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# ECONOMICS OF INTERSECTIONS AND TRANSPORTATION POLICIES

Proefschrift voorgedragen tot
het behalen van de graad van
Doctor in de
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door

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# Abstract

Transportation Economics –the use of economic models and theory to further efficient transportation – has become highly relevant in today's congested world. This PhD-thesis contributes to the field of Transportation Economics by focusing on the efficient use of traffic lights (Part I) and on the implementation of efficient transportation policies (Part II). Central to the first part of this thesis is the regulation of intersections. In particular, three issues on this subject are analyzed: Part I begins by studying the optimal regulation of intersections, where the intersection can be regulated by traffic lights or a priority rule, and tolls can be levied. Next, Part I compares the efficiency of traffic-responsive signal control and anticipatory signal control. Part I ends by analyzing whether traffic lights can achieve the same results as tolls. Part II of this thesis focuses on a case study of efficient transportation policies and more particularly on the effectiveness and welfare effects of alternative transport policies designed to reduce urban traffic externalities in a medium sized city.

# Samenvatting

Door de toenemende congestie is de studie van transporteconomie - het gebruik van economische theorie en modellen om efficiënte mobiliteit te bevorderen - vandaag de dag uiterst relevant. Deze thesis in transporteconomie focust op het efficiënt gebruik van verkeerslichten (Deel I) en op de toepassing van efficiente transportmaatregelen (Deel II). In het eerste gedeelte van deze thesis staat de regulering van kruispunten centraal. Drie aspecten worden er geanalyseerd: Als eerste wordt de optimale regulering van kruispunten bestudeerd. Het kruispunt kan hierbij zowel door een voorrangsregel als door verkeerlichten geregeld worden. Ook kan er een tolheffing van toepassing zijn. Ten tweede wordt de efficiëntie van een proactieve verkeerslichtinstelling afgewogen tegen de efficiëntie van een reactieve verkeerslichtinstelling. En ten slotte wordt er nagegaan of verkeerslichten dezelfde resultaten kunnen behalen als een tolheffing. In het tweede gedeelte van deze thesis ligt de nadruk op een case-study omtrent efficiënt transportbeleid. In dit gedeelte wordt met name ingegaan op de doeltreffendheid en welvaartseffecten van transportmaatregelen, bedoeld om de externe kosten van vervoer te reduceren in een middelgrote stad.

# Contents

# Chapter 1

# Introduction

In many areas, a steadily increasing demand for mobility is confronting economic, social and physical constraints on transportation infrastructure. One way to meet with this increasing demand is by building new infrastructure. A more cost-effective way however would be to make better use of the existing infrastructure. In its recent Green Paper the European Commission emphasizes that to limit congestion "in certain cases new infrastructure might be needed, but the first step should be to explore how to make better use of existing infrastructure" (European Commission (2007)).

Intersections are an important component of the traffic network. Nielsen et al. (1998) states that intersection delays account for $17 - 35\%$ of the total travel time in the Municipality of Copenhagen. Arnott (1990) indicates that "the principal form of congestion in C.B.D. auto travel is signalized intersection congestion". The National Transportation Operations Coalition (NTOC) declares that delays at traffic signals constitute 5-10% of all traffic delay (NTOC (2012)). The regulation of these intersections consequently has a large impact on the efficiency of the network as a whole.

Intersections can be regulated by traffic lights or a priority rule, or they can be controlled by a roundabout. Kakooza et al. (2005) found that with light traffic, roundabouts perform better than un-signalized and signalized intersections in terms of easing congestion, but with heavy traffic, signalized intersection perform better. In this thesis we will only focus on unsignalized and signalized intersections.

In 1868, the first gas-lit traffic lights were installed outside the Houses of Parliament in

London to control the traffic in Bridge Street, Great George Street and Parliament Street. The first electric traffic light was installed on the corner of East 105th Street and Euclid Avenue in Cleveland, Ohio in 1914 (Wikipedia contributors (2015)). Roughly fifty years later, the first signal control model was studied by Webster (1958). This model assumed traffic flows to be unaffected by the signal settings. In reality, however, traffic flows will react to a changes signal settings. And these modified traffic flows subsequently require signal settings to be re-optimized.
The second generation of signal control, traffic-responsive control (Miller (1963)), takes into account this mutual dependence of traffic flow and signal settings. In traffic-responsive control, data collected from vehicle detectors located upstream is used to optimize the signal settings. Successful commercial products of this sort are the Split Cycle Offset Optimisation Technique (SCOOT, Hunt et al. (1981)) and the Sydney Coordinated Adaptive Traffic System (SCATS, Luk (1984)).

The strand in the literature that deals with traffic-responsive signal control focuses on the iterative optimization and assignment procedure. In the iterative optimization and assignment procedure the signal settings and equilibrium flow patterns are updated alternatively, until both flows are at equilibrium and signal settings are optimal given the flows (Allsop and Charlesworth (1977), Cantarella et al. (1991), Gartner et al. (1980), Lee and Hazelton (1996)).

Another approach that takes into account the interaction between traffic control and traffic assignment is known as anticipatory control. In the literature, this has been formulated as a bi-level problem, in which the upper level is the signal setting problem and the lower level is the traffic equilibrium assignment problem (Chiou (1999), Yang and Yagar (1995)), and as a Stackelberg game (Fisk (1984)).

Although traffic signal control has been the subject of numerous studies, it is widely accepted that traffic signal benefits are not fully realized and there is plenty of room for improvement (Lo (1999)). In the second chapter of this thesis, we therefore take a fresh look at the optimization of intersections.

To acquire maximum insight and understanding of the optimization problem, we focus on a simple two-road network, which allows us to obtain an optimal solution. More specifically, we optimize the regulation of an intersection of two routes connecting one origin-destination pair. This network was also used by Smith (1979b) to maximize capacity. The simplified network can represent two parallel roads (e.g. a road through an urban area and a parallel road bypassing the city) or two parallel modes (e.g. a train and a road connecting two cities).

We follow Fisk (1984) in modelling the combined assignment and control problem as a Stackelberg game. In this game the leader (traffic authority) moves first and bases his decision on the expected reaction of the follower (the drivers). The existence of congestion generates interdependencies between users' decisions, which can also be modeled using game theory. We will assume that the followers behave according to the noncooperative principle of Nash (1951).

In his paper Smith (1980) puts forward different real life cases where introducing flow-responsive traffic signals has deteriorated journey times, in some cases even by 30%. In addition, both Gershwin and Tan (1978) and Dickson (1981) have shown, albeit for a specific numerical example, that the iterative optimization and assignment procedure (Cournot) leads to a worse solution than the constrained optimization approach (Stackelberg). Despite these results, policy makers generally assume that traffic responsive signal control is the most efficient control policy. The result is that in many cities, signal control is of the traffic responsive type.

The rapid and widespread implementation of traffic-responsive signal control is strongly connected to the intuitive superiority of this control policy. An equal intuitive discourse is thus needed to challenge this inclination towards responsive signal control. The third chapter in this thesis therefore provides a clear and accessible comparison of responsive signal control versus anticipatory signal control. To provide maximum insight for policy making, we focus on a simple network and represent both the traffic responsive and the anticipatory signal setting procedure in an transparent game theoretical framework. The results are explained in a intuitive way by recognizing the presence of externalities and the first mover advantage.

Traffic lights are in the first place designed to manage vehicle conflicts at intersections, allocating green time among conflicting streams. But traffic lights can also be a powerful tool to manage traffic flow in order to provide a more efficient use of the network. In chapter 4 of this thesis, we study the extent to which traffic lights can achieve the same efficiency gains as tolls. To the best of the author's knowledge, this is the first work that analyzes under which conditions traffic lights can provide an adequate alternative to road pricing.

We focus our analysis on two different networks, one in which the traffic lights' primary objective is to regulate an intersection and another network in which traffic lights are installed with as a sole objective to influence route choice. Both networks are deliberately kept as simple as possible to allow for clear intuitive results.

We model the signal setting procedure in a first stage as a Stackelberg game. In a later stage we will use the inverse Stackelberg approach (Olsder (2009)), which is an extension of the basic Stackelberg game.

In the final chapter of this thesis, efficient urban transportation policies are at the forefront. In its communication on the Action Plan on Urban Mobility the European Commission states that "In many urban areas, ..., increasing demand for urban mobility has created a situation that is not sustainable: severe congestion, poor air quality, noise emissions and high levels of CO2 emissions"(European Commission (2015)). Chapter 5 of this thesis develops a model in the MOLINO tradition (De Palma et al. (2010), Kilani et al. (2014)), that allows policy makers to compare different transport policies designed to reduce congestion externalities, accident risk and noise and air pollution.

Unlike most models that have a high degree of detail and describe users behavior via discrete choice techniques, this model focuses on a simplified network and uses aggregate data to represent user's behavior. As such, calibration of the model requires a minimum of data allowing policy makers to quickly obtain a first impression of the efficiency of a transport policy.

The supply side of the model features a network which allows for combined trips as well as pure mode-trips. Travel costs encompass access, waiting and in-vehicle time costs. The demand side of the model consists of multiple user classes, which differ with respect to their travel preferences, incomes, and costs of travel.

The model is illustrated to the city of Leuven (Belgium). Three transport policies to reduce the traffic externalities in the city center are considered: introducing road pricing in the city center, raising parking fees in the center of Leuven and expanding public transport.

## 1.1 Scope

The study of transportation is approached from various disciplines, including engineering, economics, geography, psychology, mathematics, .... Though having the same subject, these fields are generally considered to be only distantly related. This thesis tries to perform a bridging function, with papers providing a link between transportation engineering and transportation economics.

While interacting closely with transportation engineering, the main focus of this thesis is on the contribution economics can make to the analysis of transportation. The emphasis is thus more on concepts, illustrated on simple networks, than on specific design and implementation.

Like any other branch of economics, transportation economics deals with the allocation of scarce resources. As such, transport economics and efficiency are inextricably linked. The concept of efficiency is therefore also central to this thesis. More particularly, Part I of this thesis deals with efficient traffic networks and Part II focuses on efficient transportation policies. Table 1.1 gives an overview of the different chapters.

**Table 1.1:** Overview of the different chapters

| Ch | Objective | Instruments | Model |
|---|---|---|---|
| 2 | Optimization of intersections | Priority rules, traffic lights and tolls | Stackelberg game |
| 3 | Comparison of responsive and anticipatory signal control | Traffic lights | Stackelberg game |
| 4 | Analysis of traffic lights versus road pricing | Road pricing, traffic lights | Stackelberg game, inverse stackelberg game |
| 5 | Comparison of welfare effects of alternative transport policies | Parking fees, road pricing, public transit design | multi-user, multi-period, multi-modal model in the MOLINO-tradition |

In a nutshell, the four chapters of this thesis cover the following topics:

**Chapter 2: Optimizing Intersections.** In this chapter we optimize the regulation of an intersection of two routes connecting one origin-destination pair and study the effects of priority rules, traffic lights and tolls. We show that when the intersection is regulated by a priority rule, the optimal policy is generally to block one of the two routes. When the intersection is regulated by traffic lights, it can only be optimal to leave both routes open when both routes are subject to congestion or if a toll is levied.

This chapter is joint work with Stef Proost and has been published in *Transportation Research Part B: Methodological*, 2015, volume 71, pp. 100-119.

**Chapter 3: The Puzzle of Traffic-Responsive Signal Control.** This chapter aims to show that, contrary to popular belief, traffic responsive signal control is not necessarily the most efficient control policy. More particularly, we show that for an intersection of two routes connecting one origin-destination pair where only one route is subject to congestion, anticipatory signal control performs better than traffic-responsive signal control. Furthermore, the unfolded logic behind this result suggests that the superiority of anticipatory signal control also extends to other networks.

This chapter is joint work with Stef Proost and has been published in *Transportation Research Part A: Policy and Practice*, 2015, volume 77, pp. 350-357.

**Chapter 4: Can Traffic Lights Achieve the Same Results as Tolls?** This chapter studies the extent to which traffic lights can provide an alternative to road pricing in a simple network with two routes connecting one origin destination pair. We distinguish between the case in which the main purpose of the traffic lights is to regulate the intersection and the case in which the sole objective of the traffic light is to affect route choice. For this last case, we show that road pricing performs at least as good as traffic lights. For the network in which the traffic lights regulate the intersection of the two routes, we show that the implementation of a flow-dependent signal setting makes road pricing superfluous.

**Chapter 5: Efficient transportation policies for sustainable cities.** In this paper we compare the effectiveness and welfare effects of alternative transport policies designed to reduce urban traffic externalities. We build a multimodal, multi-class, multi-period model, which allows for endogenous congestion and total demand elasticity. Pure as well as mixed modes of transport are considered, and different government perspectives are compared. The model is applied using data from a Belgian medium-size city, suggesting that the city authority will lobby for measures that are welfare decreasing from societal point of view. The results furthermore show that road pricing creates better results than increasing parking fees or expanding public transit.

This chapter is joint work with Stef Proost.

# Part I

# Efficient traffic networks

# Chapter 2

# Optimizing intersections

## 2.1 Introduction

Despite decades of research on the optimization of intersections, the poor regulation of intersections is a matter of huge frustration amongst many drivers today. The complexity of the problem makes the optimization of intersections a tough nut to crack. Understanding the causal mechanisms that govern the optimal regulation of intersections is therefore essential in dealing with complex network problems. Not least because externalities, resulting from the behaviour of drivers, can lead to results that defy intuition.

The first traffic signal control model was studied by Webster (1958). This model assumes traffic flows to be unaffected by the signal settings. This reduces the model to an isolated control problem in which signal settings are optimized for given flows on the network. The need to take into account the effects of the change in traffic light settings on the network flow was first emphasized by Allsop (1974). This insight has generated two approaches in the literature that address the interaction between control and assignment: (1) the iterative procedure and (2) the global optimization approach (Cantarella et al. (1991)).

The iterative procedure iteratively solves the signal setting problem for a fixed flow pattern and the assignment problem for fixed signal settings until two successive flow patterns or signal settings converge (Allsop and Charlesworth (1977), Cantarella et al. (1991), Gartner et al. (1980), Lee and Hazelton (1996)). A dynamic approach is proposed by Hu and Mahmassani (1997) and Lo et al. (2001). With a simple example, Dickson (1981) showed that the total network cost can increase during the iterative procedure. The iterative procedure does thus not necessarily lead to the optimal solution.

When the global optimization approach is applied, some network objective function is optimized while taking into account the equilibrium route choice behaviour of the drivers (Chiou (1999), Cipriani and Fusco (2004), Fisk (1984), Marcotte (1983), Sheffi and Powell (1983), Yang and Yagar (1995)). The global optimization problem can be modelled as a bilevel programming problem in which the upper level deals with the control problem and the lower level with the user equilibrium assignment problem. The dynamic approach is studied by Abu-Lebdeh and Benekohal (2003).

Smith (1979a) provided the necessary mathematical fundamentals of the traffic control and assignment problem by stating the conditions under which the problem has a unique and stable solution. In later work, he proposes and elaborates a local traffic control policy (called $P_0$) that maximizes network capacity (Smith (1980, 1981)). In successive papers, Smith extends his work on the combined traffic assignment and control problem (see e.g. Smith and Van Vuren (1993)).

Relatively few papers address both signal optimization and road pricing. Clegg et al. (2001) explored the use of both instruments by specifying an algorithm that continuously moves current traffic flows, green-times and road prices within the model toward locally-optimal values. Chiou (2007) proposed a globally convergent iterative scheme designed to heuristically search for a local optimum.

Fisk (1984) was the first to model the combined assignment and control problem as a Stackelberg game. Chen and Ben-Akiva (1998) developed a dynamic model dealing with the combined assignment and control problem and formulated it as a Cournot, Stackelberg and monopoly game. Overall, there have only been a limited number of authors focussing on the game theoretical perspective of the combined assignment and control problem. For an overview, see Hollander and Prashker (2006).

This paper follows Fisk's example in modelling the combined assignment and control problem as a Stackelberg game. In this game the leader (traffic authority) moves first and bases his decision on the expected reaction of the follower (the drivers). The behaviour of the drivers can also be represented as a game, because the congestion on one road is dependent upon how many users choose to use the same road. We will assume that the followers behave according to the noncooperative principle of Nash (1951).

To obtain an optimal and tractable solution for the intersection problem, we use a simple two-road intersection, that can be controlled by either a priority rule or traffic lights. This simplified network structure can represent different types of routes and different modes of transport that are either congestible or insensitive to congestion.

The remainder of this paper is organized as follows: Section 2 focuses on the case in which the intersection is regulated by a priority rule. Section 3 presents the main results when the intersection is regulated by traffic lights. Section 4 offers a comparison between the regulation by traffic lights and by a priority rule. Section 5 illustrates the theory by means of two applications and Section 6 offers a conclusion.

## 2.2 Priority

In this section, the intersection is regulated by a priority rule. In Section 2.2.2, Route 2 is considered to have unlimited capacity, while Route 1 is subject to congestion. In Section 2.2.3, we consider the case where both routes have limited capacity. The focus will be on the main results and the intuition. The complete mathematical derivations are given in Appendices. Before turning to the results, the assumptions underlying the priority model will be set out.

### 2.2.1 Assumptions underlying the priority model

N drivers want to go from point A to point B (Figure 2.1). They can either take Route 1 or Route 2. The two routes intersect at point C and the intersection is regulated by a priority rule. Six assumptions are imposed on the model representing the intersection regulated by a priority rule.

**A.1** Demand: total demand is inelastic and equals N.

**A.2** Homogeneous users: all users are identical and try to minimize their expected user cost. The stationary distribution of vehicles will be a Wardrop equilibrium (Wardrop (1952)).

**A.3** Arrival rate: the arrival rate is static.

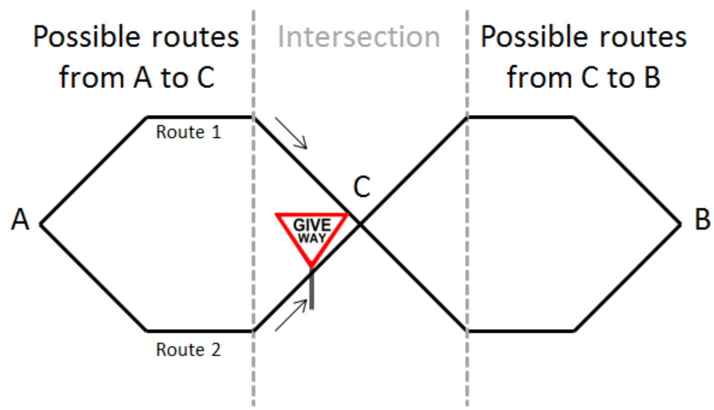


**Figure 2.1:** Outline of an intersection of two routes connecting one OD pair (AB) regulated by a priority rule.

**A.4** Undersaturated conditions: the arrivals at the intersection on Route 2 in the time interval $\frac{1}{X_1}$ don't exceed the number of Route 2-drivers that can cross the intersection in the time interval $(\frac{1}{X_1} - v)$.

**A.5** Priority: Route 1 always has priority.

**A.6** Time cost of priority: v hours before a car on Route 1 passes C, cars on Route 2 already wait until the car on Route 1 has passed.

**Proposition 1.** *When the intersection of two routes connecting one OD pair is regulated by a priority rule, the optimal policy is generally to block one of the two routes. The only exception is the case where the marginal congestion cost on the minor route is greater than half of the square of the marginal waiting cost ($a_2 > \frac{v^2}{2}$). In this scenario, the optimal policy is to leave both routes open.*

### 2.2.2 Only one route subject to congestion

Let $a_1$ be the increase in average cost on Route 1 when one vehicle is added, we call it the sensitivity to congestion; let $X_i$ equal the flow on route $i$; let $\omega$ be the resource cost for a trip from A to C; let $\phi_i$ stand for the minimal time cost from A to C of route $i$; and let $\frac{v^2 X_1}{2}$ be the expected waiting time cost[1] on Route 2 at the intersection. When two parallel routes connect one OD pair, the government can use three possible policies to maximize

[1] The waiting time cost function is derived in Appendix E.

total welfare: to block Route 1, to block Route 2, or to leave both routes open. The users in turn can react in three ways to any chosen policy: to only take Route 1, to only use Route 2, or to use both routes. If the government decides to block Route 1, then all drivers need to take Route 2 and the total cost will equal $(\omega + \phi_2) N$. If, on the other hand, the government blocks Route 2, then the user equilibrium will be $X_1 = N$ and the total cost will be $(a_1 N + \omega + \phi_1) N$. If, however, the government decides to leave both routes open, the equilibrium reaction of the drivers will be $X_1 = N$ if $a_1 N + \phi_1 < \phi_2 + \frac{v^2(N-1)}{2}$, $X_2 = N$ if $\phi_1 > \phi_2$ and the drivers will use both routes if $\phi_1 \leq \phi_2 \leq 1N + \phi_1 - \frac{v^2(N-1)}{2}$. In this last case the Wardrop equilibrium implies $\left(\omega + \phi_2 + \frac{v^2 X_2^e}{2}\right) N$ as total cost. The user equilibrium in which both routes are used is, however, never optimal from the government's point of view. Therefore, it is always optimal to block one of the two routes.

Which route to block depends on the relative cost of both routes: a rational authority minimizing the social cost closes Route 1 if $a_1 N + \phi_1 > \phi_2$, and Route 2 if $a_1 N + \phi_1 < \phi_2$. Remark that if the interior equilibrium exists, it is always optimal to block the route with limited capacity. This result can be explained intuitively. If both routes are used, drivers on the minor (uncongested) route incur a waiting cost, whereas if only the minor (uncongested) route were to be used, no waiting cost would be incurred and, compared to the interior solution, no other additional costs are incurred. If on the other hand only the congested route were to be used, then the total cost would be higher than for the interior solution due to additional congestion costs. We have shown this result for linear average cost functions, but this result holds more generally for any travel cost function in which the running time is a continuous nonlinear nondecreasing function of the flow. Indeed, the main driver of the result is the additional waiting time externality that is imposed when using both Route 1 and Route 2. If there is an internal solution where both user costs are equal, it is always optimal to have all users only using Route 2 as this cost is always lower by avoiding the priority waiting costs.

In the absence of a government intervention blocking one road, the driver will often make the sub-optimal choice. Indeed, whenever $a_1 N + \phi_1 - \frac{v^2(N-1)}{2} < \phi_2 < a_1 N + \phi_1$, the user equilibrium is $X_1 = N$, while $X_2 = N$ would be optimal. On the other hand, whenever $\phi_1 < \phi_2 < a_1 N + \phi_1 - \frac{v^2(N-1)}{2}$ there will be an equilibrium in which both routes are used, while it would be optimal to have all drivers on Route 2. Figure 2.2 illustrates this second situation. The Wardrop equilibrium is given by the intersection of the average cost-curves of Route 2 and Route 1 (point G).[2] It is clear that the total cost for the interior solution

[2]Note that for interior solutions every additional user on Route 1 imposes an extra waiting cost for the

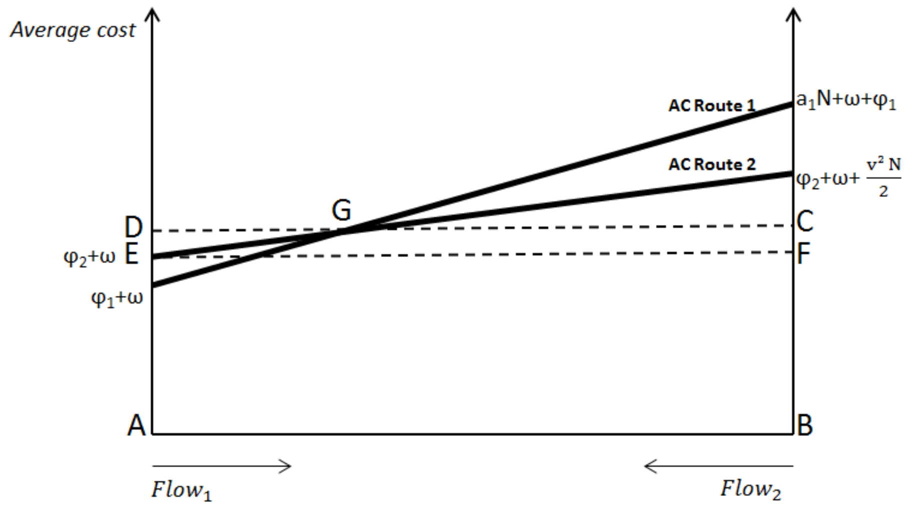


**Figure 2.2:** The solution in which all travellers use Route 2 (E) is optimal. However, without intervention, the interior solution (G) will be the user equilibrium.

equals ABCD, while the total cost would only equal ABFE if Route 1 had been blocked.

This can be seen as an illustration of the Braess paradox (Braess (1968)). In the Braess paradox, adding one additional link can increase total travel cost. Braess' paradox occurs because the congestion externality is not taken into account by the drivers. Here, we also add a link and, in this case, it is the external waiting cost that users on the main road impose on the users of the minor road that can increase the total travel cost.

### 2.2.3 Both routes subject to congestion

The suboptimality of an interior solution continues to hold for the case in which both routes are subject to congestion and $a_2 \leq \frac{v^2}{2}$ (with $a_2$ the sensitivity to congestion of Route 2). However, when $a_2 \geq \frac{v^2}{2}$, the total cost can be at lowest when both routes are used in equilibrium.

In the Stackelberg game the traffic authority (leader) moves first and bases her decision on the expected reaction of the drivers (follower). Figure 2.3 illustrates this sequential game and shows the different options for the government and the possible reactions of the drivers.

drivers on Route 2. This explains the upward sloping AC curve of Route 2 for increasing $X_1$.

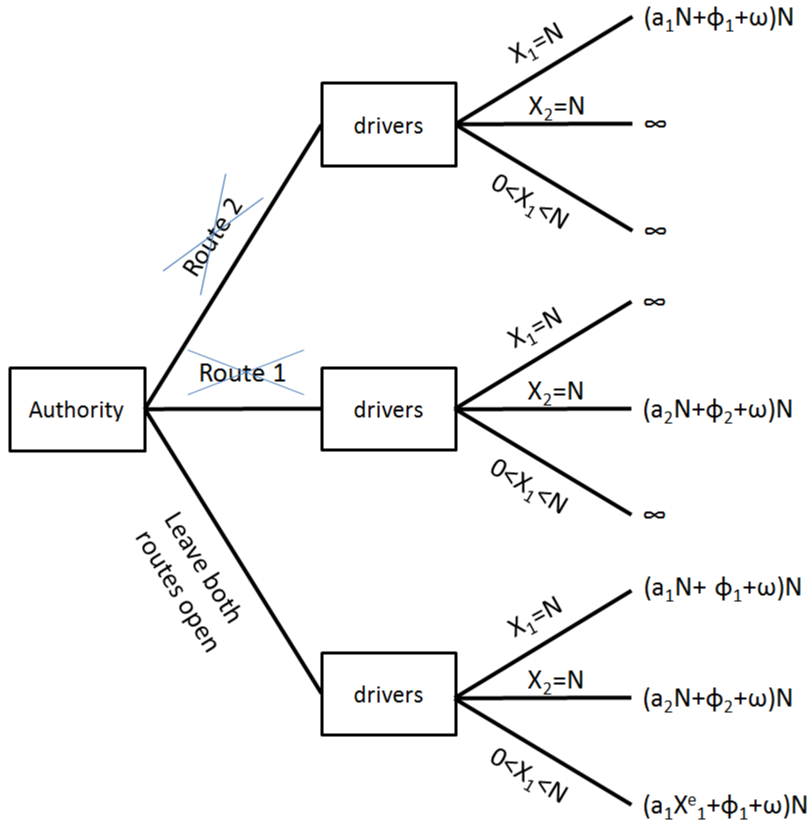


**Figure 2.3:** The total travel costs resulting from a Stackelberg game.

When the government decides to close Route 1, the user equilibrium will be $X_2 = N$, and the total cost will be $(a_2 N + \omega + \phi_2) N$. When only Route 1 is accessible, $X_1 = N$ will be the only equilibrium, and the total cost will amount to $(a_1 N + \phi_1 + \omega)N$. When both routes are accessible, the user equilibrium that will be in place depends on the relative value of the parameters: $X_2 = N$ if $a_2 N + \phi_2 < \phi_1$; $X_1 = N$ if $a_1 N + \phi_1 < \phi_2 + \frac{v^2}{2}(N-1)$; and $0 < X_1^e < N$ in all other cases.

Taking into account the reaction of the drivers, the government will block one of the two routes if $a_2 < \frac{v^2}{2}$, and leave both routes open if $a_2 \geq \frac{v^2}{2}$. The optimal policy thus depends on the ratio of the congestion coefficient ($a_2$) to the reaction time ($v$). If $a_2 < \frac{v^2}{2}$, the situation is similar to the case where only one route is subject to congestion and then it is always optimal to block one of the two routes. If $a_2 \geq \frac{v^2}{2}$ and the government leaves both routes open, it can be shown that in the user equilibrium the lowest cost will be attained.

In graphical terms, $a_2 < \frac{v^2}{2}$ boils down to an upward sloping (in $X_1$) average cost curve of Route 2, while $a_2 > \frac{v^2}{2}$ boils down to a downward sloping $AC_{route2}$-curve. The underlying logic is that if $a_2 \geq \frac{v^2}{2}$, the extra congestion cost of having all travellers on Route 2 is more costly than the saving in waiting costs. If, however, $a_2 < \frac{v^2}{2}$, the opposite is true.

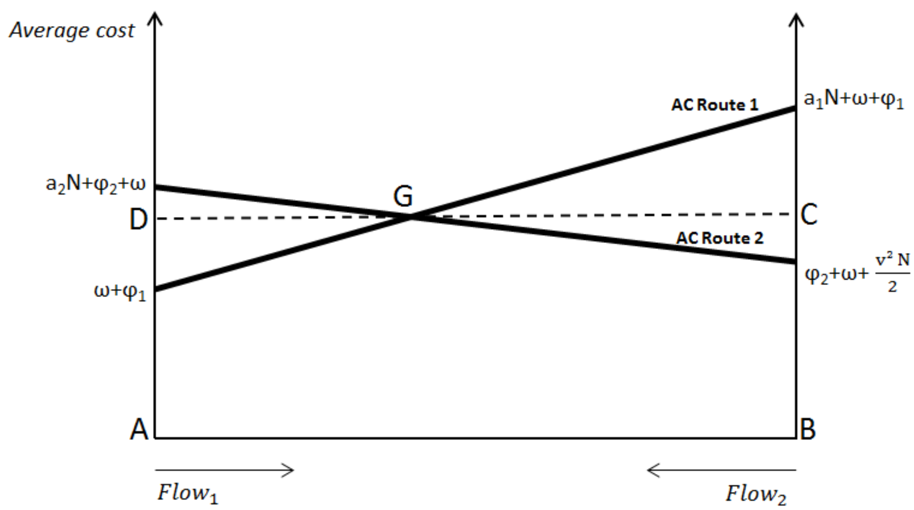


**Figure 2.4:** The interior solution (G) is optimal. No government intervention is needed to reach the optimal solution.

## 2.3 Traffic lights

In this section, the two-road intersection is regulated by traffic lights. Following the same approach as in the previous section, first only one route is considered to have limited capacity, and subsequently both routes are considered to have limited capacity.

### 2.3.1 Assumptions underlying the traffic lights model

Five assumptions are imposed on the model representing the intersection regulated by traffic lights.

**A.1** Demand: total demand is inelastic and equals N. This assumption is relaxed in Section 2.3.3.2.

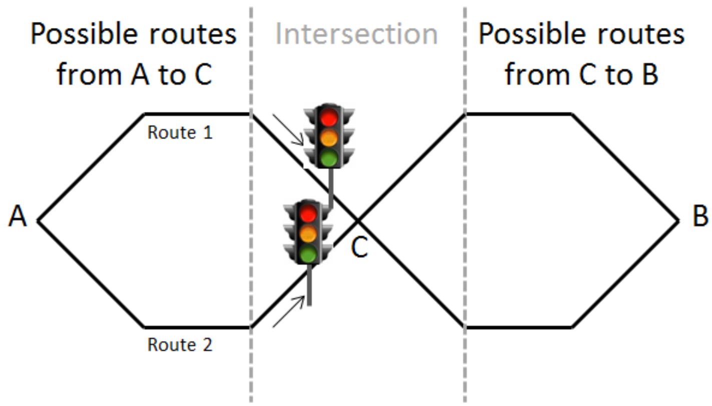


**Figure 2.5:** Outline of an intersection of two routes connecting one OD pair (AB) regulated by traffic lights.

**A.2** Homogeneous users: all users are identical and try to minimize their expected user cost. The stationary distribution of vehicles will be a Wardrop equilibrium.

**A.3** Arrival rate: the arrival rate is static.

**A.4** Undersaturated conditions: vehicle queues are only created during red phases, and fully dissipate during green phases. This assumption is relaxed in Section 2.3.2.1.

**A.5** Cycle time: to simplify matters, the cycle time[3] 'c' is held fixed. Hence, it follows that including intergreen time in the analysis is not relevant and will thus be ignored.

In this paper, the variable 'r' represents the red time on Route 2 and will be the main control variable. The corresponding green time on Route 2 will thus be (c-r) and a reverse scenario holds for Route 1. As there will be alternating red times to avoid collisions at the intersection, both routes will experience an expected traffic light waiting time cost $(T_1(c,r), T_2(c,r))$. The traffic light waiting cost functions are increasing in the red time and decreasing in the green time ($\frac{\partial T_1(c,r)}{\partial r} < 0, \frac{\partial T_2(c,r)}{\partial r} > 0$). Furthermore, the expected traffic light waiting functions are discontinuous and jump to infinity when it is always red.

### 2.3.2 Only one route subject to congestion

In this section, only one route is considered to have limited capacity. In Section 2.3.2.1 the optimal policy is determined for the case in which the traffic authority can only control the

[3]That is the duration of the sum of green time and red time.

traffic lights. In Section 2.3.2.2 the traffic authority can use both signal settings and a toll to minimize total costs.

#### 2.3.2.1 Traffic lights without road pricing

**Proposition 2.** *When the intersection of two routes connecting one OD pair is regulated by traffic lights and only one of the two routes is congested, a signal setting whereby drivers choose to use both routes can never be an optimal policy.*

**Table 2.1:** The total travel cost for every (r, UE)-combination

| Signal setting | $X_1 = N$ | $X_2 = N$ | $0 < X_1 < N$ |
|---|---|---|---|
| $r = c$ | $(a_1 N + \phi_1 + \omega) N$ | $\infty$ | $\infty$ |
| $r = 0$ | $\infty$ | $(\omega + \phi_2) N$ | $\infty$ |
| $0 < r < c$ | $(a_1 N + \omega + \phi_1 + T_1) N$ | $(\omega + \phi_2 + T_2) N$ | $(\omega + \phi_2 + T_2) N$ |

Table 2.1 shows that a rational government will never decide on an alternating signal setting when both routes are substitutes. Indeed, let $T_i(c, r)$ be the expected waiting time cost on route $i$ at the intersection.[4] As $T_i(c, r)$ is positive when $0 < r < c$, the total travel cost for an alternating signal setting will always be higher than for $r = c$ or $r = 0$. Which of the two non-alternating signal settings will be optimal is dependent on the values of the parameters $a_1, N, \phi_1$ and $\phi_2$. When $a_1 N + \phi_1 < \phi_2$, $r = c$ is the optimal solution and when $a_1 N + \phi_1 \geq \phi_2$, $r = 0$ will be implemented.[5]

The suboptimality of an alternating signal ($0 < r < c$) in case the intersection is regulated by traffic lights and only one route is subject to congestion, can be explained intuitively. When the user equilibrium is $X_2 = N$ or $X_1 = N$ (i.e. $\phi_2 + T_2(c, r) < \phi_1 + T_1(c, r)$ or $a_1 N + \phi_1 + T_1(c, r) < \phi_2 + T_2(c, r)$ respectively), drivers have to wait at the traffic light while there is no one crossing the intersection. The intuition behind the suboptimality of an alternating signal setting when the user equilibrium is $0 < X_1^e < N$ is shown in Figure 2.6. If the duration of red light for Route 2 is reduced, then the expected average waiting time on Route 1 increases, indicated by an upward shift of the $AC_{route1}$ curve in Figure 2.6. At the same time, the expected average waiting time for Route 2 decreases, corresponding

[4] We will assume that the saturation flow s is very large in comparison to the arrival rate $X_i$, so that the traffic light waiting time due to departure delay is negligible.

[5] The optimality of only one route (mode) connecting one OD pair remains valid in the case where both routes have unlimited capacity. The total minimal cost then equals $(\omega + \phi_i) N$ with $i$ the route index that procures the lowest minimal time cost.

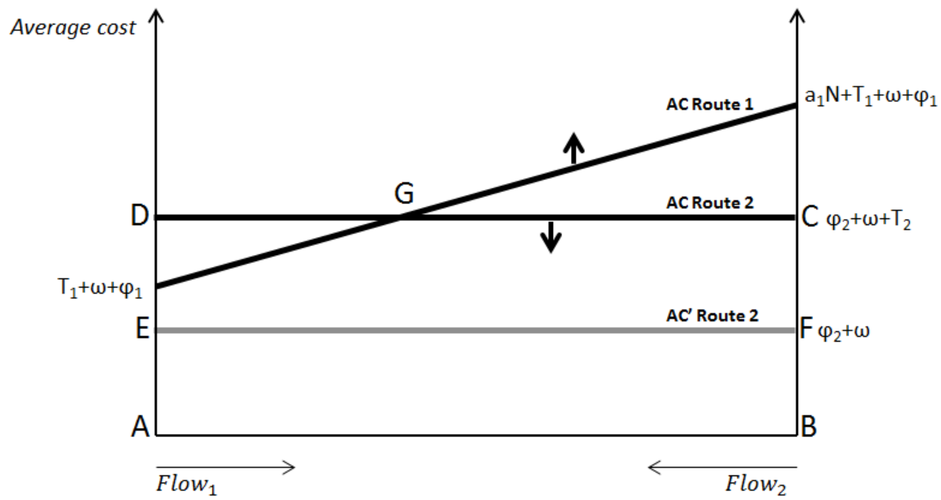


**Figure 2.6:** A corner solution constitutes the optimal solution; in this case: $X_2 = N$.

to a downward shift of the AC$_{route2}$ curve. This forces the switching point G to the left, hence less people travel on Route 1 with a simultaneous decrease in total cost. From the graph it is clear that the lowest cost (area ABFE) will be achieved when it is always green for drivers on Route 2 (grey line). It is noted that the AC$_{route1}$ curve would lie infinitely high in this situation.

This can be vivified by the following example: consider the situation in which demand from point A to point B is relatively inelastic. Suppose A and B are connected by a tram[6] and a road plagued by traffic congestion. Users are indifferent between the two modes, only the user cost matters. The tram line intersects the road trajectory, and this intersection is regulated by traffic lights. In this situation, even though counterintuitive, the optimal policy would either be to close the road and only maintain the tram, or to remove the tram and only keep the road, depending on the relative costs of the two scenarios.

Proposition 2 can be generalized to average cost function in which the running time component is a continuous nonlinear nondecreasing function of the flow. Indeed, the main driver of the result is that any interior solution can be improved by either giving only green to Route 2 or giving only green to Route 1. So the result does not depend on the precise

[6] The tram is assumed to be relatively insensitive to congestion and a substitute for the car.

curvature of the average cost function of Route 1. Furthermore, when we relax the assumption that the waiting time due to departure delay is negligible,[7] the result obtained in this subsection is still valid, since any interior solution is more costly than the solution for either $r = 0$ or $r = c$.

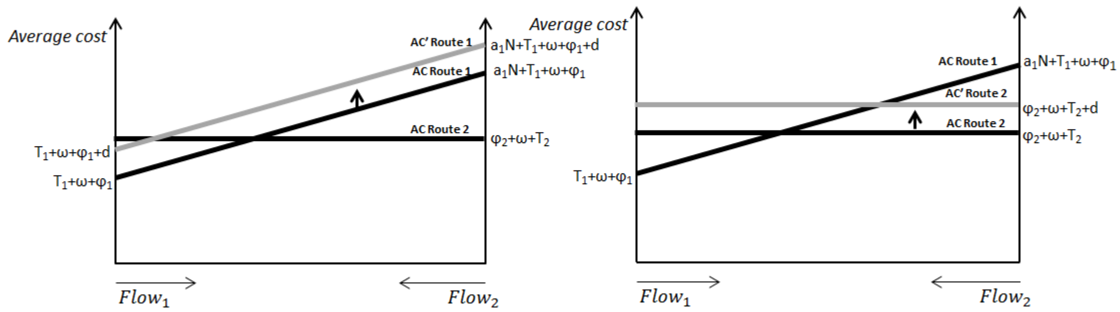


**Figure 2.7:** The total cost when traffic conditions are saturated is at least as high as when conditions are undersaturated.

Finally, we can show that Proposition 2 can also be generalized to saturated traffic conditions. Remark first that saturated traffic conditions can only occur for alternating signal settings. Indeed, if $r = 0$ or $r = c$, the traffic light can not be the restricting factor. Suppose that the demand and the relative time and resource costs are such that there exists a signal setting for which the amount of drivers arriving at the intersection on Route 1 is larger than the amount of drivers that can exit the intersection. That is, there is an r for which $X_1^e c > s_1 r$, with c the cycle time, $X_1^e$ the equilibrium flow on Route 1, $s_1$ the saturation flow and $r$ the green time for Route 1 per cycle. In this case, a queue will develop on Route 1, the time cost for Route 1 will go up, and the drivers will shift from Route 1 to Route 2. If the capacity on Route 2 is not sufficient to accommodate the excess demand of Route 1, then the total demand (N) will decrease,[8] which results in a loss of surplus. If Route 2 has enough spare capacity to accommodate the shifting Route 1 drivers, then we know from Yang and Yagar (1995) that in the steady state, $X_1^e c$ equals $s_1 r$ and the queue waiting time cost will be such that the user cost on Route 1 and Route 2 is equal. From

[7] In this case, the user cost for Route 1 is $a_1 X_1 + \phi_1 + \omega + \frac{(c-r)^2}{2c\left(1-\frac{X_1}{s_1}\right)}$ and the user cost for Route 2 equals $\phi_2 + \omega + \frac{r^2}{2c\left(1-\frac{X_2}{s_2}\right)}$.

[8] Inelastic demand was assumed here. This, however, is always relatively inelastic, because if the user cost becomes infinitely large on both routes, the drivers will refrain from making the trip.

Figure 2.7 (graph on the left-hand side) it is clear that the total cost in this case is as high as the total cost for the same signal setting in the undersaturated case. Figure 2.7 (graph on the right-hand side) furthermore shows that if Route 2 has the limiting capacity, the total cost is higher than the total cost for the same signal setting in the undersaturated case.

From the analysis of the undersaturated case, we know that the total cost for an alternating signal setting is always higher than for either $r = 0$ or $r = c$. If traffic conditions can be saturated, then the optimal signal setting can lead to either saturated or undersaturated conditions. Given that we have shown that for saturated conditions, the total cost is at least as high as for the same signal setting in the undersaturated case, we can conclude that also in the saturated case it is optimal to use only one of the two routes.

#### 2.3.2.2 Traffic lights and road pricing

In this subsection, both signal settings and a toll ($\tau$) are instruments the authorities can use to minimize total costs. The toll is levied on the route subject to congestion, and the toll revenues are subtracted from the total cost. Note that with fixed demand, a toll on Route 1 is equivalent to a subsidy to the users of Route 2.

**Proposition 3.** *When the intersection of two routes connecting one OD pair is regulated by traffic lights and only one route is subject to congestion, then an alternating signal setting can be optimal if a toll is possible.*

**Table 2.2:** The total travel cost for every ($\tau$, r, UE)-combination

| Signal & toll | $X_1$=$N$ | $X_2$=$N$ | 0<$X_1$<$N$ |
|---|---|---|---|
| $r$=$c$ | $(\mathrm{a}_1N+\omega+\phi_1)N$ | $\infty$ | $\infty$ |
| $r$=0 | $\infty$ | $(\phi_2+\omega)N$ | $\infty$ |
| 0<$r$<$c$, $\tau$>0 | $(\mathrm{a}_1N+\omega+\phi_1+T_1)N$ | $(\phi_2+\omega+T_2)N$ | $(a_1X_1^e+\omega+\phi_1+T_1)\,N+\tau X_2^e$ |

From Table 2.2 it is clear that a rational authority will only implement an alternating signal setting if both routes are used in the user equilibrium. It can be shown that if there exists an optimal alternating signal setting and an optimal toll (i.e. if $2a_1N > c$ and $2a_1N - \frac{c}{2} + \phi_1 > \phi_2 > \phi_1 + \frac{c}{2}$), both routes are used. So, the optimal (r, $\tau$) combination can be obtained as the solution of the following minimization problem:

$$\min_{X_1,X_2,r,\tau} (a_1X_1 + \omega + \phi_1 + T_1\,(c,r) + \tau)\,X_1 + (\omega + \phi_2 + T_2\,(c,r))\,X_2 - \tau X_1 \qquad (2.1)$$

s.t.

$$X_1 + X_2 = N \tag{2.2}$$

$$a_1 X_1 + \omega + \phi_1 + T_1(c, r) + \tau = \omega + \phi_2 + T_2(c, r) \tag{2.3}$$

$$0 \leq r \leq c \tag{2.4}$$

$$X_1 > 0 \tag{2.5}$$

$$X_2 > 0 \tag{2.6}$$

$$\tau > 0 \tag{2.7}$$

Now that a toll can be levied, the external costs on Route 1 are taken into account in the user equilibrium. The optimal toll is (Appendix A):

$$\tau = \frac{\phi_2 - \phi_1 + T_2(c, r) - T_1(c, r)}{2} \tag{2.8}$$

Assuming undersaturated traffic conditions, i.e. queues at the intersection are only created during the red phases and dissolved during the green phases, the traffic light functions take the following form for $0 < r < c$ (see Appendix E):

$$T_1(c, r) = \frac{(c - r)^2}{2c} \tag{2.9}$$

$$T_2(c, r) = \frac{r^2}{2c} \tag{2.10}$$

The optimal toll can then be written as follows:

$$\tau^* = \frac{\phi_2 - \phi_1 - \frac{c}{2} + r}{2} \tag{2.11}$$

As $\partial\tau/\partial r > 0$, the optimal toll on Route 1 is increasing in r (the green time on Route 1). The intuition is the following: if the green time increases on the congested route, people will switch from Route 2 to Route 1 to take advantage of the extra green time. However, this causes Route 1 to be even more congested. As individuals do not take into account the effect of their switching on the existing drivers on Route 1, the toll has to increase in order to reach a social optimum for given traffic light settings.

The dependence of the optimal toll on the optimal traffic light setting drives the statement that the implementation of optimal signal settings can lead to a higher acceptance of toll roads. First, in the absence of a toll, the optimal policy is to block one of the two routes. When a toll becomes available on Route 1, it can be optimal to open both routes, which increases the possibilities for the drivers. Second, consider a suboptimal signal setting without toll where the congested Route 1 receives a very low green time. When a toll can be implemented on Route 1, one can increase the green time on Route 1.

The optimal r is defined by the following equation in which N represents the total amount of vehicles, $\tau$ the toll, $a_1$ the sensitivity to congestion on Route 1, and $T_i(c, r)$ the expected waiting time cost on route $i$ at the intersection (A).

$$\left(-N + \frac{\tau}{a_1}\right) \frac{\partial T_2(c,r)}{\partial r} = \frac{\tau}{a_1} \frac{\partial T_1(c,r)}{\partial r} \tag{2.12}$$

Combining equations (2.8) and (2.12) allows us to identify the optimal r as the solution of the following equation:

$$(\phi_2 - 2a_1 N - \phi_1 + T_2 - T_1) \frac{\partial T_2(c,r)}{\partial r} = (\phi_2 - \phi_1 + T_2 - T_1) \frac{\partial T_1(c,r)}{\partial r} \tag{2.13}$$

Assuming undersaturated traffic conditions and using the results from Appendix E, the optimal r as a function of the exogenous parameters can be obtained from equation (2.13):

$$r^* = \frac{\left(\phi_1 - \phi_2 + \frac{c}{2}\right) c}{-2a_1 N + c} \tag{2.14}$$

When $2a_1N - c < 0$, the optimal $(r, \tau)$ combination is a saddle point, and consequently the minimum will be near the boundary (Appendix A). On the other hand, if $2a_1N - c \geq 0$, the optimal $(r, \tau)$ combination provides the minimum attainable costs.

To identify the optimal policy, the total cost needs to be compared for all three policies. When the user equilibrium is $X_2 = N$ or $X_1 = N$, tolls and traffic lights are not needed. Therefore, the total costs equal $(\omega + \phi_2) N$ in case $r = 0$, and $(a_1 N + \omega + \phi_1) N$ in case $r = c$.

If the optimal alternating signal setting exists, i.e. if $2a_1N > c$ and $2a_1N - \frac{c}{2} + \phi_1 > \phi_2 > \phi_1 + \frac{c}{2}$, then the corresponding cost is always lower than the cost for $r = 0$ or $r = c$ (Appendix G). If an optimal alternating signal setting does not exist, then it is optimal to implement $r = c$ if $\phi_2 > \phi_1 + a_1 N$, whereas it is optimal for Route 2 if $\phi_2 \leq \phi_1 + a_1 N$.

Figure 2.8 schematically depicts the situation in which a toll is levied on the limited capacity route in a two-road network and the intersection is regulated by traffic lights. In

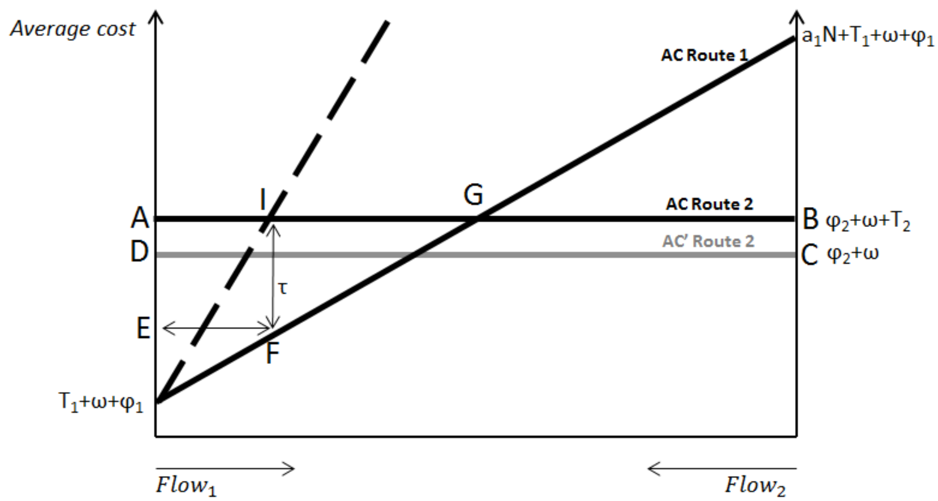


**Figure 2.8:** The interior solution (I) is optimal.

this graph the signal settings are considered optimal. It is clear that an interior solution exists (point I). A switch toward the corner solution $X_2 = N$ is accompanied by a cost reduction equal to the area ABCD. This equals:

$$N\frac{r^{*2}}{2c} = N\frac{\left(\phi_1 - \phi_2 + \frac{c}{2}\right)^2 c}{2\left(-2a_1 N + c\right)^2} \tag{2.15}$$

However, the toll revenue obtained at the interior solution equals area AIFE. This is:

$$X_1\tau(X_1) = \frac{\left(\phi_2 - \phi_1 + r^* - \frac{c}{2}\right)^2}{4a_1} \tag{2.16}$$

In Appendix F, it is shown that area AIFE is larger than area ABCD. Therefore, in this case, the interior optimum is better than the corner solution $X_2 = N$. This confirms the previous statement that if an interior optimum exists, it leads to lower costs than the corner solutions.

Figure 2.8 also shows that, if the optimal $\tau$ is levied and if the traffic light settings are optimal, the user equilibrium corresponds to the social optimum. This makes sense, as the externality is internalized by the toll.

### 2.3.3 Both routes subject to congestion

We will first focus on the case in which demand is inelastic and then extend the analysis to include elastic demand.

#### 2.3.3.1 Inelastic demand

**Proposition 4.** *When the intersection of two congested routes connecting one OD pair is regulated by traffic lights, the optimal alternating signal setting is independent of the total flow and is given by* $r = \frac{a_2 c}{a_1 + a_2}$.

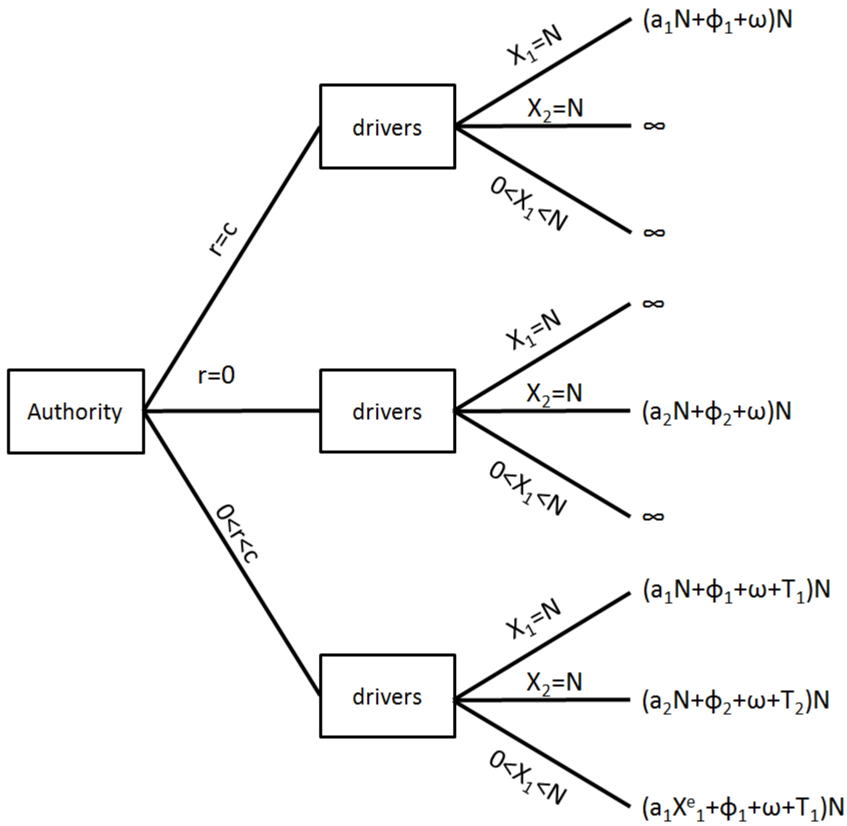


**Figure 2.9:** The total costs when the combined assignment and control problem is modelled as a Stackelberg game.

The government first decides on the signal settings and the drivers subsequently determine which route to take. Figure 2.9 illustrates this sequential game and shows all the feasible signal settings and the reaction of the drivers to each of these signal settings.

The third branch, representing the decision of the government to implement an alternating signal setting ($0 < r < c$), considers all red times between zero and c.

It can be shown that, if there exists an alternating signal setting for which the total cost is lower than for any non-alternating r ( i.e., if $a_1 X_1^e + T_1(c,r) < a_1 N$ and $a_2 X_2^e + T_2(c,r) < a_2 N$), then the user equilibrium for this r is the one in which both routes are used. Hence, the optimal alternating signal setting is the solution of the following optimization problem:

$$\min_{X_1, X_2, r} \left(a_1 X_1 + \omega + \phi_1 + T_1(c,r)\right) X_1 + \left(a_2 X_2 + \omega + \phi_2 + T_2(c,r)\right) X_2 \tag{2.17}$$

s.t.

$$X_1 + X_2 = N \tag{2.18}$$

$$a_1 X_1 + \omega + \phi_1 + T_1(c,r) = a_2 X_2 + \omega + \phi_2 + T_2(c,r) \tag{2.19}$$

$$0 < r < c, \qquad X_1 > 0, \qquad X_2 > 0 \tag{2.20}$$

The solution will be determined by:

$$a_1 \frac{\partial T_2(c,r)}{\partial r} = -a_2 \frac{\partial T_1(c,r)}{\partial r} \tag{2.21}$$

Assuming undersaturated traffic conditions, the explicit waiting time functions are given by equations (2.9) and (2.10). Inserting these expressions in equation (2.21), and solving for r, the proportion of red time for Route 2 equals:

$$\frac{r^*}{c} = \frac{a_2}{a_1 + a_2} \tag{2.22}$$

Even though the minimal time costs add to the total costs and we would therefore expect them to appear in the formula, they are not part of the optimal signal setting-formula. This can be explained by observing that the drivers themselves take the minimal time cost into account when choosing a route, while they omit the external congestion cost in their decision criterium. This omission is corrected by the optimal signal setting. The optimal red time on Route 2 increases in $\frac{a_2}{a_2 + a_1}$. The more congestible Route 2 is compared to Route 1, the more users will take Route 1 and thus a larger cost reduction is expected from an increase in green time on Route 1.

The total cost that the optimal traffic light settings produce is given by the following equation:

$$\left( \frac{2a_1a_2\left(a_1+a_2\right)N+2a_1\left(a_1+a_2\right)\left(\phi_2-\phi_1\right)+a_1a_2c}{\left(a_1+a_2\right)^2 2}+\phi_1+\omega \right) N \tag{2.23}$$

If this cost is lower than $\left(a_1N+\omega+\phi_1\right)N$ and $\left(a_2N+\omega+\phi_2\right)N$, then the optimal policy is to implement r=$\frac{a_2c}{a_1+a_2}$. If, on the other hand, the parameters are such that $\left(a_1N+\omega+\phi_1\right)N$ is the lowest cost, then the optimal policy would be to only give green to Route 1. Finally, if $\left(a_2N+\omega+\phi_2\right)N$ is the lowest cost a rational authority would implement $r=0$.

#### 2.3.3.2 Elastic demand

**Proposition 5.** *When the intersection of two congested routes connecting one OD pair is regulated by traffic lights, the optimal signal setting is independent of the elasticity of demand.*

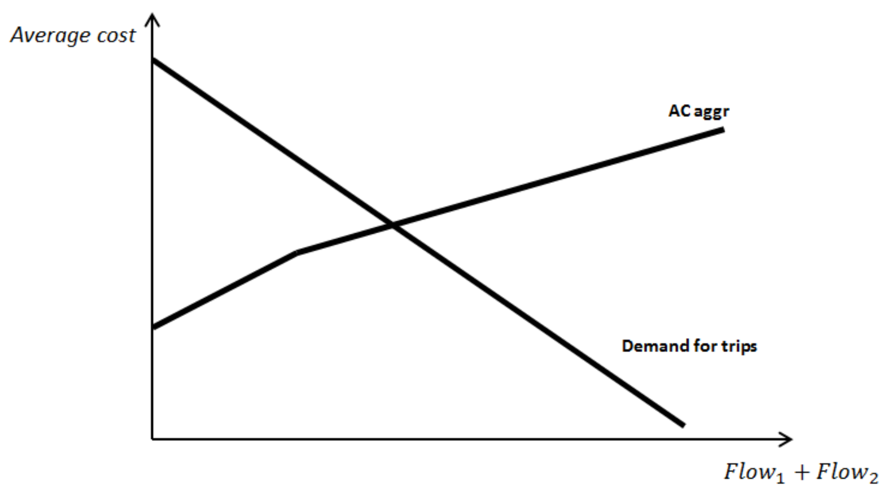


**Figure 2.10:** Equilibrium with elastic demand curve.

Regardless of the number of drivers that go from A to C, both routes will be used in the user equilibrium that comes about when the authorities implement an alternating signal setting. Taking this into account, we can construct an aggregate average cost function (Appendix C). The optimal alternating signal setting is the solution of the following maximization problem with $X_T$ the total flow in the network and $(\delta - \pi X_T)$ the demand function:

$$\max_{r,X_T} \int_0^{X_T} \left( (\delta - \pi X_T) - \left( 2\frac{a_1 a_2}{a_1+a_2} X_T + 2\frac{a_1\phi_2 + a_2\phi_1}{a_1+a_2} + 2\omega + \frac{a_1 T_2(c,r) + a_2 T_1(c,r)}{a_1+a_2} \right) \right) dX_T \tag{2.24}$$

s.t.

$$\frac{2a_1a_2X_T}{a_1+a_2} + \frac{2(a_1\phi_2 + a_2\phi_1)}{a_1+a_2} + 2\omega + \frac{a_1T_2(c,r) + a_2T_1(c,r)}{a_1+a_2} = \delta - \pi X_T \tag{2.25}$$

$$X_T \geq 0 \tag{2.26}$$

$$c > r > 0 \tag{2.27}$$

$X_T$ is determined by equalizing the elastic demand function $(\delta - \pi X_T)$ and the aggregate average cost function (2.25) and can be written as follows:

$$X_T = \frac{(a_1+a_2)\delta - 2\omega(a_1+a_2) - 2a_1\phi_2 - 2a_2\phi_1 - a_1T_2(c,r) - a_2T_1(c,r)}{(a_1+a_2)\pi + 2a_1a_2} \tag{2.28}$$

Assuming traffic conditions are undersaturated, equations (2.9) and (2.10) hold, and can be introduced in the optimization function. The optimal red time is then given by :

$$r* = \frac{a_2 c}{a_1+a_2} \tag{2.29}$$

This equation shows that the optimal red time is independent of the total flow within the network and yields the same result as in the case with inelastic demand. Therefore, it can be concluded that the optimal alternating signal settings are independent of the elasticity of demand when both routes have limited capacity.

## 2.4 The choice between traffic lights and a priority rule

**Proposition 6.** *If only one route is subject to congestion, then for all* $r < \sqrt{v^2 c \frac{\phi_2 - \phi_1}{a_1 - \frac{v^2}{2}}}$ *traffic lights are better than a priority rule. Furthermore, the higher the total number of drivers, the larger the cost advantage of traffic lights compared to a priority rule.*

If for some reason both routes have to be used and if r can be chosen such that $r < \sqrt{v^2 c \frac{\phi_2 - \phi_1}{a_1 - \frac{v^2}{2}}}$ and $0 < r < c$, then traffic lights are the better choice. This can be seen as follows: if both routes have to be used when traffic lights are present, the total cost amounts to $(\omega + \phi_2 + \frac{r^2}{2c})N$. Comparing this with the total cost in a priority rule situation, $\left(\omega + \phi_2 + \frac{v^2 X_1^e}{2}\right) N$, it is clear that if $\frac{r^2}{2c} < \frac{v^2 X_1^e}{2}$ (this comes down to $r < \sqrt{v^2 c X_1^e}$), traffic lights reduce the total cost. The cost savings that accompany the transfer to traffic light regulation thus equal $(\frac{v^2 \frac{\phi_2 - \phi_1}{a_1 - \frac{v^2}{2}}}{2} - \frac{r^2}{2c})N$. If this value exceeds the additional annualized investment cost, traffic lights are optimal.

A further examination on the condition on r ($r < \sqrt{v^2 c \frac{\phi_2 - \phi_1}{a_1 - \frac{v^2}{2}}}$) shows that traffic lights become more interesting when drivers are more careful (higher v). This is explicable, a higher v increases the lost time when the intersection is regulated by a priority rule, leading to a favourable regulation of the intersection by traffic lights.

## 2.5 Two applications

In this section, some of the theoretical results obtained in the previous section are illustrated with an example.

The first example applies a result of Section 2.2.2: when the intersection of two routes connecting one OD pair, of which only one is congestible, is regulated by a priority rule and the interior equilibrium exists, then the optimal policy is to block the congested route. The second example extends the results obtained in Section 2.3.2 by including local traffic within the problem setting.

### 2.5.1 A low-traffic city center

Consider a city where the inhabitants live on the edge of the city and work in the city center. There is a bike path, as well as a congestible road connecting work and home, and both are currently being used for commuting trips. In the city center, cyclists always have to give way to cars. Applying the results of Section 2.2.2 to this situation, we can conclude that for this city, the optimal policy would be to make the city center a car-free zone.

A city center in which no motorized traffic at all is allowed, is however, unrealistic. After all, shops have to be provisioned and emergency vehicles have to be able to enter the center. A simple solution, already adopted in many cities, is to allow only certain vehicles to enter the city center. This can be implemented using, for example, automatic rising bollards.

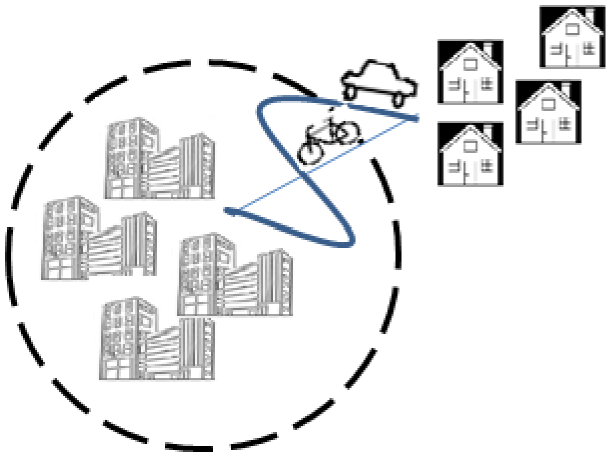

**Figure 2.11:** Commuters can take the car or the bicycle.

### 2.5.2 A bypass and a city road

A well-known situation in which one OD pair is connected by two parallel roads is represented in Figure 2.12. Here, transit traffic can choose between Route 1 or Route 2 to reach point C, while local traffic can only take Route 2. Let Route 1 (the bypass) have a large capacity. Furthermore, we will assume that both local traffic and cut-through traffic contribute to the city road congestion. Suppose that the traffic lights are regulated by a federal authority whose objective is to minimize the total cost of all drivers.

Let

$X_b$ be the number of transit drivers taking the bypass per hour;

$X_v$ be the number of transit drivers taking the city road per hour;

$a_v$ be the congestion sensitivity of the city road;

$\phi_v$ be the minimal time cost to get to point C using the city road;

$\phi_b$ be the minimal time cost to get to point C using the bypass;

R be the local traffic per hour;

N be the total transit traffic per hour;

$T_i\left(c, r\right)$ be the total waiting time cost at the traffic light on route $i$;

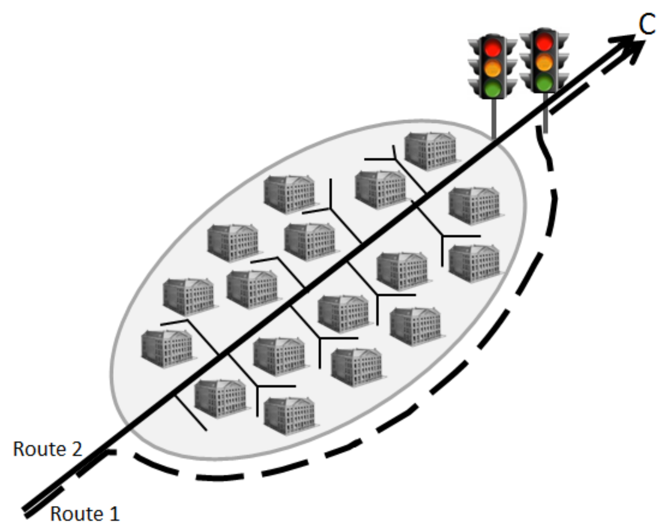


**Figure 2.12:** Transit traffic will either take the city road or the bypass depending on the signal settings.

and r be the red time for the city road.

**Table 2.3:** The total travel cost for every (r,UE)-combination in a bypass situation

| Signal setting | $X_b = N,$ | $X_v = N$ | $0 < X_b < N$ |
|---|---|---|---|
| $r = c$ | $\infty$ | $\infty$ | $\infty$ |
| $r = 0$ | $\infty$ | $(\phi_v + a_v(N+R))(N+R)$ | $\infty$ |
| $0 < r < c$ | $(\mathrm{a}_v R + \phi_v + T_v)R$ $+(\phi_b + T_b)N$ | $(a_v(N+R) + \phi_v + T_v)(N+R)$ | $(\phi_b + T_b)(N+R)$ |

In Table 2.3, the total cost is shown for every combination of policy and user equilibrium. A glance at the table shows that a rational authority would never implement $r = c$. When the city road always has green ($r = 0$), the only Nash equilibrium is $X_v = N$. The total cost in this case amounts to $(\phi_v + a_v\,(R+N))\,(R+N)$. Furthermore, the total cost for the combination ($X_v = N$, $0 < r < c$) is always larger than the total cost for the combination ($X_v = N$, $r = 0$). Finally, it can be shown that the FOC of $(\phi_b + T_b)(N+R)$ w.r.t. r is always negative. As a consequence, the lowest cost when both routes are used occurs at the signal setting for which transit traffic is indifferent between using both routes and using only the bypass. The total cost curve when $X_b = N$ is convex. Here, the minimum of this total cost curve equals $r = \frac{Nc}{N+R}$.

The previous observations narrow down the candidate solutions to either $r = 0$, $r = -a_v R - \phi_v + \phi_b + \frac{c}{2}$ or $r = \frac{Nc}{N+R}$. In order to determine the optimal signal setting, the government has to compare the cost of the different solutions.

Suppose now that the city road has limited capacity, then how will this change the results? From Section 2.3.2.1, we know that if a signal setting leads to saturated conditions on the city road and both routes are used, the total cost will be the same as in the under-saturated case. Consequently, the first order condition of the total cost function will again be negative and the optimal solution will be the signal setting for which transit traffic is indifferent between using both routes and using only the bypass.

The minimal green time for which all the local drivers are below capacity ($r = c - \frac{Rc}{s_v}$) will be an element of the interval for which $X_b = N$. If $r = c - \frac{Rc}{s_v}$ is greater than $\frac{Nc}{N+R}$, then either $\frac{Nc}{N+R}$, $r = -a_v R - \phi_v + \phi_b + \frac{c}{2}$, or $r = 0$, will be the optimal solution. If on the other hand $r = c - \frac{Rc}{s_v}$ is lower than $\frac{Nc}{N+R}$, then either $r = c - \frac{Rc}{s_v}$, $r = -a_v R - \phi_v + \phi_b + \frac{c}{2}$, or $r = 0$, will be the optimal solution.[9]

It is clear that a local government, preferring minimal transit traffic in its city, would try to avoid the ($X_v = N$) outcome. The local government can do this by increasing $a_v$ or $\phi_v$.[10] Increasing $a_v$ or $\phi_v$ raises the cost of the ($X_v = N$, $r = 0$) combination relatively more,[11] which decreases the likelihood of the federal govenment implementing $r = 0$.
Today, many cities already apply this strategy. Indeed, speed bumps and speed limits are put in place to increase $\phi_v$ and local governments limit the capacity of roads to increase $a_v$ (De Borger and Proost (2013)).

## 2.6 Concluding remarks

In this paper, we studied the effects of a priority rule, traffic lights, and a toll on an intersection of two routes connecting one OD pair. We derived the intersection regulation that minimizes total travel cost, taking into account Wardrop's principles and the delay at the intersection.

[9] The demand function of the local drivers is assumed inelastic, so it is relatively expensive to drive back the demand. So even if for $\frac{Nc}{N+R}$ the total cost is minimal, the loss in consumer surplus of the drivers that no longer make the trip still reduces welfare.

[10] We only mention the parameters the local government can influence.

[11] $\left(\frac{dTC_{r=0}}{d\phi_v} > \frac{dTC_{r=\frac{Nc}{N+R}}}{d\phi_v} \text{ and } \frac{dTC_{r=0}}{da_v} > \frac{dTC_{r=\frac{Nc}{N+R}}}{da_v}\right)$

We have four major results. First, if the intersection is regulated by a priority rule, the optimal policy is generally to block one of the two routes. Second, if the intersection is regulated by traffic lights, and only one route is congestible, the optimal policy is again to block one route. However, the addition of a toll allows for an optimal alternating signal setting. Third, if the intersection is regulated by traffic lights, the optimal alternating signal setting is always independent of the elasticity of demand. Finally, if only one route is subject to congestion, the superiority of a regulation by traffic lights over a priority rule becomes more likely the lower the reaction time of the drivers, and the higher the cycle time.

These results are important for three reasons. First, the counter-intuitive nature of these results confirms the importance of a good understanding of the causal mechanisms that govern the optimal regulation. Second, these insights allow to solve larger networks more efficiently as well as more effectively. More efficient, because the increased insight in the location of the optimal solution allows for a reduction in computation time. More effective, because local optima can be detected more easily. Finally, the obtained results can be applied in practice. Our results can be useful in different contexts. We primarily think about two parallel roads (e.g. a road through an urban area and a parallel road bypassing the city) or two parallel modes (e.g. a train and a road connecting two cities).

The results in this paper can be applied to solve one particular larger network problem. In this network problem the two routes are a chain of individual components similar to the one solved in this paper. If, for every component, it is optimal to use one and the same link,[12] it can be concluded that it is optimal to maintain only one route. As this composition technique can only be applied to a certain type of network problems, one future research line is to extend the model to larger networks. Other future work includes the extension of the model towards multiple government levels.

## Acknowledgements

This work was supported financially by the OT/11/068-project. The authors wish to thank three anonymous referees, the Editor-in-Chief Hai Yang as well as C. Tampère, F. Viti and participants at the ITEA meeting (Northwestern 2013) for helpful suggestions on a previous draft.

[12] We can use the results from this paper to solve this problem on the component level.

# Glossary

**Parameters**

$\delta$ maximum willingness-to-pay for a trip from A to B

$\omega$ resource cost per trip from A to C

$\phi_1$ minimal time cost to go from A to C using route 1

$\phi_2$ minimal time cost to go from A to C using Route 2

$\pi$ marginal willingness-to-pay for a trip from A to B

$a_1$ increase in average cost on Route 1 when one vehicle is added

$a_2$ increase in average cost on Route 2 when one vehicle is added

$c$ cycle length

$N$ total inelastic demand from A to B

$s_1$ saturation flow of Route 1

$s_2$ saturation flow of Route 2

$v$ time gap in which the users of the minor road are stationary at the intersection before the crossing of a main road user

**Control variables**

$\tau$ toll fee

$r$ duration of red per cycle given to Route 2

**Variables**

$X_1$ flow on Route 1

$X_2$ flow on Route 2

$X_T$ total flow in the network

# Chapter 3

# The puzzle of traffic-responsive signal control: why common sense does not always make sense.

## 3.1 Introduction

A lot of the current signal control systems are based on traffic-responsive control. This type of control allocates green time in proportion to the relative magnitude of the flow. In this paper, we show that, though intuitively superior, this type of control is not necessarily the most efficient. The main reason is that in traffic networks the user equilibrium is often not optimal. Blindly attempting to accommodate to the volume of traffic on a link with congestion, by adding capacity or by giving more green time, is a widespread problem, which can, more generally, be ascribed to the phenomenon of "induced demand".

The theory of induced demand asserts that improvements in the transportation infrastructure attract new traffic. The available literature has largely centered around the demand-inducing and traffic diversion effects of particularly road expansion (Downs (1962), Braess (1968), Noland (2001)). This literature has provided a basis for a major rethinking of road-expansion policies. Recognition of the generated traffic effects of traffic-responsive control can influence policy making in the same way.

In many cities, signal control is of the traffic responsive type. The control system SCOOT, for example, has been implemented in more than 250 towns and cities (Hamilton et al. (2013)). The rapid and widespread implementation of traffic-responsive signal control is strongly connected to the intuitive superiority of this control policy, which made it politically more acceptable. An equal intuitive discourse is needed to challenge this inclination towards responsive signal control. The objective of this paper is therefore to provide a clear and accessible comparison of responsive signal control versus anticipatory signal control, which provides insight for policy making.

Already in 1974, the need to take into account the interaction between route choice and signal control was pointed out by Allsop (1974). In most papers nowadays the importance of this interaction is recognized and the interaction is thus included in the model. The way this interaction is modelled differs, however, from paper to paper. Here, two specific ways to model this interaction, i.e. anticipatory signal control and responsive signal control, are considered.

Miller (1963) was the first to introduce the notion of traffic-responsive control. In traffic-responsive control, data collected from vehicle detectors located upstream is used to optimize the signal settings.

The strand in the literature that deals with traffic-responsive signal control focuses on the iterative optimization and assignment procedure. In the iterative optimization and assignment procedure the signal settings and equilibrium flow patterns are updated alternatively, until both flows are at equilibrium and signal settings are optimal given the flows (Allsop and Charlesworth (1977), Cantarella et al. (1991), Gartner et al. (1980), Lee and Hazelton (1996)).

In the case of anticipatory signal control the road authority anticipates the reaction of the drivers to a change in the signal settings and thus optimizes the signal settings taking into account the reaction of the drivers. In the literature, this has been formulated as a bi-level problem, in which the upper level is the signal setting problem and the lower level is the traffic equilibrium assignment problem (Chiou (1999), Yang and Yagar (1995)), and as a Stackelberg game (Fisk (1984)).

A few papers have touched on the shortcomings of responsive signal control. Both Gershwin and Tan (1978) and Dickson (1981) have solved the combined traffic assignment and control problem for a specific numerical example, using on the one hand the iterative optimization and assignment procedure and on the other hand a constrained optimization approach. For their specific examples, both show that the iterative optimization and assignment leads to a worse solution.

In this paper we use a game theoretical perspective to model both the anticipatory and the responsive signal setting procedure for a simple network. This results in clear theoretical results, which allow to give insights in the underlying mechanisms. The remainder of this paper is organized as follows: First, the problem at hand is described in Section 2. Subsequently, the outcomes of the traffic-responsive and anticipatory framework are compared and discussed in Section 3. Section 4, finally, offers a conclusion.

## 3.2 Problem formulation

The model we use to compare the performance of anticipatory signal control with the performance of traffic-responsive signal control is the simple two-road model represented in Figure 3.1. We assume that, per time unit, N people want to go from A to B.[1] Furthermore, we limit the model to undersaturated traffic conditions, i.e. queues at the intersection are only created during the red phases and dissolved during the green phases. To go from A to B, drivers can either take a congestible route (Route 1) or an uncongestible route (Route 2).[2] In this paper, the red phase on Route 2 is represented by 'r', and will be the main control variable. The corresponding green phase on Route 2 will thus be 'c-r', and a reverse scenario holds for Route 1. The duration of the sum of the red and the green phase is the cycle time 'c', which, to simplify matters, is held fixed. Hence, it follows that including intergreen time in the analysis is not relevant and will thus be ignored.

As there will be alternating red times to avoid collisions at the intersection, drivers on both routes will experience an expected traffic light waiting time cost $(T_1(c,r), T_2(c,r))$. It is clear that the expected traffic light waiting cost functions are increasing in the red time and decreasing in the green time ($\frac{\partial T_1(c,r)}{\partial r} < 0, \frac{\partial T_2(c,r)}{\partial r} > 0$). Assuming undersaturated

[1] The arrival rate is thus inelastic, static and deterministic.
[2] Up to N drivers per hour, the time cost curve for this route is horizontal.

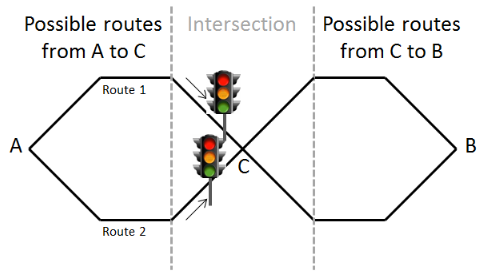


**Figure 3.1:** Outline of an intersection of two routes connecting one OD pair (AB) regulated by traffic lights.

traffic conditions, the expected traffic light functions take the following form for $0 < r < c$:[3]

$$T_1(c,r) = \frac{(c-r)^2}{2c} \tag{3.1}$$

$$T_2(c,r) = \frac{r^2}{2c} \tag{3.2}$$

When it is always red ($r = c$ for Route 2 and $r = 0$ for Route 1 ) the expected traffic light waiting function jumps to infinity.

We will model both the anticipatory and the traffic-responsive control and assignment problem as a Stackelberg game (Von Stackelberg (1934)). The Stackelberg game is a sequential game in which the leader moves first and the follower acts sequentially. In this paper, the traffic authority is the leader when the signal control is anticipatory and the traffic authority is the follower when the signal control is traffic-responsive. The objective of the traffic authority is to maximize welfare. The drivers in turn represent the follower when the signal control is anticipatory and the leader when the signal control is traffic-responsive. We will assume that all drivers are identical and try to minimize their expected travel cost.

The behaviour of the drivers can also be represented as a game, because the congestion on one road is dependent upon how many users choose to use the same road. In this paper, we will assume that the drivers behave non-cooperatively (Nash (1951)).

[3] We will assume that the saturation flow rate g is very large in comparison to the arrival rate $X_i$, so that the traffic light waiting time due to departure delay is negligible.

## 3.3 Results

### 3.3.1 Traffic-responsive signal control

When the signal control is traffic-responsive, the signal settings respond to the current traffic conditions measured by a vehicle detector. This situation is represented as a Stackelberg game in Figure 3.2. The game tree shows all the possible distributions of the drivers over the two routes and all the policies the government can implement in reaction to the drivers' choice.

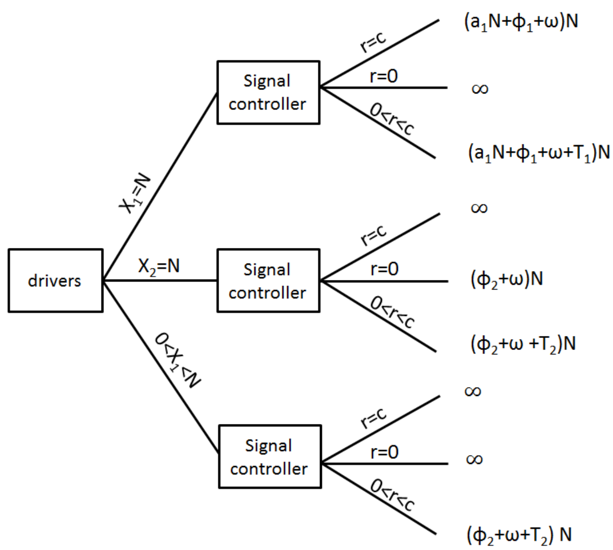


**Figure 3.2:** Total costs when signal control is traffic-responsive.

To predict the outcome of this game, we will first determine the best response of the road authority to every possible distribution of the drivers over the two routes. Assuming that the government wants to maximize welfare, their best response to any distribution over the two routes is to maximize welfare taking as given this distribution. From the game tree, it is clear that the traffic authority will always give green to Route 1 when all drivers choose Route 1. When all drivers choose Route 2, the rational decision for the traffic authority is to give always green to Route 2. When the drivers divide themselves over the 2 routes, the optimal response of the traffic authority is the solution of the following optimization problem.

$$\min_r \left(a_1 X_1 + \omega + \phi_1 + T_1\left(r\right)\right) X_1 + \left(\omega + \phi_2 + T_2\left(r\right)\right) X_2 \tag{3.3}$$

$$0 < r < c \tag{3.4}$$

In this optimization problem $a_i$ represents the sensitivity to congestion of route $i$, $X_i$ equals the flow on route $i$,[4] $\phi_i$ stands for the minimal time cost of route $i$, $\omega$ is the resource cost for a trip from A to C on either route and $T_i\left(c, r\right)$ is the expected waiting time cost on route $i$ at the intersection.

Taking the derivative of (3.3) to r and taking into account that $X_1 + X_2 = N$, we find that the optimal strategy for the government is to implement the following red time:

$$r^* = \frac{X_1 c}{N} \tag{3.5}$$

Making this trip day in day out, the drivers will come to learn the optimization formula of the government, which is easy to understand as the red time is inversely proportional to the flow. As a result, their private cost functions have the following form for $0 < X_1 < N$:

$$a_1 X_1 + \phi_1 + \omega + \frac{cX_2^2}{2N^2} \tag{3.6}$$

for Route 1 and

$$\phi_2 + \omega + \frac{cX_1^2}{2N^2} \tag{3.7}$$

for Route 2. Remark that $\frac{cX_2^2}{2N^2}$ is the expected traffic light waiting cost for Route 1 ($\frac{(c-r^*)^2}{2c}$, with $r^* = \frac{X_1 c}{N}$) and $\frac{cX_1^2}{2N^2}$ is the expected traffic light waiting cost for Route 2 ($\frac{r^{*2}}{2c}$, with $r^* = \frac{X_1 c}{N}$).

Drivers will individually seek to minimize their private cost and will consequently change routes until unilaterally changing increases their private cost. At that point the stationary distribution of vehicles in the network, i.e. the equilibrium, is reached. And this stationary distribution will thus be the outcome of the drivers' part of the game which, together with the government's strategy, will determine the total cost of responsive signal control.

Remark that, even though the signal setting policy of the traffic authority provides the drivers with an opportunity to manage the actions of the traffic authority, they can not

[4] This is the flow measured by the vehicle detector.

exploit this advantage as the drivers do not cooperate.

Depending on the parameter values and the amount of drivers on Route 1, the average cost curve is either downward sloping ($\frac{dAC_1}{dX_1} < 0$) or upward sloping ($\frac{dAC_1}{dX_1} > 0$). The average cost on Route 1 can be downward sloping because an increase in volume on Route 1 implies a decrease in volume on Route 2. With responsive signals this implies a longer green phase on Route 1 which can outweight the increased congestion on Route 1. In this paper, we will determine the equilibria for only one instance: the waiting cost always outweights the congestion cost ($\forall X_1 : \frac{dAC_1}{dX_1} < 0$). We can restrict ourselves to this instance, as it suffices to show the superiority of anticipatory control.

When the waiting cost outweights the congestion cost for all possible distributions of vehicles over the two routes, we can distinguish between three cases: either the average cost of Route 1 is always larger than the average cost of Route 2 or the other way around, or the average cost curves intersect. We will determine the equilibria for each of these cases separately.

*Case 1. If for every distribution of vehicles over the two routes $AC_1$ is larger than $AC_2$, then the only equilibrium is $X_2 = N$.*

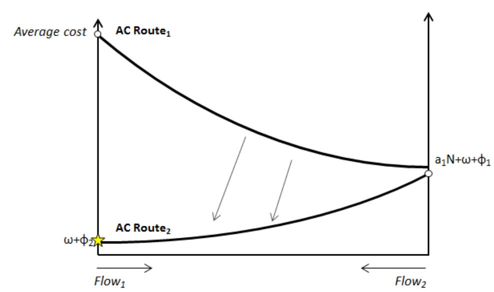


**Figure 3.3:** The only stationary distribution is $X_2 = N$

The search for potential equilibria in the set of possible distributions of vehicles over the 2 routes is greatly simplified by Wardrop's first principle (Wardrop (1952)): In a user equilibrium, all used routes for an OD pair should have equal generalized prices, and there are no unused routes with lower generalized prices. For this case, the first part of the principle, eliminates all distributions in which both routes are used. The second part of the principle, eliminates the outcome where all drivers are on Route 1. The only distribution left, $X_2 = N$, satisfies the definition of a Nash equilibrium. The total cost in this case equals $(\omega + \phi_2) N$.

*Case 2. If there exists a distribution of vehicles over the two routes for which $AC_1 = AC_2$, then there are two potential equilibria: $X_2 = N$ or $X_1 = N$.*

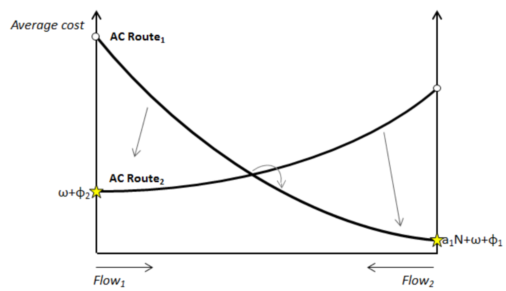


**Figure 3.4:** There are two stationary distributions: $X_2 = N$ and $X_1 = N$

In this case, Wardrop's first principle leaves us with three potential equilibria: $X_2 = N$, $X_1 = N$, and $X_1 = \frac{\left(\frac{c}{2}+\phi_1-\phi_2\right)N}{c-a_1N}$. When the distribution of vehicles is such that the average cost of both routes is the same, then a driver on Route 1 could lower his cost by unilaterally changing to Route 2 (or a driver on Route 2 could lower his cost by unilaterally changing to Route 1). So depending on the initial distribution of vehicles over the two routes, either $X_2 = N$ or $X_1 = N$ will be the stationary distribution. The corresponding total cost equals $(\omega + \phi_2) N$ when the equilibrium is $X_2 = N$ and $(a_1N + \omega + \phi_1) N$ when $X_1 = N$.

*Case 3. If for every distribution of vehicles over the two routes $AC_2$ is larger than $AC_1$, then the only equilibrium is $X_1 = N$.*

If for every possible distribution of vehicles over the two routes, the private cost when taking Route 2 is higher than the private cost of taking Route 1, then all drivers will take Route 1. This dominant strategy leads us to the equilibrium distribution $X_1 = N$. The private cost every driver will incur equals $a_1N + \omega + \phi_1$, and the total cost thus amounts to $(a_1N + \omega + \phi_1) N$.

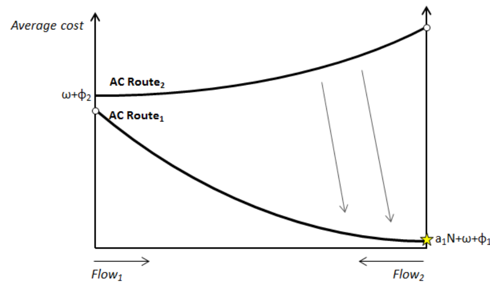


**Figure 3.5:** The only stationary distribution is $X_1 = N$

### 3.3.2 Anticipatory signal control

When signal control is anticipatory, the traffic authority moves first and bases its decision on the expected reaction of the drivers. Figure 3.6 shows the different options for the government, and the possible reactions of the drivers.

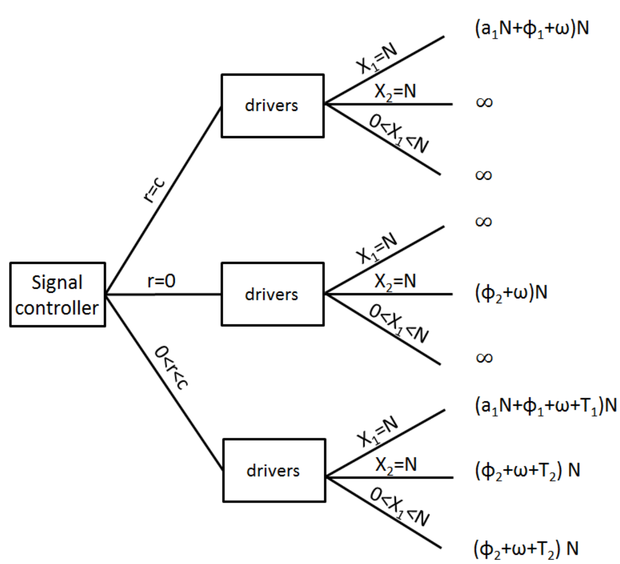


**Figure 3.6:** Total costs when signal control is anticipatory.

The traffic authority has three possible policies: to grant always green to Route 1, to grant always green to Route 2, or to implement an alternating signal setting.[5] The users in turn can react in three different ways to any chosen policy: to only take Route 1, to only use Route 2, or to use both routes.

If the traffic authority decides to implement always red for Route 1, then all drivers will take Route 2 and the total cost will equal $(\omega + \phi_2) N$. If, on the other hand, the traffic authority grants always green to Route 1, then the user equilibrium will be $X_1 = N$ and the total cost will be $(a_1 N + \omega + \phi_1) N$. If, however, the government decides to implement an alternating signal setting, the equilibrium reaction of the drivers will be $X_1 = N$ if $a_1 N + \phi_1 + T_1(c, r) < \phi_2 + T_2(c, r)$, and $X_2 = N$ if $\phi_1 + T_1(c, r) > \phi_2 + T_2(c, r)$. If $\phi_1 + T_1(c, r) < \phi_2 + T_2(c, r) < a_1 N + \phi_1 + T_1(c, r)$, the drivers will use both routes, and the Wardrop equilibrium implies $(\omega + \phi_2 + T_2(c, r)) N$ as total cost.

A glance at Figure 3.6 reveals that a rational government will never decide on an alternating signal setting when both routes are substitutes. Indeed, let $T_i(c, r)$ be the expected waiting time cost on route $i$ at the intersection. As $T_i(c, r)$ is positive when $0 < r < c$, the total travel cost for an alternating signal setting will always be higher than for $r = c$ or $r = 0$. Which of the two non-alternating signal settings will be optimal depends on the values of the parameters $a_1, N, \phi_1$ and $\phi_2$. Whenever $a_1 N + \phi_1 < \phi_2$, $r = c$ is the optimal solution and whenever $a_1 N + \phi_1 \geq \phi_2$, $r = 0$ will be implemented.

### 3.3.3 Anticipatory versus traffic-responsive signal control

From the analysis in Section 3.3.2, we know that when $a_1 N + \phi_1 < \phi_2$, the total cost amounts to $(a_1 N + \omega + \phi_1) N$ and when $a_1 N + \phi_1 \geq \phi_2$, the total cost equals $(\omega + \phi_2) N$ when traffic control is anticipatory. The outcome in each of the two scenarios is not so straightforward in case traffic-responsive control is implemented,[6] so it is rather cumbersome to directly compare total costs. However, the following line of reasoning allows to asses the relative performance of anticipatory and traffic-responsive control in an indirect way.

Comparing the outcomes of anticipatory signal control (Figure 3.6) to the possible outcomes of traffic responsive signal control (Figure 3.2), it is clear that the performance

[5] The third branch, representing the decision of the government to implement an alternating signal setting $(0 < r < c)$, is a clubbing of all red times between zero and c.

[6] As it depends on the relative values of some of the parameters.

of traffic responsive signal control can only be equally well or worse than anticipatory signal control. Consequently, if there exists one case for which the performance of traffic responsive signal control is worse, we can conclude that the overall expected perfomance of traffic-responsive signal control is worse than anticipatory signal control.

Take the scenario in which $a_1N + \phi_1 < \phi_2$, then out of the three cases we have dealt with in section 3.3.1, Case 2 and Case 3 can occur. If Case 2 occurs the total cost is either $(a_1N + \omega + \phi_1)\,N$ or $(\omega + \phi_2)\,N$. If Case 3 occurs, the total cost is $(a_1N + \omega + \phi_1)\,N$.

Remember that in this case the total cost with anticipatory signal control equals $(a_1N + \omega + \phi_1)\,N$. The possible outcome of the stationary distribution $X_2 = N$, resulting in a signal setting $r = 0$ and total cost $(\omega + \phi_2)\,N$, when $a_1N + \phi_1 < \phi_2$ thus proves that there exists at least one case for which the performance of traffic responsive signal control is worse than the performance of anticipatory signal control. As a result, we can assert that for our model, the performance of anticipatory signal control is superior to the performance of traffic-responsive signal control.

This result can be explained by recognizing the presence of externalities and the first mover advantage. Because of externalities, the drivers' individual choices are not socially optimal. Or, putting it differently, every driver minimizes his own cost, but this does not necessarily minimize the cost of all drivers. The traffic authority's objective is to minimize the cost of all drivers, so its unconstrained decisions are socially optimal. However, in both the traffic responsive control problem and the anticipatory control problem the traffic authority's optimization problem is constrained, to a greater or lesser extent, by the behaviour of the drivers. When the traffic authority is the leader, she can act so as to elicit the most favorable response of the driver. However, when the traffic authority is the follower, her influence on the drivers' behaviour is more restricted. Analytically, when signal control is anticipatory the leader's optimization problem is constrained by the drivers' reaction function, while when signal control is traffic-responsive the leader's optimization problem is constrained by the drivers' individually optimized distribution over the different routes. It is clear that the traffic authority's constraint is much more restricting when signal control is traffic-responsive. This also becomes apparent in Figure 3.7 below, in which the total cost is compared in case the traffic authority is the leader and when the traffic authority is the follower.

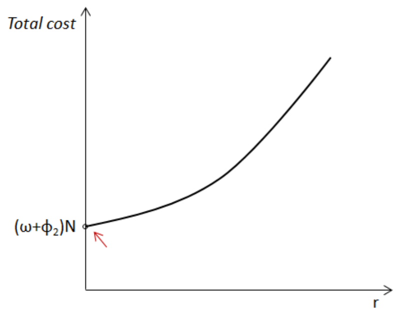


**Figure 3.7:** Comparison of the total cost when traffic control is anticipatory and traffic-responsive for an interior equilibrium.

When both routes are used in equilibrium, the anticipatory signal setting will always be the lowest red time possible (on Route 2), resulting in the lowest costs (see red arrow in Figure 3.7). However, when signal control is responsive, the equilibrium signal setting equals $r = (\frac{\phi_2 - \phi_1 - \frac{c}{2}}{a_1 N - c})c$, which, depending on the parameter values, results in a total cost that is at least as high as the lowest cost.

Our discussion allows to extend the obtained results to other networks. Indeed, the above reasoning can be applied as well to other situations in which signal settings can influence route choice. If there is no route choice, then the traffic-responsive control coincides with anticipatory control.

## 3.4 Conclusion

In this paper we have shown that for an intersection of two routes connecting one OD pair where only one route is subject to congestion (1) traffic responsive signal control can only perform just as well or worse than anticipatory signal control and that (2) the expected performance of traffic responsive signal control is worse than the performance of anticipatory signal control. The game theoretic perspective taken in this paper furthermore suggests that these results can also be extended to larger instances.

These results have important implications for policy. The counter-intuitiveness of these results indicates that great attention needs to be given to the accuracy of the appraisal of signal control investments. Since both the costs of road transportation infrastructure and user costs are large, policy should be based on careful analysis rather than on intuition alone. The allocation of public money to the intuitively superior traffic-responsive signal control, may actually make society worse off as the money could be more efficiently spent on anticipatory signal control. This paper furthermore intends to raise awareness that policies based on intuition alone can have unintended consequences in the hope that these can be recognized and avoided.

A final note on the results in this paper concerns the deterministic nature of demand in this paper. An interesting extension would deal with stochastic demand as the flexible nature of traffic-responsive signal control could mitigate the advantage of anticipatory control in this case.

## Acknowledgements

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# Chapter 4

# Can traffic lights achieve the same results as tolls?

## 4.1 Introduction

One of the main constraints in the optimization of networks is the socially suboptimal behavior of drivers. This suboptimal behavior manifests itself in an excessive amount of drivers on the road, or in an inefficient distribution of vehicles over alternative routes. In this paper, the focus will solely be on this second type of inefficiency.

The main reason behind this inefficiency is a congestion externality often present in transportation. As drivers do not take into account their delaying effect on other users when making a decision about which road to take, an inefficient situation arises.

The solution proposed by Pigou (1920) and Knight (1924) is to internalize the externality by introducing road pricing. Travellers are then confronted with their marginal social cost, rather than their average cost, and will consequently make the socially optimal route choice. This paper analyzes the conditions under which traffic lights can provide an adequate alternative to road pricing.

The suggestion for traffic lights as an alternative to road pricing is based on the observation that traffic lights have the potential to influence route choice.

Already in 1974, Allsop pointed out that signal settings can affect route choice. A change in the signal settings influences the average cost of the routes, leading to a change in route choice. This insight has generated two approaches in the literature to optimize a network with traffic lights: (1) the iterative procedure and (2) the global optimization approach (Cantarella et al. (1991)).

The iterative procedure iteratively solves the signal setting problem for a fixed flow pattern and the assignment problem for fixed signal settings until two successive flow patterns or signal settings converge (Allsop and Charlesworth (1977), Gartner et al. (1980), Cantarella et al. (1991)).

When the global optimization approach is applied, some network objective function is optimized while taking into account the equilibrium route choice behaviour of the drivers. The global optimization problem can be modelled as a bilevel programming problem (Yang and Yagar (1995)) or as a Stackelberg game (Fisk (1984)).

Compared to the iterative approach, the Stackelberg approach enables the traffic authority to exert more control over the equilibrium route choice of the drivers. As this paper aims at using traffic lights to influence route choice, applying the Stackelberg approach seems the logical choice.

First, we will model the signal setting procedure as a Stackelberg game. In a later stage, we will use the inverse Stackelberg approach (Olsder (2009)), which is an extension of the basic Stackelberg game. In the basic Stackelberg game the leader chooses an action after which the follower determines his optimal response. In the inverse Stackelberg game the leader action is generalized from making a direct decision to determining a function that maps the followers' decision space into the leader's decision space. As such, the inverse Stackelberg approach allows to fully control the route choice behaviour of the drivers.

The following example, adopted from Groot et al. (2012), illustrates the inverse Stackelberg concept.

**Example 1.** *Consider the following simple static, single-leader single-follower situation. Let the objective functions of leader and follower be respectively:*

$$J_L\left(u_L, u_F\right) = \left(u_F - 5\right)^2 + u_L^2,$$

$$J_F(u_L, u_F) = u_L^2 + u_F^2 - u_L u_F,$$

*with decision variables* $u_L \in \Re, u_F \in \Re$. *The leader's global optimum is* $\left(u_L^d, u_F^d\right) = (0, 5)$. *In the original Stackelberg game formulation, the follower's response to the desired variable* $u_L^d = 0$ *would be the suboptimal* $u_F^* = 1/2u_L = 0$.
*However, under the leader function*

$$u_L = \gamma_L(u_F) = 2u_F - 10,$$

*the follower's response will be:*

$$\arg\min_{u_F} J_F(u_F) = \arg\min_{u_F} (2u_F - 10)^2 + u_F^2 + (2u_F - 10)\, u_F = 5.$$

We focus our analysis on two different types of networks, one in which the traffic lights' primary objective is to regulate an intersection and another network in which traffic lights are installed with as a sole objective to influence route choice. Both networks are deliberately kept as simple as possible to allow for clear, intuitive results.

The remainder of this paper is organized as follows: in Section 4.2 the different networks are described. In Section 4.3 the relative performance of road pricing and traffic lights is compared for the network with two parallel routes. In Section 4.4, we focus on the network in which the main purpose of the traffic lights is to avoid collisions. Section 4.5 offers a discussion and Section 4.6 concludes this paper.

## 4.2 Basic set-up: network, demand, equilibrium conditions

Per time unit N users wish to travel from a single origin (A) to a single destination (B). The drivers can choose between two alternative routes indexed by $i \in \{1, 2\}$. Let $f_1$ ($f_2$) be the minimum travel time from A to B via Route 1 (Route 2). Both routes are congestible, and the congestion is represented by a the variable travel time that is an increasing linear function of the number of users, $X_i$, on this route. The route's sensitivity to congestion is denoted by $a_i$.

In this paper, we distinguish between two different networks of two routes connecting an origin A to a destination B. In the first network (Figure 4.1) the capacity of road CB is such that the merge of Route 1 and Route 2 drivers can occur without hindrance. In that case, no additional cost is incurred at point C and the average cost when using Route $i$ equals:

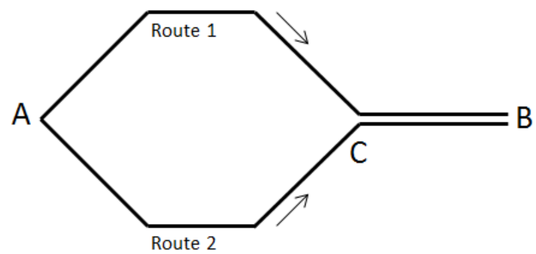


**Figure 4.1:** A network without intersection

$$AC_i = f_i + a_i X_i \tag{4.1}$$

In the second network (Figure 4.2) the intersection is regulated by traffic lights. In that case, the two routes can not have simultaneous right of way, and the drivers on both routes will experience an expected traffic light waiting time cost $(T_1(c,r), T_2(c,r))$.

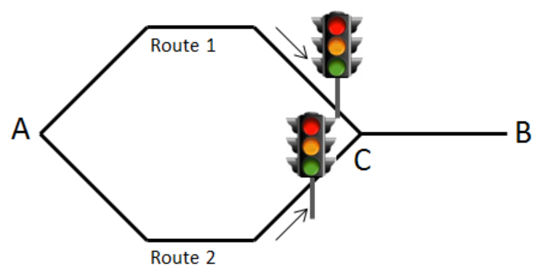


**Figure 4.2:** A network with an intersection

The average cost on Route $i$ is given by the sum of the fixed and variable time cost and the expected waiting time cost at the traffic light $(T_i(r,c))$:

$$AC_i = f_i + a_i X_i + T_i(r,c) \tag{4.2}$$

In this paper, $r_2$ is the red time for Route 2. The duration of the sum of the red and the green phase is the cycle time ‘c’, which, to simplify matters, is held fixed. Hence, it follows that including intergreen time in the analysis is not relevant and will thus be ignored. The red time for Route 1 will be $(c - r_2)$.

It is clear that the expected traffic light waiting cost functions are increasing in the red time and decreasing in the green time ($\frac{\partial T_1(c,r_2)}{\partial r_2} < 0, \frac{\partial T_2(c,r_2)}{\partial r_2} > 0$). When it is always red ($r_2 = c$ for Route 2 and $r_2 = 0$ for Route 1) the expected traffic light waiting cost is infinitely high. For simplicity, we will assume that the queue that builds up during the red time fully dissipates during the green time and that the cost associated with it is negligible. In that case the expected traffic light functions take the following form for $0 < r_2 < c$:

$$T_1\left(c, r_2\right) = \frac{\left(c - r_2\right)^2}{2c} \tag{4.3}$$

$$T_2\left(c, r_2\right) = \frac{r_2^2}{2c} \tag{4.4}$$

All drivers are assumed identical and try to minimize their expected travel cost. The equilibrium concept used in this paper is known as the user equilibrium. It was introduced by Bernstein and Smith (1994) and used by e.g. De Palma and Nesterov (1998). In the user equilibrium no arbitrarily small portion of drivers on a route can lower its private cost by deviating to another route that connects the same origin destination pair.[1] When the private cost functions are continuous, the user equilibrium reduces to the Wardrop equilibrium (Wardrop (1952)). With two routes, the Wardrop equilibrium has either all drivers on Route 1, or all drivers on Route 2, or a distribution of drivers over the two routes such that the average cost on both routes is equal.

## 4.3 A network without intersection

In this section, the focus is on a network in which traffic lights are not strictly necessary, in the sense that there are no conflicting traffic streams. We first determine the minimal total cost when route choice is influenced by road pricing (Section 4.3.1). In this same section, we also work out the minimum cost when traffic lights are used to influence route choice (Section 4.3.2). In both sections, the optimization process is modelled as a Stackelberg game in which the traffic authority is the leader and the drivers are the follower. In Section 4.3.3 the different policies are compared.

**Proposition 7.** *In a network with two parallel routes, a traffic light with an optimal signal setting determined by the Stackelberg game can not provide an adequate alternative for road pricing.*

[1] See Section 4.4.2 for a formal definition.

### 4.3.1 Road pricing to influence route choice

When demand is inelastic, the inefficiency caused by the socially suboptimal behaviour of drivers only manifests itself in a socially suboptimal route-choice. Therefore, a toll or a subsidy on only one of the two routes suffices to account for this suboptimality, even when both routes are congested (see Appendix H).

For the network in Figure 4.1, we will assume that a toll is levied on Route 1. The total cost for this network then equals the sum of the average cost of all Route 1 and Route 2-drivers minus the toll revenue, i.e. $(a_1X_1 + f_1 + \tau)X_1 + (a_2X_2 + f_2)X_2 - \tau X_1$. The minimization of this cost through road pricing has two stages. First the traffic authority determines the toll value. Second, the drivers make their route choice. To find the optimal toll value, we work backward.

Depending on the toll value, the Wardrop equilibrium[2] will either be $X_1 = N$, $X_2 = N$ or $0 < X_1 < N$ such that the average cost is equal on both routes. Suppose the traffic authority implements $\tau$, then all drivers will take Route 1 in equilibrium if $a_1N + f_1 + \tau < f_2$. The total cost then equals $(f_1 + a_1N)\,N$. If $a_2N + f_2 < f_1 + \tau$ all drivers will take Route 2 in equilibrium and the total cost is $(f_2 + a_2N)\,N$. Finally, if the value of $\tau$ is such that $a_1N + f_1 + \tau > f_2$ and $a_2N + f_2 < f_1 + \tau$ then the drivers will distribute themselves over the two routes, resulting in a total cost of $\left(f_1 + \frac{a_1(a_2N+f_2-f_1)}{a_1+a_2} + \frac{a_2\tau}{a_1+a_2}\right)N - \tau X_1^e$ (Appendix I).

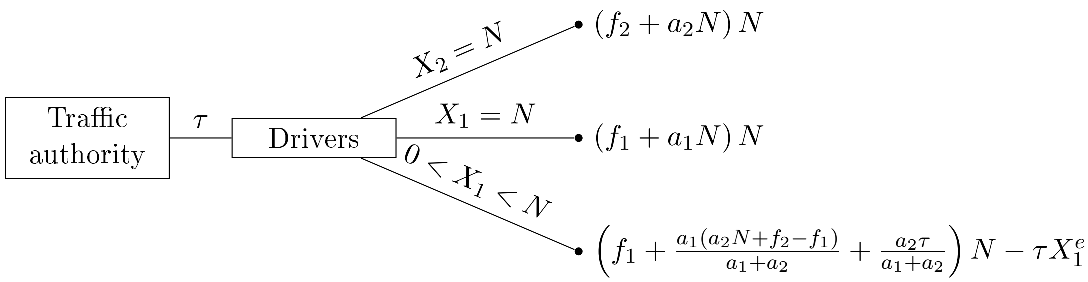


**Figure 4.3:** Total costs when road pricing is applied

Taking into account the equilibrium behaviour of the drivers, the traffic authority

[2]Both equation (4.1) and (4.2) are continuous in $X_i$ and so the equilibrium distribution of vehicles will be a Wardrop equilibrium.

determines the optimal $\tau$-value. Figure 4.3 shows that the total cost is not affected by the toll value when all drivers take one route in equilibrium. So if $(f_1 + a_1N)\,N$ $((f_2 + a_2N)\,N)$ is the lowest cost, a rational traffic authority will implement a toll that satisfies $a_1N + f_1 + \tau < f_2$ $(a_2N + f_2 < f_1 + \tau)$.

When $a_1N + f_1 + \tau > f_2$ and $a_2N + f_2 > f_1 + \tau$, the drivers will use both routes. In this case, the toll value influences the total cost, and the total cost is minimized when the toll value equals $\frac{f_2-f_1}{2}$.
It can be shown that if the optimal toll value is feasible,[3] then the associated minimal cost is lower than $(f_1 + a_1N)\,N$ and $(f_2 + a_2N)\,N$. It can furthermore be shown that if $\tau = \frac{f_2-f_1}{2}$ is such that $X_1^e < 0$ $(X_1^e > N)$, $(a_2N + f_2)N$ $((a_1N + f_1)N)$ is the lowest cost and the equilibrium distribution is $X_2 = N$ $(X_1 = N)$ when $\tau = \frac{f_2-f_1}{2}$ is implemented. Thus, independently of the parameter values, the optimal policy for the traffic authority would always be to set the toll equal to $\frac{f_2-f_1}{2}$.

### 4.3.2 Traffic lights to influence route choice

Consider the network in Figure 4.4 in which, instead of road pricing, a traffic light is installed on Route 1 to influence route choice.

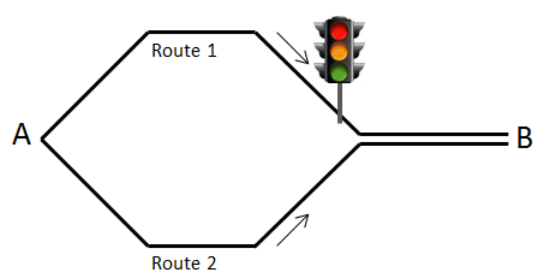


**Figure 4.4:** A traffic light to influence route choice

Suppose the authority implements the signal setting $r_1$. Then the equilibrium distribution will be $X_1 = N$ when $f_2 > a_1N + f_1 + T_1$, $X_2 = N$ $f_1 + T_1 > a_2N + f_2$ and the drivers will use both routes in the equilibrium when $f_2 \leq a_1N + f_1 + T_1$ and $f_1 + T_1 \leq a_2N + f_2$.

[3] That is, if for the optimal toll value the equilibrium amount of drivers on Route 1 and Route 2 is positive.

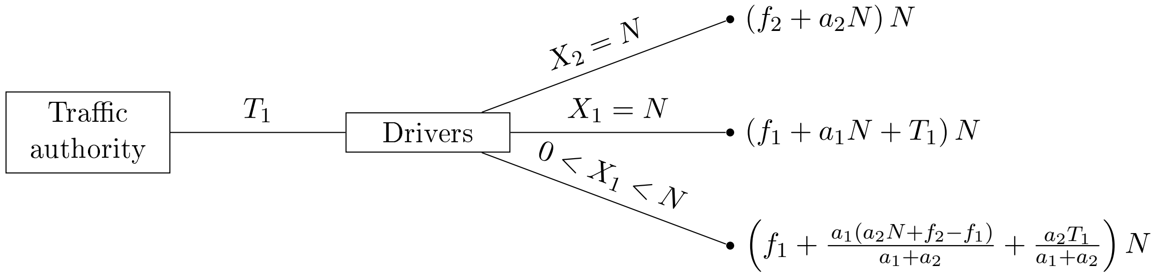


**Figure 4.5:** Total costs when a traffic light is installed

If all drivers take Route 1, the total cost can be minimized by setting the traffic light always to green. The total cost then equals $(a_1N + f_1)N$. If all drivers take Route 2 in equilibrium, the total cost will be $(f_2 + a_2N)N$. When both routes are used in the equilibrium, the total cost is minimal for $T_1 = 0$,[4] as illustrated in Figure 4.6.

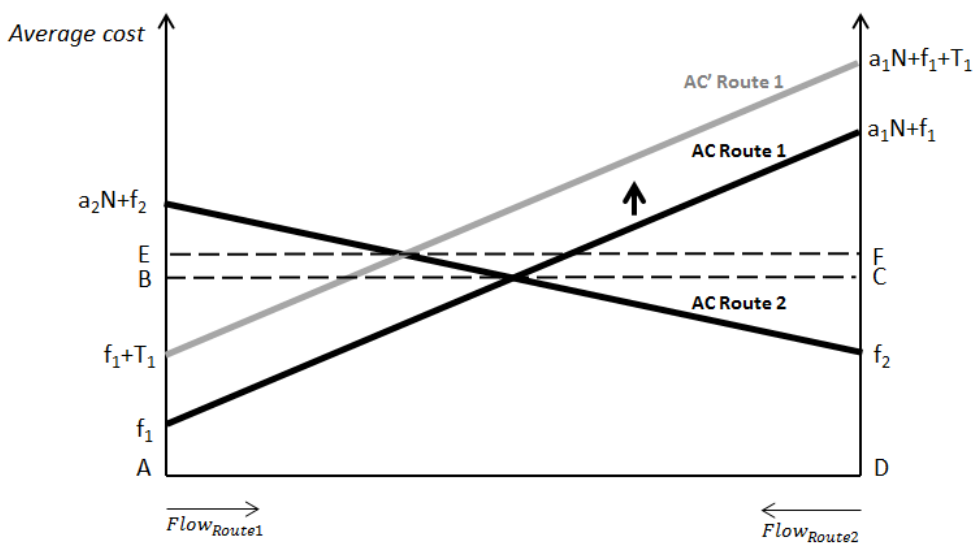


**Figure 4.6:** The total cost is minimal when the traffic light is always green.

If $T_1 = 0$, the total cost equals area ABCD. If a traffic light is installed on Route 1

[4]The cost decrease resulting from a better route choice is outweighted by the cost increase due to the additional waiting time.

and the expected traffic light waiting cost is positive, then the average cost curve of Route 1 will shift up (the grey curve in Figure 4.6). The total cost will then equal the area AEFD, which is larger than the area ABCD. Given that the average cost curve of Route 2 is upward sloping, it is clear that any $T_1 > 0$ will result in a higher total cost than $T_1 = 0$.

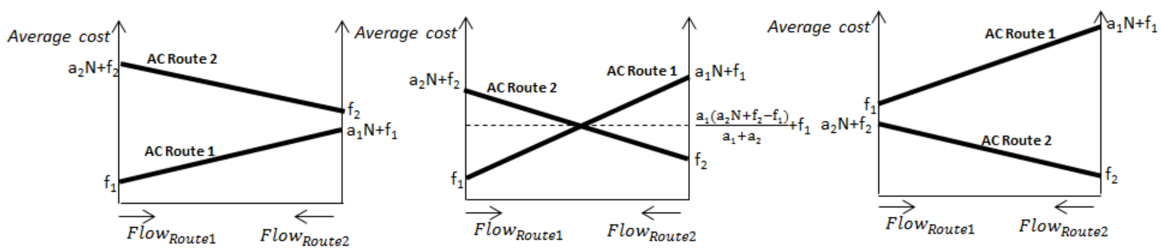


**Figure 4.7:** The cost minimizing user equilibrium always comes about.

To obtain the lowest cost, the reaction of the drivers to the optimal signal setting also has to be such that the cost minimizing user equilibrium comes about. In Figure 4.7, the three possible scenarios, dependent on the parameter settings, are depicted. In the first graph, the situation in which $(a_1N + f_1)N$ is the lowest cost is represented. It is clear from the graph that $X_1 = N$ will always be the Wardrop equilibrium in this case. In the middle graph the parameter values are such that $\left(f_1 + \frac{a_1(a_2N+f_2-f_1)}{a_1+a_2}\right) N$ is the lowest cost. In this case, both routes are used in equilibrium. In the graph on the right $(f_2 + a_2N)N$ is the lowest cost. Here, $X_2 = N$ is the user equilibrium.

Remark finally, that even though the optimal signal setting for all cases in this section is to give always green, we can not conclude that installing a traffic light solely to influence route choice can not be a good policy.[5] Indeed, a well placed traffic light can sidestep the Braess paradox (Braess (1968)). In the Braess paradox adding a road to a congested network can increase overall journey time. Consider the network in Figure 4.8 in which the addition of link v-t increases the total social cost. The installation of a traffic light on link v-t in the network (b) in Figure 4.8 thus allows to lower total costs.

[5] I am indebted to André de Palma for pointing this out to me.

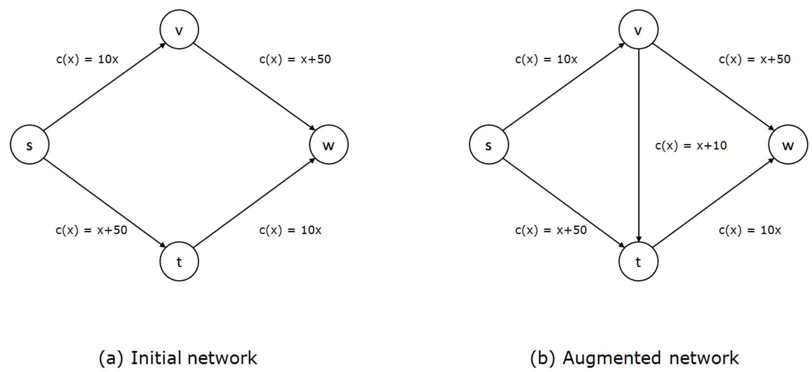


**Figure 4.8:** Braess' paradox

### 4.3.3 Policy comparison: minimum costs and flexibility of the instruments

In Table 4.1 the minimal costs that can be obtained with the different instruments are summarized. The minimal cost that can be obtained when all the drivers take Route 1 is the same for both instruments. When $(f_1 + a_1 N)\,N$ is the lowest cost, the traffic authority can attain this lowest cost both with a traffic light and with a toll.

**Table 4.1:** The total travel cost for each user equilibrium and policy measure

| | $X_1 = N$ | $X_2 = N$ | $0 < X_1 < N$ |
|---|---|---|---|
| $\tau$ | $(f_1 + a_1 N)\,N$ | $(f_2 + a_2 N)\,N$ | $\frac{-\left(\frac{f_1 - f_2}{2}\right)^2 + a_2 f_1 N + a_1 f_2 N + a_1 a_2 N^2}{a_1 + a_2}$ |
| $T_1$ | $(f_1 + a_1 N)\,N$ | $(f_2 + a_2 N)\,N$ | $\frac{a_1 a_2 N^2 + a_1 f_2 N + a_2 f_1 N}{a_1 + a_2}$ |

The minimal cost that can be obtained by directing all drivers to Route 2 is equal in both cases. Both instruments are equally flexible in directing the traffic towards this user equilibrium. Consequently, if the parameter values are such that $(f_2 + a_2 N)\,N$ is the lowest cost, then both instruments can always attain the lowest cost in the network.

If in the user equilibrium the drivers distribute themselves over the two routes, then the total cost when road pricing is applied is at least as low as when a traffic light is installed. Indeed, $\frac{a_1 a_2 N^2 + a_1 f_2 N + a_2 f_1 N}{a_1 + a_2} - \frac{\left(\frac{f_1 - f_2}{2}\right)^2}{a_1 + a_2} \leq \frac{a_1 a_2 N^2 + a_1 f_2 N + a_2 f_1 N}{a_1 + a_2}$. This result is intuitive. A traffic light can affect route choice, but any intervention comes at a cost, namely the waiting cost at the traffic light. A toll can also affect route choice, but, as the toll revenue flows back to society, the toll is not a cost. Combining these insights with the knowledge that the optimal toll can always direct the drivers towards the lowest cost equilibrium, it can be concluded that road pricing performs better than traffic lights.

## 4.4 A network with intersection

In this section, the focus is on a network in which a traffic light is essential for reasons of traffic safety. For this network, we first determine the minimal cost that can be attained when road pricing is applied in conjunction with traffic lights (Section 4.4.1). Subsequently, we compare these results with the minimal costs obtained when road pricing can not be implemented and only fixed signal settings are possible (Section 4.4.2). Next, we determine the conditions the signal settings have to satisfy to obtain the same results as road pricing. Finally, a numerical example is presented to illustrate the effectiveness of the proposed methodology.

**Proposition 8.** *In a network with two parallel routes that intersect, a traffic light with an optimal signal setting determined by an inverse Stackelberg game can provide an adequate alternative for road pricing.*

### 4.4.1 The optimal solution when road pricing is applied

We will model the signal and toll setting procedure for the network in Figure 4.9 as a Stackelberg game.

The traffic authority who controls the signal settings and toll, can either give always green to Route 1, or give always green to Route 2 or implement an alternating signal setting. The toll is levied on Route 1, and can be positive or negative. In deciding upon his optimal control (optimal in the sense that it minimizes total cost), the traffic authority will take into account the reaction of the drivers. The drivers can react to a certain signal and toll-combination by taking all together Route 1, or Route 2, or dividing themselves over the two routes in equilibrium.

If $r_2 = 0$, the Wardrop equilibrium is $X_2 = N$. As a consequence, all combinations of variables for which $r_2 = 0$ and $0 < X_1 \leq N$ are not part of the feasible set. Following

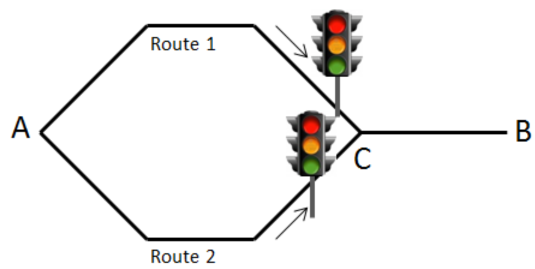


**Figure 4.9:** Road pricing applied to a network with an intersection

the same reasoning, if $r_2 = c$ the only feasible flow variable is $X_1 = N$. As road pricing does not affect the objective function, the value of the objective function will be the same for solutions ($r_2 = c$, $X_1 = N$, $\tau > 0$) and ($r_2 = c$, $X_1 = N$, $\tau = 0$) and for solutions ($r_2 = 0$, $X_2 = N$, $\tau > 0$) and ($r_2 = 0$, $X_2 = N$, $\tau = 0$). Given that the implementation of road pricing is costly[6]only the candidate solutions ($r_2 = 0$, $X_2 = N$, $\tau = 0$) and ($r_2 = c$, $X_1 = N$, $\tau = 0$) are retained.

For $0 < r_2 < c$, the objective function will only be lower than the objective function of the two previous candidate solutions if $0 < X_1 < N$.[7] As a consequence, the third candidate solution will be the solution of the following minimization problem.

$$\min_{r,X_1,X_2,\tau} \left(f_1 + a_1X_1 + T_1\left(c, r_2\right) + \tau\right) X_1 + \left(f_2 + a_2X_2 + T_2\left(c, r_2\right)\right) X_2 - \tau X_1 \tag{4.5}$$

s.t.

$$f_1 + a_1X_1 + T_1\left(c, r_2\right) + \tau = f_2 + a_2X_2 + T_2\left(c, r_2\right) \tag{4.6}$$

$$X_1 + X_2 = N \tag{4.7}$$

$$X_1 > 0 \tag{4.8}$$

$$X_2 > 0 \tag{4.9}$$

$$r_2 > 0 \tag{4.10}$$

$$r_2 < c \tag{4.11}$$

[6]For simplicity, this has not been included in the objective function.

[7]Indeed, for a signal setting r $\in ]0, c[$, $\left(f_1 + a_1N + T_1\left(c, r_2\right)\right) N$ is always larger than $\left(f_1 + a_1N\right) N$ and $\left(f_2 + a_2N + T_2\left(c, r_2\right)\right) N$ is always larger than $\left(f_2 + a_2N\right) N$.

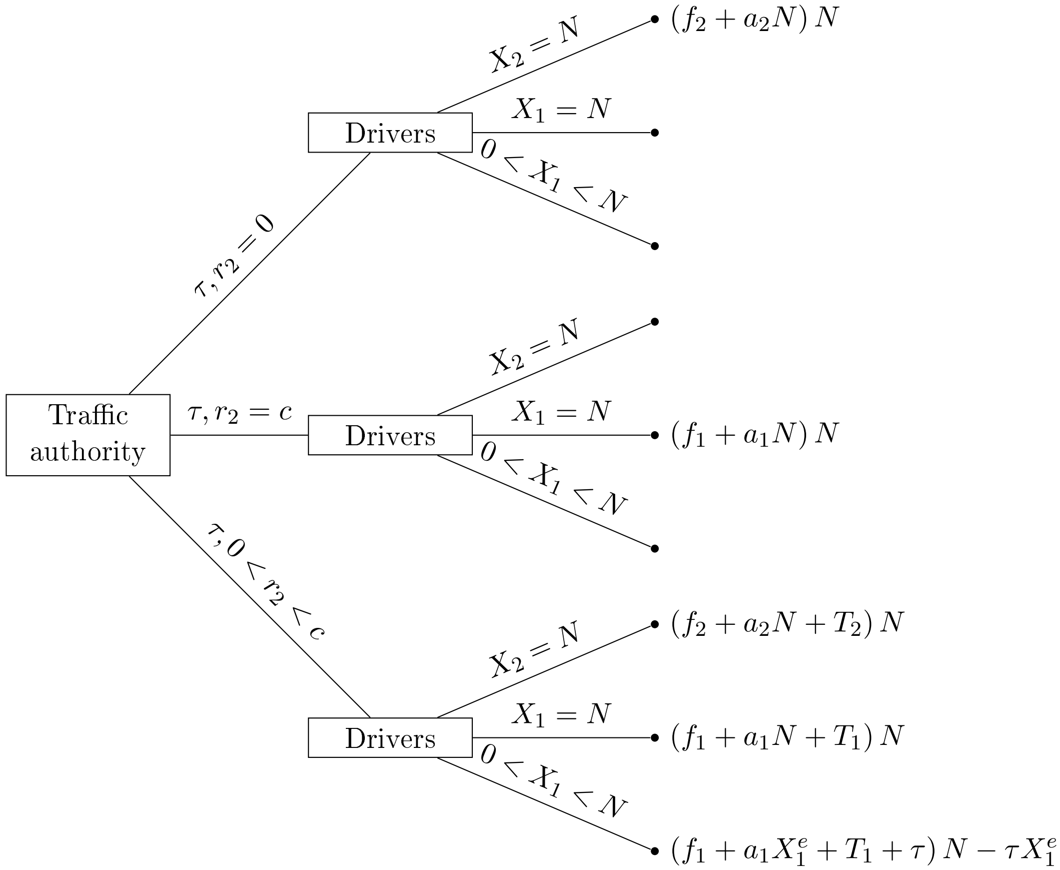


**Figure 4.10:** Total costs when road pricing is applied in conjunction with traffic lights

In this optimization problem $a_i$ represents the sensitivity to congestion of route $i$, $X_i$ equals the flow on route $i$, $f_i$ stands for the minimal time and resource cost of route $i$, $\tau$ is the value of the toll levied on Route 1 and $T_i(c, r_2)$ is the expected waiting time cost on route $i$ at the intersection.

For simplicity we will assume that the parameter values are such that the total cost function is convex ($2\,(a_1 + a_2)\,N - c \geq 0$). Then the solution of this minimization problem is the global minimum. Furthermore, we will assume that the parameter values are such that $0 < r_2^* < c$.

Solving the first order conditions of the Langragian function associated with this optimization problem, yields the optimal flow pattern, toll value and signal setting (Appendix K). The optimal distribution of vehicles ($X^{int}$) can be found where the marginal cost curves of both routes intersect:

$$f_1 + 2a_1X_1 + T_1\,(c, r_2) = f_2 + 2a_2X_2 + T_2\,(c, r_2) \tag{4.12}$$

The optimal toll ($\tau^{int}$) equals the difference between the marginal external congestion cost of Route 1 and the marginal external congestion cost of Route 2.

$$\tau = a_1X_1 - a_2X_2 \tag{4.13}$$

The optimal signal setting ($r_2^{int}$) is determined by the following equation:

$$\frac{\delta T_1\,(c, r_2)}{\delta r_2}X_1 = -\frac{\delta T_2\,(c, r_2)}{\delta r_2}X_2 \tag{4.14}$$

It can be shown that there exist parameter values for which the interior solution is feasible and the associated total cost ($TC^{int}$) is lower than $(a_1N + f_1)\,N$ and $(a_2N + f_2)$. As a consequence, the interior solution is a candidate solution.

Depending on the parameter values, the desired choice for the traffic authority will thus be one of the three following combinations: either Route 1 always has green, no toll is levied and all drivers take Route 1 or Route 2 receives always green, no toll is levied and all drivers take Route 2 or the optimal alternating signal setting ($r_2^{int}$) is implemented, the optimal toll ($\tau^{int}$) is levied and the drivers are distributed over the two routes such that $MC_1 = MC_2$.

The total costs associated with these solutions are the lowest attainable in this network if there are to be N drivers per time unit going from A to B. The network itself is indeed only restricted by the fact that the signal settings have to be such that collisions are avoided. The lowest costs for the network are thus associated with the solutions of the following optimization problem

$$\min_{r_2, X_1} \left(f_1 + a_1 X_1 + T_1\left(c, r_2\right)\right) X_1 + \left(f_2 + a_2\left(N - X_1\right) + T_2\left(c, r_2\right)\right)\left(N - X_1\right) \tag{4.15}$$

s.t.

$$0 \leq X_1 \leq N \tag{4.16}$$

$$0 \leq r_2 \leq c \tag{4.17}$$

An inspection of the corner solutions already provides two candidate optima: $(r_2 = c, X_1 = N)$ and $(r_2 = 0, X_1 = 0)$. The total cost associated with these solutions is $(a_1 N + f_1)N$ and $(a_2 N + f_2)N$ respectively. The third candidate solution is the interior solution $(0 < r_2 < c$ and $0 < X_1 < N)$. In Appendix K it is shown that the flow pattern of the optimal interior solution $(X_1^*)$ is determined by $MC_1 = MC_2$. Remark that when the optimal toll is levied, the flow distribution equals the socially optimal flow pattern. This implies that, for this network, road pricing allows to completely control route choice. The optimal interior signal setting $(r_2^*)$ is determined by $\frac{dT_1(c,r_2)}{dr_2} X_1 = -\frac{dT_2(c,r_2)}{dr_2} X_2$, which equals equation (4.14). Given that $X_1^*$ equals $X_1^{int}$, $r_2^*$ is equal to $r_2^{int}$ and the toll value does not affect the value of the objective function, the total cost will equal $\text{TC}^{int}$.

Suppose now that for the network in Figure 4.9, the toll cannot be levied anymore. In the previous section, we found that the deployment of traffic lights increases costs, as the delays caused by the traffic light are added to the social cost. However, contrary to the network in Section 4.3, the traffic light is now part of the network. So the delays caused by the traffic light are not added to the social cost, but are already part of the social cost also when road pricing is applied. Would it for this network then be possible to obtain the same result with traffic lights alone?

### 4.4.2 The optimal solution when road pricing can not be applied

Figure 4.11 shows the different options for the traffic authority and the possible reactions of the drivers when the signal setting procedure is modelled as a Stackelberg game. From Evers and Proost (2015), we know that, depending on the parameter values, the minimal cost for

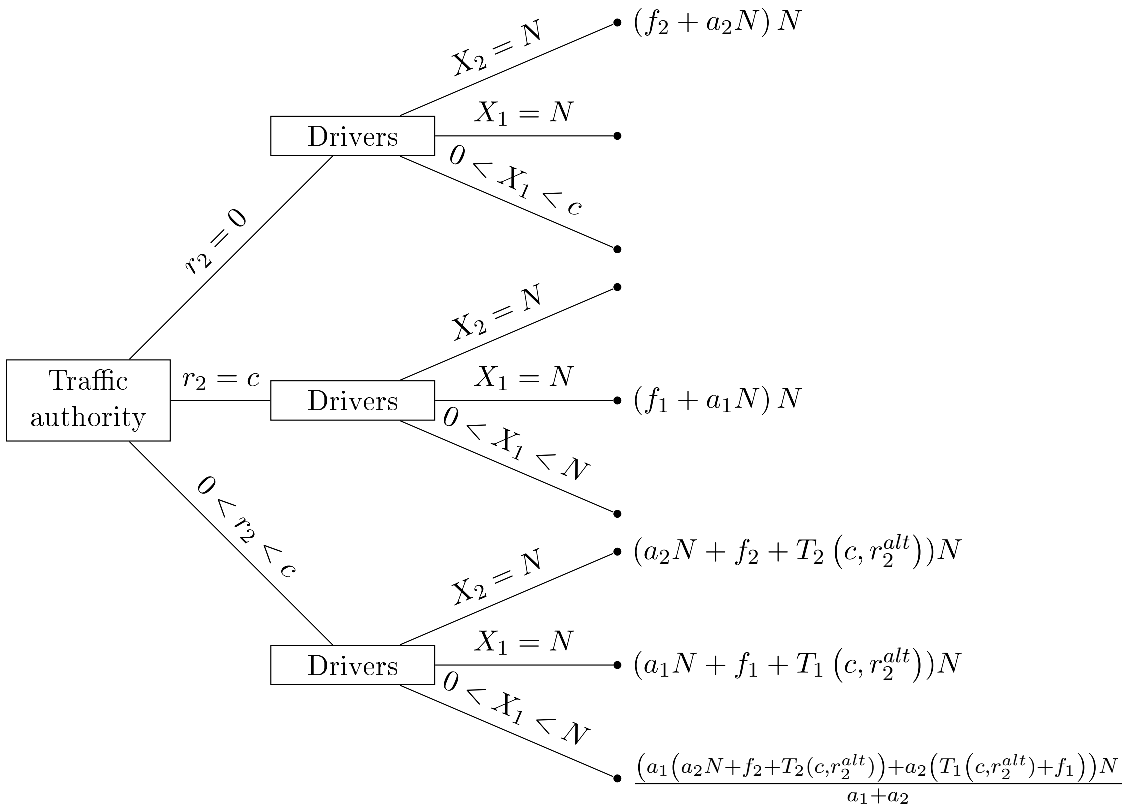


**Figure 4.11:** Total costs when traffic lights are the only instrument

the network is given by $(a_2N+f_2)N$, $(a_1N+f_1)N$ or $\left(\frac{2a_1a_2(a_1+a_2)N+2a_1(a_1+a_2)(f_2-f_1)+a_1a_2c}{(a_1+a_2)^2 2} + f_1\right)N$. The associated optimal signal settings are $r_2 = 0$, $r_2 = c$ and $r_2 = \frac{a_2c}{a_1+a_2}$ respectively.[8]

Given that we now have fewer instruments to solve the same problem, these results will be equally as good or worse than when a toll was also available. The focus will be on the conditions under which deviations occur and on the sources of these deviations.

When the parameter values are such that $(a_1N + f_1)N$ is the lowest cost attainable with road pricing, then the simple implementation of $r_2 = c$ allows to obtain the same cost. When $(a_2N + f_2)N$ is the minimum, then the signal setting $r_2 = 0$ provides this solution. When $TC^{int}$ is the minimal cost, then the strategy to obtain this cost is not so

[8] Remark that the optimal alternating signal setting $r_2 = \frac{a_2c}{a_1+a_2}$ only equals $r_2^*$ for those parameter values for which the flow that solves $AC_1 = AC_2$ also solves $MC_1 = MC_2$.

straightforward. Indeed, when the traffic authority implements $r_2 = 0$ or $r_2 = c$, then the total cost equals $(a_2N + f_2)N$ respectively $(a_1N + f_1)N$. These costs are by assumption larger than $TC^{int}$. When the traffic authority implements the optimal alternating signal setting $r_2 = \frac{a_2 c}{a_1+a_2}$ the minimal cost is also at least as large as $TC^{int}$. Indeed, at the end of Section 4.4.2 we have shown that $TC^{int}$ is equal to the value of the total cost associated with the optimal interior solution of an unconstrained optimization problem. The minimal cost associated with $r_2 = \frac{a_2 c}{a_1+a_2}$ is the solution of the exact same optimization problem, yet constrained by the equilibrium reaction of the driver. This last observation indicates that the real constraint in obtaining the lowest cost is the route choice of the drivers. If the traffic authority could exert more control over the route choice of the drivers, the lowest cost would be attainable. This insight prompts us to shift to an approach with more influence over route choice to see whether this allows traffic lights to achieve the same results as tolls.

Compared to the Stackelberg approach, the inverse Stackelberg approach allows to exert more control over the choices of the follower. Therefore, we will now model the signal setting procedure for the network in Figure 4.9 as an inverse Stackelberg game.

Whereas in the basic Stackelberg game the traffic authority implements a flow-independent signal setting, in the inverse Stackelberg game the signal setting can be a function of the flow pattern. In the following, we will determine what this function should look like to obtain the lowest cost.[9]

We know that when $(f_1 + a_1N)\,N$ $((f_2 + a_2N)\,N)$ is the lowest cost, the signal setting $r_2 = c$ ($r_2 = 0$ respectively) allows to obtain this minimum cost. A constant function thus suffices in these cases. However, we also have shown that when $TC^{int}$ is the lowest cost a flow-independent signal setting is not sufficient. In fact, to reach $TC^{int}$ the signal setting policy has to be such that when $r_2^{int}$ is set, the user equilibrium that comes about coincides with the system equilibrium. The constraints a valid function has to satisfy thus depend on the properties of the user equilibrium. We therefore first give a formal definition of the user equilibrium.

**Definition 1.** A feasible flow pattern $f^{UE}$ is a user equilibrium if for any OD pair and all

[9]Remark that in Section 4.4 we have shown that the results obtained when both road pricing and traffic lights are available are the lowest attainable in the network. We will thus use the terms lowest cost and $TC^{int}$ interchangeably.

routes connecting this OD pair

$$AC_r(f^{UE}) \leq \lim_{\epsilon \to 0} \inf\{AC_s(f^{UE} + \alpha 1_r - \alpha 1_s) : 0 < \alpha < \min(\epsilon, f_s)\} \tag{4.18}$$

with $s$ any route connecting the same OD pair as route $r$. In this definition $1_i$ denotes the vector with a '1' in position $i$ and a '0' elsewhere.

The traffic authority wants the social equilibrium to coincide with the user equilibrium. And to obtain the lowest cost, the traffic authority will implement the socially optimal signal setting ($r_2^*$) when the flow pattern is socially optimal. Taking all this into account, a valid function has to satisfy the following constraints.

$$AC_1(X_1^*, r_2^*) \leq \lim_{\epsilon \to 0} \inf\{AC_2(X_2^* + \alpha 1, r_2) : 0 < \alpha < \min(\epsilon, X_1^*)\} \tag{4.19}$$

$$AC_2(X_2^*, r_2^*) \leq \lim_{\epsilon \to 0} \inf\{AC_1(X_1^* + \alpha 1, r_2) : 0 < \alpha < \min(\epsilon, X_2^*)\} \tag{4.20}$$

Here, $(X_1^*, X_2^*)$ is the socially optimal flow, ($r_2^*$) is the socially optimal signal setting and $r_2$ is the function we are looking for. When the signal setting is such that equations (4.19) and (4.20) are satisfied, the socially optimal flow is a user equilibrium.

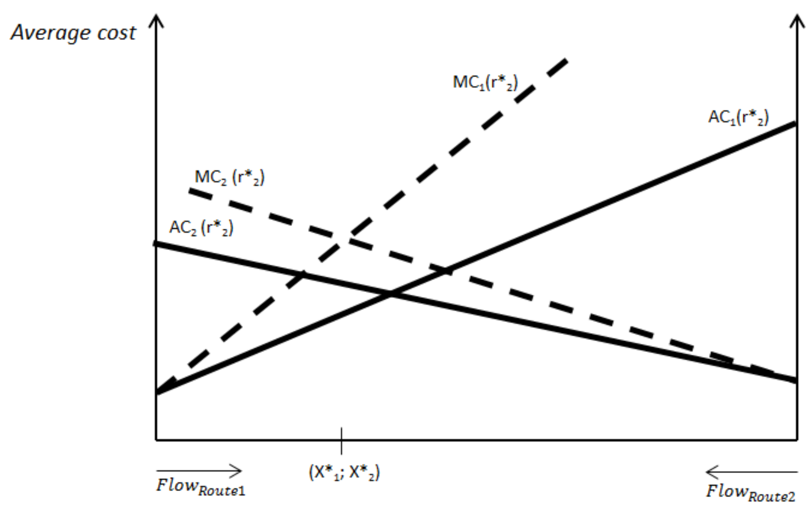


**Figure 4.12:** The average cost in the optimum is different on both routes

Remark that $AC_1(X_1^*, r_2^*)$ and $AC_2(X_2^*, r_2^*)$ are generally not equal. Equations (4.19) and (4.20) thus indicate that the function that maps the flow distribution in a signal setting has to be discontinuous.

When the flow is not in equilibrium, e.g. due to a shock, the drivers will change routes to minimize their private cost. This process has to converge to the equilibrium flow. As a consequence, a valid function also has to satisfy the following constraints:

$$AC_1(X_1) < AC_2 \qquad \forall X_1 < X_1^* \tag{4.21}$$

$$AC_1(X_1) > AC_2 \qquad \forall X_1 > X_1^* \tag{4.22}$$

There exist many functions that can satisfy these constraints. However, given that the objective of this paper is only to show that there exists a function that satisfies the constraints, we limit ourselves to the elaboration of the simplest case with a fixed signal setting $r_2^h$ for all $X_1 > X_1^*$ and a fixed signal setting $r_2^l$ for all $X_1 < X_1^*$.

### 4.4.3 Characterization of the fixed signal settings and numerical example

As mentioned before, $r_2$ has to equal $r_2^*$, when the flow pattern is socially optimal. This thus determines the signal setting when $X_1 = X_1^*$. Using the constraints in Section 4.4.2, we will also characterize the fixed signal setting for $X_1 < X_1^*$ and for $X_1 > X_1^*$.

$$f(X_1) = \begin{cases} r_2^l & \text{if } X_1 < X_1^* \\ r_2^* & \text{if } X_1 = X_1^* \\ r_2^h & \text{if } X_1 > X_1^* \end{cases}$$

We first focus on the area right of the social optimum. With fixed signal settings, undersaturated traffic conditions and linear congestion $AC_1$ is linearly increasing in $X_1$. So any $r_2^h$ satisfying $AC_2(X_2^*, r_2^*) \leq AC_1(X_1^*, r_2^h)$ will also satisfy equation (4.20). With $AC_1$ linearly increasing in $X_1$ and $AC_2$ linearly decreasing in $X_2$, any $r_2^h$ satisfying $AC_1(X_1^*, r_2^h) > AC_2(X_2^*, r_2^h)$ will also satisfy equation (4.24) (see Figure 4.13). The constraints that thus determine a feasible $r_2^h$ can be written as follows:

$$f_1 + a_1 X_1^* + \frac{\left(c - r_2^h\right)^2}{2c} \geq a_2 X_2^* + f_2 + \frac{\left(r_2^*\right)^2}{2c} \tag{4.23}$$

$$f_1 + a_1 X_1^* + \frac{\left(c - r_2^h\right)^2}{2c} > a_2 X_2^* + f_2 + \frac{\left(r_2^h\right)^2}{2c} \tag{4.24}$$

Equation (4.23) reduces further to:

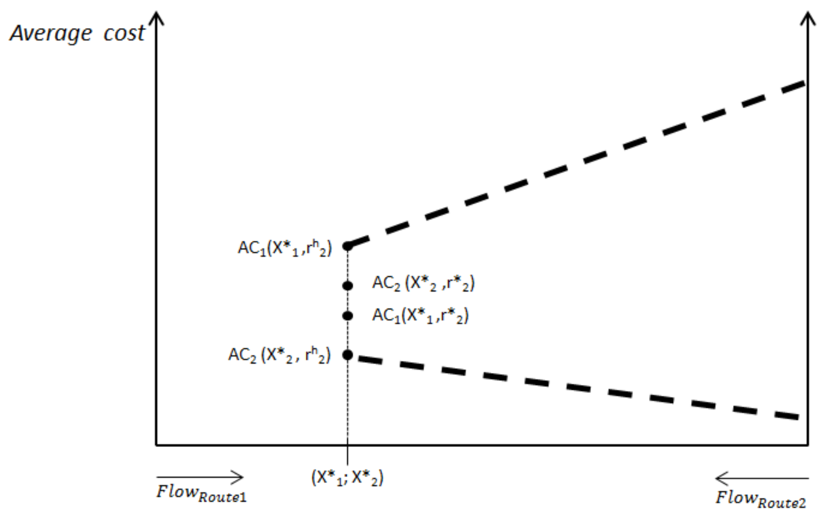


**Figure 4.13:** Average cost functions associated with $r_2^h$ that satisfy the constraints

$$r_2^h \leq c - \sqrt{\left(f_2 - f_1 + (a_2 - a_1)X_1^* + a_2 N + \frac{(r_2^*)^2}{2c}\right) 2c} \tag{4.25}$$

and equation (4.24) reduces to:

$$r_2^h < f_1 - f_2 - (a_2 - a_1)X_1^* - a_2 N + \frac{c}{2} \tag{4.26}$$

In Appendix L we show that if $AC_2(X_2^*, r_2^*) > AC_1(X_1^*, r_2^*)$, then equation (4.25) implies equation (4.26) and if $AC_2(X_2^*, r_2^*) < AC_1(X_1^*, r_2^*)$, equation (4.26) implies equation (4.25) for all $X_1 > X_1^*$.

Next, we analyse the area left of the social optimum. Also here, there are two constraints that have to be satisfied.

With fixed signal settings $AC_2$ is linearly decreasing in $X_1$ and equation (4.19) will always be satisfied when $AC_1(X_1^*, r_2^*) \leq AC_2(X_2^*, r_2^l)$ is satisfied. Equation (4.21) always holds for any $r_2^l$ that satisfies $AC_1(X_1^*, r_2^l) < AC_2(X_2^*, r_2^l)$ (see Figure 4.14). A feasible $r_2^l$

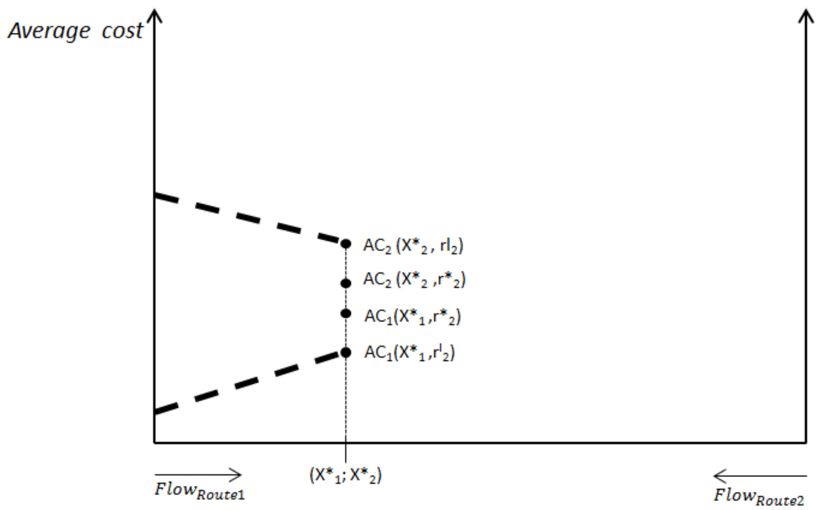


**Figure 4.14:** Average cost functions associated with $r_2^l$ that satisfy the constraints

will thus satisfy the following constraints:

$$a_2X_2^* + f_2 + \frac{\left(r_2^l\right)^2}{2c} \geq f_1 + a_1X_1^* + \frac{(c - r_2^*)^2}{2c} \tag{4.27}$$

$$a_2X_2^* + f_2 + \frac{\left(r_2^l\right)^2}{2c} > f_1 + a_1X_1^* + \frac{\left(c - r_2^l\right)^2}{2c} \tag{4.28}$$

Equation (4.27) reduces to

$$r_2^l \geq \sqrt{\left(f_1 - f_2 + (a_1 + a_2)X_1^* - a_2N + \frac{(c - r_2^*)^2}{2c}\right) 2c} \tag{4.29}$$

and equation (4.28) reduces to

$$r_2^l > f_1 - f_2 + \frac{(r^*)^2}{2c} + (a_1 + a_2)X_1^* - a_2N + \frac{c}{2} \tag{4.30}$$

If $AC_1(X_1^*, r_2^*) > AC_2(X_2^*, r_2^*)$, then equation (4.29) implies equation (4.30) and if $AC_2(X_2^*, r_2^*) > AC_1(X_1^*, r_2^*)$, equation (4.30) implies equation (4.29).

Suppose there are 30,000 drivers per time unit (N) that want to go from A to B. The minimal time and resource cost equals 22 euro for Route 1 ($f_1$), and 3 euro for Route 2 ($f_2$).

The parameters $a_1$ and $a_2$ are 0.003 and 0.0005 euro time units per driver respectively. The total duration of red and green time (c) equals 50 time units.

**Table 4.2:** When road pricing is applied, the optimal result is obtained when both routes are used in the equilibrium.

| | $(r_2 = 0; \tau = 0)$ | $(r_2 = 50; \tau = 0)$ | $(r_2^{int} = 7.5; \tau^{int} = 0.75)$ |
|---|---|---|---|
| Total cost | $1,110,000$ | $2,790,000$ | $1,056,000$ |

For these parameter values, the total cost when the traffic authority implements the in Section 4.4.1 calculated optimal alternating signal setting and toll is lower than the total cost when the traffic authority implements either $r_2 = 0$ or $r_2 = c$ (see Table 4.2). The traffic authority will thus implement a signal setting that varies with the flow distribution to obtain this cost without having to implement road pricing. We will use the framework developed in Section 4.4.2 to deduce the flow-dependent signal setting $f(X_1)$ for this specific example.

The optimal red time for Route 2 equals 7.5 time units. The optimal flow on Route 1 is 4,500 vehicles per time unit, and thus 25,500 vehicles per time unit on Route 2. For this optimal combination of red time and flow distribution, $AC_2(25,500; 7.5)$ is larger than $AC_1(4,500; 7.5)$. Consequently, the constraint that determines $r_2^h$ for all $X_1 > X_1^*$ is given by equation (4.25). For the given parameter values, this equation reduces to:

$$0 < r_2^h \leq 1.72 \tag{4.31}$$

The constraint that determines $r_2^l$ for all $X_1 < X_1^*$ is given by equation (4.30). Here, this equation reduces to:

$$c > r_2^l > 6.75 \tag{4.32}$$

The signal setting $f(X_1)$ can then be determined as follows:

$$f(X_1) = \begin{cases} 10 & \text{if } X_1 < 4,500 \\ 7.5 & \text{if } X_1 = 4,500 \\ 1 & \text{if } X_1 > 4,500 \end{cases}$$

Remark that even though for all the flow patterns left of the equilibrium ($X_1 < X_1^*$) there are not enough Route 1 drivers compared to the amount of Route 1 drivers in the equilibrium, equation (4.32) does not specify that the red time for Route 1 has to decrease for all $X_1 < X_1^*$ (compared to the red time for Route 1 in the equilibrium). A deviation to Route 2 from the equilibrium is thus not necessarily punished by an increase in the red time for Route 2. This is because the other cost components are such that even for $7.5 \geq r_2^l > 6.75$ equation (4.19) and equation (4.21) are still satisfied.

**Table 4.3:** Contrary to the optimal fixed signal setting, the flow dependent signal setting allows to obtain the same result as road pricing.

| | $r_2 = \frac{a_2 c}{a_1+a_2}$ | $f(X_1)$ | $(r_2^{int} = 7.5; \tau^{int} = 0.75)$ |
|---|---|---|---|
| Total cost | $1,056,120$ | $1,056,000$ | $1,056,000$ |

The total costs in Table 4.3 show that $f(X_1)$ indeed allows to obtain the same result as road pricing. The optimal flow-independent alternating signal setting, by contrast, has a higher total cost.

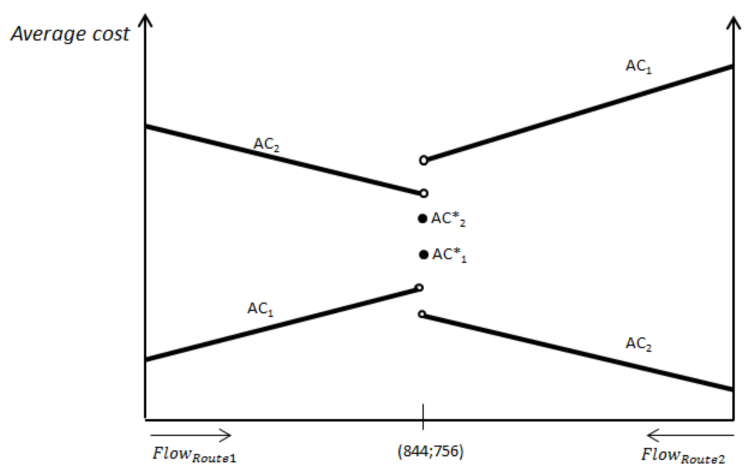


**Figure 4.15:** Shape of the average cost functions for a feasible flow-dependent signal setting

With $f(X_1)$ the flow dependent signal setting, the average cost curve of Route 2 is given by:

$$AC_2(X_2) = \begin{cases} 23 + 0.0005X_2 & \text{if } X_1 < 4,500 \\ 35.31 & \text{if } X_1 = 4,500 \\ 22.01 + 0.0005X_2 & \text{if } X_1 > 4,500 \end{cases}$$

The average cost curve of Route 1 equals:

$$AC_1(X_1) = \begin{cases} 19 + 0.003X_1 & \text{if } X_1 < 4,500 \\ 34.56 & \text{if } X_1 = 4,500 \\ 27.01 + 0.003X_1 & \text{if } X_1 > 4,500 \end{cases}$$

Remark that for all $X_1 > 4,500$ $AC_1$ is always greater than 35.31 and $AC_1$ is always greater than $AC_2$. Similarly, for all $X_1 < 4,500$, $AC_2$ is always greater than 34.56 and $AC_2$ is always greater than $AC_1$.

## 4.5 Practical considerations

In the previous section, we have shown that a signal setting determined by the inverse Stackelberg approach allows to obtain the same results as when the traffic authority has both traffic lights and a toll to work with. In the user equilibrium that is brought about when the inverse Stackelberg approach is used, the average cost on Route 1 and Route 2 is generally not equal. This might raise concerns regarding the stability of this user equilibrium. To determine the effect on the stability of the equilibrium, we distinguish between the situation in which drivers learn the average cost by experience and the situation in which the average cost is common knowledge.

If the average cost is known only be experience, then a driver who uses the highest cost route in equilibrium, might experience a lower cost if he deviates from the equilibrium distribution at the same moment as another driver who takes the other route in the equilibrium. This could then result in swapping behaviour. At a certain point, however, the drivers who take the route with the lowest cost in equilibrium will not deviate anymore. Consequently, the drivers on the route which has the highest cost in equilibrium will always experience a higher cost when deviating and will also stop deviating.

Currently many cars are equipped with route guidance and information systems. These devices can provide information or guide drivers to certain routes. If the average cost functions are known to all drivers, the traffic authority could obtain the socially optimal distribution of vehicles over the two routes by each day randomly assigning $X_1^*$ drivers to Route 1 and $X_2^*$ drivers to Route 2. As for every individual driver, the resulting time averaged cost is lower than the time averaged cost when the signal setting is determined by a Stackelberg approach (which is the second best alternative in this case), a rational driver has no incentive to deviate from the assigned route. The assumption of rationality could, however, be very strong in this case. When this assumption is relaxed, the optimal policy for the authority would be to randomly assign $X_1^*$ drivers to Route 1 and $X_2^*$ drivers to Route 2 and fine each driver who deviates from his assigned route. The optimal fine would then equal the incurred cost of all the other drivers due to the deviation of this driver, which would then be redistributed among the other drivers.

Another line of approach can be found in Jahn et al. (2005). This paper adopts a system optimum approach but honors the individual needs by imposing additional constraints to ensure the drivers are assigned to acceptable paths only. Acceptable paths are determined by their level of fairness with respect to the user cost in the user equilibrium and several notions of unfairness are introduced. Jahn et al. (2005) show that their model leads to a significantly better utilization of a traffic network.

## 4.6 Conclusion

In this paper, we have compared the performance of traffic lights and tolls for two different networks. For the network with two parallel routes, we have modeled both the signal setting procedure and the toll setting procedure as a Stackelberg game. We find that tolls generally perform better than traffic lights.

For the network in which traffic lights regulate the intersection of two routes connecting one OD pair, we have used a Stackelberg approach as well as an inverse Stackelberg approach. We have derived the conditions a signal setting has to satisfy to be able to obtain the same results as when both traffic lights and a toll can be used to optimize the network. With a numerical example, this paper showed that it is possible to find a flow-dependent signal setting that can render road pricing redundant.

This study is designated as an initial step towards finding flow-dependent signal settings that can render road pricing redundant. A possible extension to this paper would then study different functional forms that allow flow-dependent signal setting to reach the same result as road pricing.

# Part II

# Efficient transportation policies

# Chapter 5

# Efficient transportation policies for sustainable cities

## 5.1 Introduction

In the context of sustainability, many European cities want to increase the liveability of their city center. Different cities have explored alternative ways to curb traffic. London charges a congestion fee for commuters who drive into the city center, Copenhagen is creating bicycle superhighways to connect the suburbs to the city, Hamburg is working on a "Green network" that would eliminate the need for cars within the city, ... What is now the most efficient transport policy to reduce congestion externalities, accident risk and noise and air pollution? It is to this question that this paper formulates an answer.

To analyze the effects of alternative transport policies, this paper develops a model in a multi-user multi-period multimodal context, keeping locations fixed. This model fits in the tradition of MOLINO-models (De Palma et al. (2010), Kilani et al. (2014)).

The supply part of the model is defined by several fixed origin destination pairs that are connected by combinations of links. The generalized cost of a link contains several components: a monetary cost, taxes levied by the government, and a time cost which can be a function of the number of users. In urban transportation, many trips use more than one mode of transport. The model therefore allows for combined trips as well as unimodal trips.

Consumers choose between alternative ways of transportation on the basis of their subjective preferences and the perceived generalized cost of the different transport alternatives. For each user category and each OD pair we compose an aggregate nested CES utility function with three levels: choice between transport and consumption of a composite commodity (first nest), choice between peak and off-peak period (second nest), and choice between the transport alternatives (third nest). As in many cities a large share of all transport externalities comes from incoming traffic (such as commuting traffic) and from pure through traffic (i.e., traffic that has neither origin nor destination within the city), the model distinguishes between different types of transport users. Furthermore, multiple user classes are considered, as fares can differ according to the purpose of the trip.

The model can be used in two ways. First, it can be used to measure the effects of different types of transport policies and trade off their total welfare effects. Second, the model can be used to maximize social welfare by adjusting transit design variables, network pricing and changing network design.

The model is illustrated to the city of Leuven, Belgium. Three transport policies to reduce the traffic externalities in the city center are considered: introducing road pricing in the city center, raising parking fees in the center of Leuven and expanding public transport.

The remainder of the paper is organized as follows. Section 2 gives an overview of the relevant literature. Section 3 develops the theoretical model. Section 4 discusses the properties of the numerical model and the characteristics of the reference equilibrium in Leuven. Section 5 presents the results and Section 6 offers a conclusion.

## 5.2 Literature

The available urban transport studies differ in terms of the amount of spatial, temporal and modal detail in the representation of urban transport, as well as in the transport cost components that are taken into account.

The most common approach in the urban transportation literature is to deal with each mode of transport separately. Mohring (1972) developed a model that considers optimal frequency and spacing of stops. He found the optimal frequency to be proportional to the square root of demand. Jansson (1980) extended the square root principle to a model in

which service frequency is optimized simultaneously with bus size. Ahn (2009) extended Mohring's work to the case where buses share the congestion interaction road with other automobiles. Tirachini (2014) reconsidered the problem of choosing the number of bus stops along urban routes.
Going beyond systems with a single mode of transportation, some papers concentrate on the bimodal problem. Basso and Jara-Díaz (2012) focus on the analytical properties of optimal prices and design of transport services in a bimodal context. In a two-mode system, De Borger and Wouters (1998) study the joint optimisation of transport prices and supply decisions of urban transport services. This paper extends this literature by focusing on a multimodal setting.

From spatial point of view, many papers focus on a single corridor setting. Tirachini and Hensher (2011), for instance, study the impact of fare payment technology on a bus corridor. Tirachini et al. (2014) develops a model that allows to analyse the interplay between congestion and externalities in the design of public transport services. The model is applied to a single transport corridor in Sydney, Australia. Unlike previous literature, this paper takes a network approach.

Another paper that takes a network approach is Tirachini et al. (2010). Using data from Australian cities, this paper compares light rail, heavy rail, and BRT on a radial transit network. The analysis, however, only focuses on costs, which does not allow to make welfare comparisons between different scenarios, as the consumer utility is not taken into account. Unlike this paper, we also account for changes in consumer surplus.

Among the models that include environmental, accident or noise externalities in their analysis we can cite De Borger et al. (1996), Proost and Van Dender (2008) and Parry and Small (2009).

## 5.3 The model

The case study results presented in Section 5 are based on a multimodal model of the transport system. This section develops this model theoretically by first focusing on the demand side, subsequently unfolding the supply side, and finally constructing the different components of the welfare function.

### 5.3.1 Demand

The demand for travel is subdivided into M classes of representative users with different socio-economic characteristics. These classes differ with respect to their travel preferences, incomes, and costs of travel. For every user class and every OD pair, we calibrate a nested CES utility function with four levels as illustrated in Figure 5.1.

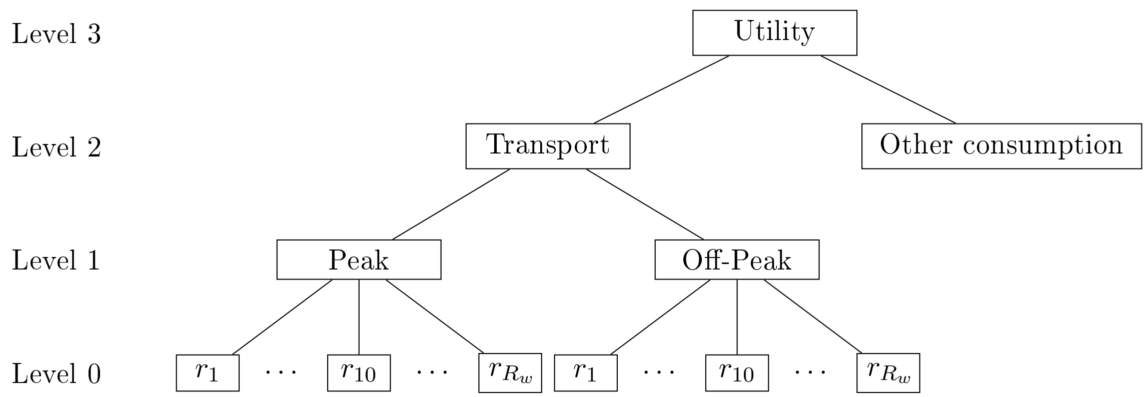


**Figure 5.1:** CES implementation of the utility function

The travelers can decide on the number of trips (first nest), whether to travel in the peak or off-peak period (second nest) and which combination of links to use to go from their origin to their destination (third nest). The elasticity of substitution for every branching of the choice tree will determine the ease of substitution between different transport alternatives. When travelers consider different routes to be perfect substitutes, the Wardrop equilibrium (Wardrop (1952)) prevails. In this paper we focus on the more realistic stochastic user equilibrium (Daganzo and Sheffi (1977)), which departs from the assumption that travelers try to minimize their perceived travel cost.

The main advantage of the nested CES formulation is its ease of calibration. If the elasticities of substitution at each branch plus the total quantities and prices at the lowest level of the utility tree are available, the shareparameters $\psi_{i,e}$ can be calculated to exactly reproduce this exogenous baseline. In this way, the CES functions are calibrated.

When the preferences and behaviour of the travelers are described by a nested CES utility function, it can be shown that the demand functions take the following form (Keller (1976)):

$$q_{0,i} = \frac{I}{P_3} \prod_{e=1}^{3} \psi_{e-1,i} (\frac{P_{e,i}}{P_{e-1,i}})^{\sigma_{e,i}} \qquad \forall i \tag{5.1}$$

where $\psi$ is a share parameter, $\sigma$ is the elasticity of substitution, $P$ is the generalized price, $I$ equals total income, index $i$ stands for the different commodities that affect consumer utility and $e$ denotes the level. These demand functions are the solution of a constrained utility maximization problem in which all the utility components are linear homogeneous functions of the associated components at the next lower level.

The demand for transport services $q_{0,i}$ corresponds to the number of trips on route $r$ from OD pair $w$ from user class $m$ in period $p$. We will further denote this demand by $Y_{r,w}^{m,p}$. From equation (5.1) it is clear that demand is a function of the generalized price. In the next section, we will discuss the generalized price in detail.

### 5.3.2 Supply

A directed transportation network G(N;L) is defined by a set N of nodes and a set L of directed links (denoted l=1,. . . ,L). Let W be the set of all OD pairs in the network (denoted w=1, . . . , W), and $R_w$ be the set of routes between OD pair w $\in$ W. We furthermore define a set of dummy indicators $\delta_{lr}$ to denote whether link $l$ is part of route $r$.
This paper defines a link by the two nodes it connects and by the mode that is used. The set L thus includes the set of automobile links $L^U$, the set of transit links $L^Z$, the set of bike links $L^B$ and the set of pedestrian links $L^V$. Each link in the multimodal network has an associated travel cost which consists of a monetary cost and a time cost. These two cost components are discussed in detail in the next subsections.

#### 5.3.2.1 Monetary cost

For travelers on a public transport link, the monetary cost equals the fare the user pays. We assume this fare to vary between the peak and the off-peak period, and between different public transport modes. Furthermore, we assume the fare to be charged proportionally to the travel distance.

The monetary cost for a car link is the sum of the resource cost and, if applicable, the congestion toll and the parking cost. The resource cost includes fuel costs, maintenance and operational costs, and investment costs. We assume that these costs are shared by the occupants of the car.

For a bicycle link the monetary cost equals the resource cost, which includes the purchasing price of the bicycle and the maintenance cost. For a trip on foot, there is no monetary cost.

Defining a dummy indicator $\lambda_{li}$ that equals 1 if link $l$ is associated with mode $i$, the monetary cost for link $l$ in period $p$ is given by

$$mc_l^p = \left( \sum_z^Z h_z^p \, \lambda_{lz} + \frac{\tau_l^p + rc_u}{n} \, \lambda_{lu} + rc_b \, \lambda_{lb} \right) a_l, \tag{5.2}$$

where $h_z^p$ is the fare in period $p$ for public transport mode $z$ [€/$km$], $rc_u$ is the resource cost for a car [€/$km$], $\tau_l^p$ is the congestion toll in period $p$ for a car on link $l$ [€/$km$], $n$ is the car occupancy, $rc_b$ is the resource cost for a bicycle [€/$km$] and $a_l$ is the length of link $l$ [$km$].

#### 5.3.2.2 Time cost

The generalized travel time cost is a weighted sum of in-vehicle time, waiting time and walking time. First, consider in-vehicle time. When formulating the in-vehicle time, we follow Basso and Silva (2014) and distinguish between the case in which infrastructure is shared between different modes and the case in which infrastructure is separate. The time it takes to travel one km on link $l$ in period $p$ when infrastructure is separated is given by:

$$ivt_l^p = free_l \left( 1 + \alpha \left( \frac{ce_l^p}{C_l} \right)^{\beta} \right) \tag{5.3}$$

With parameter values $\alpha = 0.15$ and $\beta = 4$, this function is known as the BPR (Bureau of Public Roads) function. In the above equation $free_l$ is the free-flow travel time on link $l$ [$h/km$], $C_l$ is the capacity of the infrastructure measured in equivalent car trips per hour, and $ce_l^p$ represents the flow on link $l$ (in car equivalents per hour) in period $p$.

Depending on the mode that operates on a link, the formulation of the flow differs. The car flow is given by:

$$ce_l^p = \frac{\sum_{w \in W} \sum_{r \in R_w} \sum_{m \in M} \delta_{lr} \, Y_{r,w}^{m,p}}{n \, H^p} \qquad \forall l \in L^U \tag{5.4}$$

where $Y_{r,w}^{m,p}$ is the demand on route $r$ from OD pair $w$ and user class $m$ in period $p$, $H^p$ is the period duration [$h$], $n$ is the car occupancy and $\delta_{lr}$ is a dummy indicator that equals 1 if route $r$ uses link $l$ and 0 otherwise.

For public transit links the flow is given by:

$$ce_l^p = f_l^p\, \mu \qquad \forall l \in L^Z \tag{5.5}$$

where $f_l^p$ [veh/h] is the frequency on link $l$ in period $p$, and $\mu$ is an equivalence factor.

For bike and pedestrian links, we assume the car equivalents to be zero. The flow on link $l$ in period $p$ can thus be modelled as follows:

$$ce_l^p = \frac{\sum_{w \in W} \sum_{r \in R_w} \sum_{m \in M}\ \delta_{lr}\ Y_{r,w}^{m,p}}{n\ H^p}\ \lambda_{lu} + f_l^p\ \mu\ \lambda_{lz} \tag{5.6}$$

When traffic conditions are mixed, we have to consider congestion across modes. The in-vehicle time for link $l$ in period $p$ is then represented as follows:

$$ivt_l^p = free_l \left(1 + \alpha \left(\frac{ce_l^p + \sum_{\rho \neq l}^{L} ce_\rho^p\, \gamma_{l\rho}}{C_l}\right)^{\beta}\right) \tag{5.7}$$

In this equation $\gamma_{l\rho}$ is a dummy indicator that equals one if link $l$ shares its infrastructure with link $\rho$.

Next, consider waiting time. For travel by car, foot or bicycle there is no need to wait, so only trips by public transit have a waiting cost component. When modeling the waiting time for public transit links, we follow Tirachini et al. (2010) in distinguishing between services with a low frequency and services with a high frequency.

For high frequency services[1] we can assume a uniform arrival distribution of passengers at the station. Consequently, the waiting time can be modeled as a fraction ($\theta_i$) of the headway. For low frequency services, the waiting time consists of two components. First, passengers incur schedule delay, as they have to travel either earlier or later than they would like to. The schedule delay cost increases proportionally with the headway, so it can be modeled as a fraction of the headway. Besides schedule delay cost, travelers also incur a safety waiting cost. Indeed, travelers generally calculate in some additional safety time at the station. The average waiting time can then be modeled as follows:

$$wait_l^p = \left(\epsilon_l \left(\frac{\theta_1}{f_l^p}\right) + (1 - \epsilon_l)\left(s + \kappa \frac{\theta_2}{f_l^p}\right)\right)_{lz} \tag{5.8}$$

[1] For the applications we will follow Tirachini et al. (2010) and use 12 minutes as threshold headway.

where $f_l^p$ is the frequency of the transit mode on link $l$ in period $p$, $\kappa$ is a factor indicating that waiting time at the station outweighs waiting time at home, $\epsilon_l$ is a dummy indicator that equals 1 if the frequency is high and 0 otherwise and $s$ is the safety time.

Finally, consider access time. It is clear that only public transit link costs have an access time component. We assume this to be a fixed component dependent on the link, $access_l$.

When we multiply the different time components with their respective value of time, we get a monetary cost. The generalized cost for link $l$ in period $p$ is thus given by:

$$gtc_l^p = VOT_{ivt}\, ivt_l^p\, a_l + VOT_{wait}\, wait_l^p + VOT_{access}\, access_l \tag{5.9}$$

where $VOT_{ivt}$ is the value of in-vehicle time [€/$h$], $ivt_i^p$ is the in-vehicle time in period $p$ for link $l$ [$h$], $VOT_{wait}$ is the value of waiting time [€/$h$], $wait_l^p$ stands for the waiting time on link $l$ in period $p$ [$h$], $VOT_{access}$ is the value of access time [€/$h$] and $access_l$ represents the time needed to access the transport mode [$h$].

The travel time on route $r \in R_w$ in period $p$ between OD pair w ∈ W is given by the sum of the generalized costs $gc$ of the links that are used:

$$c_r^{w,p} = \sum_{l \in Lt_l} \delta_{lr}^w\, gc_l^p \qquad \forall r \in R_w, \forall w \in W \tag{5.10}$$

where $\delta_{lr}^w = 1$ if route $r$ between OD pair $w$ uses link $l$, and 0 otherwise.

### 5.3.3 Welfare function

The objective function includes consumer surplus, external costs, government revenue, transit agency net revenues, operational transit costs, and implementation costs of policies.

Total consumer surplus is given by the sum of the consumer surplus of all types of users and all OD pairs:

$$\sum_{w \in W} \sum_{m \in M} U_{m,w} \tag{5.11}$$

We consider the external costs of congestion, accidents, air pollution and noise. The external congestion costs enter the utility function via the generalized consumer prices. The external costs of accidents, air pollution and noise are considered separately. We assume these costs to be constant per vehicle kilometer. As a result the total external cost per day is given by:

$$\sum_{l}^{L}\sum_{p}^{P}(ec_l\, ce_l^p)a_l \tag{5.12}$$

In the above equation, $ec_l$ represents the external cost [$€/km$], $ce_l^p$ is the flow, in car equivalents per hour, on link $l$ in period $p$ and $a_l$ is the length of link $l$.

The government revenues per day consist of the sum of the daily parking and toll revenues.

$$\sum_{l\in L^{U^{toll}}}\sum_{p}^{P}\tau_l^p\, ce_l^p + \sum_{l\in L^{U^{park}}}\sum_{p}^{P} par_l\, ce_l^p \tag{5.13}$$

where $U^{toll}$ contains the car links that are part of a tolling zone, $U^{park}$ is the set of car links that have a parking component and $par_l$ stands for the parking fee on link $l$ [$€/vehicle$]. The changes in toll revenue receive a different weight than the changes in consumer and producer surplus to account for the marginal cost of public funds.

The transit agency collects the daily fare revenues:

$$\sum_{z}^{Z}\sum_{p}^{P}\sum_{w}^{W}\sum_{m}^{M}\sum_{r\in R_w}(h_z^p\,\lambda_{lz})\,a_l\,\delta_{lr}^w\,Y_{r,w}^{n,p} \qquad \forall l\in L^Z \tag{5.14}$$

In equation (5.14) $h_z^p$ is the fare of public transit mode $z$ in period $p$, $\lambda_{lz}$ is a dummy indicator that equals 1 if link $l$ is associated with mode $z$, $Y_{r,w}^{m,p}$ is the demand on route $r$ from OD pair $w$ and user class $m$ in period $p$, $\delta_{lr}^w = 1$ if route $r$ between OD pair $w$ uses link $l$ and $a_l$ is the length of link $l$.

The operating costs of the transportation system encompass vehicle maintenance costs (fuel, tires, vehicle servicing), vehicle operation costs (operating staff wages, administration, ticketing and fare collection, system security) and non-vehicle maintenance costs (roadway, track, signals, and stations). In equation (5.15) below, the first term captures the daily distance-related costs, the second term represents the daily time-related costs and the third term route-related costs.

$$\sum_z \sum_p \sum_l lenght_l\, H^p\, f_l^p\, vmc_z\, \lambda_{lz} + \sum_l \sum_p lenght_l\, \frac{H^p\, f_l^p}{speed_l^p}\, voc_l + \sum_l lenght_l\, nvmc_l \quad (5.15)$$

The daily vehicle-kilometers are the sum of the vehicles-kilometers in both periods: $\sum_l \sum_p H^p\, f_l^p\, length_l$, where *length* is the length of the route on link $l$. $vmc_z$ is the vehicle maintenance cost [€$/vehkm$] for public transit mode $z$. $voc_l$ is the vehicle operation cost [€$/vehh$] which is multiplied by the total daily vehicle hours: $\sum_l \sum_p length_l\, \frac{H^p\, f^p}{speed_l^p}$. Finally, $nvmc_l$ is the non-vehicle maintenance cost per route kilometer.

We model the net investment cost of public transit vehicle capacity as a function of the fleet (FL) in the following way:

$$(1 + soft)\,(impl_z\, length_l\, \lambda_{lz} + FL\, price) - RV \quad (5.16)$$

In this equation, *soft* stands for the fraction of soft costs, $impl_z$ is the implementation cost for public transit mode $z$[€$/km$], $length_l$ is the length of the track $[km]$, price represents the purchasing price per vehicle and RV is the residual value of the investment at time T, where T is determined by the technical life of the vehicle. The required fleet is determined by the following equation:

$$FL = \sum_l^L \frac{dis_l}{0,85} \max_p \frac{f_l^p}{speed_l^p} \quad (5.17)$$

where $dis_l$ is the total distance that a vehicle covers before starting a new cycle and $speed_l^p$ is the speed on link $l$ in period $p$. To account for a fleet reserve capacity of 15%, this term is divided by 0.85.

To allow for comparison between different scenario's, the net present value is calculated.

## 5.4 Case study

The geographical area covered by the case study is the zone encompassing the conurbation of Leuven. Figure 5.2 represents this area schematically by distinguishing four regions: the center of Leuven (C), the periphery (PER), the suburban employment center (SEC), and the surrouding region (O). Like many European cities, Leuven has a radial structure. The model assumes the parts in between the radials in the surrounding region to be symmetrical in terms of demand, capacity, costs, etc.

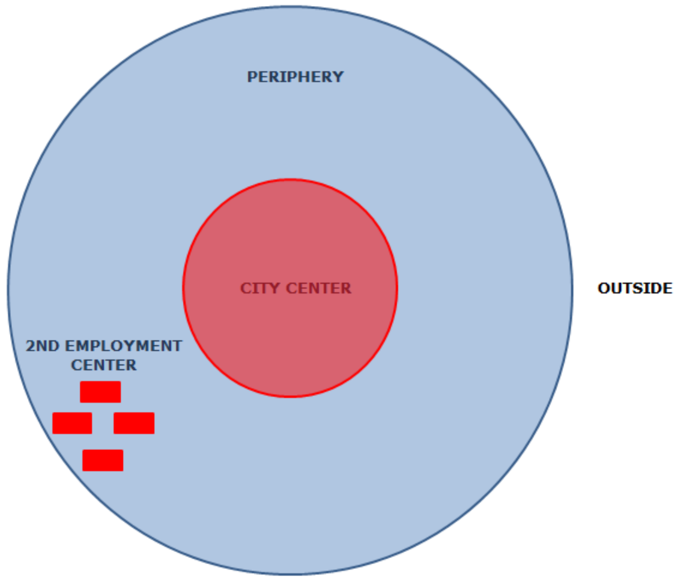


**Figure 5.2:** Schematical representation of the conurbation of Leuven

In the morning, travelers from the outside region travel to the center or the suburban employment center. Travelers living in the center either go to a destination in the center or travel to the suburban employment center. And travelers residing in the periphery either go to the center or the suburban employment center. In Table 5.1 the flow between the different OD pairs in the reference year 2009 is presented.

**Table 5.1:** Number of passengers per hour in the reference year 2009

| | O→ $C$ | C→ $C$ | PER→ $C$ | C→ $SEC$ | O→ $SEC$ |
|---|---|---|---|---|---|
| Peak | 11,002 | 32,660 | 14,515 | 9,081 | 4,421 |
| Off-peak | 6,348 | 21,154 | 8,707 | 5,400 | 2,288 |

Users can choose to travel by car, train, bus, bike or they can walk to their destination. In Table 5.2 the different modes that can be used to go from an origin to a destination are listed. Both unimodal and multi-modal trips are considered. Car and bus can be used for all trips. Only trips in the city center can be done on foot. The train is only used for trips between the city and the outside region. The bike can be used for all trips except for the trips between the city and the outside region. In Table 5.3 the trip purpose shares per mode in the city center are given for the reference year 2009.

**Table 5.2:** Model combinations considered

| | O→ $C$ | C→ $C$ | PER→ $C$ | C→ $SEC$ | O→ $SEC$ |
|---|---|---|---|---|---|
| car | x | x | x | x | x |
| walk | | x | | | |
| bike | | x | x | x | |
| bus | x | x | x | x | x |
| train+walk | x | | | | |
| train+bus | x | | | | x |
| train+bike | x | | | | x |
| bus+bike | x | | | | x |
| bus+walk | x | | | | |

**Table 5.3:** Mode choice shares and trip purpose shares in the city center for the reference year 2009

| | Car | Bus | Bike | Pedestrian | Total |
|---|---|---|---|---|---|
| School | 54% | 85% | 60% | 66% | 61% |
| Work | 40% | 14% | 39% | 20% | 32% |
| Other | 6% | 1% | 1% | 14% | 7% |

Figure 5.3 shows the multimodal network. This hypernetwork consists of five sub-networks: the auto network, the train network, the bike network, the bus network and the pedestrian network. The hypernetworks or supernetworks combine networks of various modes (car, bike, train, etc) and include special links to interconnect them (represented by the dashed lines in Figure 5.3). To these links transfer costs are attributed. A path trough the network including such a transfer link then represents a multimodal trip. Apart from transfer links, transit links and private transport links, we also include embarking links and alighting links. These links capture the relevant waiting and access time.

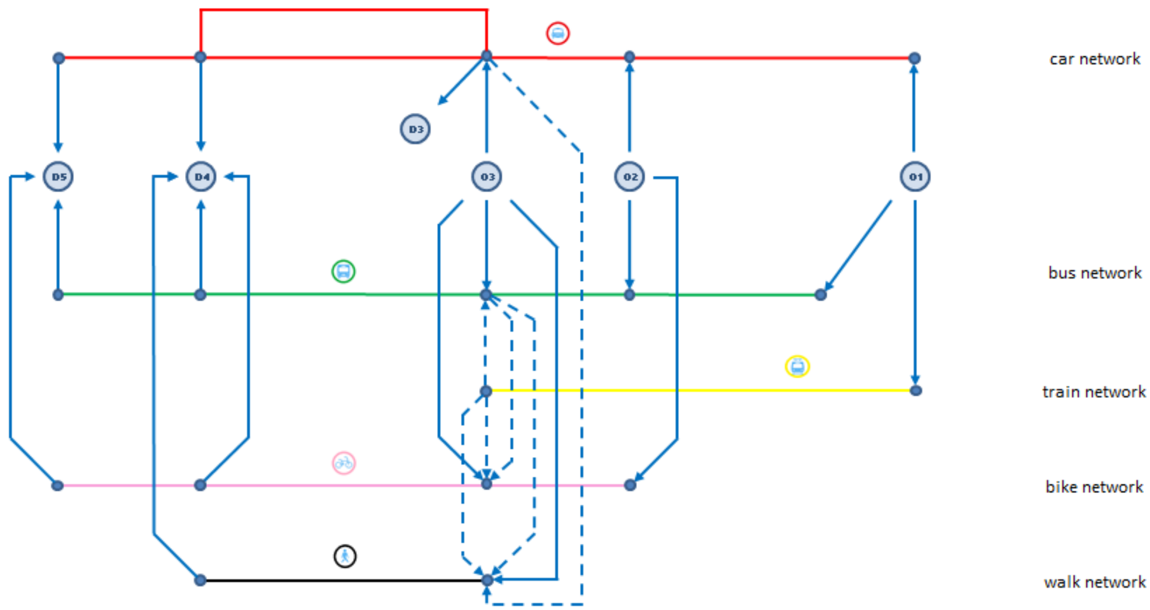


**Figure 5.3:** Network representation of the conurbation of Leuven

The model is implemented by adapting the MOLINO-model and calibrated for the reference year 2009 using observed prices and quantities for all transport modes together with information on the ease of substitution between transport and other goods as well as between the different means of transport.

Observed quantities per OD pair and per trip purpose are obtained from Vlaams Verkeerscentrum (See Appendix M). Data per mode share is calculated using mode share data in Janssens et al. (2012). Observed prices consist of a monetary and a time cost. Table 5.4 lists the resource cost for the different modes.

**Table 5.4:** Resource costs

| | resource cost |
|---|---|
| car | 0.32 euro/vehiclekm |
| bike | 0.23 euro/km |
| train | 0.06 euro/personkm |
| bus | 26.51 euro/vehiclekm |

Source: Delhaye et al. (2010)

To calculate the time costs in the reference case we have used observed speed data. Table 5.5 shows the travel time per mode in the city center for commuters in the reference case.

**Table 5.5:** Travel time in the center (min)

| | car | pedestrian | bus | bike |
|---|---|---|---|---|
| work peak | 3.00 | 16.67 | 4.20 | 3.33 |
| work off-peak | 2.10 | 16.67 | 3.30 | 3.33 |

Remark that, although cars and buses are assumed to use the same infrastructure, the travel time when driving is lower than when taking the bus. This is because we have taken into account the delay at bus stops.

The calibrate the nested CES functions we need two more inputs; for each level we need to specify the elasticity of substitution (Table 5.6) and we need to determine the share of income per user class (Table 5.7).

**Table 5.6:** The elasticities of substitution

| | work | other | school |
|---|---|---|---|
| transport/other | 1.4 | 1.4 | 1.4 |
| peak/off-peak | 1.2 | 2 | 1.5 |
| paths | 2 | 2 | 2 |

Source: Kilani et al. (2014)

**Table 5.7:** Percentage of income spent on transportation

| work | school | other |
|---|---|---|
| 5% | 10% | 7% |

The capacity of each link is chosen to fit the observed speeed-flow data. As such, truck traffic is also incorporated, though as a constant.

For the city of Leuven, 3 scenarios to reduce externalities in the city center are considered: introducing road pricing, expanding public transport and raising parking fees. In the next section, the effects of these three scenarios is assessed.

## 5.5 The effects of alternative policies

This section presents the impact of three transport policies on traffic volumes, external costs, welfare, etc. We first assess the effects of the introduction of road pricing. Second, we describe the consequences of an increase in parking fees in the city center. Third, we examine the impact of the construction of a tram lane. And finally, we compare the welfare effects of the different policies from the viewpoint of the city authority on the one hand, and from societal perspective, on the other hand.

### 5.5.1 Introducing zonal pricing

In the first scenario, we analyze the effect of introducing a zonal congestion charge in the city center for car traffic. The toll comprises the full external cost and amounts to 0.28 euro in the peak period and 0.06 euro in the off peak period. The toll revenues are collected by the city authority.

Introducing road pricing reduces flows by approximately 10% in the city center, which leads to travel speed increases of more than 20% (see Table 5.8). The resulting decrease in time cost, can however, not compensate for the price increase due to the toll levy. The increase in generalized price will thus lower consumer surplus which induces travelers to divert from the city center roads to the ringroad, or to divert from car use to public transit or soft means of transportation (biking, walking). The increase in patronage for public transit increases the frequency and reduces the waiting time, which, on its turn, attracts new users.[2] The introduction of a road toll on cars also reduces the total amount of travelers by 3%. As drivers either refrain from travelling, or switch to lower externality modes, the total noise, accidents and pollution cost decreases by approximately 10%.

### 5.5.2 Increasing the cost of parking

In the second scenario, the parking fee is increased by the weighted average of the externality in peak and off-peak, where the different weights are given by the relative elasticity. In the reference scenario the parking cost was 2.1 euro per hour.[3] After the increase the parking fee amounts to 2.24 euro per hour. We assume commuters to have parking provided by their employer. As such they are not affected by the increase in parking fees.

[2] We do not take into account congestion discomfort costs.
[3] We will assume that drivers park on average for three hours.

Due to the increase in parking fees, the number of cars in the city center decreases with approximately 3%. This results in a decrease of accident, pollution and noise externalities by more than 3% as drivers change from a mode with a large externality to a mode with a small (or positive) externality (public transit, soft modes).

The effects are similar to the road pricing case, though smaller in magnitude. This can be attributed to the fact that a large group of travelers, i.e. commuters, is not affected by the increase in parking fees.

### 5.5.3 Expanding public transit

As there was mention of a tram in Leuven (Rijnders (2014)), we assess the impact of the construction of a separated tram lane in the city center and between the city center and the suburban employment center. The total investment cost amounts to 154.625.250 euro and is based on data from Van Oppens (2013) and RebelGroup Advisory Belgium NV (2013). When a tram line runs between the city center and the SEC, buses are banned out of the city center and also the buses between the city center and the SEC are dispensed with. The capacity that has become available on the car links, as a result of the absence of buses on the road and the construction of a separated tram lane, is ocupied by additional cars. As a result, the speed in the city remains approximately at the same level as does the travel cost for car travelers.

**Table 5.8:** Effects of the different scenarios in the city center

| | Reference equilibrium | Full external cost pricing | Parking charges | Tram |
|---|---|---|---|---|
| | | *Change w.r.t.* | *the reference (%)* | |
| NPV (mio euro) | | | | |
| *Total delay cost* | 0.577 | -44 | -11 | -33 |
| *Total noise, accidents and pollution cost* | 23.06 | -10.34 | -3.11 | -2.35 |
| Traffic flow (CE/day) | 221,266 | -10.66 | -3.21 | -0.008 |
| Speed (km/h) | | | | |
| *Peak* | 19.99 | +20.06 | +3.05 | +0.15 |
| *Off-peak* | 28.5 | +0.7 | +0.7 | +0.14 |

### 5.5.4 Welfare effects

The welfare functions of the city authority divert from the welfare function of society. The city authority takes into account the consumer surplus of the local citizens, the accidents, pollution and noise in the city and the revenues for the city. The welfare function for society, on the other hand, includes consumer surplus and accidents, pollution and noise costs for all those involved, also in the outside region. It furthermore includes revenues for all authorities, local and central, and the financial result of the transit agency.

**Table 5.9:** Welfare effects of the different scenarios

| | Reference equilibrium | Full external cost pricing | Parking charges | Tram |
|---|---|---|---|---|
| | *NPV (mio euro)* | *Change w.r.t. the reference (%)* | | |
| Welfare city | 42,425.97 | +0.49 | -0.06 | +0.001 |
| CS city | 40,379.98 | -0.16 | -0.04 | +0.002 |
| Parking city | 2,082.12 | +0.58 | -0.45 | -0.03 |
| Externalities city | 44.13 | -9.83 | -2.68 | 1.37 |
| Welfare society | 166,777.36 | +0.04 | -0.08 | -0.03 |

Table 5.9 summarizes the welfare effects of the different scenarios and compares them with the reference situation. We find that the first choice for the city authority would be to implement road pricing in the city center as this increases local welfare by 0.49%. The result for society is also positive (+0.04%), though smaller. This can be explained by the fact that the benefits of tolling are now balanced by a larger amount of people who see their consumer surplus decrease.

Suppose that the city authority is impeded from introducing road pricing by political reasons. In this case the city authority will opt for the construction of a tram lane, as this will increase city welfare by 0.001%. The construction of a tram line is, however, welfare decreasing from societal point of view. This bifurcation does not surprise as the city authority does not take into account the investment cost of the project.

Even though the price of driving through the city increases on average by the same amount when parking charges are increased as when a toll is levied, total welfare decreases in this case (as opposed to the total welfare increase in the toll case). This loss in welfare can be attributed to different factors. First, commuters are not affected by this increase in parking fees, as employers are assumed to provide parking for their employees. Second, travelers who drive through the city without parking are not affected either. Third, we have assumed the parking fees to be constant throughout the day. Efficiency is lost because parking fees are not differentiated by time. Further simulations have shown that when parking fees can be differentiated between peak and off-peak periods, the welfare effect of an increase in parking fees becomes positive.

## 5.6 Conclusion

In this paper, we have compared the effects and welfare changes of alternative urban transport policies. We have built a stochastic, multimodal, multi-class, multi-period model, which allows for endogenous congestion and total demand elasticity. Pure as well as mixed modes of transport are considered, and different government perspectives are compared. For the conurbation of Leuven, we find that Leuven would profit from a introduction of road pricing in the city center. Another finding is that the expansion of public transit in Leuven implies a welfare increase from the point of view of the city authority, while it is welfare decreasing from the perspective of society. As a result we conclude that caution is recommended for when transit agencies are state-owned as city authorities can lobby for projects which are welfare decreasing from societal point of view.

# Chapter 6

# Epilogue

How can transportation be organized as efficiently as possible? From different angles, transport economists contribute to answering this question by studying various aspects of efficiency in the context of transportation. This PhD-thesis within the field of transportation economics also does its part by focusing on the efficient use of traffic lights and on the implementation of efficient transportation policies.

In Chapter 2 of this thesis, we studied the effects of a priority rule, traffic lights, and a toll on an intersection of two routes connecting one O-D pair. We derived the intersection regulation that minimizes total travel cost, taking into account Wardrop's principles and the delay at the intersection.

We have four major results. First, if the intersection is regulated by a priority rule, the optimal policy is generally to block one of the two routes. Second, if the intersection is regulated by traffic lights, and only one route is congestible, the optimal policy is again to block one route. However, the addition of a toll allows for an optimal alternating signal setting. Third, if the intersection is regulated by traffic lights, the optimal alternating signal setting is always independent of the elasticity of demand. Finally, if only one route is subject to congestion, the superiority of a regulation by traffic lights over a priority rule becomes more likely the lower the reaction time of the drivers, and the higher the cycle time.

These results are important for three reasons. First, the counter-intuitive nature of these results confirms the importance of a good understanding of the causal mechanisms that govern the optimal regulation. Second, these insights allow to solve larger networks more efficiently as well as more effectively. More efficient, because the increased insight in the location of the optimal solution allows for a reduction in computation time. More effective, because local optima can be detected more easily. Finally, the obtained results can be applied in practice. Our results can be useful in different contexts. We primarly think about two parallel roads (e.g. a road through an urban area and a parallel road bypassing the city) or two parallel modes (e.g. a train and a road connecting two cities).

In Chapter 3 of this thesis, we compared two different regulations for an intersection of two routes connecting one O-D pair where only one route is subject to congestion. In particular, we contrasted traffic-responsive control with anticipatory control.

We have shown that (1) traffic responsive signal control can only perform just as well or worse than anticipatory signal control and that (2) the expected performance of traffic responsive signal control is worse than the performance of anticipatory signal control. The game theoretic perspective taken in this paper furthermore suggests that these results can also be extended to larger instances.

These results have important implications for policy. The counter-intuitiveness of these results indicates that great attention needs to be given to the accuracy of the appraisal of signal control investments. Since both the costs of road transportation infrastructure and user costs are large, policy should be based on careful analysis rather than on intuition alone. The allocation of public money to the intuitively superior traffic-responsive signal control, may actually make society worse off as the money could be more efficiently spent on anticipatory signal control. This paper furthermore intends to raise awareness that policies based on intuition alone can have unintended consequences in the hope that these can be recognized and avoided.

In Chapter 4 of this thesis, we studied the extent to which traffic lights can provide an alternative to road pricing in a simple network with two routes connecting one O-D pair. We have compared the performance of traffic lights and tolls for two different networks. For the network with two parallel routes, we have modeled both the signal setting procedure and the toll setting procedure as a Stackelberg game. We find that tolls generally perform better than traffic lights.

For the network in which traffic lights regulate the intersection of two routes connecting one OD pair, we have used a Stackelberg approach as well as an inverse Stackelberg approach. We have derived the conditions a signal setting has to satisfy to be able to obtain the same results as when both traffic lights and a toll can be used to optimize the network. With a numerical example, this paper showed that it is possible to find a flow-dependent signal setting that can render road pricing redundant.

In Chapter 5 of this thesis, we have compared the effects and welfare changes of alternative urban transport policies. We have built a stochastic, multimodal, multi-class, multi-period model, which allows for endogenous congestion and total demand elasticity. Pure as well as mixed modes of transport are considered, and different government perspectives are compared. For the conurbation of Leuven, we find that Leuven would profit from a introduction of road pricing in the city center. Another finding is that the expansion of public transit in Leuven implies a welfare increase from the point of view of the city authority, while it is welfare decreasing from the perspective of society. As a result we conclude that caution is recommended for when transit agencies are state-owned as city authorities can lobby for projects which are welfare decreasing from societal point of view.

# Appendix A

# Derivation of $\tau^*$ and r* and second order conditions for $X_1 > 0$, $X_2 > 0$ (only 1 route liable to congestion)

$$\min_{X_1,X_2,r,\tau} \left(a_1 X_1 + \omega + \phi_1 + T_1(c,r) + \tau\right) X_1 + \left(\omega + \phi_2 + T_2(c,r)\right) X_2 - \tau X_1 \tag{A.1}$$

s.t.

$$X_1 + X_2 = N \tag{A.2}$$

$$a_1 X_1 + \omega + \phi_1 + T_1(c,r) + \tau = \omega + \phi_2 + T_2(c,r) \tag{A.3}$$

$$0 < r < c \tag{A.4}$$

$$X_1 > 0 \tag{A.5}$$

$$X_2 > 0 \tag{A.6}$$

$$\tau > 0 \tag{A.7}$$

This can be rewritten as follows:

$$\min_{r,\tau} \left(\omega + \phi_2 + T_2(c,r)\right) N - \tau \left(\frac{\phi_2 - \phi_1 + T_2(c,r) - T_1(c,r) - \tau}{a_1}\right) \tag{A.8}$$

$$\phi_2 - \phi_1 + T_2(c,r) - T_1(c,r) - \tau > 0 \tag{A.9}$$

$$a_1 N - \phi_2 + \phi_1 - T_2(c,r) + T_1(c,r) + \tau > 0 \tag{A.10}$$

$$0 < r < c \tag{A.11}$$

$$-\tau < 0 \tag{A.12}$$

$$L(r,\tau) = (-\omega - \phi_2 - T_2(c,r))N + \tau\left(\frac{T_2(c,r) - T_1(c,r) + \phi_2 - \phi_1 - \tau}{a_1}\right) \tag{A.13}$$

$$-\lambda_1(-T_2(c,r) + T_1(c,r) + \phi_1 - \phi_2 + \tau) \tag{A.14}$$

$$-\lambda_2(-\tau + T_2(c,r) - T_1(c,r) - a_1 N + \phi_2 - \phi_1) \tag{A.15}$$

$$-\lambda_3(r - c) - \lambda_4(-r) - \lambda_5(-\tau) \tag{A.16}$$

Derivation to $\tau$

$$\frac{\partial L}{\partial \tau} = \left(\frac{T_2(c,r) - T_1(c,r) + \phi_2 - \phi_1 - \tau}{a_1}\right) - \frac{\tau}{a_1} - \lambda_1 + \lambda_2 + \lambda_5 = 0 \tag{A.17}$$

$$\lambda_1 = 0, \quad \phi_2 - \phi_1 + T_2(c,r) - T_1(c,r) > \tau, \tag{A.18}$$

$$\lambda_1(-T_2(c,r) + T_1(c,r) + \phi_1 - \phi_2 + \tau) = 0 \tag{A.19}$$

$$\lambda_2 = 0, \quad a_1 N - \phi_2 + \phi_1 - T_2(c,r) + T_1(c,r) > -\tau, \tag{A.20}$$

$$\lambda_2(-\tau + T_2(c,r) - T_1(c,r) - a_1 N + \phi_2 - \phi_1) = 0 \tag{A.21}$$

$$\lambda_5 = 0, \quad -\tau < 0, \tag{A.22}$$

$$\lambda_5(-\tau) = 0 \tag{A.23}$$

For $0 < r < c$:

$$\tau = \frac{\phi_2 - \phi_1 + T_2(c,r) - T_1(c,r)}{2} \tag{A.24}$$

Derivation to r

$$\frac{\partial L}{\partial r} = -\frac{\partial T_2(c,r)}{\partial r}N + \left(\frac{\tau}{a_1} + \lambda_1 - \lambda_2\right)\left(\frac{\partial T_2(c,r)}{\partial r} - \frac{\partial T_1(c,r)}{\partial r}\right) - \lambda_3 + \lambda_4 = 0 \tag{A.25}$$

$$\lambda_1 = 0, \quad -T_2(c,r) + T_1(c,r) < -\tau + \phi_2 - \phi_1, \tag{A.26}$$

$$\lambda_1(-T_2(c,r) + T_1(c,r) + \phi_1 - \phi_2 + \tau) = 0 \tag{A.27}$$

$$\lambda_2 = 0, \quad T_2(c,r) - T_1(c,r) < \tau + a_1 N - \phi_2 + \phi_1, \tag{A.28}$$

$$\lambda_2(-\tau + T_2(c,r) - T_1(c,r) - a_1 N + \phi_2 - \phi_1) = 0 \tag{A.29}$$

$$\lambda_3 = 0, \quad r < c, \quad \lambda_3(r - c) = 0 \tag{A.30}$$

$$\lambda_4 = 0, \quad -r < 0, \quad \lambda_4(-r) = 0 \tag{A.31}$$

The second order conditions are given by the following equations:

$$\frac{\partial^2 L}{\partial r^2} = -\frac{N}{c} < 0 \tag{A.32}$$

$$\frac{\partial^2 L}{\partial \tau^2} = -\frac{2}{a_1} < 0 \tag{A.33}$$

$$\frac{\partial^2 L}{\partial r \tau} = \frac{1}{a_1} \tag{A.34}$$

It is clear that if $2a_1 N \geq c$, then $\frac{2}{a_1}\frac{N}{c} - \frac{1}{a_1^2} \geq 0$ and so the optimal $(\tau$, r) is a minimum. However, if $N < \frac{c}{2a_1}$ the optimal $(\tau$, r) is a saddle point. In this case, the lowest point will be near the corner.

# Appendix B

# Derivation of r* for $X_1 > 0$, $X_2 > 0$ (both routes liable to congestion)

$$\min_{X_1, X_2, r} \left(a_1 X_1 + \omega + \phi_1 + T_1(c,r)\right) X_1 + \left(a_2 X_2 + \omega + \phi_2 + T_2(c,r)\right) X_2 \tag{B.1}$$

s.t.

$$X_1 + X_2 = N \tag{B.2}$$

$$a_1 X_1 + \omega + \phi_1 + T_1(c,r) = a_2 X_2 + \omega + \phi_2 + T_2(c,r) \tag{B.3}$$

$$0 < r < c \tag{B.4}$$

$$X_1 > 0 \tag{B.5}$$

$$X_2 > 0 \tag{B.6}$$

This can be written as follows:

$$\min_{r} \left(a_1 \left(\frac{a_2 N + \phi_2 - \phi_1 + T_2(c,r) - T_1(c,r)}{a_1 + a_2}\right) + \omega + \phi_1 + T_1(c,r)\right) N \tag{B.7}$$

s.t.

$$a_2 N + \phi_2 - \phi_1 + T_2(c,r) - T_1(c,r) > 0 \tag{B.8}$$

$$a_1 N - \phi_2 + \phi_1 - T_2(c,r) + T_1(c,r) > 0 \tag{B.9}$$

$$0 < r < c \tag{B.10}$$

The corresponding Lagragian is given by:

$$L = \left( -a_1 \left( \frac{a_2 N + \phi_2 - \phi_1 + T_2(c,r) - T_1(c,r)}{a_1 + a_2} \right) - \omega - \phi_1 - T_1(c,r) \right) N \quad \text{(B.11)}$$

$$-\lambda_1 \left( -a_2 N - \phi_2 + \phi_1 - T_2(c,r) + T_1(c,r) \right) \quad \text{(B.12)}$$

$$-\lambda_2 \left( -a_1 N + \phi_2 - \phi_1 + T_2(c,r) - T_1(c,r) \right) \quad \text{(B.13)}$$

$$-\lambda_3 (r - c) - \lambda_4 (-r) \quad \text{(B.14)}$$

The FOC are as follows:

$$\frac{\partial L}{\partial r} = -\frac{a_1 N}{a_1 + a_2} \left( \frac{\partial T_2(c,r)}{\partial r} - \frac{\partial T_1(c,r)}{\partial r} \right) - \frac{\partial T_1(c,r)}{\partial r} N \quad \text{(B.15)}$$

$$-\lambda_1 \left( -\frac{\partial T_2(c,r)}{\partial r} + \frac{\partial T_1(c,r)}{\partial r} \right) - \lambda_2 \left( \frac{\partial T_2(c,r)}{\partial r} - \frac{\partial T_1(c,r)}{\partial r} \right) \quad \text{(B.16)}$$

$$-\lambda_3 + \lambda_4 = 0 \quad \text{(B.17)}$$

$$-a_2 N - \phi_2 + \phi_1 - T_2(c,r) + T_1(c,r) < 0, \quad \lambda_1 = 0, \quad \text{(B.18)}$$

$$\lambda_1 \left( -a_2 N - \phi_2 + \phi_1 - T_2(c,r) + T_1(c,r) \right) = 0 \quad \text{(B.19)}$$

$$-a_1 N + \phi_2 - \phi_1 + T_2(c,r) - T_1(c,r) < 0, \quad \lambda_2 = 0, \quad \text{(B.20)}$$

$$\lambda_2 \left( -a_1 N + \phi_2 - \phi_1 + T_2(c,r) - T_1(c,r) \right) = 0 \quad \text{(B.21)}$$

$$r < c, \quad \lambda_3 = 0, \quad \lambda_3 (r - c) = 0 \quad \text{(B.22)}$$

$$-r < 0, \quad \lambda_4 = 0, \quad \lambda_4 (-r) = 0 \quad \text{(B.23)}$$

If the optimal r is inserted in constraints (B.8) and (B.9), then it becomes clear that the interior solution is only feasible if:

$$2(a_1 + a_2)(a_2 N + \phi_2 - \phi_1) > a_1 c - a_2 c \quad \text{(B.24)}$$

and

$$2(a_1 + a_2)(a_2 N - \phi_2 + \phi_1) > a_2 c - a_1 c \quad \text{(B.25)}$$

# Appendix C

# Derivation of the aggregate AC

We will assume the properties of the routes connecting CB to be the same as those connecting AC.

$AC_{1ac} = a_1 X_{1ac} + \omega + \phi_1 + T_1(c, r)$

$AC_{2ac} = a_2 X_{2ac} + \omega + \phi_2 + T_2(c, r)$

$AC_{1cb} = a_1 X_{1cb} + \omega + \phi_1$

$AC_{2cb} = a_2 X_{2ac} + \omega + \phi_2$

**demand** $= \delta - \pi (X_1 + X_2)$

$X_T = X_{1cb} + X_{2cb} = X_{1ac} + X_{2ac}$

$$X_{1ac} = \frac{AC_{ac} - \omega - \phi_1 - T_1(c, r)}{a_1} \tag{C.1}$$

$$X_{2ac} = \frac{AC_{ac} - \omega - \phi_2 - T_2(c, r)}{a_2} \tag{C.2}$$

$$a_1 a_2 X_{Tac} = (a_1 + a_2) AC_{ac} - (a_1 + a_2)\omega - a_1(\phi_2 + T_2(c, r)) - a_2(\phi_1 + T_1(c, r)) \tag{C.3}$$

$$AC_{ac} = \frac{a_1 a_2 X_{Tac}}{a_1 + a_2} + \omega + \frac{a_1}{a_1 + a_2}(\phi_2 + T_2(c, r)) + \frac{a_2}{a_1 + a_2}(\phi_1 + T_1(c, r)) \tag{C.4}$$

$$AC_{cb} = \frac{a_1 a_2 X_{Tcb}}{a_1 + a_2} + \omega + \frac{a_1}{a_1 + a_2}(\phi_2) + \frac{a_2}{a_1 + a_2}(\phi_1) \tag{C.5}$$

$$AC_{cb} + ACac = \frac{2 a_1 a_2 X_T}{a_1 + a_2} + \omega + \frac{2a_1}{a_1 + a_2}(\phi_2) + \frac{2a_2}{a_1 + a_2}(\phi_1) + \frac{a_1}{a_1 + a_2}(T_2(c, r)) + \frac{a_2}{a_1 + a_2}(T_1(c, r)) \tag{C.6}$$

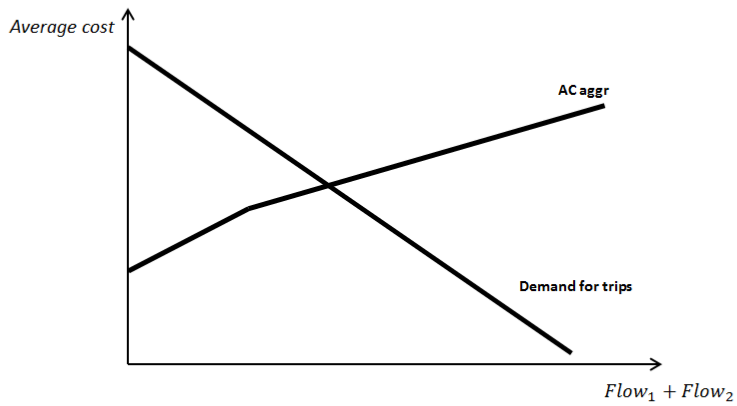


**Figure C.1:** The aggregate cost

In Figure C.1 the aggregated cost curve is shown. The kink in the cost curve is due to the change from using only one route, to using both routes.

# Appendix D

# Derivation of the total cost for an intersection regulated by a priority rule (both routes liable to congestion)

$$\min_{X_1,X_2} (a_1 X_1 + \omega + \phi_1) X_1 + \left( a_2 X_2 + \omega + \phi_2 + \frac{v^2 X_1}{2} \right) X_2 \tag{D.1}$$

s.t.

$$X_1 + X_2 = N \tag{D.2}$$

$$a_1 X_1 + \omega + \phi_1 = a_2 X_2 + \omega + \phi_2 + \frac{v^2 X_1}{2} \tag{D.3}$$

$$X_1 > 0 \tag{D.4}$$

$$X_2 > 0 \tag{D.5}$$

The total cost at the interior solution equals:

$$(a_1 X_1^e + \omega + \phi_1) N \tag{D.6}$$

The total cost at the corner solution $(X_1 = N)$ equals:

$$(a_1 N + \omega + \phi_1) N \tag{D.7}$$

The total cost at the corner solution $(X_2 = N)$ equals:

$$(a_2 N + \omega + \phi_2) N \tag{D.8}$$

## D.1 Derivation of r* when demand is elastic (both routes liable to congestion)

To find r*, the derivative of the following equation to r is taken. After which it is checked if r* satisfies the constraints.

The total cost equals:

$$\delta X_T^e - \frac{\pi}{2}(X_T^e)^2 - \frac{a_1 a_2}{a_1+a_2}(X_T^e)^2 - 2\omega X_T^e - \frac{2(a_1\phi_2 + a_2\phi_1)}{a_1+a_2} X_T^e - \frac{a_1 T_2\left(c,r\right) + a_2 T_1\left(c,r\right))}{a_1+a_2} X_T^e \tag{D.9}$$

The derivation to r gives us the following condition:

$$\frac{\partial X_T^e}{\partial r} X_T^e \left(\frac{\pi}{2} + \frac{a_1 a_2}{a_1+a_2}\right) = 0 \tag{D.10}$$

The signal setting for which $X_T^e = 0$, would imply that no driver would make the trip and the welfare would be equal to zero. In this case, the road could just as well be closed off. The signal setting for which $\frac{\partial X_T^e}{\partial r} = 0$ equals:

$$\frac{a_2 c}{a_1+a_2} \tag{D.11}$$

# Appendix E

# Waiting time computation for one route

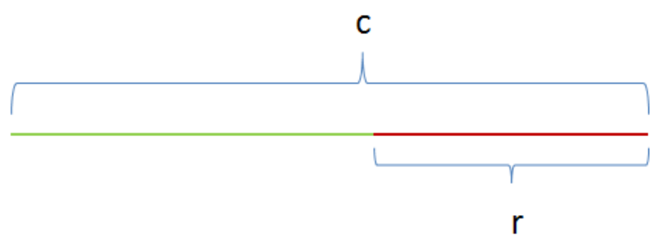


**Figure E.1:** Composition of the cycle time

It is assumed that traffic conditions are undersaturated. Then the average waiting time for drivers arriving when the traffic light is red is equal to $\frac{r}{2}$. Given that the chance thar drivers arrive when it is red is $\frac{r}{c}$, the average waiting time for all drivers equals:

$$\frac{r^2}{2c} \tag{E.1}$$

and the waiting time for Route 1 (with c-r red) equals:

$$\frac{(c-r)^2}{2c} \tag{E.2}$$

In cases where the intersection is regulated by a priority rule, the red time is replaced by v (v seconds before a car on the main road passes the intersection, cars on the minor road already wait until the car on the main road has passed the intersection), and the cycle time is replaced by the interarrival time of cars on the main road ($\frac{1}{X_1}$). The waiting time for cars on the minor road then equals:

$$\frac{v^2 X_1}{2} \tag{E.3}$$

# Appendix F

# Area AIFE > area ABCD

Area AIFE- area CDEF >0

$$\frac{\left(\phi_2 - \phi_1 - \frac{c}{2}\right)^2}{4a_1} + \frac{\left(\phi_2 - \phi_1 + r* - \frac{c}{2}\right) r*}{2a_1} + \frac{r*^2}{4a_1} - N\frac{r*^2}{2c} > 0 \tag{F.1}$$

$$\frac{\left(\phi_2 - \phi_1 - \frac{c}{2}\right)^2}{2a_1} > 0 \tag{F.2}$$

# Appendix G

# Comparison of the interior solution and the corner solutions

TC(interior*) $< TC$($X_2$=N)

$$\left(\frac{(\phi_1 - \phi_2 + \frac{c}{2})^2}{2\,(-2a_1N + c)} + \omega + \phi_2\right) N < (\omega + \phi_2)\, N \tag{G.1}$$

$$\frac{(\phi_1 - \phi_2 + \frac{c}{2})^2}{2\,(-2a_1N + c)} < 0 \tag{G.2}$$

As the left hand side is negative, this is always satisfied.
TC(interior*)$< TC$($X_1$=N)

$$\left(\frac{(\phi_1 - \phi_2 + \frac{c}{2})^2}{2\,(-2a_1N + c)} + \omega + \phi_2\right) N < (a_1N + \omega + \phi_1)\, N \tag{G.3}$$

$$\frac{(\phi_1 - \phi_2 + \frac{c}{2})^2}{2\,(-2a_1N + c)} + \phi_2 - \phi_1 < a_1N \tag{G.4}$$

As $2aN - \frac{c}{2} + \phi_1 > \phi_2$ for an interior optimum, this is always satisfied.

# Appendix H

# One instrument can account for the inefficiency in a parallel network with inelastic demand

$$\min_{\tau_1, X_1, X_2, \tau_2} (a_1 X_1 + f_1 + \tau_1) X_1 + (a_2 X_2 + f_2 + \tau_2) X_2 - \tau_1 X_1 - \tau_2 X_2 \tag{H.1}$$

s.t.

$$a_1 X_1 + f_1 + \tau_1 = a_2 X_2 + f_2 + \tau_2 \tag{H.2}$$

$$X_1 + X_2 = N \tag{H.3}$$

$$X_1 > 0 \tag{H.4}$$

$$X_2 > 0 \tag{H.5}$$

$$\tau_1 > 0 \tag{H.6}$$

$$\tau_2 > 0 \tag{H.7}$$

The optimal $\tau_2$ is given by

$$\tau_2 = \tau_1 + \frac{f_1 - f_2}{2} \tag{H.8}$$

The optimal $\tau_1$ equals

$$\tau_1 = \tau_2 + \frac{f_2 - f_1}{2} \tag{H.9}$$

It is clear that only the difference between $\tau_1$ and $\tau_2$ is uniquely determined. Remark that in equation (H.10), $\tau_1 - \tau_2$ can be replaced by $\tau$.

$$(a_1(\frac{a_2N + f_2 - f_1 - \tau}{a_+a_2}) + f_1 + \tau)N - \tau(\frac{a_2N + f_2 - f_1 - \tau}{a_+a_2}) \tag{H.10}$$

The optimal $\tau$ then equals

$$\tau = \frac{f_2 - f_1}{2} \tag{H.11}$$

# Appendix I

# Computation of the total cost for a network without intersection with road pricing

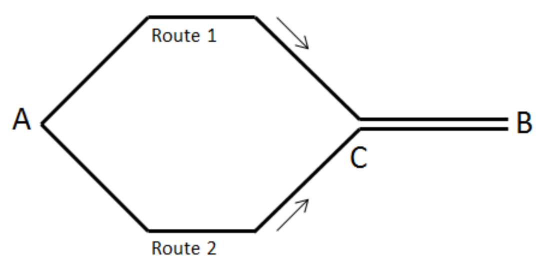


**Figure I.1:** Road pricing to influence route choice

Suppose that in the network in Figure I.1 a toll is levied on Route 1. Then all drivers will take Route 1 in equilibrium if $f_2 > a_1N + f_1 + \tau$. If however, $f_1 + \tau > a_2N + f_2$ then all drivers will take Route 2 in equilibrium. Finally, if $f_2 \leq a_1N + f_1 + \tau$ and $f_1 + \tau \leq a_2N + f_2$, then the drivers will use both routes in equilibrium. When both routes are used in equilibrium, the average cost of the two routes has to be equal.

$$f_1 + a_1X_1 + \tau = a_2X_2 + f_2 \tag{I.1}$$

Substituting this constraint in the total cost function, and using $X_1 + X_2 = N$, the total cost takes the following form:

$$\left(f_1 + \frac{a_1(a_2N + f_2 - f_1)}{a_1 + a_2}) + \frac{a_2\tau}{a_1 + a_2}\right)N - \tau(\frac{a_2N + f_2 - f_1 - \tau}{a_1 + a_2}) \tag{I.2}$$

# Appendix J

# Computation of the total cost for a network without intersection with traffic lights

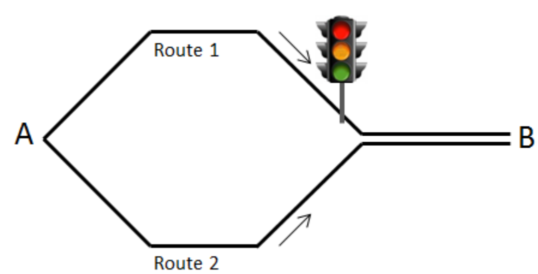


**Figure J.1:** A traffic light to influence route choice

Consider the network in Figure J.1. All drivers will take Route 1 in the equilibrium if $f_2 > a_1N + f_1 + T_1$. If, however, $f_1 + T_1 > a_2N + f_2$ then all drivers will take Route 2 in the equilibrium. Finally, if $f_2 \leq a_1N + f_1 + T_1$ and $f_1 + T_1 \leq a_2N + f_2$, then the drivers will use both routes in the equilibrium. If all drivers take Route 1, then the total cost is the sum of the minimal time cost, the congestion cost and the traffic light waiting cost for every driver. In this case, the total cost can be minimized by setting the traffic light always to green. The total cost then equals $(a_1N + f_1)N$. On Route 2 there is no traffic light, so the average cost will be the sum of the minimal time cost and the congestion cost. If all drivers take Route 2 in equilibrium, the total cost will thus be $(f_2 + a_2 N)N$. When both routes are used in equilibrium, the average cost of both routes has to be equal:

$$f_1 + a_1 X_1 + T_1 = a_2 X_2 + f_2 \tag{J.1}$$

Substituting the result of this equation together with $X_1 + X_2 = N$ in the total cost $(f_1 + a_1 X_1 + T_1(c, r)) X_1 + (f_2 + a_2 X_2) X_2$, we obtain:

$$\left( f_1 + \frac{a_1(a_2 N + f_2 - f_1)}{a_1 + a_2} ) + \frac{a_2 T_1}{a_1 + a_2} \right) N \tag{J.2}$$

# Appendix K

# Optimization of the network with road pricing and traffic lights

$$\min_{r,X_1,X_2,\tau} \left(f_1 + a_1X_1 + T_1\left(c, r_2\right) + \tau\right)X_1 + \left(f_2 + a_2X_2 + T_2\left(c, r_2\right)\right)X_2 - \tau X_1 \tag{K.1}$$

s.t.

$$f_1 + a_1X_1 + T_1\left(c, r_2\right) + \tau = f_2 + a_2X_2 + T_2\left(c, r_2\right) \tag{K.2}$$

$$X_1 + X_2 = N \tag{K.3}$$

$$X_1 > 0 \tag{K.4}$$

$$X_2 > 0 \tag{K.5}$$

$$r_2 > 0 \tag{K.6}$$

$$r_2 < c \tag{K.7}$$

The associated Langragian is given by:

$$L = \left(f_1 + a_1X_1 + T_1 + \tau\right)X_1 + \left(f_2 + a_2X_2 + T_2\right)X_2 - \tau X_1 + \lambda_1(N - X_1 - X_2) + \lambda_2(f_2 - f_1 - a_1X_1 - T_1 + T_2$$

(K.8)

The FOC are the following:

$$\frac{dL}{dX_1} = a_1X_1 + f_1 + a_1X_1 + T_1(c, r_2) - \lambda_1 - \lambda_2 a_1 = 0 \tag{K.9}$$

$$\frac{dL}{dX_2} = a_2X_2 + f_2 + a_2X_2 + T_2(c, r_2) - \lambda_1 - \lambda_2 a_2 = 0 \tag{K.10}$$

$$\frac{dL}{d\lambda_1} = N - X_1 - X_2 = 0 \tag{K.11}$$

$$\frac{dL}{d\lambda_2} = f_2 + a_2X_2 + T_2(c, r) - f_1 - a_1X_1 - T_1(c, r) - \tau = 0 \tag{K.12}$$

$$\frac{dL}{d\tau} = -\lambda_2 = 0 \tag{K.13}$$

$$\frac{dL}{dr_2} = \frac{dT_1(c, r_2)}{dr_2}X_1 + \frac{dT_2(c, r_2)}{dr_2}X_2 + \lambda_2\left(\frac{dT_2(c, r_2)}{dr_2} - \frac{dT_1(c, r_2)}{dr_2}\right) = 0 \tag{K.14}$$

Combining $\lambda_2 = 0$ and the first order conditions for $X_1$ and $X_2$, we find that $MC_1 = MC_2$ in the optimum.
If we combine $\lambda_2 = 0$, the first order conditions for $X_1$ and $X_2$ and the first order condition for $\lambda_2$, we obtain the optimal toll:

$$\tau = a_1X_1 - a_2X_2 \tag{K.15}$$

Finally, if we combine $\lambda_2 = 0$ and the first order condition for $r_2$, we find that the optimal signal setting is determined by the following equation:

$$\frac{dT_1(c, r_2)}{dr_2}X_1 = -\frac{dT_2(c, r_2)}{dr_2}X_2 \tag{K.16}$$

The optimal variables are then:

$$\tau = \frac{\left(f_2 - f_1 - \frac{c}{2}\right)(a_1 + a_2)N + a_2Nc}{2(a_1 + a_2)N - c} \tag{K.17}$$

$$r = \frac{\left(f_2 - f_1 - \frac{c}{2} + 2a_2N\right)c}{2(a_1 + a_2)N - c} \tag{K.18}$$

$$X_1^e = \frac{\left(f_2 - f_1 - \frac{c}{2} + 2a_2N\right)N}{2(a_1 + a_2)N - c} \tag{K.19}$$

And the minimum cost equals:

$$TC^{int} = \frac{N(c^2 + 4c(f_1 - f_2) - 4(4a_2Nf_1 - (f_1 - f_2)^2 + 4a_1N(a_2N + f_2)}{8(c - 2(a_1 + a_2)N)} \tag{K.20}$$

## Appendix L

# Implications of $AC_2(X_2^*, r_2^*) > AC_1(X_1^*, r_2^*)$ and $AC_2(X_2^*, r_2^*) < AC_1(X_1^*, r_2^*)$

If $AC_2(X_2^*, r_2^*) > AC_1(X_1^*, r_2^*)$, then equation (4.25) implies equation (4.26) for all $X_1 > X_1^*$. Indeed, if there exists a $r_2^h$ for which $AC_1(X_1^*, r_2^h) \geq AC_2(X_2^*, r_2^*)$ and $AC_2(X_2^*, r_2^*) > AC_1(X_1^*, r_2^*)$, then $AC_1(X_1^*, r_2^h) > AC_1(X_2^*, r_2^*)$, which implies that $r_2^h < r^*$. As a consequence, $AC_2(X_1^*, r_2^h) < AC_2(X_2^*, r_2^*)$, and $AC_1(X_1^*, r_2^h) > AC_2(X_1^*, r_2^h)$.

If, on the other hand, $AC_2(X_2^*, r_2^*) < AC_1(X_1^*, r_2^*)$, then equation (4.26) implies equation (4.25).

From Figure L.1 is clear that for $r_2^*$ equation (4.23) and equation (4.24) are satisfied. If $r_2^h$ increases, then the average cost curve of Route 1 shifts down ($AC_1'$ in Figure L.1) and the average cost curve of Route 2 shifts up ($AC_2'$ in Figure L.1). Figure L.1 shows that there exists a $r_2^h$ for which equation (4.24) is violated while equation (4.23) is still satisfied. For $r_2^h < r_2^*$, both equations (4.23) and (4.24) are always satisfied.

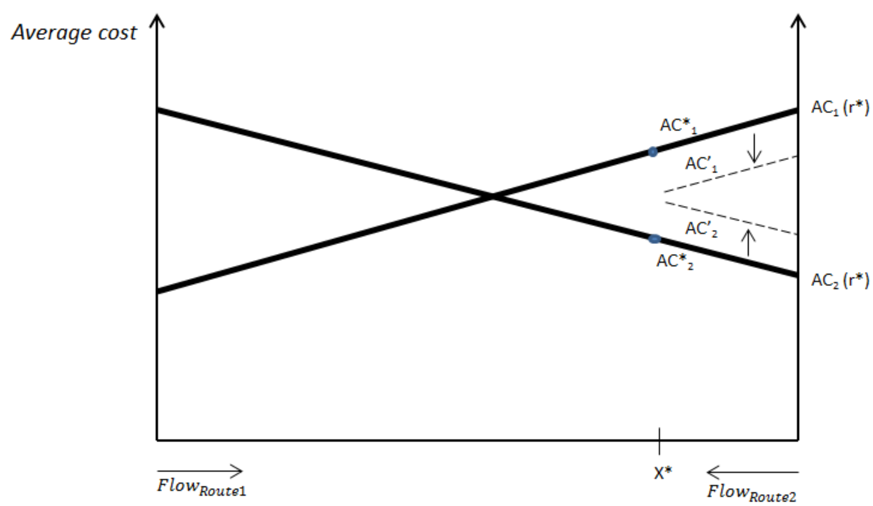


**Figure L.1:** If $AC_2^* < AC_1^*$, then equation (4.26) implies equation (4.25).

# Appendix M

# Data

### M.0.1 Quantities

Road traffic (cars) between 7h-8h in 2009:

**Table M.1:** Flow (number of cars/h)

| | work | school | other |
|---|---|---|---|
| O $\rightarrow$ C | 2729.1 | 2671.63 | 167.83 |
| O $\rightarrow$ SEC | 1727.77 | 573.33 | 66.84 |
| C $\rightarrow$ SEC | 1173.75 | 1465.88 | 146.09 |
| PER $\rightarrow$ C | 1794.2 | 2641.66 | 254.32 |
| C $\rightarrow$ C | 1730.05 | 3700.9 | 649.59 |

Source: Computations on basis of data from Verkeerscentrum Vlaanderen

This data needs to be converted into passengers per OD pair and per user class for each period. The data we need is represented in the following tables:

**Table M.2:** O → C (number of passengers in %)

| | work | school | other |
|---|---|---|---|
| bike | 9.89 | 13.29 | 4.7 |
| bus/bike | 0.33 | 1.06 | 0.39 |
| bus/bus | 4.87 | 15.51 | 5.74 |
| bus/pedestrian | 3.78 | 12.02 | 4.44 |
| car/parking center | 80.04 | 54.63 | 79.44 |
| car/parking periphery | 0 | 2.88 | 4.18 |
| train/bike | 0.15 | 0.58 | 0.07 |
| train/bus | 0.18 | 0.71 | 0.09 |
| train/pedestrian | 1.70 | 6.61 | 0.80 |

Source: computations on the basis of Janssens et al. (2012).

**Table M.3:** O → SEC (number of passengers in %)

| | work | school | other |
|---|---|---|---|
| bike | 9.89 | 13.29 | 4.7 |
| bus/bike | 0.41 | 1.30 | 0.48 |
| bus/bus | 4.87 | 15.51 | 5.74 |
| car | 80.04 | 57.5 | 83.62 |
| train/bike | 1.19 | 4.63 | 0.56 |
| train/bus | 1.19 | 4.63 | 0.56 |

Source: computations on the basis of Janssens et al. (2012).

**Table M.4:** C → SEC (number of passengers in %)

| | work | school | other |
|---|---|---|---|
| bike | 40.08 | 39 | 16.12 |
| bus | 2.21 | 15.6 | 3.41 |
| car | 53.85 | 40 | 72.52 |
| pedestrian | 0 | 1.39 | 6.4 |

Source: computations on the basis of Janssens et al. (2012).

**Table M.5:** PER → C (number of passengers in %)

| | work | school | other |
|---|---|---|---|
| bike | 40.08 | 33.43 | 16.12 |
| bus | 2.21 | 18 | 3.41 |
| car/center | 51.16 | 42.75 | 68.89 |
| car/periphery | 2.69 | 2.25 | 3.63 |
| pedestrian | 0 | 1.39 | 6.4 |

Source: computations on the basis of Janssens et al. (2012).

**Table M.6:** C → C (number of passengers in %)

| | work | school | other |
|---|---|---|---|
| bike | 39.83 | 32.20 | 17.44 |
| bus | 2.34 | 5.97 | 0.34 |
| car | 31.14 | 27.18 | 32.45 |
| pedestrian | 25.54 | 31.53 | 49 |

Source: computations on the basis of Janssens et al. (2012).

**Table M.7:** Peak/off-peak ratio

| | work* | other* | school** |
|---|---|---|---|
| peak | 0.7 | 0.47 | 0.6 |
| off-peak | 0.3 | 0.53 | 0.4 |

*Source: Van Der Loo and Proost (2010)
**Source: author's own estimate

**Table M.8:** Duration peak/off-peak (h)

| peak | off-peak |
|---|---|
| 6 | 18 |

**Table M.9:** Occupancy

| | car |
|---|---|
| peak | 1.58 |
| off-peak | 1.68 |

Source: Delhaye et al. (2010)

### M.0.2 Generalized prices

To calculate the time cost, we need the following data:

**Table M.10:** Observed speed (km/h)

| | observed speed car/bus |
|---|---|
| outside* | 60 |
| in PER* | 60 |
| in city center** | 20 |
| on ringroad** | 40 |
| between SEC and city center** | 50 |

*Source: Verkeerscentrum (2014)

**Source: author's own estimate

**Table M.11:** Average distance of trips (km)

| Average distance of trips between: | | | | |
|---|---|---|---|---|
| O-C | C-C | PER-C | C-SEC | O-SEC |
| 11 | 1 | 3 | 3.5 | 14.5 |

**Table M.12:** Stop-time bus (hours/km)

| center | outside | SEC |
|---|---|---|
| 0.02 | 0.0067 | 0.0017 |

Source: authors' calculations based on timetables of De Lijn

**Table M.13:** Value of time (euro/h)

| | work | school | other |
|---|---|---|---|
| car | 7.71 | 6.47 | 6.47 |
| bus/tram/metro | 5.48 | 4.59 | 4.59 |
| pedestrian/bike | 6.71 | 5.62 | 5.62 |

Source: Hertveldt et al. (2009)

The frequencies of bus and tram are determined assuming that the transit agency only increases frequency when all vehicles are full.

**Table M.14:** Free flow speed (km/h)

| | free flow speed car/bus |
|---|---|
| outside | 90 |
| in PER | 90 |
| in city center | 30 |
| on ringroad | 50 |
| between SEC and city center | 60 |

**Table M.15:** Free flow speed (km/h)

| | pedestrian* | bike** | train*** | tram |
|---|---|---|---|---|
| free flow speed | 3.6 | 18 | 50 | $30^{\pm}/50^{\pm\pm}$ |

*Source: Basso and Silva (2014)

**Source:van der Steenhoven and Borgman (2009)

***Source: timetables NMBS

$^{\pm}$Speed in center

$^{\pm\pm}$Speed between SEC and center

**Table M.16:** Capacity (number of passengers)

| bus* |
|---|
| 70 - 100 |

*Source: author's own estimate

**Table M.17:** Access time (h)

| train | bus |
|---|---|
| 0.15 | 0.1-0.14 |

Source: author's own estimate

**Table M.18:** Frequency train (vehicles/h)

| | frequency train |
|---|---|
| peak | 6.84 |
| off-peak | 3 |

Source: author's computation based on the timetables of NMBS

### M.0.3 Welfare

The operation, maintenance and investment costs are listed in Table M.19

**Table M.19:** Cost

| | tram | bus |
|---|---|---|
| cost per vehiclekm (euro) | 3.78 | 2.77 |
| cost per vehiclehour (euro) | 111.14 | 82.83 |
| cost per routekm (euro) | 264.88 | / |
| cost per vehicle (euro) | / | 99.1 |

Source: Van Oppens (2013)

The external congestion cost is included in the generalised cost. The other external costs are the environmental, noise and accident costs in Table 4.15

**Table M.20:** Marginal external cost (euro)

| | $bus^{*}$ | $car^{*}$ | $tram^{**}$ |
|---|---|---|---|
| marginal external cost | 0.233 | 0.022 | 0.57 |

*Source: Delhaye et al. (2010)

**Source: Delhaye et al. (2010)/De Lijn

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