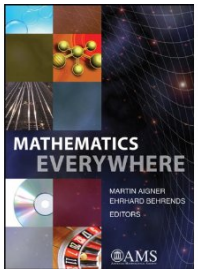
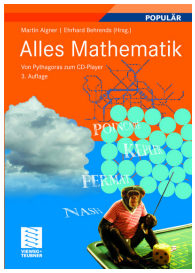
Mathematics Everywhere *Martin Aigner and Ehrhard Behrends (eds.)*, AMS, 2010 (xiv+330 p.), soft cover, ISBN 978-0-8218-4349-9.


M. Aigner






E. Behrends

The Urania Society in Berlin has a history that goes back to Alexander von Humboldt who gave in 1827/1828 public "cosmos lectures" intended for a general public. Urania became a formal society in 1888 with in its statutes the paradigm of "Spreading the knowledge, achievements, and joy of (the 'new') Sciences". Today it has over 2000 members and is one of the oldest and largest non-profit societies residing in Berlin.

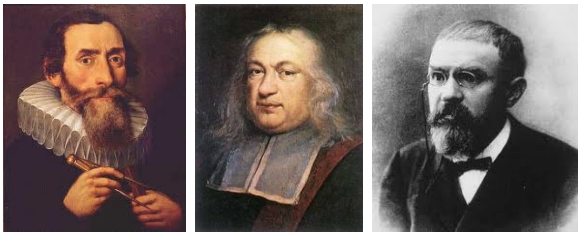


Urania, Berlin

Besides a successful film-festival, and many other activities, one of its initiatives is, as it was at the start, still to organize generally understandable lectures concerning current questions of nature and *Geisteswissenschaften*. By 1990, the lectures treated all kind of subjects but "there wasn't a single one dealing with mathematics". The classical false premises were used: that mathematics are "too abstract, too dry, and too hard" for the layman. In a world where "mathematics are everywhere", it was decided that this should change. By 2000 some fifty lectures had been discussing mathematical topics, and although they were about mathematics (and that includes also the equations and formulas!) they were presented in a lively way, explaining sometimes difficult mathematical topics using a step by step approach and illustrating them in an environment of every-day life or presenting them in a story-telling format or a gaming situation. The original German version of this book was entitled *Alles Mathematik* and appeared in 2000 (*Vieweg*). It contained a selection of elaborated texts of some of the lectures that were given in the Urania initiative. In subsequent second (2002) and third (2008) German editions new lectures were added.

This English translation contains 21 chapters, each one written by well known researchers. They are organized in three groups: 'Case Studies', 'Current Topics', and 'The Central Theme'. There is a 'Prologue' by a science journalist (G. von Randow) and and 'Epilogue' by a mathematician-philosopher (Ph. J. Davis).

The prologue has an author that is obviously convinced of the 'joy of mathematics", and that mathematics gains a booming popularity in party-conversations. It is the reviewer's experience that this is still the privilege of a happy-few enthusiasts, and that in most cases it is still a no-go zone if you want to socialize with non-mathematicians.


Kepler, Fermat, Poincaré

The epilogue is an interesting read. It gives excerpts of a lecture given in 1998 at the International Mathematical Congress in Berlin and discusses "The prospects for mathematics in a multi-media civilization" but the message is broader than just the multi-media aspects. Twelve years later, it is quite interesting to (re-)read it and see how much of the content has been realized and how much has faded away.

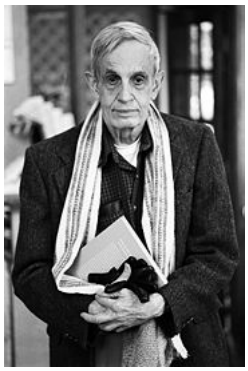
The other chapters, being written by different authors, have different styles and lengths and they also differ in the amount of the mathematical details. In the group of "case studies" we find some topics that are somewhat predictable like the encoding of CD's, different aspects of image processing in medical applications, shortest path and other graph theoretical problems and their applications. But there are also some chapters that I didn't encounter before as being popularizing math topics

like Turing instability and spontaneous pattern forming phenomena in nonlinear dynamical systems. The nice thing about this chapter is that the reader is brought a long way by an analogy with the love-life of Romeo and Juliet and their twin-siblings Roberto and Julietta. Similarly computer tomography is introduced using the game of battleship (where one has to find out blindly where the opponent has placed his battleships on a grid) while the chapter eventually becomes involved in nanotechnology. The chapter about "intelligent materials" stays at the surface of mathematics, and so does the chapter on reflections of hinged mirrors, spherical mirrors and hyperbolic geometry, but the latter of course can be very nicely illustrated. Being the written summary of lively presentations, it is clear that all chapters have ample occasion of visualizing illustrations.



Fisher Black, Myron Scholes

The group of chapters on "current topics" aren't too much different. There is one on the role of mathematics in the financial markets, and this isn't inspired by the recent global crisis, but treats things such as the role of stochastics, arbitrage, and the Black-Scholes formula. The next chapter deals with electronic money. After all coins and bank notes are just symbols, that do not have a value as such. Similarly electronic money is just a string of bits. So the problem is to distinguish between strings that are just information and others that have economic value just like money, which brings the reader to the subject of cryptography. The huge computational challenge lying in the simulation of the global dynamical system that leads to climate change is another such topic that fits into a decor of a changing world leading to catastrophic effects in an ever faster succession. Another chapter is about sphere packing, which is of a more entertaining subject, but yet has lead to a new and deep mathematical machinery in a sequence of efforts to solve Kepler's conjecture. If spheres are packed like we see then piled up on display at fruit markets, then the density of the space covered by the spheres is $\pi/\sqrt{18}$. Kepler conjectured that one can not do better. The chapter is dealing with the history of this conjecture and even formulates theorems and uses formulas. It brings us up to the computer proof of Hales in 1998. Other mathematical problems with a long history are dealt with in a chapter on Fermat's last theorem and one about the Nash equilibrium. The remaining chapter in this group is a short one on quantum computing.



John Forbes Nash

The latter ties up with the first of the next five chapters that are classified in the group "central theme". It explains how huge prime numbers, or rather the factorization of huge numbers into its prime factors, forms the heart of our current cryptosystems, but if ever we succeed in getting a quantum computer to work, then we break down the bounding walls of current computability and hence another basis for cryptography will have to be invented. The next two chapters, although they have serous applications and involve some good mathematics, will probably be perceived by a broader audience as being related to mathematical recreation. The first gives some insight into knot theory. This is often what is needed to design or solve some three-dimensional puzzles. Also the other chapter on the geometry and physics of soap bubbles is a fun-subject for many. Not so remote from these subjects is the much more fundamental subject of the structure of space and the Poincaré conjecture (every closed simply connected three-dimensional space is topologically equivalent to a three-dimensional sphere). More generally it gives the complete classification of *all* three-dimensional spaces as elaborated by Thurston, Hamilton and Perelman, and this depends on the theory of heat diffusion. This is a relatively long chapter which dives a bit deeper into the mathematics. The final chapter in this group is about the roots and applications of probability, which entered mathematics at a rather late stage of its evolution.





sphere packing, soap bubbles

Adhemar Bultheel