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Characterization of the full viscoelastic tensor of anisotropic multilayered material plates by means of the Ultrasonic Polar Scan

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Abstract
Recent studies have shown that inversion algorithms using data from the Ultrasonic Polar Scan (UPS) can be successfully used to characterize relevant mechanical properties of solid materials (e.g. aluminum) or statically homogenized composites (e.g. unidirectional carbon/epoxy). Despite the success of this procedure for large classes of materials, the current methodology is not applicable for composites where the thickness of the laminae is large compared to the wavelength of the impinging wave, thereby violating the static homogenization theory. This is for instance the case in a carbon/epoxy cross-ply laminate. To extend the utilization of the technique, we first investigated the performance of our previously developed two-stage inversion algorithm to determine the homogenized effective stiffness tensor from simulated harmonic UPS data on multilayered media in the low frequency range. The resulting frequency dependent viscoelastic tensors are compared to the effective moduli obtained by means of a Floquet wave based approach. In the second part of the study, the stiffness tensor of the individual lamina of a multilayered material is probed, while simultaneously inferring information on the stacking sequence (orientations) and the lamina thickness.

Keywords
Ultrasonic Polar Scan, homogenization, viscoelastic tensor characterization, composites

I. Introduction
Currently, the full exploitation of novel composite material is inhibited because of their complex nature, mainly expressed by way of a high degree of anisotropy, making it difficult to either characterize the material properties, or to asses internal defects and the origins of the observed damage. In response to this complexity, several authors developed non-destructive characterization techniques, mainly based on the propagation of ultrasonic, to assess the underlying structures. In order to overcome the shortcomings of bulk wave based techniques[1]-[4], the present team of authors developed a novel inversion technique based on measurements using the Ultrasonic Polar Scan (UPS). During a UPS experiment, a specific material spot is insonified by an ultrasonic beam (harmonic or pulsed signal, 1-5MHz range) at multiple combinations of out-of-plane and in-plane incident angles $\Psi(\theta,\phi)$ on a hemisphere centered above the spot (Fig. 1a). The amplitude, time-of-flight or phase are then extracted from the recorded transmitted signals and mapped on a polar diagram, with θ and ϕ the radial and circumferential direction respectively, resulting in a fingerprint of the investigated material (Fig 1b).[5] In combination with a two-step inversion algorithm, this new technique has proven successful in the extraction of the viscoelastic tensor from experimental UPS recordings on aluminum and unidirectional carbon/epoxy plates.[6] However, the methodology is currently only applicable on homogeneous materials or materials with a easily homogenized stiffness tensor, such as unidirectional composites for which the homogenization breakdown is only noticeable in the high frequency range (50MHz). For carbon/epoxy cross-ply laminates on the other hand, where each individual lamina corresponds to a rotated C-tensor and has a small thickness, the limit of the homogenization typically lies in the region of 1.5MHz, exactly in the range of frequencies used by the UPS method. Representing these materials by a single (homogenized) stiffness tensor is therefore only applicable for measurements below this frequency. Moreover, in the region of the breakdown frequency, the stiffness tensor tends to be highly dynamic ($C_{ijkl}(\omega) = C^R_{ijkl}(\omega) - iC^I_{ijkl}(\omega)$), implying that only UPS measurements using harmonic input signals can be used.[7], [8] For inversions based on broadband inputs, the breakdown frequency limit will be most likely violated and, in addition, the actual dynamic relation should be explicitly known.

In this contribution, we present a numerical UPS data inversion study on a carbon/epoxy cross-ply, from which, in a first case-study, we will extract the homogenized tensor for several frequencies and subsequently compare the results to the inversion based on Floquet wave propagation. In the second case-study, we use the same cross-ply simulation to simultaneously extract the single layer stiffness tensor and the corresponding stacking sequence and laminae thicknesses.

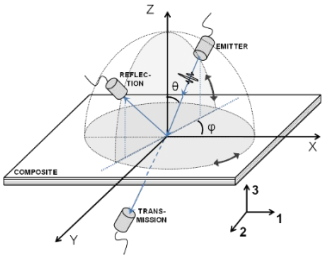


Figure 1a: Graphical representation of the UPS setup.

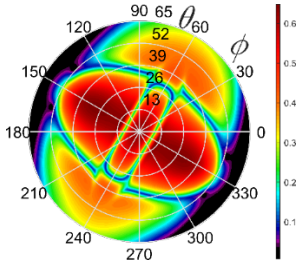


Figure 1b: Pulsed UPS experiment on a UD C/E composite plate of thickness 1mm.

II. Numerical modeling

In order to model the scattering coefficients of composite plates, we consider a cross-ply laminated plate consisting of homogeneous laminae with interfaces parallel to the surfaces. From the general theory of wave propagation in homogeneous anisotropic media it is known that the displacement field in each ply can be represented by a superposition of six fundamental waves for which the polarization vector and normal wavenumber component are obtained via the Christoffel equation.[9] For laminae having at least orthotropic symmetry, we can couple the six waves in three groups of two, where one set represents the upwards propagating waves and the other set the downwards ones with the same normal wavenumber components but opposite sign. Doing so, the displacement ($u = (u_1, u_2, u_3)$) and normal stress ($\sigma = (\sigma_{33}, \sigma_{23}, \sigma_{13})$) field can be represented by,

$$\begin{bmatrix} u \\ \sigma \end{bmatrix} = \begin{bmatrix} P^+ & P^- \\ D^+ & D^- \end{bmatrix} \begin{bmatrix} E^+ & 0 \\ 0 & E^- \end{bmatrix} \begin{bmatrix} A^+ \\ A^- \end{bmatrix} \quad (1)$$

where the '+' and '-' notation respectively denotes the upward and downward going waves. The P, D and E matrices correspond to the polarization vectors, normal stress components and the propagator matrix. The A vectors contain the amplitudes of the fundamental wave. These matrices can be calculated for each layer individually, leaving only the coupling to be performed by means of the boundary conditions at the interfaces. Here, we used the hybrid matrix method which is known to be unconditionally stable.[10], [11] In each layer, the hybrid matrix links the displacement at the top interface and the normal stress at the bottom interface of the layer to the displacement and normal stress at respectively the bottom and top surface, thereby eliminating the unknown A vectors of the layer. Iteration over all layers while incorporating the boundary conditions results in a general hybrid matrix for the composite structures and provides a model for the scattering coefficients of the composite plate. This calculation is repeated for all combinations of out-of-plane and in-plane incident angles $\Psi(\theta, \phi)$.

The hybrid matrix method also provides information about the Floquet waves which can be identified as the bulk waves travelling inside a homogenized plate. The potential Floquet modes are obtained by solving a generalized eigenvalue problem. If the value of the normal wavenumber component of a Floquet mode is imaginary the mode does not travel, whereas in the case of a real value the mode is propagating. Calculating the Floquet spectrum reveals a complex structure of bands with alternating propagating and non-propagating modes, where the true behavior of a homogeneous material can only be observed for frequencies smaller than the first Floquet stop-band. Therefore, we can use this mark as the limit frequency of the homogenization validity in the inversion.

III. Inversion procedure

For the inversions based on the UPS data we employ a two-stage characterization procedure. During the first stage of the inversion the squared error function between simulated and experimental UPS phase data is minimized to arrive at an estimation of the elasticity constants (nine independent variables). Indeed, phase data is linked to the wave speed which is mainly defined by the material elasticity and to a lesser extent to the viscosity. The minimization is

performed by means of a particle swarm optimizer as this method is capable of efficiently minimizing objective functions with a considerable amount of inversion variables. We also include an additional inversion parameter which measures the deviation between the experimental and material symmetry frame. The second part of the inversion procedure uses the UPS amplitude data to infer information on both the viscosity and elasticity. In contrast to the phase data, the amplitude data is mainly shaped by the viscosity as the amplitude level is linked to the amount of energy dissipation. The estimation of the elasticity from the first stage is used to reduce the size of the search space, making the inversion procedure more efficient.

In the next section, the inversion methodology is applied to two case studies on a carbon/epoxy cross-ply laminated plate. First, we perform an inversion assuming the plate is a single homogeneous layer, hence omitting the multilayer part in the hybrid matrix. We compare the results to the Floquet wave inversion by matching the Floquet slowness curves with those of the bulk wave of a homogeneous material. Note however that this procedure gives no information on the viscosity, nor on the out-of-plane constants e.g. σ_{33}. In the second study, the full hybrid matrix method is employed as we are simulating the multilayered character of the composite structure. Moreover, we are adding additional parameters for the laminae thickness and stacking sequence.

IV. Homogenization study

We employed the two-stage inversion procedure on data from a UPS simulation (virtual experiment) of a 1mm thick cross-ply material with sequence $[0,90]_S$, with each layer having equal thickness (250µm) and the same underlying C-tensor. The inversion for a homogenized C-tensor is performed at several operating frequencies below the homogenization limit (<1.5MHz) to assess the dynamic property of the homogenized tensor. The results of the two-stage approach (200 runs) and of the Floquet inversions for selected elastic constants are given in figure 2 alongside the static homogenized constants. As viscosity values cannot be obtained through Floquet wave inversions they are left out of the analysis. The graphs clearly illustrate the dynamic nature of the elastic constants. Even though clear differences between the UPS and Floquet inversion are noticeable, more confidence is found in the UPS inversions as it exactly matches the static homogenized tensor at low frequencies and it automatically includes adjustments stemming from the viscosity. The larger errors observed for the C_{44} parameter at low frequencies can be attributed to the difficult excitation of the waves governed by these parameters.



Figure 2: Inversion results for selected (homogenized) elastic C-tensor constants, C_{11} and C_{44}, as function of frequency employing either the UPS two-stage approach (red, with statistics for 200 runs) or the Floquet wave based procedure (blue) on a 1 mm thick cross-ply material. The static homogenized constants are shown in black.

V. Single layer extraction

In this section we employed the two-stage approach using the full hybrid matrix method at its core to obtain the C-tensor of a single lamina in the same cross-ply structure that we considered in the previous section. As all laminae have the same C-tensor but oriented differently, we can invert for only one C-tensor. Further, as the central 90° layers are adjoining and have the same C-tensor, we can add them together. This implies that we only have three layers to model during the inversion. Aside from the C-tensor inversion, we also added three parameters for the orientations and two for the thickness (the third thickness can be inferred from the total thickness). Note that these parameters are introduced as deviations from an expected value. For the orientation for instance, the expected values are 0° for the top and bottom layer and 90° for the central layer. In the virtual experiment however, we have added an overall shift of 23 degrees to the forward simulation, thus the orientation 'deviation'

parameter upon inversion should give 23° as an output for all layers. For the thicknesses, the expected values are 250µm for the top and bottom layer and 500µm for the central one. The overall results for the inversions are given in table 1 and show that the C-tensor of a single lamina is found to a high degree of satisfaction and confidence, as the relative deviation from the input parameters is below 0.08±0.06% for the elasticity and 1.80±1.21% for the viscosity. For the stacking sequence, we find a value for the deviation from the expected orientation values of 23° for all layers which is an excellent match. Finally, for the thickness inversions we find a mean 'thickness deviation' value close to zero, in agreement with the expectations. The large variance for the top and bottom layer (both are linked to each other) can probably be attributed to the symmetry present in the structure, where the top and bottom layer are equal.

Table 1: Inversion results of the UPS single lamina C-tensor performed on a cross-ply $[0,90]_S$ simulation. The stacking sequence and thickness (expressed as deviations from an expected value) are also inverted for. The inversion are performed 200 times for statistics.

Variable Name	Variable Value (GPa , ° or µm)	Variable Variance (GPa, ° or µm)
C11	122.701-8.631i	0.154-0.179i
C12	6.58-0.472i	0.085-0.097i
C13	6.56-0.443i	0.037-0.084i
C22	13.459-0.964i	0.026-0.069i
C23	6.547-0.472i	0.028-0.038i
C33	13.449-0.937i	0.067-0.022i
C44	3.396-0.234i	0.007-0.012i
C55	5.859-0.418i	0.009-0.019i
C66	6.25-0.438i	0.003-0.005i
phie_1	23.026	0.365
phie_2	23	0.009
phie_3	22.975	0.368
d_1	0.095	1.653
d_2	0.134	0.461
d_3	-0.229	1.728

VI. Conclusion

We have extended the two-stage inversion procedure based on UPS data to include the dynamic homogenization characterization of cross-ply like materials. A numerical test case on a harmonic UPS simulation of a C/E cross-ply $[0,90]_S$ showed that the UPS technique is capable of obtaining the dynamic tensor, and that it is superior to inversions based on Floquet waves. However, large deviations may occur at low frequency for certain elastic parameters, due to the difficult excitation of the corresponding waves. In addition, a numerical test-case on the same $[0,90]_S$ composite has been successfully employed to extract the stiffness tensor of a single layer together with the stacking sequence and thickness of all individual laminae.

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