. They let m vary from 0 to 1 in a continuous manner. The results of their computations are shown on Fig. 4. The variation of the real and the imaginary parts of the eigenvalue are shown when m changes from $m = 0$ to $m = 1$ for four values of the magnetic Reynolds number. Of course, only the end points of the curves have a physical meaning since m is an integer. Fig. 4 clearly shows that the fast mode gets damped and that the damping becomes independent of dissipation for small dissipation.

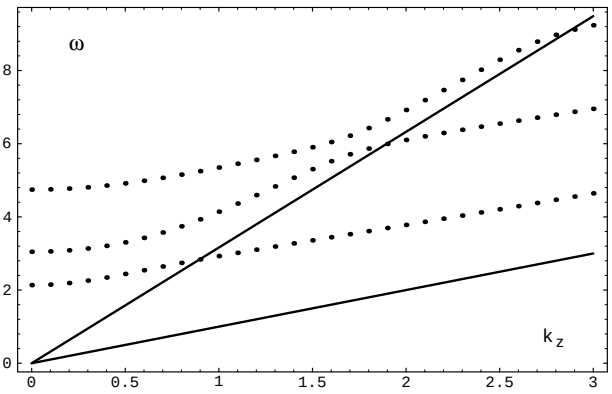

Fig. 3. Fast modes in Alfvén continuum, $m = 0$.

The important conclusion is that resonantly damped quasi-modes emerge naturally when non-uniformity is allowed for and that their damping is independent of dissipation. A similar conclusion was reached by Poedts & Kerner (1991). These authors studied the stabilization of the external kink mode instability in a plasma-vacuum system by shifting the wall towards the plasma column. They found that once the kink mode becomes stabilized, its frequency shifts into the Alfvén continuum producing a damped quasi-mode with its damping independent of dissipation.

Let us focus on the fundamental kink mode. The approximate expression for its frequency was used by Nakariakov (2000) to estimate B in oscillating coronal loops. He found the value of the magnetic field to be between 10 and 30 G. The fundamental kink mode has its frequency

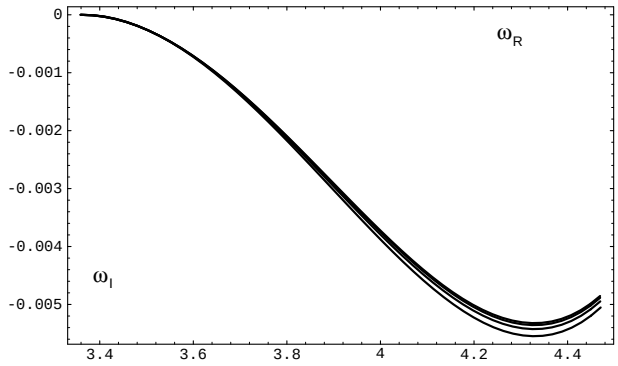

Fig. 4. Fundamental fast mode in Alfvén continuum when m=0 $\rightarrow$ m=1 for 4 values of R_m : $10^7, 10^8, 10^9, 10^{10}$.

in the Alfvén continuum. Since observations of oscillations in coronal loops and also in prominences yield

Reynolds numbers in order to explain the observations. Finally, the models used for explaining and interpreting the observations, especially those for coronal loop oscillations, are very simple. There is an obvious need to compute MHD wave spectra for non-uniform (even 1-d) models where the non-uniformity is not confined to a "thin" layer.

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