BASIC REACTOR PHYSICS PROBLEMS IN FLUID-FUEL RECIRCULATED REACTORS

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Abstract - The basic problems of the neutron physics of fluid-fuel multiplying systems are introduced in slug flow conditions and in the presence of delayed emissions from fission. The critical problem is presented and solved. The importance balance and the adjoint equations are then derived and the consistent point kinetics model is formulated. A few selected results are presented and discussed.

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1. Introduction

The physics of systems fuelled by a fluid neutron multiplying material constitutes a very interesting chapter in nuclear engineering. The interest in these systems is due to recent proposals for actinide and long-lived radioactive nuclide transmutation [1].

The physics of fluid-fuel systems is largely different from that of standard reactors. Therefore, it seems worthwhile to take into consideration some basic reactor physics problems, to get some insight into the physical problem. To that purpose, analytical solutions are particularly suitable.

Historically, fluid core systems were studied in the early stage of nuclear engineering [2,3] on a very simplified manner. More recently, numerical models have been proposed [4] and applied to assess some engineering aspects of subcritical systems [5].

The present work considers some aspects of fluid-core systems in presence of a slug-flow motion. In particular, after a discussion of the physical model, the criticality theory is presented. Results show the effect of delayed emissions from fission in conjunction with the fuel motion. Afterwards, the importance theory is outlined and its possible use for the generation of consistent *kinetic parameters* is examined in the framework of a standard point kinetics model.

2. Direct and adjoint problems in two-direction neutron transport

The simplest physical model to describe neutron transport in a fluid-fuel recirculating reactor can be constructed in a two discrete direction approach. In slab one-dimensional geometry and stationary conditions, this is equivalent to diffusion theory [6]. In the following, isotropic emissions and a materially homogeneous slab configuration are further assumed. Consequently, the balance equations for one-velocity neutrons and one-family delayed precursors read:

$$\begin{cases} \dfrac{1}{v}\dfrac{\partial\varphi(x,t)}{\partial t} + \mu\dfrac{\partial\varphi(x,t)}{\partial x} + \sigma\varphi(x,t) = \dfrac{\nu\sigma_f(1-\beta)+\sigma_s}{2}[\varphi(x,t)+\psi(x,t)] + \dfrac{1}{2}\lambda C(x,t) + \dfrac{1}{2}S(x,t) \\ \dfrac{1}{v}\dfrac{\partial\psi(x,t)}{\partial t} - \mu\dfrac{\partial\psi(x,t)}{\partial x} + \sigma\psi(x,t) = \dfrac{\nu\sigma_f(1-\beta)+\sigma_s}{2}[\varphi(x,t)+\psi(x,t)] + \dfrac{1}{2}\lambda C(x,t) + \dfrac{1}{2}S(x,t) \\ \dfrac{\partial C(x,t)}{\partial t} + u\dfrac{\partial C(x,t)}{\partial x} + \lambda C(x,t) = \beta\nu\sigma_f\ [\varphi(x,t)+\psi(x,t)] \end{cases} \tag{1}$$

or, in compact matrix form:

$$\hat{V}\mathbf{X} + \hat{\Theta}\mathbf{X} = \hat{F}\mathbf{X} + \mathbf{S} \tag{2}$$

where the following operators are introduced:

$$\hat{\Theta} = \begin{pmatrix} \mu & 0 & 0 \\ 0 & -\mu & 0 \\ 0 & 0 & u \end{pmatrix} \frac{\partial}{\partial x} + \begin{pmatrix} \sigma - \frac{\sigma_s}{2} & -\frac{\sigma_s}{2} & -\frac{\lambda}{2} \\ -\frac{\sigma_s}{2} & \sigma - \frac{\sigma_s}{2} & -\frac{\lambda}{2} \\ 0 & 0 & \lambda \end{pmatrix} \equiv \hat{T}\frac{\partial}{\partial x} + \hat{\Sigma} \tag{3}$$

$$\hat{V} = \begin{pmatrix} 1/v & 0 & 0 \\ 0 & 1/v & 0 \\ 0 & 0 & 1 \end{pmatrix} \frac{\partial}{\partial t} \qquad \hat{F} = \begin{pmatrix} \frac{1}{2}\nu\sigma_f(1-\beta) & \frac{1}{2}\nu\sigma_f(1-\beta) & 0 \\ \frac{1}{2}\nu\sigma_f(1-\beta) & \frac{1}{2}\nu\sigma_f(1-\beta) & 0 \\ \beta\nu\sigma_f & \beta\nu\sigma_f & 0 \end{pmatrix}$$

and

$$\mathbf{X} = \begin{pmatrix} \varphi(x,t) \\ \psi(x,t) \\ C(x,t) \end{pmatrix} \qquad \mathbf{S} = \begin{pmatrix} \frac{1}{2}S(x,t) \\ \frac{1}{2}S(x,t) \\ 0 \end{pmatrix} \tag{4}$$

are the (unknown) state and (known) source vectors, respectively. The direction parameter μ, assumed to be positive, identifies the directions of neutrons and u is the slug velocity of the multiplying material. φ and ψ are the angular fluxes in the μ and $-\mu$ directions, respectively. It is worth observing that also the balance equation for precursors assumes the typical structure of a transport equation, where streaming takes place only along the x-axis at velocity u. The other symbols appearing in Eq. (1) have their standard meaning in reactor physics.

Standard vacuum boundary conditions for neutrons are used, while for delayed precursors it is required that their number entering the reactor at $x = 0$ be equal to the fraction $\exp(-\lambda\tau)$ of those exiting the opposite boundary of the reactor at $x = a$, namely:

$$\varphi(0,t) = 0, \qquad \psi(a,t) = 0, \qquad C(0,t) = C(a,t-\tau)e^{-\lambda\tau} \equiv C(a,t-\tau)\xi \tag{5}$$

The factor $\xi = \exp(-\lambda\tau)$ represents the fraction of precursors that remains undecayed after transit time τ in the external circuit.

The time-dependent problem (1) requires the specification of proper initial conditions, assigning the initial neutron distribution, $\varphi(x,0)$ and $\psi(x,0)$, as well as the initial precursor density, $C(x,0)$.

The multiplication eigenvalue problem is considered next for a system characterized by material and velocity data identified by subscript 0: hence, no external source is included and the eigenvalue k_0 is introduced to modify the fission terms in the neutron and precursor equations. The problem can be written as:

$$\hat{\Theta}_0\mathbf{X}_0 = \frac{1}{k_0}\hat{F}_0\mathbf{X}_0 \tag{6}$$

with boundary conditions:

$$\varphi_0(0) = 0, \qquad \psi_0(a) = 0, \qquad C_0(0) = C_0(a)\xi \tag{7}$$

A physically significant (non-everywhere vanishing and non-negative) solution $\mathbf{X}_0$ associated to eigenvalue k_0 is determined in the next section.

In reactor physics it is significant and very helpful in many applications to introduce the concept of neutron importance for a critical stationary system [7]. In this case, the importance of a neutron at a certain position and direction can be defined, as usual in the literature [7], as the total number of fissions produced asysmptotically in the critical system by a neutron injected at that position and direction.

In the present model, the criticality of the system is not independent on the presence of delayed emissions. Therefore the concept of importance has to be extended to delayed neutron precursors, as the number of fissions that are to be produced asymptotically in the system by a precursor nucleus appearing at a certain spatial position.

The balance for the importance φ_0^+ (ψ_0^+) of neutrons requires to equate the importance of neutrons moving in the direction μ ($-\mu$) at a certain spatial position x to the importance of the fraction $(1-(\sigma_0/\mu)dx)$ of uncollided neutrons at spatial position $x+dx$ ($x-dx$) in the same direction, increased by the importance of the neutrons generated in both directions by scatterings and prompt fissions and the importance associated to the delayed precursors generated along the path. The balance for delayed precursor importance requires to equate the importance at a spatial position x to the importance of the fraction $(1-(\lambda/u)dx)$ of undecayed precursors at position $x+dx$, increased by the difference between the importance of the neutrons emitted in both directions by the decay process through the time dx/u and the importance associated to the delayed precursor produced through the fission process. The following equations are obtained:

$$\begin{cases} -\mu\dfrac{\partial\varphi_0^+(x)}{\partial x} + \sigma_0\varphi_0^+(x) = \dfrac{[\nu\sigma_f(1-\beta)/k + \sigma_s]_0}{2}[\varphi_0^+(x) + \psi_0^+(x)] + (\beta\nu\sigma_f/k)_0 C_0^+(x) \\ \mu\dfrac{\partial\psi_0^+(x)}{\partial x} + \sigma_0\psi_0^+(x) = \dfrac{[\nu\sigma_f(1-\beta)/k + \sigma_s]_0}{2}[\varphi_0^+(x) + \psi_0^+(x)] + (\beta\nu\sigma_f/k)_0\lambda C_0^+(x) \\ -u_0\dfrac{\partial C_0^+(x)}{\partial x} + \lambda C_0^+(x) = \dfrac{\lambda}{2}[\varphi_0^+(x) + \psi_0^+(x)] \end{cases} \tag{8}$$

Equations (8) can be written in matrix form as

$$\hat{\Theta}_0^+\mathbf{X}_0^+ = \frac{1}{k_0}\hat{F}_0^+\mathbf{X}_0^+ \tag{9}$$

with straightforward definitions for the operators involved.

Proper boundary conditions have to be introduced, stating that the importance of neutrons moving along an exiting direction must vanish and that the importance of delayed precursors exiting the reactor is equal to the importance of the undecayed fraction $\exp(-\lambda\tau)$ re-entering the opposite boundary of the reactor after transit time τ in the external system circuit, namely

$$\psi_0^+(0) = 0, \qquad \varphi_0^+(a) = 0, \qquad C_0^+(a) = C_0^+(0)\xi \tag{10}$$

It is worth noticing that Eqs. (8), associated to boundary conditions (10), are the formal mathematical adjoint of Eqs. (6), once the appropriate inner product is defined to include a scalar product for vectors and a space integration, as

$$(g,f) \equiv \sum_j \int_0^a g_j(x) f_j(x) dx \equiv \sum_j < g_j \mid f_j > \tag{11}$$

3. Solution of the critical problem

The critical problem (6) can be rewritten in the form:

$$\frac{\partial}{\partial x}\mathbf{X}_0 = \hat{T}^{-1}\left(\frac{1}{k_0}\hat{F}_0 - \hat{\Sigma}_0\right)\mathbf{X}_0 \tag{12}$$

The solution of such a homogeneous (no source) problem can be searched for in the form:

$$\mathbf{X}_0 = \sum_{j=1}^{3} \mathcal{A}_j e^{\omega_j x}\mathbf{V}_j \tag{13}$$

where $\mathbf{V}_j$ are the eigenvectors of the matrix $\hat{T}^{-1}\left(\frac{1}{k_0}\hat{F}_0 - \hat{\Sigma}_0\right)$ corresponding to the complex algebraic eigenvalues ω_j. The multiplication eigenvalue k_0 is determined through the imposition of the boundary conditions (7) for the components of the state vector $\mathbf{X}_0$, explicitly:

$$\begin{aligned}
&\sum_{j=1}^{3} \mathcal{A}_j V_j^{(1)} = 0 \\
&\sum_{j=1}^{3} \mathcal{A}_j V_j^{(2)} e^{\omega_j a} = 0 \\
&\sum_{j=1}^{3} \mathcal{A}_j V_j^{(3)}\left(1 - e^{-\lambda\tau} e^{\omega_j a}\right) = 0
\end{aligned} \tag{14}$$

where $V_j^{(m)}$ denotes the m-th component of the j-th eigenvector. Equations (14) constitute a homogeneous linear system for $\mathcal{A}_j$, which admits a physically significant solution only if the determinant of its coefficients vanishes, i.e.:

$$det\begin{pmatrix} V_1^{(1)} & V_2^{(1)} & V_3^{(1)} \\ V_1^{(2)} e^{\omega_1 a} & V_2^{(2)} e^{\omega_2 a} & V_3^{(2)} e^{\omega_3 a} \\ V_1^{(3)}\left(1 - e^{-\lambda\tau} e^{\omega_1 a}\right) & V_2^{(3)}\left(1 - e^{-\lambda\tau} e^{\omega_2 a}\right) & V_3^{(3)}\left(1 - e^{-\lambda\tau} e^{\omega_3 a}\right) \end{pmatrix} = 0 \tag{15}$$

As a numerical example, Table I reports the multiplication eigenvalues for a subcritical system as a function of the recirculation velocity and the fraction of delayed neutron precursors. The dependence of k_0 on β is due to the decay of a fraction of the delayed neutron precursors outside the multiplying system and it is important for any engineering assessment of nuclear reactors. For the case $\beta = 0.007$ and $u = 10 mfp/s$ the direct fluxes and precursor concentration are

shown in Fig. 1. Figure 2 reports the adjoint functions for the same case. As can be seen, the precursor distributions are completely different from what can be expected in solid-fuel critical reactors. It is also worth noticing that direct and adjoint functions are symmetrical with respect to the midplane, for this case of a homogeneous system.

4. Consistent point model and definition of the kinetic parameters

A critical ($k_0 = 1$) stationary system is assumed as a reference, in the usual point kinetics sense. Therefore, in the following, the time-dependent solution of problem (1) in absence of external source is searched for through a separation-projection procedure [9], starting from the solution of the reference system as initial condition.

The neutron population and delayed neutron concentrations are factorized as a product of a constant shape and a time-dependent amplitude function:

$$\varphi = \varphi_0 P; \qquad \psi = \psi_0 P; \qquad C = C_0 \Gamma \tag{16}$$

using the solution φ_0, ψ_0, C_0 of the reference critical reactor. The time evolution of the amplitude of the flux P and of the delayed neutron concentration Γ of the perturbed reactor is obtained with a standard projection method, using the adjoint functions φ_0^+, ψ_0^+, C_0^+ as weights. Multiplying each of the three Eqs. (1) by ppthe corresponding adjoint function, it follows easily:

$$\begin{cases} < \varphi_0^+ \mid \varphi_0 > \dfrac{1}{v}\dfrac{dP}{dt} = \left[< \varphi_0^+ \mid \dfrac{\delta(c\sigma)}{2}\Phi_0 > - < \varphi_0^+ \mid \delta\sigma\varphi_0 > \right] P + \dfrac{1}{2}\lambda < \varphi_0^+ \mid C_0 > (\Gamma - P) + \dfrac{1}{2} < \varphi_0^+ \mid S > \\ < \psi_0^+ \mid \psi_0 > \dfrac{1}{v}\dfrac{dP}{dt} = \left[< \psi_0^+ \mid \dfrac{\delta(c\sigma)}{2}\Phi_0 > - < \psi_0^+ \mid \delta\sigma\psi_0 > \right] P + \dfrac{1}{2}\lambda < \psi_0^+ \mid C_0 > (\Gamma - P) + \dfrac{1}{2} < \psi_0^+ \mid S > \\ < C_0^+ \mid u_0 \dfrac{\partial C_0}{\partial x} > \Gamma + < C_0^+ \mid \delta u \dfrac{\partial C_0}{\partial x} > \Gamma + < C_0^+ \mid C_0 > \left(\lambda\Gamma + \dfrac{d\Gamma}{dt} \right) = < C_0^+ \mid \beta\nu\sigma_f\Phi_0 > P \end{cases} \tag{17}$$

where $\Phi_0 = \varphi_0 + \psi_0$ is the total flux, and $\delta\sigma$, $\delta(c\sigma)$, δu are the perturbations of the corresponding parameters of the stationary reactor. The number of secondaries per collision $c = (\nu\sigma_f + \sigma_s)/\sigma$ is also introduced.

By summing the first two equations, an equation for the flux amplitude P is obtained:

$$\Lambda_n \frac{dP}{dt} = (\Delta_P - \beta^*)P + \lambda\mathcal{G} + \mathcal{S} \tag{18}$$

where the following definitions for the kinetic parameters are used:

$$\begin{aligned} \Lambda_n &\equiv \frac{1}{v}\frac{< \varphi_0^+ \mid \varphi_0 > + < \psi_0^+ \mid \psi_0 >}{\mathcal{F}} \\ \Delta_P &\equiv \frac{\frac{1}{2} < \Phi_0^+ \mid \delta(c\sigma)\Phi_0 > - < \varphi_0^+ \mid \delta\sigma\varphi_0 > - < \psi_0^+ \mid \delta\sigma\psi_0 >}{\mathcal{F}} \\ \beta^* &\equiv \frac{\frac{1}{2} < \Phi_0^+ \mid \lambda C_0 >}{\mathcal{F}} \\ \mathcal{F} &\equiv \frac{\lambda}{2} < \Phi_0^+ \mid C_0 > + \frac{1}{2} < \Phi_0^+ \mid \nu\sigma_f(1-\beta)\Phi_0 > \end{aligned} \tag{19}$$

and the modified precursor distribution amplitude $\mathcal{G}$ and effective source $\mathcal{S}$ are introduced as:

$$\mathcal{G} \equiv \frac{\frac{1}{2} < \Phi_0^+ \mid C_0 >}{\mathcal{F}} \Gamma$$
$$\mathcal{S} \equiv \frac{\frac{1}{2} < \Phi_0^+ \mid S >}{\mathcal{F}} \tag{20}$$

The last of Eqs. (17) gives the evolution of the precursor population, through the equation for $\mathcal{G}$:

$$\Lambda_C \frac{d\mathcal{G}}{dt} = (\beta^* + \Delta_{PC})P - (\lambda + \Delta_u)\mathcal{G} \tag{21}$$

where the following additional kinetic parameters are introduced:

$$\Lambda_C \equiv \frac{< C_0^+ \mid C_0 >}{\frac{1}{2} < \Phi_0^+ \mid C_0 >}$$
$$\Delta_{PC} \equiv \frac{< C_0^+ \mid \delta(\beta \nu \sigma_f)\Phi_0 >}{\mathcal{F}} \tag{22}$$
$$\Delta_u \equiv \frac{< C_0^+ \mid \delta u \frac{\partial C_0}{\partial x} >}{\frac{1}{2} < \Phi_0^+ \mid C_0 >}$$

The coupled Eqs. (18) and (21) govern the evolution of the flux and precursor concentration amplitudes for a recirculated reactor, given the generalized point kinetic coefficient defined in Eqs. (19) and (22). Note that the present formulation reduces correctly to the Henry point kinetic model once the limit for $u \to 0$ is taken. In that case, all system variables φ_0, ψ_0, C_0 and the corresponding adjoint variables φ_0^+, ψ_0^+, C_0^+ have the same shape and the point kinetic parameters become identical to the classical ones. In that case, the effective delayed neutron fraction, as defined by the usual point kinetic equations, is the sum of two contributions, as $\beta_{eff} = \beta^* + \Delta_{PC}$. When the effects of the recirculation are included the two contributions are separated, as some of the delayed neutron precursors decay outside the core and cannot contribute to the neutron population.

Figure 3 shows the dependence of the kinetic parameters introduced above with respect to the recirculation velocity. The neutron lifetime is not influenced by the recirculation of the fissile material, while a significant effect can be observed for the delayed neutron lifetime, as can be expected on physical grounds. An important effect is observed also on the effective delayed neutron fraction, which is important in determining the response time in a transient situation.

Figure 4 presents the transient behaviour in a subcritical system injected by a time-constant source following a step change in the kinetic parameters. Three transients are considered. A step change in Δ_{PC} causes a prompt jump response in the precursor amplitude. Also, a reduction of the circulation velocity corresponds to a reduction of the power level of the system, due to a reduction of the delayed neutron source.

5. Conclusions

Existing standard nuclear reactor physics books do not devote much space to the presentation of the problem of circulating fuel reactors. It is worth to extend present-day reactor physics investigations to cover these up-to-date themes. The paper attempts to formulate some classic problems in the physics of fluid-fuel neutron to deepen some important physical aspects. It is straightforward to generalize the present treatment to the subcritical source injected systems which are proposed for transmutation of actinides and long-lived radioactive wastes.

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Figure Captions

Fig. 1 - Direct neutron fluxes and precursor concentration for a critical system with material data as Table I and $u = 10 mfp/s$, $\beta = 0.007$.

Fig. 2 - Neutron and precursor importances for the same problem as in Fig. 1.

Fig. 3 - Dependence of the kinetic parameters on fuel velocity for a system with the same material data as in Table I.

Fig. 4 - Power and precursor transients following a step change in the kinetic parameters. Material data are assumed as in Table I and $u = 0.5 mfp/s$, $\beta = 0.002$.

Table I

Multiplication eigenvalue k_0 for a system characterized by $\sigma = 1$, $\sigma_s = 0.9$, $\nu\sigma_f = 0.18$, and $a = 5$ *mfp*. The recirculation time is such that $\xi = 0.1$. As a reference, the multiplication constant for the solid-fuel system (no motion and no recirculation) with the same material and geometry is 0.95866.

β	$u = 1[mfp/s]$	$u = 3[mfp/s]$	$u = 5[mfp/s]$	$u = 10[mfp/s]$
0.002	0.95719	0.95691	0.95685	0.95680
0.003	0.95645	0.95604	0.95594	0.95586
0.004	0.95571	0.95516	0.95503	0.95493
0.005	0.95497	0.95429	0.95412	0.95400
0.006	0.95424	0.95341	0.95322	0.95307
0.007	0.95350	0.95254	0.95231	0.95213

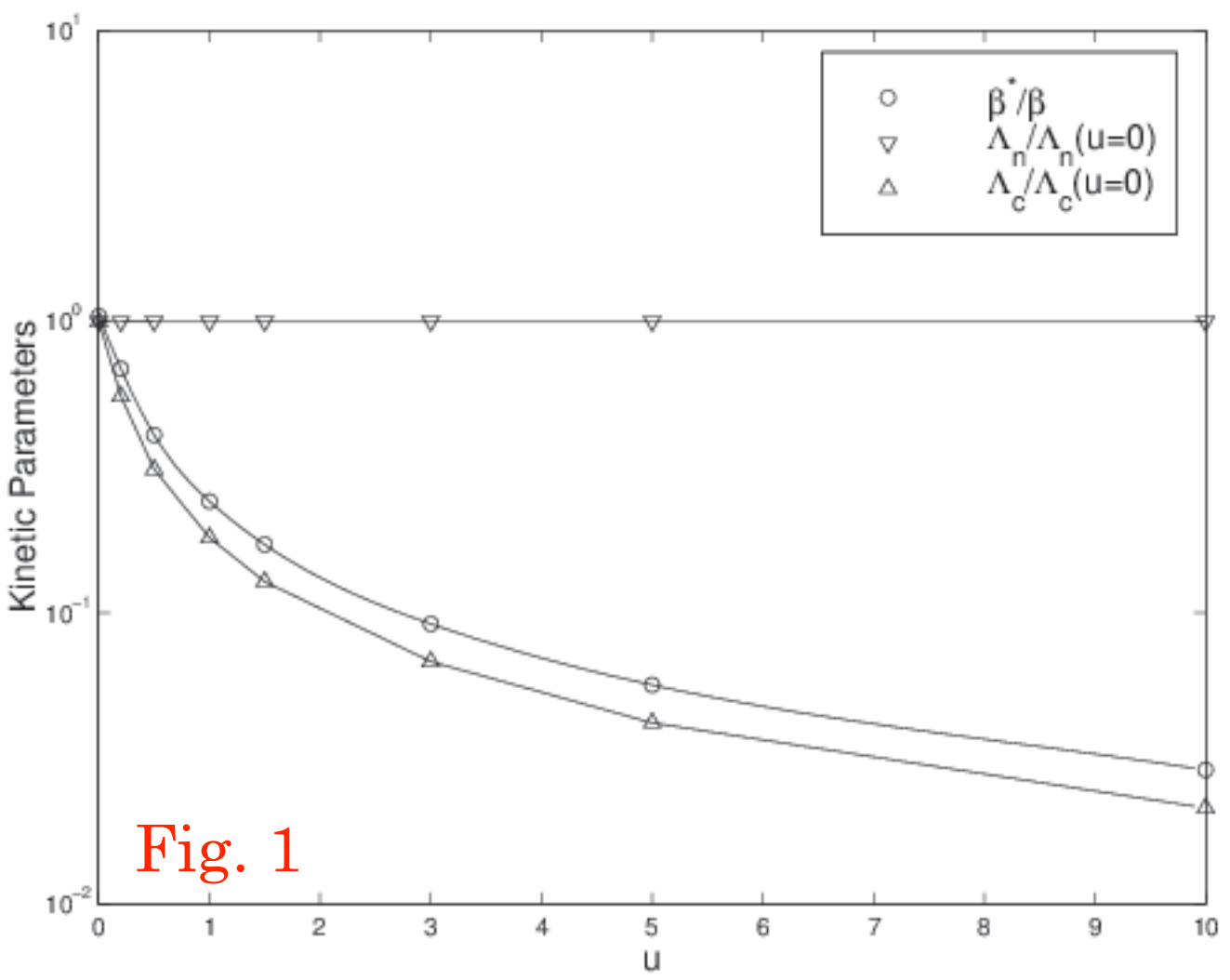
10^{1}
10^{0}
10^{-1}
10^{-2}
Kinetic Parameters
0
1
2
3
4
5
6
7
8
9
10
u
β^{*}/β
$\Lambda_n/\Lambda_n(u=0)$
$\Lambda_c/\Lambda_c(u=0)$

Fig. 1

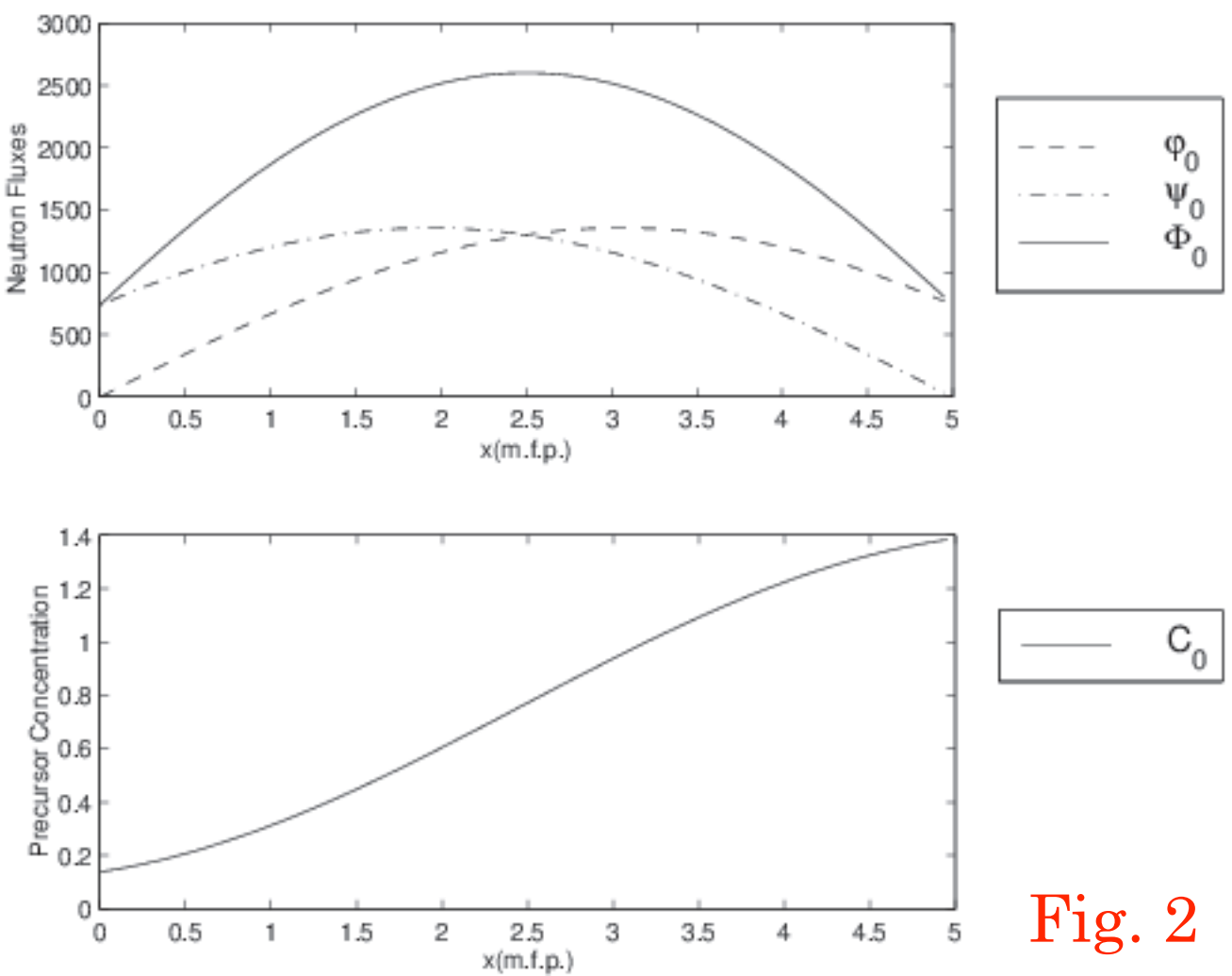
Neutron Fluxes
3000
2500
2000
1500
1000
500
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
x(m.f.p.)
φ0
ψ0
Φ0
Precursor Concentration
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
x(m.f.p.)
C0

Fig. 2

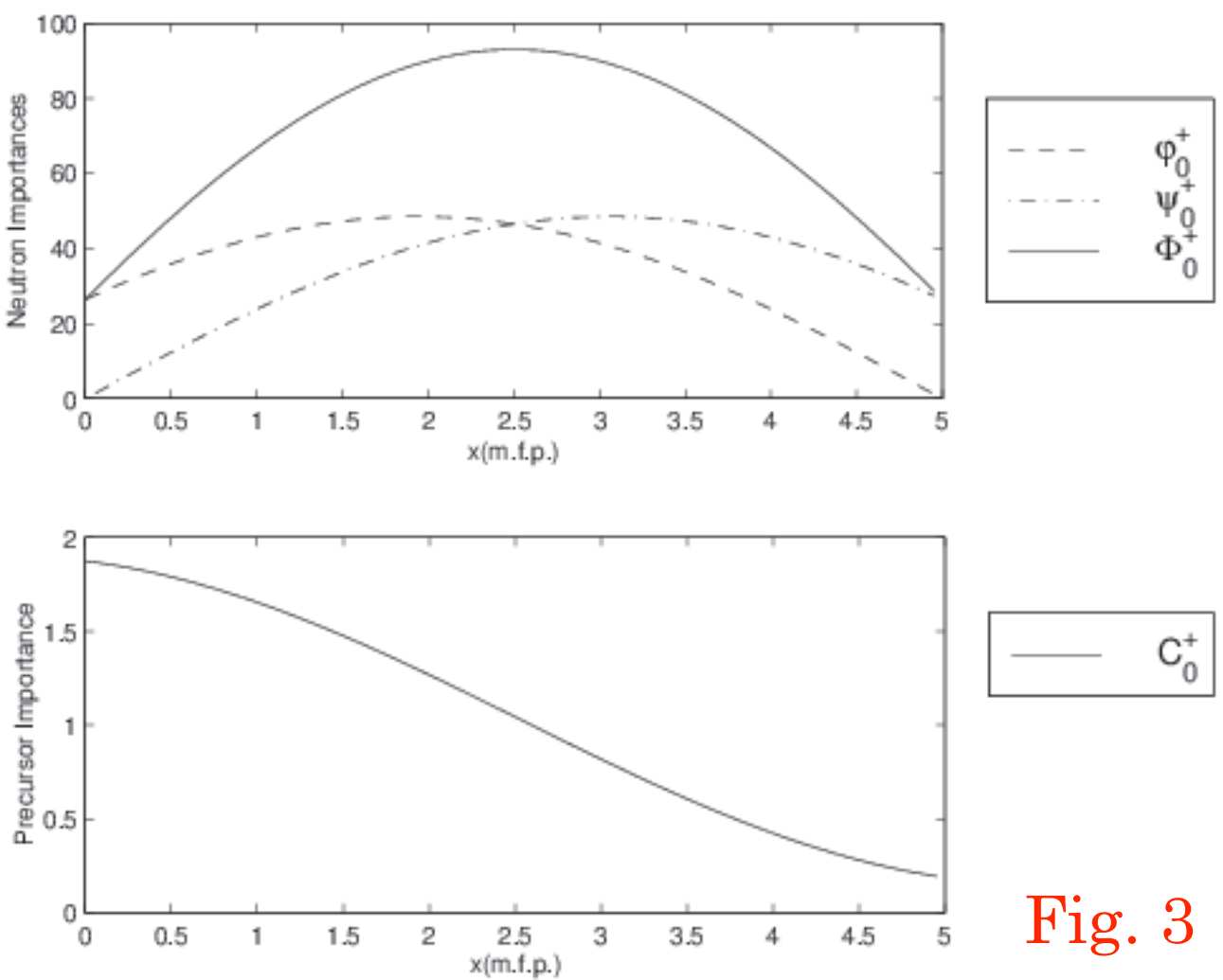
Neutron Importances
x(m.f.p.)
φ_0^+
ψ_0^+
Φ_0^+
Precursor Importance
x(m.f.p.)
C_0^+

Fig. 3

Fig. 4