

Optimizing cost-effectiveness in a stochastic Markov manpower planning system under control by recruitment

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Abstract The present paper proposes a mathematical model and algorithm for optimizing cost-effectiveness in a stochastic manpower planning system under control by recruitment. More specifically, we suggest a multi-objective model that simultaneously addresses two objectives, namely minimizing the cost and maximizing the desirability degree of the attained personnel structure. In a stochastic environment, the uncontrollable parameters of the manpower model (i.e. internal transitions and wastage) are random variables. We suggest a scenario approach in order to cope with this uncertainty. The optimization problem under study is formulated as a mixed integer program. Further, in order to decrease the computing time in solving the optimization problem, we show that the optimal recruitment strategy has some properties that enable narrowing the solution space.

Keywords Manpower planning · Markov models · Fuzzy sets · Stochastic models

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1 Introduction

The operations research literature presents a variety of mathematical models and methods to support organizations in several domains of personnel planning (Khoong 1996). In the domain of personnel rostering, optimization methods are developed to assign the available personnel to work shifts, subject to legal constraints, contractual agreements and/or personal employees preferences (Bilgin et al. 2012; Burke et al. 2004; Ernst et al. 2004; Smet et al. 2014). In the domain of staffing, on the other hand, methods are suggested to determine the personnel level that is required to cover the organization's workload (Komarudin et al. 2013; Ozcan 2009). Staffing decisions determine the personnel that will be available for rostering. However, staffing does not include providing personnel strategies that ensure that eventually the desired personnel is available. Instead, this problem is covered in the domain of manpower planning.

An extensive body of research on quantitative models for manpower planning is based on Markov theory (Bartholomew et al. 1991; De Feyter & Guerry 2011; Ugwuowo & McClean 2000; Vassiliou 1997). In Markov manpower planning models, the employees are classified into exclusive subgroups based on work-related and/or personal characteristics (De Feyter 2006). The personnel structure represents the number of employees in each of the subgroups. The objective of Markov manpower planning is to attain a desired personnel structure whereby the dynamics of the personnel system is regulated by recruitments, internal flows between the subgroups and wastage. These employee flows are the parameters of the manpower models.

In prior work, many descriptions of the Markov manpower planning problem exist, differing in the assumptions on the model parameters. On the one hand, models can be distinguished based on the parameters that are considered as decision variables. The evolution of the personnel structure can be controlled by recruitment and/or by internal flows (Bartholomew et al. 1991; De Feyter 2007; De Feyter & Guerry 2011; Guerry & De Feyter 2011; Nilakantan & Raghavendra 2005). Wastage, on the other hand, is generally assumed to be uncontrollable, because, for example, voluntary wastage is a free choice of the employee and is therefore not under control of management. In this way, the desired personnel structure objective is accompanied by hard constraints on the uncontrollable parameters of the model. On the other hand, models can be distinguished based on the time-dependency of its parameters. Manpower systems can be modeled by time-homogeneous, non-homogeneous and semi-Markov approaches (McClean 1991; McClean et al. 1997; Papadopoulou & Vassiliou 1994; Vassiliou 1981; Vassiliou 1982; Vassiliou & Papadopoulou 1992; Yadavalli et al. 2002).

Early academic contributions studied the manpower planning problem by mathematically describing the set of attainable personnel structures, given some model constraints (Bartholomew 1977; Davies 1975; Georgiou & Vassiliou 1992; Tsantas & Georgiou 1997; Vassiliou & Tsantas 1984; Vassiliou & Georgiou 1990; Vassiliou et al. 1990). Each personnel structure of this set is attainable by means of at least one personnel strategy. This way, the set of attainable personnel structures defines the solution space in which there can be searched for the most suitable personnel structure and a related personnel strategy. However, previous work shows that the desired personnel structure is not always attainable, given the model constraints (Davies 1982; Guerry 1993; Guerry & De Feyter 2012).

An obvious approach to deal with this problem is allowing constraint violations and constructing an optimization model that includes the objective of minimizing the constraint violations. Unfortunately, since promotions are a specific type of internal transitions, this approach is not always preferable in real-world applications. More specifically, in manpower systems under control by recruitment, and in which the internal flows originally were not considered as control variables, changing the promotion opportunities may lead to employees' job dissatisfaction (Klehe et al. 2011).

Another approach for coping with unattainable desired personnel structures is opting for a personnel strategy that respects the model constraints and results in a personnel structure most similar to the desired one (Komarudin et al. 2015; Guerry 1999). This approach corresponds to a maximization problem with a certain similarity measure as objective function, whereby the solution space is defined by the region of attainable personnel structures. In interesting work by Dimitriou et al. (2013) and Georgiou & Tsantas (2002), such an optimization problem was extended by including cost minimization as supplementary objective. Because such a problem is multi-objective, the solution might not be necessarily the cheapest possible personnel strategy. Nevertheless, we consider the solution to be cost-effective, because it balances the goals of minimizing operational costs and attaining the desired personnel structure to cover the organization's workload as good as possible.

In the present paper, we propose a mathematical model and algorithm for optimizing cost-effectiveness in a manpower planning system under control by recruitment. In real-world applications, the uncontrollable model parameters regarding wastage and internal flows are often subject to uncertainty (Chattopadhyay & Gupta 2007; De Feyter & Guerry 2009; Papadopoulou & Tsaklidis 2007; Shapiro et al. 2009; Vassiliou & Gerontidis 1985). Therefore, in contrast to similar previous work (Dimitriou et al. 2013; Georgiou & Tsantas 2002), we consider the stochastic nature of the manpower planning problem. In practice, to cope with uncertainty, decision makers often consider several possible scenarios for the unknown parameters in the decision model (Komarudin et al. 2016; Shapiro et al. 2009). Likewise, in comparing the cost-effectiveness of recruitment strategies, we suggest a scenario approach that takes into account a wide range of possible scenarios for internal flows and wastage. In Section 2, we introduce two measures that can serve as objectives to evaluate the cost-effectiveness of a recruitment strategy. The cost-effectiveness of a recruitment strategy is expressed analytically by the cost ratio and the desirability degree. In Section 3, we present an optimization model and algorithm to find the most cost-effective recruitment strategy. The scenario approach allows us to formulate the problem under study as a mixed integer program. To identify the solution space, we could rely on the results in prior work (Bartholomew et al. 1991), which mathematically describe the set of attainable personnel structures and the related personnel strategies, given the hard constraints in our model. In Section 4, however, we show that the optimal solution of the mixed integer program has some interesting additional properties that allow us to further narrow the solution space. In this way, we are able to tighten the mixed integer program. Our illustration, in Section 5, shows that those properties enable us to significantly decrease the computation time in solving the optimization problem for cost-effectiveness.

2 Cost-effectiveness of a recruitment strategy

Consider a stochastic manpower system under control by recruitment and assume that the personnel of the organization is divided into k subgroups. The personnel structure $\mathbf{n}(t) = (n_i(t))$ is the row vector of which the i -th component $n_i(t)$ refers to the number of employees in subgroup $i \in \{1, \dots, k\}$ at time t . Let $\mathbf{R} = (r_i)$ be a non-negative row vector of which the i -th component r_i is referring to the number of personnel recruited in subgroup i at time t . The aim is to evaluate recruitment strategy \mathbf{R} , taking into account the uncertainty about the wastage and internal flows between the subgroups in time interval $[t-1, t)$.

Let $f_{ij}(t)$ be the number of employees that move from subgroup i to subgroup j in time interval $[t-1, t)$ and let p_{ij} be the transition probability from subgroup i to subgroup j in a unit time interval. We denote by $\bar{f}_{ij}(t)$ the expected flow from subgroup i to subgroup j in the same time interval. In stochastic manpower planning, it is well known that $\bar{f}_{ij}(t)$ is given by $n_i(t-1)\hat{p}_{ij}$, with \hat{p}_{ij} being the maximum likelihood estimation of the probability for an employee to move from subgroup i at time $t-1$ to subgroup j at time t (Bartholomew et al. 1991; De Feyter & Guerry 2009). The transition probabilities \hat{p}_{ij} (for $i, j \in \{1, \dots, k\}$) are usually gathered into the internal transition matrix $\hat{\mathbf{P}} = (\hat{p}_{ij})$. Since the transition probabilities p_{ij} are unknown, previous work suggests to estimate $\mathbf{P} = (p_{ij})$ and its probability distribution, based on historical data of internal transitions (Bartholomew 1977; Bartholomew et al. 1991; De Feyter 2006; De Feyter & Guerry 2009).

In evaluating the cost-effectiveness of a recruitment strategy, our model takes into account S possible scenarios for the unknown internal transition matrix $\mathbf{F}(t) = (f_{ij}(t))$. Each scenario $s \in \{1, \dots, S\}$ is characterized by a flow matrix $\mathbf{F}_s(t) = (f_{sij}(t))$. Remark that the wastage (as unknown model parameter) is included in the scenario for the internal transitions. Because a member of subgroup i at time $t-1$ has either left the organization at time t or made an internal transition to subgroup $j \in \{1, \dots, k\}$ during time interval $[t-1, t)$, a scenario s , characterized by the flow matrix $\mathbf{F}_s(t) = (f_{sij}(t))$, by definition implies a wastage scenario.

There are several possibilities for generating scenarios. From the estimated internal transition matrix $\hat{\mathbf{P}}$, Monte Carlo methods can randomly generate S scenarios. However, the historical dataset can also be used for generating scenarios based on bootstrapping (Rubinstein & Kroese 2008). Given scenario s and recruitment strategy \mathbf{R} , the number of members $n_{si}(t)$ in subgroup i at time t is the result of the sum of the internal flows $f_{sji}(t)$ into subgroup i and the recruitments r_i into subgroup i (Eq. 1).

$$n_{si}(t) = \sum_{j=1}^k f_{sji}(t) + r_i \quad \forall i \quad (1)$$

To evaluate the cost-effectiveness of a recruitment strategy, both costs and outcomes should be taken into account. A cost-effective recruitment strategy is the cheapest way to effectively cover the organization's workload as good as possible. Therefore, we propose two objectives that should be addressed simultaneously in the pursuit of cost-effectiveness: minimizing the cost ratio and maximizing the desirability degree.

2.1 The cost ratio $\alpha_s(\mathbf{R})$

Following the related manpower planning literature (Dimitriou et al. 2013; Georgiou & Tsantas 2002; Georgiou & Vassiliou 1997; Vajda 1978), we consider two kinds of costs to express the cost ratio related to recruitment strategy \mathbf{R} : the operational costs corresponding to the employment of the members of the personnel system (e.g. wages) and the costs due to the employee flows. According to Eq. 1, given personnel structure $\mathbf{n}(t-1)$ and internal transitions $f_{sij}(t)$, recruitment strategy \mathbf{R} results in a particular personnel structure $\mathbf{n}_s(t) = (n_{si}(t))$. Scenario s and recruitment strategy \mathbf{R} result in a cost that is composed of three components, i.e. the cost of the personnel in correspondence with the number of members $n_{si}(t)$, the cost of the internal flows $f_{sij}(t)$ and the cost of the recruitment flows r_i according to strategy \mathbf{R} . Remark that we assume that the costs regarding wastage are implicitly included in the recruitment costs.

Regarding time interval $[t-1, t)$, let $c_i^p(t)$ be the cost of an employee of subgroup i , $c_{ij}^f(t)$ be the cost of an internal transition of an employee from subgroup i to subgroup j , and $c_i^r(t)$ be the cost of recruiting an employee into subgroup i . Then, the total cost $c(\mathbf{n}_s(t))$ of personnel structure $\mathbf{n}_s(t)$, corresponding to recruitment strategy \mathbf{R} , can be formulated as in Eq. 2.

$$c(\mathbf{n}_s(t)) = \sum_{i=1}^k c_i^p(t)n_{si}(t) + \sum_{i=1}^k \sum_{j=1, j \neq i}^k c_{ij}^f(t)f_{sij}(t) + \sum_{i=1}^k c_i^r(t)r_i \quad (2)$$

Let $\mathbf{n}^c(t)$ be the personnel structure with i -th component $n_i^c(t) = \sum_{j=1}^k \bar{f}_{ji}(t)$. Then, the total cost $c(\mathbf{n}^c(t))$ of the personnel structure $\mathbf{n}^c(t)$ can be formulated as in Eq. 3. Since $\mathbf{n}^c(t)$ corresponds to the personnel structure at time t in the case of no recruitments, it is the cheapest possible personnel structure for the expected flows $\bar{f}_{ij}(t)$.

$$c(\mathbf{n}^c(t)) = \sum_{i=1}^k c_i^p(t)n_i^c(t) + \sum_{i=1}^k \sum_{j=1, j \neq i}^k c_{ij}^f(t)\bar{f}_{ij}(t) \quad (3)$$

For a recruitment strategy \mathbf{R} that results in a personnel structure $\mathbf{n}_s(t)$, we introduce the cost ratio $\alpha_s(\mathbf{R})$ as the ratio of the cost $c(\mathbf{n}_s(t))$, related to the personnel structure $\mathbf{n}_s(t)$, to the cost $c(\mathbf{n}^c(t))$ related to $\mathbf{n}^c(t)$. We formulate the cost ratio $\alpha_s(\mathbf{R})$ as in Eq. 4.

$$\alpha_s(\mathbf{R}) = \frac{c(\mathbf{n}_s(t))}{c(\mathbf{n}^c(t))} \quad (4)$$

2.2 The desirability degree $\beta_s(\mathbf{R})$

The second objective in finding a cost-effective recruitment strategy is attaining at time t a personnel structure that is as similar as possible to the desired personnel structure. We denote the desired personnel structure at time t by $\mathbf{n}^d(t)$. Based on a fuzzy set approach, De Feyter & Guerry (2009) introduced the desirability degree in order to express the discrepancy between a personnel structure $\mathbf{n}(t)$ and the desired personnel structure $\mathbf{n}^d(t)$. We denote the desirability degree of

$\mathbf{n}(t)$ by $\delta(\mathbf{n}(t))$. With respect to subgroup i , the extent in which the number of members $n_i(t)$ differs from the desirable number $n_i^d(t)$ can be expressed by the fuzzy triangular membership function $\delta_i(n_i(t))$ as in Eq. 5. We define this function based on the lower and the upper limit of the number of personnel in subgroup i , respectively denoted as $n_{LL,i}$ and $n_{UL,i}$. This allows the decision maker to set a maximum upward and downward deviation from the desired number of employees in each subgroup i , as in Dimitriou et al. (2013) and Georgiou & Tsantas (2002). We use the fuzzy min-operator to obtain the desirability degree of the vector $\mathbf{n}(t) = (n_i(t))$ as in Eq. 6.

$$\delta_i(n_i(t)) = \begin{cases} 0, & \text{if } n_i(t) < n_{LL,i} \text{ or } n_i(t) > n_{UL,i} \\ \frac{n_i(t) - n_{LL,i}}{n_i^d(t) - n_{LL,i}}, & \text{if } n_{LL,i} \leq n_i(t) \leq n_i^d(t) \\ \frac{n_i(t) - n_{UL,i}}{n_i^d(t) - n_{UL,i}}, & \text{if } n_i^d(t) \leq n_i(t) \leq n_{UL,i} \end{cases} \quad (5)$$

$$\delta(\mathbf{n}(t)) = \min_i \delta_i(n_i(t)) \quad (6)$$

According to Eq. 1, given the personnel structure $\mathbf{n}(t-1)$ and the internal transitions $f_{sij}(t)$, the recruitment strategy \mathbf{R} results in a particular personnel structure $\mathbf{n}_s(t)$. Therefore, for scenario s , the degree of desirability of \mathbf{R} can be expressed as equal to $\delta(\mathbf{n}_s(t))$. Let us denote for scenario s the desirability degree of recruitment strategy \mathbf{R} as $\beta_s(\mathbf{R})$ satisfying Eq. 7.

$$\beta_s(\mathbf{R}) = \delta(\mathbf{n}_s(t)) \quad (7)$$

2.3 Cost-effectiveness

In order to evaluate the cost-effectiveness of a recruitment strategy, the aim is to simultaneously consider the cost ratio and the desirability degree. Let w_1 and w_2 be the weights that denote the relative importance of respectively the cost and desirability criterium. Then, for a given scenario s , the cost-effectiveness $\gamma_s(\mathbf{R})$ of recruitment strategy \mathbf{R} can be expressed as in Eq. 8. For evaluating a recruitment strategy, given the internal transitions $f_{sij}(t)$, Eqs. 2-8 provide measures to evaluate the outcome of a particular \mathbf{R} .

$$\gamma_s(\mathbf{R}) = w_1 \alpha_s(\mathbf{R}) - w_2 \beta_s(\mathbf{R}) \quad (8)$$

Remark that the cost-effectiveness $\gamma_s(\mathbf{R})$ is a function that simultaneously incorporates two aspects, namely the cost and the desirability. Furthermore the desirability degree $\beta_s(\mathbf{R})$ is expressed by a fuzzy membership function and consequently takes values between 0 and 1. For that reason it is more appropriate to consider besides the degree of desirability, the cost ratio $\alpha_s(\mathbf{R})$ instead of the total cost.

The cost ratio $\alpha_s(\mathbf{R})$ as well as the desirability degree $\beta_s(\mathbf{R})$ depend on the personnel structure $\mathbf{n}_s(t)$. Since the personnel structure at time t is the result of the recruitment strategy \mathbf{R} and the internal transitions $f_{sij}(t)$ (see Eq. 1), both the cost ratio and the desirability degree vary with the scenario for the internal flows. To handle the uncertainty about the internal flows in a stochastic environment, our model considers S possible scenarios, with each scenario $s \in \{1, 2, \dots, S\}$ being

characterized by its own flow matrix $\mathbf{F}_s(t) = (f_{sij}(t))$. Therefore, the evaluation of a particular recruitment strategy \mathbf{R} should rely on the expected cost-effectiveness $E[\gamma(\mathbf{R})]$, which can be expressed as in Eqs. 9-11 .

$$E[\gamma(\mathbf{R})] = w_1 E[\alpha(\mathbf{R})] - w_2 E[\beta(\mathbf{R})] \quad (9)$$

$$E[\alpha(\mathbf{R})] = \frac{1}{S} \sum_{s=1}^S \alpha_s(\mathbf{R}) \quad (10)$$

$$E[\beta(\mathbf{R})] = \frac{1}{S} \sum_{s=1}^S \beta_s(\mathbf{R}) \quad (11)$$

3 Optimizing cost-effectiveness

In order to find a recruitment strategy that optimizes cost-effectiveness, we propose a model and algorithm that simultaneously minimize the cost ratio and maximize the desirability degree. An appropriate overall objective function to optimize cost-effectiveness is given by Eq. 9. The aim is to minimize the function $E[\gamma(\mathbf{R})]$.

The scenario approach allows us to formulate the optimization problem under study as a mixed integer program (MIP), as presented in Eqs. 12-24. The model consists of the overall objective function $E[\gamma(\mathbf{R})]$, constraints on the cost ratio $\alpha_s(\mathbf{R})$ and the desirability degree $\beta_s(\mathbf{R})$ and some real-world restrictions on the variables in the model.

1. The overall objective function $E[\gamma(\mathbf{R})]$

Eq. 12 expresses the goal of minimizing the overall objective function $E[\gamma(\mathbf{R})]$ that is considered to be the average of the cost-effectiveness $\gamma_s(\mathbf{R})$ over the different scenarios $s \in \{1, \dots, S\}$. Since each scenario s is characterized by internal transitions $f_{sij}(t)$, Eq. 8 can be used to quantify $\gamma_s(\mathbf{R})$ based on the cost ratio $\alpha_s(\mathbf{R})$ and the desirability degree $\beta_s(\mathbf{R})$. Then, for each scenario s , the cost-effectiveness is expressed as in Eq. 13.

$$\text{minimize } E[\gamma(\mathbf{R})] = \frac{1}{S} \sum_{s=1}^S \gamma_s(\mathbf{R}) \quad (12)$$

$$\gamma_s(\mathbf{R}) = w_1 \alpha_s(\mathbf{R}) - w_2 \beta_s(\mathbf{R}), \forall s \quad (13)$$

2. Constraints on the cost ratio $\alpha_s(\mathbf{R})$

For scenario $s \in \{1, \dots, S\}$, the number of members $n_{si}(t)$ in subgroup $i \in \{1, \dots, k\}$ at time t is the result of the sum of the internal transitions $f_{sji}(t)$ into subgroup i and the number of recruitments r_i into subgroup i (as expressed by Eq. 14). Let $\bar{f}_{ji}(t) = n_j(t-1)\hat{p}_{ji}$ be the expected internal flow from subgroup j to subgroup i . Then, by applying Eq. 2 for the internal transitions $\bar{f}_{ij}(t)$ and the number of recruitments equal to 0, the total cost $c(\mathbf{n}^c(t))$ can be expressed as $\sum_{i=1}^k c_i^p(t)n_i^c(t) + \sum_{i=1}^k \sum_{j=1, j \neq i}^k c_{ij}^f(t)\bar{f}_{ij}(t)$ (Eq. 3). Eq. 4 can be used to quantify $\alpha_s(\mathbf{R})$. Eq. 15 expresses the cost ratio $\alpha_s(\mathbf{R})$ for each scenario s .

$$n_{si}(t) = \sum_{j=1}^k f_{sji}(t) + r_i \quad \forall s, i \quad (14)$$

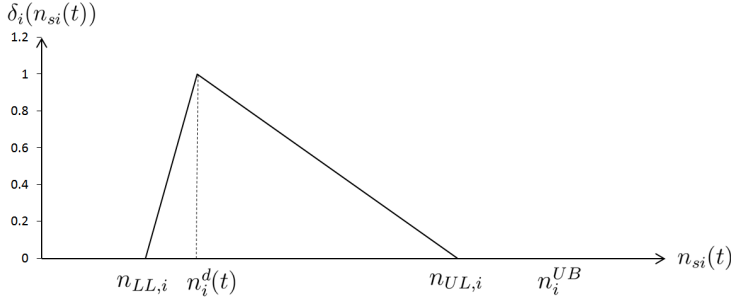


Fig. 1 Fuzzy triangular membership functions $\delta_i(n_{si}(t))$

$$\alpha_s(\mathbf{R}) = \frac{\sum_{i=1}^k c_i^p(t)n_{si}(t) + \sum_{i=1}^k \sum_{j=1, j \neq i}^k c_{ij}^f(t)f_{sij}(t) + \sum_{i=1}^k c_i^r(t)r_i}{\sum_{i=1}^k c_i^p(t)n_i^c(t) + \sum_{i=1}^k \sum_{j=1, j \neq i}^k c_{ij}^f(t)\bar{f}_{ij}(t)} \quad \forall s \quad (15)$$

3. Constraints on the desirability degree $\beta_s(\mathbf{R})$

According to Eqs. 5 and 6, the degree of desirability $\beta_s(\mathbf{R})$ regarding scenario $s \in \{1, \dots, S\}$ and recruitment vector \mathbf{R} is expressed by a min-operator and by fuzzy triangular membership functions $\delta_i(n_{si}(t))$ with $i \in \{1, \dots, k\}$. The functions $\delta_i(n_{si}(t))$ are piecewise linear. More specifically $\delta_i(n_{si}(t))$ is linear on each of the four segments: (1) $n_{si}(t) < n_{LL,i}$ (2) $n_{LL,i} \leq n_{si}(t) \leq n_i^d(t)$ (3) $n_i^d(t) \leq n_{si}(t) \leq n_{UL,i}$ and (4) $n_{si}(t) > n_{UL,i}$ (see example in Figure 1). A situation in which, for an index $i \in \{1, \dots, k\}$, the value of $n_{si}(t)$ belongs to the first or the fourth segment results in $\beta_s(\mathbf{R}) = 0$. The second segment corresponds to the increasing part of the fuzzy triangular function, and the third segment corresponds to the decreasing part of the fuzzy triangular function.

As in Williams (1999), we transform the piecewise linear membership function into mixed integer constraints. To this end, let us introduce binary variables q_{sil}^b (for $l \in \{1, \dots, 4\}$) and continuous variables q_{sil}^c (for $l \in \{1, \dots, 5\}$), in order to describe $\beta_s(\mathbf{R})$ as mixed integer constraints. Eqs. 16-18 denote the relation between q_{sil}^c and q_{sil}^b . Particularly, q_{sil}^c can only be greater than zero if $q_{sil}^b = 1$. Eq. 19 ensures that only one segment can be active at one time. Eqs. 20-21 denote that the i -th component of the personnel structure, $n_{si}(t)$, is a convex combination of the variables $n_{LL,i}$, $n_i^d(t)$, $n_{UL,i}$, and n_i^{UB} . The notation n_i^{UB} refers to an upper bound for the number of members in the i -th subgroup. Then, the degree of desirability as a result of the min-operator is represented in Eq. 22. Eq. 23 expresses that the values of the continuous variables q_{sil}^c are restricted to $[0, 1]$ and that q_{sil}^b are binary variables taking the values 0 and 1.

$$q_{si1}^c \leq q_{si1}^b \quad \forall s, i \quad (16)$$

$$q_{sil}^c \leq q_{si,l-1}^b + q_{sil}^b \quad \forall l \in \{2, 3, 4\}, \forall s, i \quad (17)$$

$$q_{si5}^c \leq q_{si4}^b \quad \forall s, i \quad (18)$$

$$\sum_{l=1}^4 q_{sil}^b = 1 \quad \forall s, i \quad (19)$$

$$n_{si}(t) = q_{si2}^c n_{LL,i} + q_{si3}^c n_i^d(t) + q_{si4}^c n_{UL,i} + q_{si5}^c n_i^{UB} \quad \forall s, i \quad (20)$$

$$\sum_{l=1}^5 q_{sil}^c = 1 \quad \forall s, i \quad (21)$$

$$\beta_s(\mathbf{R}) \leq q_{si3}^c \quad \forall s \quad (22)$$

$$q_{sil}^c \in [0, 1]; q_{sil}^b \in \{0, 1\} \quad \forall s, i, l \quad (23)$$

4. Real world restrictions on the model variables

We require $n_{si}(t)$ and r_i to be integer numbers to enhance the accuracy of the computations (Guerry 2008). Eq. 24 expresses the fact that the variables $n_{si}(t)$ and r_i are restricted to the set \mathbb{N} of integer numbers.

$$n_{si}(t), r_i \in \mathbb{N} \quad \forall s, i \quad (24)$$

4 Properties of the optimal recruitment strategy

In the previous section, we formulated a mixed integer program in order to simultaneously optimize the cost ratio and the desirability degree. In the current section, we study the properties of the optimal solution, in order to decrease the computation effort in solving the problem.

4.1 Determining an upper bound for the optimal value of the objective function

By calculating the value of the objective function for any arbitrary chosen recruitment vector, we can obtain an upper bound for the optimal value of the overall objective function $E[\gamma(\mathbf{R})]$. This enables us to tighten the solution space in which there can be searched for the optimal solution. Since it concerns a minimization problem, the optimal solution will result in a value of the objective function that is smaller or equal to the value of the objective function for this arbitrary chosen recruitment vector.

However, to determine an upper bound of the optimal value of the objective function, we suggest not to use an arbitrary chosen recruitment vector. Instead, an upper bound for the optimal value of the objective function $E[\gamma(\mathbf{R})]$ can be obtained by relaxing the formulation in Eqs. 12-24. In particular, instead of finding an optimal recruitment vector $\mathbf{R} = (r_1, \dots, r_l, \dots, r_k)$ regarding the k subgroups, we can decompose the problem by considering the subgroups one-by-one. In this way, the initial problem with k variables $r_1, \dots, r_l, \dots, r_k$ is first examined by focusing on the variables r_l one at a time.

Let personnel structure $\mathbf{n}_s^l(t)$ be the result of r_l recruitments in subgroup l and no recruitments in the other subgroups, i.e. $r_h = 0$ for $h \neq l$. According to Eq. 1, given scenario s , for $h \neq l$, the h -th component of vector $\mathbf{n}_s^l(t)$ equals $n_{sh}^l(t) = \sum_{j=1}^k f_{sjh}(t) = n_h^c(t)$ and the l -th component equal to $n_{sl}^l(t) = \sum_{j=1}^k f_{sjl}(t) + r_l$.

The number of recruitments r_l only affects the l -th component of the personnel structure $\mathbf{n}_s^l(t)$, being $n_{sl}^l(t)$. The extent in which the number $n_{sl}^l(t)$ differs from the

desirable number $n_i^d(t)$ is expressed by $\delta_i(n_{sl}^l(t))$ as defined in Eq. 5. The resulting cost ratio, on the other hand, is denoted by $E[\alpha(r_l)]$ and is defined as in Eq. 25 based on the recruitment vector \mathbf{R}^l , with l -th component equal to r_l and all the other components equal to 0:

$$E[\alpha(r_l)] = E[\alpha(\mathbf{R}^l)] \quad (25)$$

For each subgroup l , we can find the best possible value r_l^* for r_l by enumeration as in Eq. 26. The enumeration of r_l is restricted to integer valued numbers. Consecutively, an upper bound $E[\gamma(\mathbf{R})]^{UB}$ for the optimal value of the objective function $E[\gamma(\mathbf{R})]$ can be determined by considering the recruitment vector $\mathbf{R}^* = (r_l^*)$ that has for each subgroup l the best possible value r_l^* .

$$\begin{aligned} E[\gamma(\mathbf{R})]^{UB} &= E[\gamma(\mathbf{R}^*)] \\ \text{with } r_l^* &= \arg \min_{r_l} (w_1 E[\alpha(r_l)] - w_2 E[\beta_l(r_l)]) \\ E[\alpha(r_l)] &= \frac{1}{S} \sum_{s=1}^S \frac{\sum_{i=1}^k c_i^p(t) n_{si}^l(t) + \sum_{i=1}^k \sum_{j=1, j \neq i}^k c_{ij}^f(t) f_{sij}(t) + c_l^r(t) r_l}{\sum_{i=1}^k c_i^p(t) n_{si}^c(t) + \sum_{i=1}^k \sum_{j=1, j \neq i}^k c_{ij}^f(t) \bar{f}_{ij}(t)} \\ E[\beta_l(r_l)] &= \frac{1}{S} \sum_{s=1}^S \delta_l(n_{sl}^l(t)) \end{aligned} \quad (26)$$

4.2 Valid inequalities to enhance the MIP

In Section 4.2, we propose some valid inequalities on the number of recruitments r_i that restrict the feasible region without losing the global optimal solution, and that therefore can be used to tighten the MIP.

We present several valid inequalities in order to tighten the MIP in Eqs. 12-24. The valid inequalities are used to exclude some recruitment vectors \mathbf{R} from the solution space that are off-interest or proven to be not optimal. As a result, the solution space becomes smaller and the efficiency of searching the optimal recruitment vector is improved.

In what follows, the valid inequalities are classified into three classes and are formulated in the properties 1 to 3.

1. The lower and upper bound for the variable r_i based on the lower and upper limit of the fuzzy triangular function:

According to Eqs. 6-7, for $\mathbf{R} = (r_i)$ the desirability degree $\beta_s(\mathbf{R})$ is defined as $\min_i \delta_i(n_{si}(t))$ with $n_{si}(t) = \sum_{j=1}^k f_{sji}(t) + r_i$. We derive a lower and upper bound for the number of recruitments r_i based on the lower and upper limit of the fuzzy triangular function δ_i . A recruitment strategy with a desirability degree equal to 0 for all scenarios is off-interest. For this reason valid inequalities are examined for the values of r_i that satisfy $\delta_i\left(\sum_{j=1}^k f_{sji}(t) + r_i\right) > 0$ for at least one scenario s . In other words, we cut-off the number of recruitments r_i that results in $\delta_i(n_{si}(t)) = 0$ for all scenarios. This may neglect an optimal value of $E[\gamma(\mathbf{R})]$, but those corresponding recruitment vectors \mathbf{R} are off-interest since they have a desirability degree equal to zero. The resulting valid inequalities are formulated by Eq. 27 in Property 1.

Property 1 A recruitment vector $\mathbf{R} = (r_i)$ with non-zero desirability degree satisfies the following:

$$\begin{aligned} r_i^{LB^1} &\leq r_i \leq r_i^{UB^1} \\ \text{with } r_i^{LB^1} &= \min \{r_i | \exists s \in \{1, \dots, S\} \text{ with } \delta_i \left(\sum_{j=1}^k f_{sji}(t) + r_i \right) > 0\} \\ r_i^{UB^1} &= \max \{r_i | \exists s \in \{1, \dots, S\} \text{ with } \delta_i \left(\sum_{j=1}^k f_{sji}(t) + r_i \right) > 0\} \end{aligned} \quad (27)$$

The upper bound $r_i^{UB^1}$ can be used to tighten the value of n_i^{UB} in Eq. 20 by $n_i^{UB} = \max_s \sum_{j=1}^k f_{sji}(t) + r_i^{UB^1}$.

2. The lower and upper bound for the variable r_i based on $E[\gamma(\mathbf{R})]^{UB}$:
When we have the value of $E[\gamma(\mathbf{R})]^{UB}$ from Eq. 26, we can derive additional lower and upper bounds for r_i . In particular, we cut-off the number of recruitments r_i that are not potential in minimizing $E[\gamma(\mathbf{R})]$. This can be achieved by excluding the number of recruitments r_i that have for $w_1 E[\alpha(r_i)] - w_2 E[\beta_i(r_i)]$ a value greater than $E[\gamma(\mathbf{R})]^{UB}$. The inequalities are described by Eq. 28 in Property 2.

Property 2 A recruitment vector $\mathbf{R} = (r_i)$ that potentially minimizes $E[\gamma(\mathbf{R})]$, satisfies the following:

$$\begin{aligned} r_i^{LB^2} &\leq r_i \leq r_i^{UB^2} \\ \text{with } r_i^{LB^2} &= \min \{r_i | w_1 E[\alpha(r_i)] - w_2 E[\beta_i(r_i)] \leq E[\gamma(\mathbf{R})]^{UB}\} \\ r_i^{UB^2} &= \max \{r_i | w_1 E[\alpha(r_i)] - w_2 E[\beta_i(r_i)] \leq E[\gamma(\mathbf{R})]^{UB}\} \end{aligned} \quad (28)$$

Indeed, a recruitment strategy $\mathbf{R} = (r_i)$, which has for at least one subgroup i a value for $w_1 E[\alpha(r_i)] - w_2 E[\beta_i(r_i)]$ greater than $E[\gamma(\mathbf{R})]^{UB}$, can not minimize $E[\gamma(\mathbf{R})]$ since it satisfies:

$$E[\gamma(\mathbf{R})] = w_1 E[\alpha(\mathbf{R})] - w_2 E[\beta(\mathbf{R})] \geq w_1 E[\alpha(r_i)] - w_2 E[\beta_i(r_i)] > E[\gamma(\mathbf{R})]^{UB} \quad (29)$$

The reason that Eq. 29 holds is twofold: (1) The computation of $E[\alpha(r_i)]$ (according to Eq. 25) is based on the recruitment vector \mathbf{R}^i , with i -th component equal to r_i and all the other components r_j equal to 0. Furthermore the cost ratio increases when r_j (for $j \neq i$) increases and therefore $E[\alpha(r_i)] \leq E[\alpha(\mathbf{R})]$. (2) For each scenario s the value of $\beta_s(\mathbf{R})$ corresponds with the minimum over the subgroups i of $\delta_i(n_{si}(t))$ (according to Eq. 6). Besides $E[\beta_i(r_i)] = \frac{1}{S} \sum_{s=1}^S \delta_i(n_{si}^i(t))$ (according to Eq. 26) and consequently $E[\beta_i(r_i)] \geq E[\beta(\mathbf{R})]$.

3. Scenarios that result in a same value $n_{si}(t)$:

The scenario s is characterized by the matrix $\mathbf{F}_s(t) = (f_{sij}(t))$ of internal transitions. When we consider S scenarios, there may be some scenarios that result in a same value for $n_{si}(t)$. In the situation that the number of scenarios S is greater than $n_{UL,i} - n_{LL,i}$, there are two interesting situations: (1) some scenarios can result in values for $n_{si}(t)$ outside the interval determined by the lower limit $n_{LL,i}$ and the upper limit $n_{UL,i}$, and (2) some scenarios can result in a same value for $n_{si}(t)$. In the first situation, we can fix the value $\delta_i(n_i(t))$ to be zero. This is more or less similar to the first class of valid inequalities. In the second situation, we can add some valid cuts that enforce the value of some variables to be the same.

Table 1 Historical dataset (De Feyter & Guerry 2009)

t	$n_1(t)$	$n_2(t)$	$n_3(t)$	$w_1(t)$	$w_2(t)$	$w_3(t)$	n_{12}	n_{13}	n_{21}	n_{23}	n_{31}	n_{32}
1990	250	150	100	13	15	9	20	12	10	15	8	5
1991	243	163	118	10	20	14	22	14	10	19	5	8
1992	238	172	132	9	14	14	29	12	9	17	7	7
1993	236	179	145	14	16	11	22	13	8	18	7	8
1994	235	185	156	15	14	17	24	13	9	17	9	8
1995	235	190	165	16	15	13	20	15	12	19	5	8
1996	236	194	173	14	20	20	28	10	13	19	9	6
1997	237	198	179	11	22	16	28	10	15	18	8	7
1998	238	201	185	11	23	20	26	16	16	24	9	9
1999	240	204	190	10	20	20	24	18	11	20	9	10

Let $\mathbb{H}_{si} = \{v \in \{1, \dots, S\} \mid \sum_{j=1}^k f_{sji}(t) = \sum_{j=1}^k f_{vji}(t)\}$ be the set of scenarios that result in a same value for $n_{si}(t)$, the number of employees in subgroup i . Then the valid cuts are represented in Eq. 30.

Property 3 For $\mathbb{H}_{si} = \{v \in \{1, \dots, S\} \mid \sum_{j=1}^k f_{sji}(t) = \sum_{j=1}^k f_{vji}(t)\}$:

$$\begin{aligned}
 n_{si}(t) &= n_{vi}(t), \forall v \in \mathbb{H}_{si} \\
 q_{sil}^c &= q_{vil}^c, \forall v \in \mathbb{H}_{si}, l \in \{1, \dots, 5\} \\
 q_{sil}^b &= q_{vil}^b, \forall v \in \mathbb{H}_{si}, l \in \{1, \dots, 4\}
 \end{aligned} \tag{30}$$

In this way the number of variables in the model are reduced and the computation time will decrease.

5 Illustration

The data for the illustration in this section is adopted from De Feyter & Guerry (2009). It concerns a stochastic manpower system with three subgroups of employees, i.e. $k = 3$. The historical dataset is provided in Table 1. The current personnel structure is $n(\tilde{t}-1) = (200 \ 275 \ 225)$ and the desired personnel structure is $n^d(\tilde{t}) = (200 \ 260 \ 230)$. The matrix $\hat{\mathbf{P}}$ represents the maximum likelihood estimations of the transition probabilities under Markov assumptions (Bartholomew et al. 1991). The aim is to find a recruitment strategy \mathbf{R} that optimizes cost-effectiveness.

$$\hat{\mathbf{P}} = \begin{pmatrix} 0.791 & 0.102 & 0.056 \\ 0.062 & 0.739 & 0.102 \\ 0.049 & 0.049 & 0.802 \end{pmatrix}$$

In order to evaluate a particular recruitment strategy, we set the lower and the upper limits of the number of employees in the three subgroups as $(n_{LL,i}) = (195 \ 255 \ 225)$ and $(n_{UL,i}) = (220 \ 280 \ 250)$. The operational costs are given by $(c_i^p(\tilde{t})) = (1 \ 1.5 \ 2)$. The costs due to the employee flows are as follows: $c_{ij}^f(\tilde{t}) = 0$, $\forall i, j \in \{1, 2, 3\}$ and $(c_i^r(\tilde{t})) = (0.2 \ 0.1 \ 0.3)$. Further, we consider the cost and desirability criteria as being equally important ($w_1 = w_2 = 1$).

In comparing the cost-effectiveness of recruitment strategies, we considered 1000 possible scenarios for the internal flows, generated by the bootstrapping

method (Rubinstein & Kroese 2008). To construct a scenario $\mathbf{F}_s = (f_{sij})$, for each subgroup i we randomly selected a time interval $[t-1, t)$ from Table 1 and calculated the corresponding flows $f_{sij} = n_i(\tilde{t}-1) \frac{n_{ij}(t-1, \tilde{t})}{n_i(t-1)}$.

We used the CPLEX solver for solving the mixed integer program, as formulated in Section 3 (Eqs. 12-24), but the solver could not obtain a feasible solution in one hour computation time. By using the bounds for the values of the objective function as well as the valid inequalities, as presented in Section 4 (Eqs. 27-30), the solver was able to find the optimal solution within 210 seconds. We obtain the optimal recruitment vector $\mathbf{R} = (17 \ 28 \ 16)$. For this solution, the expected cost ratio $E[\alpha(\mathbf{n}(\tilde{t}))] = 1.105$, the expected desirability degree $E[\beta(\mathbf{n}(\tilde{t}))] = 0.338$ and the overall degree $E[\gamma(\mathbf{n}(\tilde{t}))] = 0.767$. Recruiting 17 employees in subgroup 1, 28 in subgroup 2 and 16 in subgroup 3 results in optimal cost-effectiveness.

6 Discussion and conclusions

The present paper introduces a mathematical model to optimize cost-effectiveness in a stochastic manpower planning system under control by recruitment. For evaluating the cost-effectiveness of a recruitment strategy, we propose to simultaneously consider two goals. On the one hand, the recruitment strategy should result in a personnel structure as similar as possible to the desired structure. On the other hand, we aim for cost minimization. To this end, we analytically formulated two criteria, on which alternative recruitment strategies should simultaneously be evaluated, i.e. the cost ratio and the desirability degree. The values on both criteria depend on the resulting personnel structure, which in turn is influenced by the wastage and internal flows in the manpower system. In a stochastic environment, those model parameters are unknown. Consequently, in taking recruitment decisions, our model evaluates the expected values of the cost ratio and the desirability degree. To estimate those expected values, as in real-world applications, we suggest to take a scenario approach. Our method considers a number of possible scenarios for the internal transitions in the manpower system. The scenario approach allows us to formulate the multi-objective optimization problem as a mixed integer program. Further, we showed that the optimal recruitment strategy satisfies some properties, which are interesting to decrease the computation effort. Our experiments show that using those properties is helpful in quickly finding a solution for a problem that otherwise could not be solved within a reasonable time.

Since the focus is on cost-effectiveness, our model and optimization algorithm can be valuable for a manpower planner who, in the current economic environment, is faced with high competition and the accompanying pressure for cost reduction. Further, our contribution helps decision makers to face the uncertainty in employee workplace behavior (Ugwuowo & McClean 2000). As in most previous work in Markov manpower planning, our model assumes wastage to be an uncontrollable model parameter, which is subject to uncertainty. Although in other fields, like work and organization psychology and business management research, many efforts are made to understand and control employee workplace behavior, voluntary wastage still depends in a great extent on internal and external factors beyond management control, like the complexity and dynamics on the current labor market. Next to wastage, our model assumes internal personnel flows to be random variables beyond management control. Indeed, depending on the way in

which the subgroups in the manpower system are defined (De Feyter 2006), internal flows are to a greater or lesser extent uncertain and beyond management control. As explained in the introduction, there are indeed important arguments against changing existing internal transition patterns.

The scenario approach, used in the current paper, offers a flexible tool for the decision makers to cope with the stochastic nature in current manpower planning. It allows the decision maker to take into account a (high) number of possible outcomes for the unknown model parameters. On the one hand, the manpower planner can identify (likely) scenarios based on his or her expectations of management interventions, explicit and tacit knowledge or even intuition. On the other hand, to develop scenarios, the decision maker could rely on stochastic methods. As illustrated in this paper, on the basis of an historical dataset, scenarios can be generated by the bootstrapping method. Also Monte Carlo methods can be used to randomly generate scenarios (Rubinstein & Kroese 2008), on the basis of maximum likelihood estimations for the transition probabilities in stochastic manpower systems (Bartholomew et al. 1991). Furthermore, based on previous work (Bartholomew 1975; De Feyter & Guerry 2009), in evaluating the expected cost-effectiveness of a recruitment strategy, it is even possible to consider probability distributions on the generated scenarios.

Although voluntary wastage is indeed strongly beyond management control, in practice, layoffs can also be part of organizations' personnel strategy. Besides recruitment, future work could consider layoffs to optimize cost-effectiveness. Likewise, recent previous work (Komarudin et al. 2015; Komarudin et al. 2016) suggests to allow but minimize changes to the internal transition patterns. Consequently, also internal flows partly become decision variables. Future work could integrate this idea in a stochastic model for optimizing cost-effectiveness.

Another interesting avenue for further research is considering a multiple period time frame in manpower planning. The optimization problem considered in the current paper is restricted to identifying the most cost-effective recruitment strategy for one single manpower planning period. Nevertheless, a manpower planner might want to take a long term perspective. The current recruitment strategy indeed strongly affects the cost-effectiveness in subsequent time periods. Future work could extend our model by incorporating desired personnel structures in subsequent time periods as objectives on the one hand, and personnel strategies in subsequent time periods as decision variables on the other hand. Such a multiple period optimization problem could be solved as a dynamic program, in which, in each step, the problem can be formulated as a mixed integer program.

References

- Bartholomew, D. J. (1975). Errors of prediction for Markov chain models. *Journal of the Royal Statistical Society Series B*, 37(3), 444–456.
- Bartholomew, D. J. (1977). Maintaining a grade or age structure in a stochastic environment. *Advances in Applied Probability*, 9(1), 1–17.
- Bartholomew, D. J., Forbes, A. F., & McClean, S. I. (1991). *Statistical techniques for manpower planning*. Chichester: John Wiley & Sons, second edition.

- Bilgin, B., De Causmaecker, P., Rossie, B., & Vanden Berghe, G. (2012). Local search neighbourhoods for dealing with a novel nurse rostering model. *Annals of Operations Research*, 194(1), 33–57.
- Burke, E., De Causmaecker, P., Vanden Berghe, G., & Van Landeghem, H. (2004). The state of the art of nurse rostering. *Journal of Scheduling*, 7(6), 441–499.
- Chattopadhyay, A. K. & Gupta, A. (2007). A stochastic manpower planning model under varying class sizes. *Annals of Operations Research*, 155(1), 41–49.
- Davies, G. S. (1975). Maintainability of structures in Markov chain models under recruitment control. *Journal of Applied Probability*, 12(2), 376–382.
- Davies, G. S. (1982). Control of grade sizes in a partially stochastic Markov manpower model. *Journal of Applied Probability*, 19(2), 439–443.
- De Feyter, T. (2006). Modelling heterogeneity in manpower planning: dividing the personnel system into more homogeneous subgroups. *Applied Stochastic Models in Business and Industry*, 22(4), 321–334.
- De Feyter, T. (2007). Modeling mixed push and pull promotion flows in manpower planning. *Annals of Operations Research*, 155(1), 25–39.
- De Feyter, T. & Guerry, M.-A. (2009). Evaluating recruitment strategies using fuzzy set theory in stochastic manpower planning. *Stochastic Analysis and Applications*, 27(6), 1148–1162.
- De Feyter, T. & Guerry, M.-A. (2011). Markov models in manpower planning: a review. In J. Varela & S. Acuña (Eds.), *Handbook Of Optimization Theory: Decision Analysis and Application*. New York: Nova Science Publishers.
- Dimitriou, V. A., Georgiou, A. C., & Tsantas, N. (2013). The multivariate non-homogeneous Markov manpower system in a departmental mobility framework. *European Journal of Operational Research*, 228(1), 112–121.
- Ernst, A. T., Jiang, H., Krishnamoorthy, M., Owens, B., & Sier, D. (2004). An annotated bibliography of personnel scheduling and rostering. *Annals of Operations Research*, 127(1-4), 21–144.
- Georgiou, A. C. & Tsantas, N. (2002). Modelling recruitment training in mathematical human resource planning. *Applied Stochastic Models in Business and Industry*, 18(1), 53–74.
- Georgiou, A. C. & Vassiliou, P.-C. G. (1992). Periodicity of asymptotically attainable structures in nonhomogeneous Markov systems. *Linear Algebra and its Applications*, 176, 137–174.
- Georgiou, A. C. & Vassiliou, P.-C. G. (1997). Cost models in nonhomogeneous Markov systems. *European Journal of Operational Research*, 100(1), 81–96.
- Guerry, M.-A. (1993). The probability of attaining a structure in a partially stochastic model. *Advances in Applied Probability*, 25(4), 818–824.
- Guerry, M.-A. (1999). Using fuzzy sets in manpower planning. *Journal of Applied Probability*, 36(1), 155–162.
- Guerry, M.-A. (2008). On the evolution of stock vectors in a deterministic integer-valued Markov system. *Linear Algebra and its Applications*, 429(8-9), 1944–1953.
- Guerry, M.-A. & De Feyter, T. (2011). An extended and tractable approach on the convergence problem of the mixed push-pull manpower model. *Applied Mathematics and Computation*, 217(22), 9062–9071.
- Guerry, M.-A. & De Feyter, T. (2012). Optimal recruitment strategies in a multi-level manpower planning model. *Journal of the Operational Research Society*, 63(7), 931–940.

- Khoong, C. M. (1996). An integrated system framework and analysis methodology for manpower planning. *International Journal of Manpower*, 17(1), 26–46.
- Klehe, U.-C., Zikic, J., Van Vianen, A. E. M., & De Pater, I. E. (2011). Career adaptability, turnover and loyalty during organizational downsizing. *Journal of Vocational Behavior*, 79(1), 217–229.
- Komarudin, De Feyter, T., Guerry, M.-A., & Vanden Berghe, G. (2016). Balancing desirability and promotion steadiness in partially stochastic manpower planning systems. *Communications in Statistics: Theory and Methods*, 45(6), 1805–1818.
- Komarudin, Guerry, M.-A., De Feyter, T., & Vanden Berghe, G. (2013). The roster quality staffing problem - A methodology for improving the roster quality by modifying the personnel structure. *European Journal of Operational Research*, 230(3), 551–562.
- Komarudin, Guerry, M.-A., Vanden Berghe, G., & De Feyter, T. (2015). Balancing attainability, desirability and promotion steadiness in manpower planning systems. *Journal of the Operational Research Society*, 66(12), 2004–2014.
- McClellan, S. I. (1991). Manpower planning models and their estimation. *European Journal of Operational Research*, 51(2), 179–187.
- McClellan, S. I., Montgomery, E., & Ugwuowo, F. I. (1997). Non-homogeneous continuous-time Markov and semi-Markov manpower models. *Applied Stochastic Models and Data Analysis*, 13(3-4), 191–198.
- Nilakantan, K. & Raghavendra, B. G. (2005). Control aspects in proportionality Markov manpower systems. *Applied Mathematical Modelling*, 29(1), 85–116.
- Ozcan, Y. (2009). *Quantitative methods in health care management: techniques and applications*. San Francisco: Jossey-Bass, second edition.
- Papadopoulou, A. A. & Tsaklidis, G. (2007). Some reward paths in semi-Markov models with stochastic selection of the transition probabilities. *Methodology and Computing in Applied Probability*, 9(3), 399–411.
- Papadopoulou, A. A. & Vassiliou, P.-C. G. (1994). Asymptotic behavior of non-homogeneous semi-Markov systems. *Linear Algebra and its Applications*, 210, 153–198.
- Rubinstein, R. Y. & Kroese, D. P. (2008). *Simulation and the Monte Carlo method*. New York: John Wiley & Sons, second edition.
- Shapiro, A., Dentcheva, D., & Ruszczyński, A. (2009). *Lectures on stochastic programming: modeling and theory*. Philadelphia: SIAM.
- Smet, P., Bilgin, B., De Causmaecker, P., & Vanden Berghe, G. (2014). Modelling and evaluation issues in nurse rostering. *Annals of Operations Research*, 218(1), 303–326.
- Tsantas, N. & Georgiou, A. C. (1997). Partial maintainability of a population model in a stochastic environment. *Applied Stochastic Models and Data Analysis*, 13(3-4), 183–189.
- Ugwuowo, F. I. & McClellan, S. I. (2000). Modelling heterogeneity in a manpower system: a review. *Applied Stochastic Models in Business and Industry*, 16(2), 99–110.
- Vajda, S. (1978). *Mathematics of manpower planning*. London: John Wiley & Sons.
- Vassiliou, P.-C. G. (1981). On the limiting behaviour of a nonhomogeneous Markov chain model in manpower systems. *Biometrika*, 68(2), 557–561.
- Vassiliou, P.-C. G. (1982). Asymptotic-behavior of Markov systems. *Journal of Applied Probability*, 19(4), 851–857.

- Vassiliou, P.-C. G. (1997). The evolution of the theory of non-homogeneous Markov systems. *Applied Stochastic Models and Data Analysis*, 13(3-4), 159–176.
- Vassiliou, P.-C. G. & Georgiou, A. C. (1990). Asymptotically attainable structures in nonhomogeneous Markov systems. *Operations Research*, 38(3), 537–545.
- Vassiliou, P.-C. G., Georgiou, A. C., & Tsantas, N. (1990). Control of asymptotic variability in nonhomogeneous Markov systems. *Journal of Applied Probability*, 27(4), 756–766.
- Vassiliou, P.-C. G. & Gerontidis, I. (1985). Variances and covariances of the grade sizes in manpower systems. *Journal of Applied Probability*, 22(3), 583–597.
- Vassiliou, P.-C. G. & Papadopoulou, A. A. (1992). Non-homogeneous semi-Markov systems and maintainability of the state sizes. *Journal of Applied Probability*, 29(3), 519–534.
- Vassiliou, P.-C. G. & Tsantas, N. (1984). Maintainability of structures in non-homogeneous Markov systems under cyclic behavior and input control. *SIAM Journal on Applied Mathematics*, 44(5), 1014–1022.
- Williams, H. P. (1999). *Model building in mathematical programming*. London: John Wiley & Sons, 4th edition.
- Yadavalli, V. S. S., Natarajan, R., & Udayabhaskaran, S. (2002). Time dependent behavior of stochastic models of manpower system - Impact of pressure on promotion. *Stochastic Analysis and Applications*, 20(4), 863–882.