Symmetry perception

Peter A. van der Helm
University of Leuven (KU Leuven), Belgium
peter.vanderhelm@ppw.kuleuven.be

To appear in
Oxford Handbook of Perceptual Organization
Oxford University Press
Edited by Johan Wagemans

Abstract
Detection of (mirror) symmetry - which is abundantly present in the world - is often assumed to be an integral part of the perceptual organization process that is applied to every visual stimulus. On the one hand, as reviewed in this article, the exact role of symmetry in perceptual organization is actually rather elusive due to its interaction with other grouping factors. On the other hand, the detectability of single and multiple symmetry is indeed extraordinary in comparison to that of repetition and Glass patterns. Empirical and theoretical findings pertaining to this detectability are discussed, focusing on converging evidence rather than on details of individual studies, and putting various findings in evolutionary perspectives. Reflecting a seeming opposition between process models and representation models, a specific question is whether symmetry detection relies on crude or precise correspondences between symmetry halves. Both stances find psychophysical support, and this article ends with a unification proposal.

Keywords
symmetry; repetition; Glass patterns; multiple symmetry; perceptual organization; psychophysical detectability laws; perception versus evolution; representation versus process

Acknowledgement
Preparation of this chapter was supported by Methusalem grant METH/08/02 awarded to Johan Wagemans (www.gestaltrevision.be).

1. Introduction

Mirror symmetry (henceforth, symmetry) is a visual regularity that can be defined by configurations in which one half is the mirror image of the other (see Figure 1a) - these halves then are said to be separated by a symmetry axis.[1] Albeit with fluctuating degrees of asymmetry, it is abundantly present in the world. For instance, the genetic blueprint of nearly every organism implies a symmetrical body - if the mirror plane is vertical, this conveniently yields gravitational stability. Furthermore, many organisms tend to organize things in their environment such that they are symmetrical - think of bird nests and human art and design (Hargittai, 1986; Shubnikov & Koptsik, 1974; Washburn & Crowe, 1988; Weyl, 1952; Wynn, 2002; van Tonder, Chapter 46; Koenderink, Chapter 47). Presumably, for organisms with symmetrical bodies, symmetrical things are practical to make and to work with (Allen, 1879). Think also of the preference which many organisms have for more symmetrical shapes over less symmetrical ones in mate selection and, by pollinators, in flower selection (Møller, 1992, 1995; Johnstone, 1994; Swaddle & Cuthill, 1993). This preference presumably favors mates and flowers with high genetic quality (Møller, 1990). Currently relevant is that it also requires a considerable perceptual sensitivity to symmetry - which many species of mammals, birds, fish, and insects indeed are known to have (Barlow & Reeves, 1979; Beck, Pinsk, & Kastner, 2005; Giurfa, Eichmann, & Menzel, 1996; Horridge, 1996).

In human perception research, detection of symmetry is in fact assumed to be an integral part of the perceptual organization process that is applied to every incoming visual stimulus (Tyler, 1996; van der Helm & Leeuwenberg, 1996; Wagemans, 1997). This assumption has been related to the idea that extraction of regularities like symmetry can be used to model the outcome of the perceptual organization process, because it would allow for efficient mental representations of patterns (for more details about this idea and its potentially underlying neuro-cognitive mechanisms, see van der Helm, Chapter 57). It has also been related to the idea that the high perceptual sensitivity to symmetry arose because the evolution of visual systems selected individual regularities on the basis of their relevance in the world (Tyler, 1996). It may, however, also have arisen because the evolution selected a general regularity-detection mechanism with sufficient survival value (cf. Enquist & Arak, 1994). The latter option suggests a package deal: to survive, a visual system's detection mechanism may pick up irrelevant regularities as long as it also picks up relevant regularities.

The foregoing indicates that perceptual organization and evolutionary

1 This definition reflects the common usage of the word symmetry. In mathematics, the word symmetry is also used to refer to any configuration that remains invariant under certain transformations; this definition is suited to classify visual regularities, but another definition is needed to model their perception (see Section 4).

relevance provide an appropriate context for an appreciation of symmetry perception. It also indicates that, to this end, it is expedient to consider symmetry in reference to other visual regularities (i.e., regularities to which the visual system is sensitive; see Figure 1). These starting points reverberate in the next evaluation of the presumed role of symmetry in perceptual organization, as well as in the subsequent review of research on symmetry perception. Notice that it would take too much space to give a detailed account of this extensive research field in which empirical evidence is based on many different experimental designs and stimuli. Evidence, however, is always evidence of something. Therefore, rather than elaborating on details of empirical studies (which readers may look up using the given references), this review focuses on the conclusions that can be drawn from them, to look for converging evidence for or against proposed ideas, theories, and models.

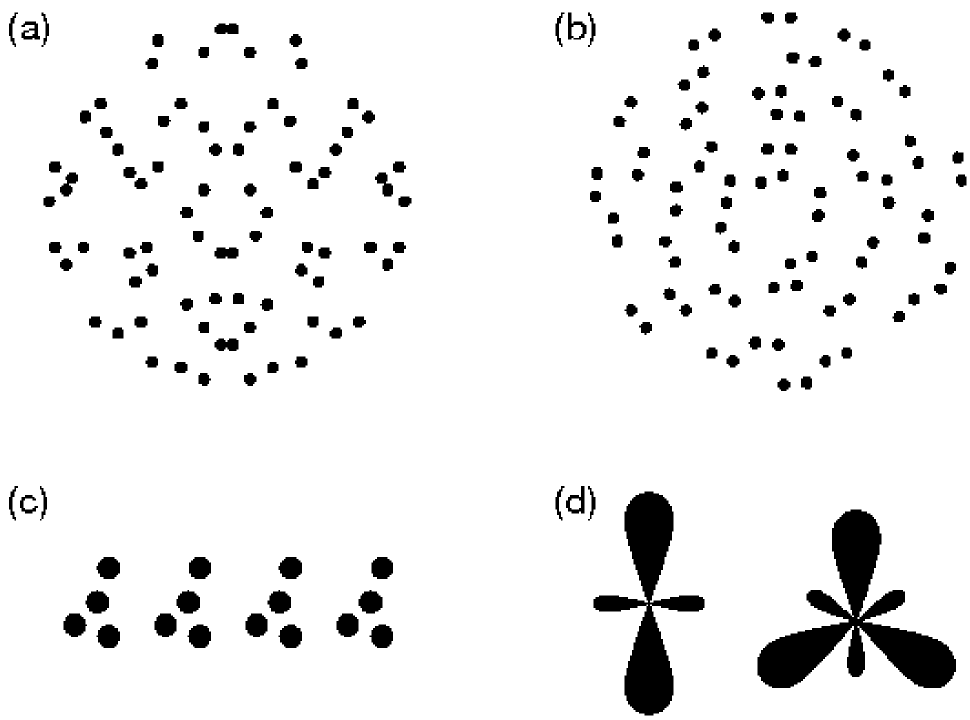

Figure 1. Visual regularity. (a) A symmetry - left and right hand halves are mirror images of each other. (b) A Glass pattern with coherently-oriented dot dipoles at random positions. (c) A repetition with four identical subpatterns (the repeats). (d) Multiple symmetries with two and three global symmetry axes, respectively.

2. The role of symmetry in perceptual organization

Mach (1886) was surely not the first to notice that symmetry is visually salient, but he is to be credited for his pioneering empirical work on the role of symmetry in visual perception. After that, for instance, the Gestalt psychologists (Koffka, 1935; Köhler, 1920; Wertheimer, 1912, 1923) identified symmetry as a factor in perceptual grouping, and Bahnsen (1928) concluded that symmetry influences figure-ground segmentation. Such seminal work triggered, in the second half of the 20-th century, an enormous increase in the number of symmetry studies.

Other reasons for that increase were not only that symmetry was recognized as being relevant in the world (see Section 1), but also that it is suited to study the mechanisms by which the visual system picks up information from stimuli. Formal process models of symmetry detection are discussed later on, but here, it is expedient to briefly address its neural basis. In this respect, notice that grouping principles seem to be effective throughout the hierarchical visual process (Palmer, Brooks, & Nelson, 2003), so that it may not be possible to assign a specific locus to symmetry detection. Indeed, various neuroscientifical studies used symmetry patterns as stimuli, but thus far, the data are too divergent to draw firm conclusions about locus and timing of symmetry detection in the brain. One thing that seems clear, however, is that the lateral occipital complex (LOC) is prominently involved (Beh & Latimer, 1997; Sasaki et al., 2005; Tyler & Baseler, 1998; Tyler et al., 2005; van der Zwan et al., 1998). The LOC in fact seems a hub where different perceptual-grouping tendencies interact, which agrees with ideas that it is a shape-selective area associated with perceptual organization in general (Grill-Spector, 2003; Malach et al., 1995; Treder & van der Helm, 2007). Hence, the neuroscientific evidence may still be scanty, but all in all, it adds to the above-mentioned idea that symmetry is relevant in perceptual organization.

In cognitive science, behavorial research into this idea yielded evidence that symmetry plays a role in issues such as object recognition (Pashler, 1990; Vetter & Poggio, 1994), figure–ground segregation (Driver, Baylis, & Rafal, 1992; Leeuwenberg & Buffart, 1984; Machilsen, Pauwels, & Wagemans, 2009), and amodal completion (Kanizsa, 1985; van Lier, van der Helm, & Leeuwenberg, 1995). It further finds elaboration in structural description approaches, that is, formal models which - using some criterion - predict preferred stimulus interpretations on the basis of view-independent specifications of the internal structure of objects. Some of these approaches work with a-priori fixed perceptual primitives like the volumetric building blocks called geons (e.g., Biederman, 1987; Binford, 1981), which is convenient for object recognition. Other approaches (e.g., Leeuwenberg, 1968, 1969, 1971; Leeuwenberg & van der Helm, 2013) allow primitives to be assessed flexibly, that is, in line with the Gestaltist idea that the whole determines what the perceived parts are. The latter is more plausible regarding object perception (Kurbat, 1994; Leeuwenberg, van der Helm, & van Lier, 1994; Palmer & Rock, 1994), but in both cases, symmetry is taken to be a crucial component of how perception imposes structure on stimuli. In Leeuwenberg's approach, for instance, symmetry is one of the regularities exploited to arrive at simplest stimulus organizations in terms of objects arranged in space (van der Helm, Chapter 57). Furthermore, in Biederman's approach, symmetry is taken to define geons because it is a so-called nonaccidental property: if present in the proximal stimulus, it is also likely to be present in the distal stimulus (Biederman, Chapter 28; Feldman, Chapter 16).

However, the proximal features of symmetry vary with viewpoint, and this drives a wedge between the perception of symmetry as such and its role in object perception (Schmidt & Schmidt, 2013; Wagemans, 1993). That is, symmetry is effective as nonaccidental property only when viewed orthofrontally - then, as discussed later on, it indeed has many extraordinary detectability properties. Yet, in structural description approaches, it is taken to be effective as grouping factor also when viewed non-orthofrontally. This touches upon the more general problem of viewpoint generalization: how does the visual system arrive at a view-independent representation of a three-dimensional (3D) scene, starting from a two-dimensional (2D) view of this scene?

Viewpoint generalization has been proposed to involve normalization, that is, a mental rotation yielding a canonical 2D view of a scene (e.g., Szlyk, Rock, & Fisher, 1995). This presupposes the generation of candidate 3D organizations which, subsequently, are normalized. However, Sawada, Li, and Pizlo (2011) not only showed that any pair of 2D curves is consistent with a 3D symmetry interpretation, but also argued that it is implausible that every such pair is perceived as being symmetrical. View-dependent coincidences, for instance, have a strong effect on how a scene is perceptually organized, and may prevent interpretations involving symmetry (van der Helm, Chapter 57). Likewise, detection of symmetry viewed in perspective or skewed (i.e., sheared plus rotated, yielding something close to perspective) seems to rely on proximal features rather than on hypothesized distal features. That is, it deteriorates as its proximal features are more perturbed (van der Vloed, Csathó, & van der Helm, 2005; Wagemans, van Gool, & d'Ydewalle, 1991).

Also when viewed orthofrontally, the grouping strength of symmetry is elusive. Symmetry is often thought to be a cue for the presence of a single object - as opposed to repetition which the Gestaltists had identified as a grouping factor too (under the umbrella of similarity), but which rather is a cue for the presence of multiple objects. However, it seems safer to say that symmetry is better detectable when it forms one object than when the symmetry halves form separate objects, and that repetition is less detectable when it forms one object than when the repeats form separate objects. At least, this is what Corballis and Roldan (1974) found for dot patterns in which grouping by proximity was responsible for the perceived objects. To tap more directly into the grouping process, Treder and van der Helm (2007) used stereopsis to assign symmetry halves and repeats to different perceived depth planes. The process of depth segregation is known to take a few hundreds of milliseconds, and they found that it interacts hardly with repetition detection but strongly with symmetry detection. This suggests that the segregation into separate objects (i.e., the depth planes) agrees with the perceptual structure of repetition but not with that of symmetry. In a similar vein, Morales and Pashler (2002) found that grouping by color interfers with symmetry detection, in a way that suggests that individual colors are attended one at a time.

The foregoing perhaps questions the grouping capability of symmetry, but above all, it shows the relevance of interactions between different grouping factors. In any case, further investigation is required to see if firmer conclusions can be drawn regarding the specific role of symmetry in the build-up of perceptual organizations. Furthermore, notice that the foregoing hardly affects considerations about the functionality of symmetry in the world - after all, this functionality takes effect once symmetry has been established. It also stands apart from the extraordinary detectability properties that are discussed next.

3. Modulating factors in symmetry detection

Whereas the foregoing sections discussed the context of research on symmetry perception, the remainder of this chapter focuses on symmetry perception as such. The essence of detecting symmetry and other visual regularities in a stimulus is that correlations between stimulus parts are to be assessed to establish if a stimulus exhibits some form of regularity. The central question therefore is: which correlations between which parts are to be assessed, and how? This question is addressed in the next sections by discussing various models and their accounts of observed phenomena. Before that, this section addresses four of the most prominent general factors that can be said to have a modulating effect on those correlations between parts, namely, absolute orientation, eccentricity, jitter, and proximity.

3.1. Absolute orientation

The absolute orientation of symmetry axes is known to be relevant (for effects of the relative orientation of symmetry axes, see Section 5.3). The effect usually found is that vertical symmetry (i.e., with a vertical axis) is more salient than horizontal symmetry which, in turn, is more salient than oblique symmetry (see, e.g., Barlow & Reeve, 1979; Baylis & Driver, 1994; Kahn & Foster, 1986; Palmer & Hemenway, 1978; Rock & Leaman, 1963). This usually found vertical-symmetry advantage has been attributed to the neural architecture of the brain (Julesz, 1971), but the evidence for that is not conclusive (Corballis, Miller, & Morgan, 1971; Herbert & Humphrey, 1996; Jenkins, 1983). Furthermore, other studies did not find this usual effect or found even an opposite effect (see, e.g., Corballis & Roldan, 1975; Fisher & Bornstein, 1982; Jenkins, 1983, 1985; Locher & Smets, 1992; Pashler, 1990; Wagemans, van Gool, & d'Ydewalle, 1992). In any case, notice that horizontal symmetry and vertical symmetry are not different regularities but are the same regularities in different absolute orientations. Hence, it might well be that effects of absolute orientation result from visuo-cognitive interactions (e.g., with the vestibular system) rather than from purely visual processes (cf. Latimer, Joung, & Stevens, 1994; Wenderoth, 1994).

3.2. Eccentricity

Detection of symmetry deteriorates as it is presented more eccentrically (Saarinen, 1988), but if scaled-up properly, it can maintain the same level of detectability (Tyler, 1999). This scaling-up compensates for the fact that eccentric receptive fields are sensitive to relatively large-scale information, as opposed to foveal receptive fields which are sensitive to relatively small-scale information. Hence, this is a general property of the visual system and not specific to symmetry which, apparently, remains equally detectable across the visual field if this factor is taken into account (see also Sally & Gurnsey, 2001).

3.3. Jitter

Jitter refers to relatively small, dynamic, displacements of stimulus elements. Then, but also in case of small static displacements, regularity detection depends on the visual system's tolerance in matching potentially corresponding elements in symmetry halves or repeats. This tolerance too is a general property of the visual system and not specific to regularity detection. In any case, Barlow and Reeves (1979) found that symmetry detection is quite resistant to jitter. Furthermore, Dry (2008) proposed Voronoi tesselation as a scale-independent mechanism yielding stimulus-dependent tolerance areas. Such a mechanism can, in any model, be adopted to account for the visual system's tolerance in matching elements.

3.4. Proximity

Proximity effects refer to the fact that stimulus elements that are closer to each other can be matched more easily (this is not to be confused with the Gestalt law of proximity, which is not about matching but about grouping). For instance, whereas detection of *n*-fold repetition (i.e., *n* juxtaposed repeats) can only start to be successful by matching elements that are one repeat apart, symmetry detection can already start to be successful by matching elements near the axis of symmetry. Jenkins (1982) in fact proposed that symmetry detection integrates information from only a limited region about the axis of symmetry: his data suggested that this integration region (IR) is a strip approximately 1 deg wide, irrespective of the size of the texture at the retina. Dakin and Herbert (1998) specified this further: their data suggested that the IR has an aspect ratio of about 2:1, and that its size scales with the spatial frequency content of the pattern. Thus, for homogeneous blob patterns for instance, the IR scales with blob size, so that it steadily covers a more or less constant number of features.

Noticing this scale invariance, however, Rainville and Kingdom (2002) proposed that the size of the IR is not determined by spatial frequency but by the spatial density of what they called "microelements": their data suggested that the IR covers about 18 such informational units regardless of their spatial separation. This agrees with studies reporting that the detectability of symmetry does not vary with the number of elements (i.e., no number effect) for symmetries with more than about 20 elements (e.g., Baylis & Driver, 1994; Dakin & Watt, 1994; Olivers, Chater, & Watson, 2004; Tapiovaara, 1990; Wenderoth, 1996a). For symmetries with less than about 20 elements, however, these studies reported opposite effects, and this hints at an explanation that takes into account that symmetry detection is an integral part of perceptual organization, as follows (see also van der Helm, 2013).

For any stimulus - including symmetry stimuli - a symmetry percept is basically just one of the possible outcomes of the perceptual organization process; it results only if it is stronger than other percepts. It is true that a symmetry percept is bound to result for a really otherwise-random symmetry stimulus, but such stimuli are rare if not impossible. A symmetry structure with many symmetry pairs is usually strong enough to overcome spurious structures, but the smaller the number of symmetry pairs is, the harder it is to construct a symmetry stimulus without spurious structures. This also implies that, in dense stimuli, such spurious structures are more prone to arise in the area near the axis. In case of small numbers of symmetry pairs, such spurious structures may have various effects on detection (see below), and in general, they may give the impression that only the area near the axis is decisive.

In sum, it is true that proximity plays a role in symmetry perception, and the area near the symmetry axis is indeed relatively important. Notice, however, that Barlow and Reeves (1979) already found that also symmetry information in the outer regions of stimuli is picked up quite effectively (see also Tyler et al., 2005; van der Helm & Treder, 2009; Wenderoth, 1995). Furthermore, even if symmetry processing would be restricted to a limited stimulus area, then this would not yet specify which stimulus information in this area is processed, and how. The latter reflects the fundamental question that formal models of symmetry detection focus on. That is, the factors discussed here can of course be taken into account in model applications, but are usually not at the heart of formal models. This is already an indication of their scope, which is next discussed further.

4. The scope of formal models of symmetry detection

Existing formal models of symmetry detection can be divided roughly into representation models and process models (these are also discussed separately in the next two sections). Whereas process models rather focus on performance (how does the detection process proceed?), representation models rather focus on competence (what is the result?). In other words, whereas process models rather focus on detection mechanisms, representation models rather focus on detectability, or salience, in terms of the strength of symmetry percepts. Of course, eventually, this difference in scope should be overcome to obtain a unified account, and a possible unification direction is discussed at the end of this chapter.

Furthermore, as a rule, formal models of symmetry detection start from ideas about the perceptual structure of symmetry, that is, about the parts that are to be correlated somehow to assess if symmetry is present in a stimulus. Models may differ fundamentally regarding these ideas (see below), but these ideas usually imply that the models are applicable only to single and nested symmetries, possibly perturbed by noise. For instance, if an experimental task involves the detection of a local symmetry among juxtaposed local symmetries, then humans perform about the same as when this context were noise (either case is also called crowding, and in either case, symmetry is known to not pop-out; Nucci & Wagemans, 2007; Olivers et al., 2004; Olivers & van der Helm, 1998; Roddy & Gurnsey, 2011). Indeed, to a particular local symmetry, juxtaposed local symmetries actually constitute noise, and this is usually also how such situations are treated by formal models of symmetry perception.

Moreover, many models are tailored specifically to symmetry (e.g., Chipman, 1977; Dakin & Watt, 1994; Dry, 2008; Masame, 1986, 1987; Yodogawa, 1982; Zimmer, 1984). Ideally, however, a model should be equally applicable to other visual regularities (i.e., repetition and Glass patterns; see Figure 1bc). To this end, one might invoke considerations about visual regularity in general. In the 20-th century, this led first to the transformational approach, and later, to the holographic approach. Both approaches propose a formal criterion for what visual regularity is, and they conclude to more or less the same visual regularities. However, they rely on fundamentally different mathematical formalizations of regularity, and as a result, they assign different structures to those visual regularities. The mathematical details are beyond the scope of this chapter, but the following gives a gist.

According to the transformational approach, visual regularities are configurations that remain invariant under certain transformations

(Palmer, 1983). This idea of invariance under motion relies on the same formalization as used in the classification of crystals and regular wall patterns (Shubnikov & Koptsik, 1974; Weyl, 1952). It holds that symmetry and repetition are visual regularities because they remain invariant under a 180° 3D rotation about the symmetry axis and a 2D translation the size of one or more repeats, respectively. Because these transformations identify entire symmetry halves or entire repeats with each other, they can be said to assign a block structure to both regularities (see Figure 2a).

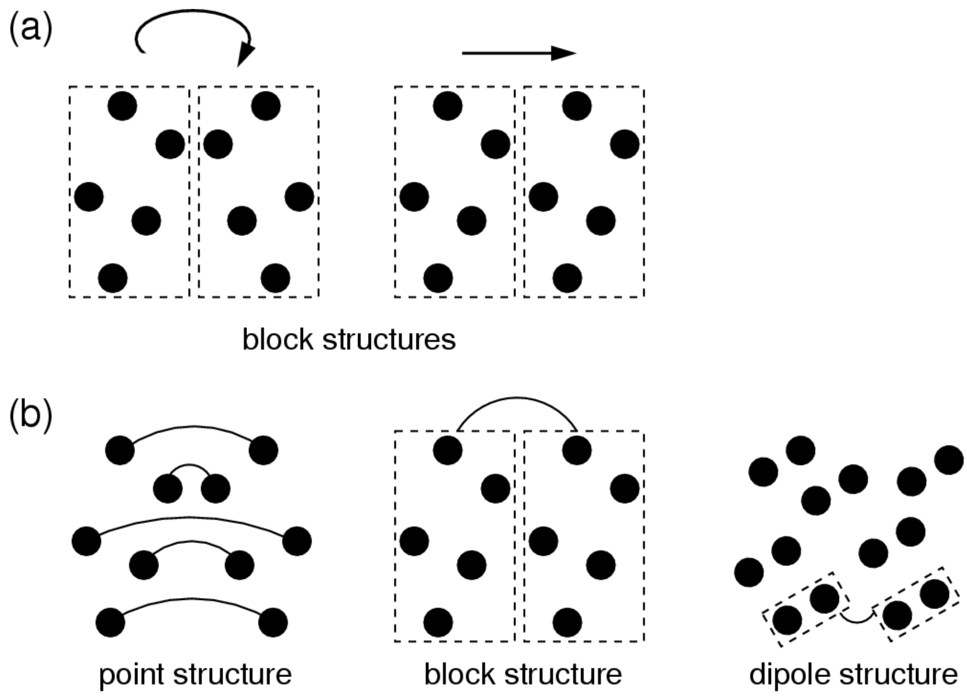

Figure 2. (a) The transformational approach relies on invariance under motion; it assigns a block structure to both symmetry (at the left) and repetition (in the middle), because entire symmetry halves and entire repeats are the units that are identified with each other by the shown transformations. (b) The holographic approach relies on invariance under growth; it assigns a point structure to symmetry, a block structure to repetition, and a dipole structure to - here, translational - Glass patterns (at the right), because symmetry pairs, repeats, and dipoles, respectively, are the units by which these configurations can be expanded preserving the regularity in them.

However, its applicability is unclear for Glass patterns (which are as detectable as symmetry; see below). Originally, Glass (1969) constructed the patterns named after him by superimposing two copies of a random dot pattern - one slightly translated or rotated with respect to the other, for instance. With the transformational approach in mind, this construction method suggests that the resulting percept too is that of a whole consisting of two overlapping identical substructures (i.e., those two copies). This also seems to comply with a grouping over muliple views as needed in case of binocular disparity and optic flow (Wagemans, van Gool, Swinnen, & van Horebeek, 1993). However, the actually resulting percept rather seems to require a framing in terms of relationships between randomly positioned but coherently oriented dot dipoles (see Section 5.2). Furthermore, in original rotational Glass patterns, the dipole length increases with the distance from the center of the pattern, but later, others consistently constructed rotational Glass patterns by placing identical dot dipoles in coherent orientations at random positions (as in Figure 1b). The two types of Glass patterns do not seem to differ in salience but, by the transformational construction above, the latter type would be a perturbed regularity. Because transformational invariance requires perfect regularity, however, the transformational approach has a problem with perturbed regularity. A formal solution might be to cross-correlate corresponding parts, but in symmetry for instance, a simple cross-correlation of the two symmetry halves does not seem to agree with human performance (Barlow & Reeves, 1979; Tapiovaara, 1990).

This unclarity regarding Glass patterns adds to the fact the transformational approach does not account for the key phenomenon - discussed later on in more detail - that symmetries and Glass patterns are about equally detectable but generally better detectable than 2-fold repetitions (notice that they all consist transformationally of the same number of corresponding parts; cf. Bruce & Morgan, 1975). Hence, the transformational approach may account for how visual regularities can be classified, but not for how they are perceived preceding classification.

This drawback does not hold for the holographic approach (van der Helm & Leeuwenberg, 1996, 1999, 2004). This approach is also based on a rigorous mathematical formalization of regularity in general (van der Helm & Leeuwenberg, 1991), but the difference is that it relies on invariance under growth (which agrees with how mental representations can be built up). To give a gist, according to this approach, symmetries, repetitions, and Glass patterns are visual regularities because, preserving the regularity in them, they can be expanded stepwise by adding symmetry pairs, repeats, and dot dipoles, respectively. This implies that these regularities can be said to be assigned a point structure, a block structure, and a dipole structure, respectively (see Figure 2b). Thereby, this mathematical formalization supports a structural differentiation that, as discussed next, seems to underlie detectability differences between visual regularities (see also Attneave, 1954; Bruce & Morgan, 1975).

5. Representation models of symmetry detection

As indicated, representation models of symmetry perception focus on detectability, or salience, in terms of the strength of symmetry percepts. As a rule, such models capitalize on the concept of weight of evidence (MacKay, 1969) - that is, they provide a measure of the weight of evidence for the presence of symmetry in a stimulus. This typically implies that the somehow quantified amount of symmetry information in a stimulus is normalized by the somehow quantified total amount of information in the stimulus. Thereby, such a measure is a metric of the strength of the symmetry percept, and can be applied to both perfect and perturbed symmetry. This also holds for the holographic model which is based on considerations about visual regularity in general but which, for symmetry, is usually not outperformed by models tailored specifically to symmetry. Therefore, here, this holographic model is taken as a robust representative. It is specified in terms of multi-element stimuli (like the dot stimuli in Figure 2), but notice that such stimuli allow for straightforward generalizations to other stimulus types.

Next, the predictive power of this holographic model is evaluated for perfect symmetry (in comparison to repetition and Glass patterns), perturbed symmetry (also in comparison to repetition and Glass patterns, and focusing on cases of noise added to a perfect regularity), and multiple or n-fold symmetry (i.e., patterns with n global symmetry axes) - all viewed orthofrontally (some examples are given in Figure 1). To this end, various detectability phenomena are considered, some of which are put in an evolutionary perspective.

5.1. Perfect symmetry

In the holographic model, the support for the presence of a regularity is quantified by the number of nonredundant relationships (E) between stimulus parts that, according to this model, constitute a regularity. Thus, for symmetry E equals the number of symmetry pairs; for repetition E equals the number of repeats minus one; and for Glass patterns E equals the number of dot dipoles minus one. Furthermore, the total amount of information in a stimulus is given by the total number of elements in the stimulus (n), so that the holographic weight-of-evidence metric (W) for the detectability of a regularity is: $W = E/n$.

A perfect symmetry on n elements is constituted by $E=n/2$ symmetry pairs, so that it gets $W=0.5$ no matter the total number of elements - hence, symmetry is predicted to show no number effect, which agrees with empirical reports (e.g., Baylis & Driver, 1994; Dakin & Watt, 1994; Olivers et al., 2004; Tapiovaara, 1990; Wenderoth, 1996a; see also Section 3.4). Furthermore, $E=n/2-1$ for a Glass pattern on n elements, so that, for large n, it is predicted to show more or less the same detectability as symmetry - empirical support for this is discussed in the next subsection. For an m-fold repetition on n elements, however, $E=m-1$, so that its detectability is predicted to depend strongly on the number of elements per repeat - hence, a number effect, which found empirical support (Csathó, van der Vloed, & van der Helm, 2003). In particular, 2-fold repetition is predicted to be generally less detectable than symmetry - which also found empirical support (Baylis & Driver, 1994, 1995; Bruce & Morgan, 1975; Csathó et al., 2003; Corballis & Roldan, 1974; Zimmer, 1984).

Hence, the foregoing shows that holographic weight of evidence accounts for the key phenomenon that symmetry and Glass patterns are about equally detectable but generally better detectable than repetition. This differentiation holds not only for perfect regularities, but as discussed next, also for perturbed regularities.

5.2. Perturbed symmetry

A perfect regularity can be perturbed in many ways, and there are of course limits to the detectability of the remaining regularity. Relevant in this respect is that the percept of an imperfect regularity results from the perceptual organization process applied to the stimulus. This means that the percept generally cannot be said to be some original perfect regularity plus some perturbation. For instance, if a perfect repetition is perturbed by randomly added noise elements (which is the form of perturbation considered here), then there may be some remaining repetitiveness depending on the location of the noise. In general, however, repetition seems quite easily destroyed perceptually - some evidence for this can be found in Rappaport (1957) and in van der Helm and Leeuwenberg (2004).

Symmetry and Glass patterns, however, are quite resistant to noise, and this is fairly independent of the location of the noise (e.g., Barlow & Reeves, 1979; Maloney, Mitchison, & Barlow, 1987; Masame, 1986, 1987; Nucci & Wagemans, 2007; Olivers & van der Helm, 1998; Troscianko, 1987; Wenderoth, 1995). In fact, both symmetry and Glass patterns exhibit graceful degradation, that is, their detectability decreases gradually with increasing noise proportion (i.e., the proportion of noise elements relative to the total number of stimulus elements). Their behavior is explicated next in more detail.

By fitting empirical data, Maloney et al. (1987) found that the detectability (d') of Glass patterns in the presence of noise follows the psychophysical law

$$d' = g/(2+N/R)$$

with R the number of dot dipoles that constitute the regularity; N the number of added noise elements; and g an empirically determined proportionality constant that depends on stimulus type and that enables more detailed data fits than rank orders. Maloney et al. (1987) concluded to this on the basis of considerations from signal detection theory, and the holographic model predicts the same law on the basis of structural considerations. In the holographic model, $W=E/n$ is proposed to be proportional to the detectability of regularity, and for Glass patterns in the presence of noise, it implies n=2R+N and E=R-1 or, for large R, approximately E=R. Substitution in W=E/n then yields the psychophysical law above.

The holographic model also predicts this psychophysical law for symmetry (with R equal to the number of symmetry pairs), and it indeed yields a near perfect fit on Barlow and Reeves' (1979) symmetry data (van der Helm, 2010). In the middle range of noise proportions, this fit is as good as that for the Weber-Fechner law (Fechner, 1860; Weber, 1834) if, in the latter, the regularity-to-noise ratio R/N is taken as signal (cf. Zanker, 1995). In both outer ranges, it is even better because, unlike the Weber-Fechner law, it accounts for floor and ceiling effects. This means that, in both outer ranges of noise proportions, the sensitivity to variations in R/N is disproportionally lower than in the middle range, so that disproportionally larger changes in R/N are needed to achieve the same change in the strength of the percept (which is also supported by Tjan & Liu, 2005, who used morphing to perturb symmetries).

Interestingly, this account of perturbed symmetry also predicts both symmetry and asymmetry effects, that is, apparent overestimations and underestimations of the symmetry in a stimulus when compared triadically to slightly more and slightly less symmetrical stimuli (Freyd & Tversky, 1984). These effects are context dependent, and the psychophysical law above suggests that they are due not to incorrect estimations of symmetry but to correct estimations of symmetry-to-noise ratios. For more details on this, see Csathó, van der Vloed, and van der Helm (2004), but notice that these effects are evolutionary relevant for both prey and predators. As discussed in van der Helm and Leeuwenberg (1996), overestimation by oneself may occur in the case of partly occluded opponents, for instance, and is helpful to detect them. Furthermore, underestimation by opponents may occur if oneself is camouflaged, for instance, and is helpful to avoid being detected. The occurrence of such opposite effects is consistent with the earlier-mentioned idea of a package deal in the evolutionary selection of a general regularity-detection mechanism. This idea is supported further by the above-established fact that symmetry and Glass patterns exhibit the same detectability properties, even though symmetry clearly has more evolutionary relevance. A further hint at such a package deal is discussed at the end of the next subsection.

5.3. Multiple symmetry

Regularities can also occur in nested combinations, and in general, additional local regularities in a global regularity enhance the detectability of this global regularity (e.g., Nucci & Wagemans, 2007). To account for this, the holographic model invokes Leeuwenberg's (1968) structural description approach, which specifies constraints for hierarchical combinations of global and local regularities in descriptive codes (which are much like computer programs that produce things by specifying the internal structure of those things). As a rule, this implies that a compatible local regularity is one that occurs within a symmetry half of a global symmetry or within a repeat of a global repetition. The general idea then is that the just-mentioned enhancement occurs only in case of such combinations. More specifically, however, it implies that local regularity in symmetry halves adds only once to the detectability of the symmetry, and that local regularity in the repeats of an *m*-fold repetition adds *m* times to the detectability of the repetition (van der Helm and Leeuwenberg, 1996). In other words, repetition is predicted to benefit more from compatible local regularities than symmetry does - as supported by Corballis and Roldan (1974).

A special case of nested regularities is given by multiple symmetry (see Figure 1d). According to the transformational approach, the detectability of multiple symmetry is predicted to increase monotonically as a function of the number of symmetry axes - which seems to agree with empirical data (e.g., Palmer & Hemenway, 1978; Wagemans et al., 1991). Notice, however, that these studies considered 1-fold, 2-fold, and 4-fold symmetries, but not 3-fold symmetries which seem to be odd ones out: they tend to be less detectable than 2-fold symmetries (Wenderoth & Welsh, 1998).

According to the holographic approach, hierarchical-compatibility constraints indeed imply that 3-fold symmetries - and, likewise, 5-fold symmetries - are not as detectable as might be expected on the basis of the number of symmetry axes alone. For instance, in a 2-fold symmetry, each global symmetry half is itself a 1-fold symmetry which, in a descriptive code, can be described as being nested in that global symmetry half. In 3-fold symmetry, however, each global symmetry half exhibits two overlapping 1-fold symmetries, and because they overlap, only one of them can be described as being nested in that global symmetry half. In other words, those hierarchical-compatibility constraints imply that all symmetry can be captured in 2-fold symmetries but not in 3-fold symmetries - and, likewise, in 4-fold symmetries but not in 5-fold symmetries. This suggest not only that 3-fold and 5-fold symmetries can be said to contain perceptually hidden regularity - which may increase their aesthetic appeal (cf. Boselie & Leeuwenberg, 1985) - but also that they are less detectable than 2-fold and 4-fold symmetries, respectively.

A study by Treder, van der Vloed, and van der Helm (2011) into imperfect 2-fold symmetries composed of two superimposed perfect 1-fold symmetries (which allows for variation in their relative orientation) showed that the relative orientation of symmetry axes can indeed have this effect. That is, though equal in all other respects and controling for absolute orientation, orthogonal symmetries (as in 2-fold symmetry) were found to be better detectable than nonorthogonal ones (as in 3-fold symmetry). This suggests that the constituent single symmetries in a multiple symmetry first are detected separately and then engage in an orientation-dependent interaction. Notice that this would be a fine example of the Gestalt motto that the whole is something else than the sum of its parts.

Evolutionary interesting, 3-fold and 5-fold symmetries are overrepresented in flowers (Heywood, 1993). Furthermore, in human designs, they are virtually absent in decorative motifs (Hardonk, 1999) but not in mystical motifs (think of triquetas and pentagrams; Forstner, 1961; Labat, 1988). This might well be due to a subconsciously attributed special status to them - caused by their special perceptual status. In flowers, this may have given them a procreation advantage (Giurfa, Dafni, & Neal, 1999). In this respect, notice that insect vision evolved 200-275 million years earlier than flowering plants (Sun, Dilcher, Wang, & Chen, 2011), so that such an perceptual effect may have influenced the distributon of flowers from the start. Furthermore, throughout human history, the special perceptual status of 3-fold and 5-fold symmetries may have made humans feel that they are more appropriate for mystical motifs than for decorative motifs (van der Helm, 2011). Such considerations are of course more speculative than those based on psychophysical data, but they do suggest a plausible two-way interaction between vision and the world: the world determines if a visual system as a whole has sufficient evolutionary survival value, but subsequently, visual systems also influence how the world is shaped (see also van der Helm, Chapter 57).

6. Process models of symmetry detection

To account for the process of symmetry detection, various spatial filtering models have been proposed (e.g., Dakin & Hess, 1997; Dakin & Watt, 1994; Gurnsey, Herbert, & Kenemy, 1998; Kovesi, 1997, 1999; Osorio, 1996; Poirier & Wilson, 2010; Rainville & Kingdom, 2000; Scognamillo, Rhodes, Morrone, & Burr, 2003). Whereas representation models usually rely on fairly precise correlations between stimulus elements to establish symmetry, spatial filtering models usually rely on fairly crude correlations. For a review, see Treder (2010), but to give an example, Dakin and Watt (1994) proposed a two-stage model: first, an image is spatially filtered yielding a number of blobs, and then a blob alignment procedure is applied to measure how well the centroids of the blobs align along a putative symmetry axis. In the brain, something like spatial filtering occurs in the lateral geniculate nucleus, that is, before symmetry perception takes place. It is more than just a modulating factor, however. In Dakin and Watt's (1994) model, for instance, the chosen spatial filtering scale in fact determines the elements that are correlated to establish symmetry in a stimulus.

The latter can be exemplified further by considering anti-symmetry, that is, symmetry in which otherwise perfectly corresponding elements have opposite properties in some dimension. For instance, in stimuli consisting of monochromatic surfaces, angles may be convex in one contour but concave in the corresponding contour (this can also be used to define anti-repetition in such stimuli; Csathó et al., 2003). Such corresponding contours have opposite contrast signs, and detection seems possible only post-perceptually (van der Helm & Treder, 2009). This also holds, in otherwise symmetrical checkerboard stimuli, for corresponding squares with opposite contrasts (Mancini, Sally, & Gurnsey, 2005). In both cases, contrast interacts with other grouping factors (grouping by color in particular). It can, however, also be considered in isolation, namely, in dot patterns in which symmetrically positioned dots can have opposite contrast polarities with respect to the background (this can also be used to define anti-repetition and anti-Glass patterns in such stimuli). This does not seem to have much effect on symmetry detection (Saarinen & Levi, 2000; Tyler & Hardage, 1996; Wenderoth, 1996b; Zhang & Gerbino, 1992). Representational models cannot account for that, because they rely on precise correspondences. In contrast, there are spatial filters (and maybe neural analogs) that filter out positional information only, thereby canceling the difference between symmetry and antisymmetry in such stimuli (Mancini et al., 2005).

In Glass patterns, spatial filtering may also be responsible for identifying the constituent dot dipoles which, after all, may blur into coherently-oriented blobs at courser scales. A potential problem here, however, is that this might not work for Glass patterns in the presence of noise given by randomly added single dots. For instance, in Maloney et al.'s (1987) experiment, each dipole dot had 6-10 noise dots closer by than its mate. Further research is needed to assess how spatial filtering might agree with the psychophysical law discussed in Section 5.2, which is based on precise correspondences and holds for Glass patterns and symmetry.

The foregoing indicates a tension between process models that rely on fairly crude spatial filtering and representation models that rely on fairly precise correlations between stimulus elements. Neither type of model alone seems able to account for all aspects of symmetry detection. Yet, unification might be possible starting from Dakin and Watt's (1994) conclusion that their human data match the performance of a fairly fine-scale filter. This empirical finding suggests that symmetry does not benefit from the presence of relatively large blobs. As elaborated in the remainder if this section, such an effect is in fact predicted by a process model that allows for effects of spatial filtering even though it relies on fairly precise structural relationships between elements (van der Helm & Leeuwenberg, 1999). This model fits in the holographic approach discussed above, but it also builds on processing ideas by Jenkins (1983, 1985) and Wagemans et al. (1993). In this respect, it is a nice example of a stepwise development of ideas - each previous step as important as the next one.

6.1. Bootstrapping

Jenkins (1983, 1985) subjected symmetry and repetition to various experimental manipulations (e.g., jitter), to investigate what the properties are that characterize these regularities perceptually. He concluded that symmetry and repetition are characterized by properties of what he called virtual lines between corresponding elements. That is, for orthofrontally viewed perfect regularities, symmetry is characterized by parallel orientation and midpoint colinearity of virtual lines between corresponding elements in symmetry halves. Likewise, repetition is characterized by parallel orientation and constant length of virtual lines between corresponding elements in repeats. Thus, both symmetry and repetition can be said to have a point structure, that is, a structure in which each element constitutes one substructure. Notice that this idea suggests a detection mechanism which connects virtual lines to assess regularity in a stimulus (see Figure 3ab, top panels).

Virtual lines between corresponding points are indeed plausible anchors for a detection mechanism, but this idea seems to be missing something. This was made clear by Wagemans et al. (1991) who found that the detectability of symmetry in skewed symmetry is hampered, even though skewing preserves the parallel orientation and midpoint colinearity of virtual lines. Wagemans et al. (1993) therefore proposed that the actual detection anchors of symmetry and repetition (and, likewise, of Glass pattern) are given by virtual trapezoids and virtual parallelograms, respectively (see Figure 3ab, top and middle panels). Notice that skewing is an appropriate manipulation to assess this for symmetry (because it perturbes the virtual trapezoids), but not for repetition (because a skewed perfect repetition is still a perfect repetition). Nevertheless, van der Vloed et al.'s (2005) study on symmetry and repetition in perspective supports the idea that such correlation quadrangles are indeed the detection anchors for both regularities. The detection process can then be modeled as exploiting these anchors in a bootstrap procedure which starts from correlation quadrangles to search for additional correlation quadrangles in order to build a representation of a complete regularity (Wagemans et al., 1993; see Figure 3ab, middle and bottom panels).

This bootstrap idea is indeed plausible, but it still seems to be missing something else. That is, just as Jenkins' idea, it is not sustained by a mathematical formalism (cf. Bruce & Morgan, 1975), and just as the transformational approach, both ideas do not yet explain detectability differences between symmetry and repetition. To the latter end, one might resort to modulating factors - in particular, to proximity. As discussed in Section 3, such factors do play a role, but as discussed next, those detectability differences can also be explained without resorting to such factors.

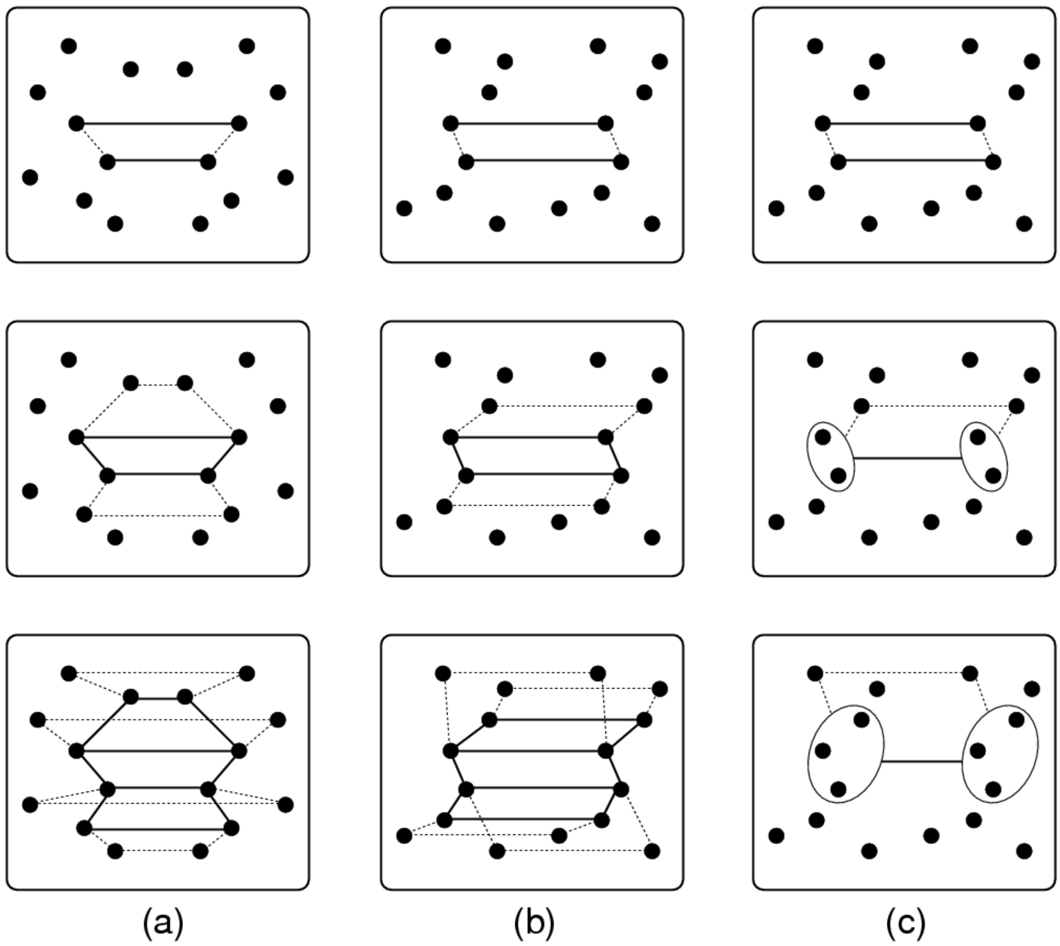

Figure 3. (a) Symmetry is characterized by parallel orientation and midpoint colinearity of virtual lines (indicated in bold in top panel) between corresponding elements in symmetry halves; two such virtual lines can be combined to form a virtual trapezoid (middle panel), from which detection can propagate in an exponential fashion (bottom panel). (b) In the original bootstrap model, the same applies to repetition, which is characterized by parallel orientation and constant length of virtual lines between corresponding elements in repeats. (c) In the holographic bootstrap model, repetition involves an intermediate stepwise grouping of elements into blocks, which implies that detection propagates in a linear fashion.

6.2. Holographic bootstrapping

In a reaction to Wagemans (1999) and consistent with the holographic approach, van der Helm and Leeuwenberg (1999) proposed that symmetry is indeed detected as proposed by Wagemans et al. (1993) but that repetition detection involves an additional step. That is, according to the holographic approach, symmetry pairs are indeed the constituents of symmetry, but repeats - rather than single element pairs - are the constituents of repetition. This suggest that repetition detection involves an intermediate step, namely, the grouping of elements into blocks that, eventually, correspond to complete repeats (see Figure 3c).

This holographic procedure implies that symmetry detection propagates exponentially, but that repetition detection propagates linearly. For Glass patterns, in which it takes the dot dipoles as constituents, it also implies that detection propagates exponentially. Thus, it again accounts for the key phenomenon that symmetry and Glass patterns are about equally detectable but better detectable than repetition. In addition, it predicts the following.

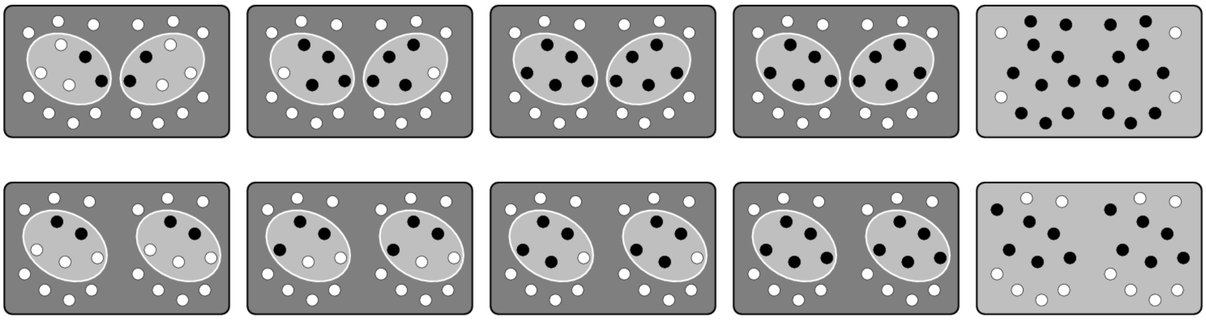

Figure 4. Holographic bootstrapping in case of split stimuli, for symmetry (top) and repetition (bottom). Going from left to right, suppose that, at a first stage, only the grey areas in the stimuli are available to the regularity detection process. Then, at first, the propagation proceeds as usual (the structure detected so far is indicated by black dots). The restriction to the grey areas, however, stops the exponentially spreading propagation in symmetry sooner than the linearly spreading propagation in repetition -- hence symmetry is hindered more by the split situation than repetition is. When, later, the rest of the stimulus becomes available, the propagation again proceeds as usual and symmetry restores its advantage over repetition.

Suppose that, for some odd reason, a restricted part of a stimulus is processed before the rest of the stimulus is processed. Then, exponentially propagating symmetry detection is hampered, whereas linearly propagating repetition detection is not or hardly hampered (see Figure 4). By way of analogy, one may think of a slow car for which it matters hardly whether or not there is much traffic on the road, versus a fast car for which it matters a lot. Such a split-stimulus situation seems to occur if the restricted part contains relative large and therefore salient blobs. Such blobs can plausibly be assumed to be processed first, namely, due to the spatial filtering difference, in the lateral geniculate nucleus, between the magnocellular pathway (which mediates relatively course structures relatively fast) and the parvocellular pathway (which mediates relatively fine structures relatively slow). Hence, then, the holographic bootstrap model predicts that symmetry detection is hampered by such blobs. Furthermore, due to the number effect in repetition (see Section 5.1), repetition detection is actually predicted to benefit from such blobs. Both predictions were confirmed empirically by Csathó et al. (2003). They are also relevant to the evolutionary biology discussion on whether symmetry or size - of sexual ornaments and other morphological traits - is the more relevant factor in mate selection (e.g., Breuker & Brakefield, 2002; Goddard & Lawes, 2000; Morris, 1998). That is, a global symmetry may be salient as such but its salience is reduced by salient local traits.

7. Conclusion

Visual symmetry will probably remain an inexhaustible topic in many research domains. It is instrumental in ordering processes that counter natural tendencies towards chaos. Thereby, it is probably also the most important regularity in the interaction between vision and the world. In vision, there is still unclarity about its exact role in perceptual organization (which depends on interactions between various grouping factors), but its detectability is extraordinary. The perceptual sensitivity to symmetry seems part of an evolutionary package deal, that is, evolution seems to have yielded a detection mechanism that includes a lower sensitivity to repetition (which is also less relevant evolutionary) but an equally high sensitivity to Glass patterns (even though these are even less relevant evolutionary). Therefore, rather than focusing on the relevance of individual regularities in the external world, it seems expedient to focus on internal perceptual mechanisms to explain these sensitivities in a unified fashion. As discussed on the basis of empirical evidence, these mechanisms seem to rely not only on fairly precise correlations between stimulus elements, but also on spatial filtering to establish what the to-be-correlated elements might be.

References

Allen, G. (1879). The origin of the sense of symmetry. *Mind, 4,* 301-316.

Attneave, F. (1954). Some informational aspects of visual perception. *Psychological Review, 61,* 183-193.

Bahnsen, P. (1928). Eine untersuchung über symmetrie und asymmetrie bei visuellen wahrnehmungen. *Zeitschrift für Psychologie, 108,* 355-361.

Barlow, H. B., & Reeves, B. C. (1979). The versatility and absolute efficiency of detecting mirror symmetry in random dot displays. *Vision Research, 19,* 783-793.

Baylis, G. C., & Driver, J. (1994). Parallel computation of symmetry but not repetition within single visual shapes. *Visual Cognition, 1,* 377-400.

Baylis, G. C., & Driver, J. (1995). Obligatory edge assignment in vision: The role of figure and part segmentation in symmetry detection. *Journal of Experimental Psychology: Human Perception and Performance, 21,* 1323-1342.

Beck, D. M., Pinsk, M. A., & Kastner, S. (2005). Symmetry perception in humans and macaques. *Trends in Cognitive Sciences, 9,* 405-406.

Beh, H. C., & Latimer, C. R. (1997). Symmetry detection and orientation perception: Electrocortical responses to stimuli with real and implicit axes of orientation. *Australian Journal of Psychology, 49,* 128-133.

Biederman, I. (1987). Recognition-by-components: A theory of human image understanding. *Psychological Review, 94,* 115-147.

Binford, T. (1981). Inferring surfaces from images. *Artificial Intelligence, 17,* 205-244.

Boselie, F., & Leeuwenberg, E. L. J. (1985). Birkhoff revisited: Beauty as a function of effect and means. *American Journal of Psychology, 98,* 1-39.

Breuker, C. J., & Brakefield, P. M. (2002). Female choice depends on size but not symmetry of dorsal eyespots in the butterfly *Bicyclus anynana*. *Proceedings of the Royal Society of London B, 269,* 1233-1239.

Bruce, V. G., & Morgan, M. J. (1975). Violations of symmetry and repetition in visual patterns. *Perception, 4,* 239-249.

Chipman, S. F. (1977). Complexity and structure in visual patterns. *Journal of Experimental Psychology: General, 106,* 269-301.

Corballis, M. C., Miller, G. A., & Morgan, M. J. (1971). The role of left-right orientation in interhemispheric matching of visual information. *Perception & Psychophysics, 10,* 385-388.

Corballis, M. C., & Roldan, C. E. (1974). On the perception of symmetrical and repeated patterns. *Perception & Psychophysics, 16,* 136-142.

Corballis, M. C., & Roldan, C. E. (1975). Detection of symmetry as a function of angular orientation. *Journal of Experimental Psychology: Human Perception and Performance, 1,* 221-230.

Csathó, Á., van der Vloed, G., & van der Helm, P. A. (2003). Blobs strengthen repetition but weaken symmetry. *Vision Research, 43,* 993-1007.

Csathó, Á., van der Vloed, G., & van der Helm, P. A. (2004). The force of symmetry revisited: Symmetry-to-noise ratios regulate (a)symmetry effects. *Acta Psychologica, 117,* 233-250.

Dakin, S. C., & Herbert, A. M. (1998). The spatial region of integration for visual symmetry detection. *Proceedings of the Royal Society London B,* 265, 659-664.

Dakin, S. C., & Hess, R. F. (1997). The spatial mechanisms mediating symmetry perception. *Vision Research, 37,* 2915-2930.

Dakin, S. C., & Watt, R. J. (1994). Detection of bilateral symmetry using spatial filters. *Spatial Vision, 8,* 393-413.

Driver, J., Baylis, G. C., & Rafal, R. D. (1992). Preserved figure-ground segregation and symmetry perception in visual neglect. *Nature, 360,* 73-75.

Dry, M. (2008). Using relational structure to detect symmetry: A Voronoi tessellation based model of symmetry perception. *Acta Psychologica, 128,* 75-90.

Enquist, M., & Arak, A. (1994). Symmetry, beauty and evolution. *Nature, 372,* 169-172.

Fechner, G. T. (1860). *Elemente der Psychophysik.* Leipzig: Breitkopf und Härtel.

Fisher, C. B., & Bornstein, M. H. (1982). Identification of symmetry: Effects of stimulus orientation and head position. *Perception & Psychophysics, 32,* 443-448.

Forstner, D. (1961). *Die Welt der Symbole* [The world of symbols]. Innsbruck: Tyriola Verlag.

Freyd, J., & Tversky, B. (1984). Force of symmetry in form perception. *American Journal of Psychology, 97,* 109-126.

Giurfa, M., Dafni, A., & Neal, P. R. (1999). Floral symmetry and its role in plant-pollinator systems. *International Journal of Plant Sciences, 160,* S41-S50.

Giurfa, M., Eichmann, B., & Menzel, R. (1996). Symmetry perception in an insect. *Nature, 382,* 458-461.

Glass, L. (1969). Moiré effect from random dots. *Nature, 223,* 578-580.

Goddard, K. W., & Lawes, M. J. (2000). Ornament size and symmetry: Is the tail a reliable signal of male quality in the Red-collared Widowbird? *The Auk, 117,* 366-372.

Grill-Spector, K. (2003). The neural basis of object perception. *Current Opinion in Neurobiology, 13,* 159-166.

Gurnsey, R., Herbert, A. M., & Kenemy, J. (1998). Bilateral symmetry embedded in noise is detected accurately only at fixation. *Vision Research, 38,* 3795-3803.

Hardonk, M. (1999). *Cross-cultural universals of aesthetic appreciation in decorative band patterns.* Ph.D. thesis, Radboud University Nijmegen, The Netherlands.

Hargittai, I. (Ed.) (1986). *Symmetry: unifying human understanding.* New

York: Pergamon.
Herbert, A. M., & Humphrey, G. K. (1996). Bilateral symmetry detection: testing a "callosal" hypothesis. *Perception, 25,* 463-480.
Heywood, V. H. (Ed.) (1993). *Flowering plants of the world.* London: Batsford.
Horridge, G. A. (1996). The honeybee (*Apis mellifera*) detects bilateral symmetry and discriminates its axis. *Journal of Insect Physiology, 42,* 755-764.
Jenkins, B. (1982). Redundancy in the perception of bilateral symmetry in dot textures. *Perception & Psychophysics, 32,* 171-177.
Jenkins, B. (1983). Component processes in the perception of bilaterally symmetric dot textures. *Perception & Psychophysics, 34,* 433-440.
Jenkins, B. (1985). Orientational anisotropy in the human visual system. *Perception & Psychophysics, 37,* 125-134.
Johnstone, R. A. (1994). Female preferences for symmetrical males as a by-product of selection for mate recognition. *Nature, 372,* 172-175.
Julesz, B. (1971). *Foundations of cyclopean perception.* Chicago: University of Chicago Press.
Kahn, J. I., & Foster, D. H. (1986). Horizontal-vertical structure in the visual comparison of rigidly transformed patterns. *Journal of Experimental Psychology: Human Perception and Performance, 12,* 422-433.
Kanizsa, G. (1985). Seeing and thinking. *Acta Psychologica, 59,* 23-33.
Koffka, K. (1935). *Principles of Gestalt psychology.* London: Routledge & Kegan Paul.
Köhler, W. (1920). *Die physischen Gestalten in Ruhe und im stationären Zustand* [Static and stationary physical shapes]. Braunschweig, Germany: Vieweg.
Kovesi, P. (1997). Symmetry and asymmetry from local phase. In *Proceedings AI'97, Tenth Australian Joint Conference on Artificial Intelligence* (pp. 185-190).
Kovesi, P. (1999). Image features from phase congruency. *Videre: A Journal of Computer Vision Research, 1.*
Kurbat, M. A. (1994). Structural description theories: Is RBC/JIM a general-purpose theory of human entry-level object recognition? *Perception, 23,* 1339-1368.
Labat, R. (1988). Manuel d'épigraphie akkadienne: signes, syllabaire, idéogrammes (6-th ed.). Paris: Imprimerie Nationale.
Latimer, C. R., Joung, W., & Stevens, C. (1994). Modelling symmetry detection with back-propagation networks. *Spatial Vision, 8,* 415-431.
Leeuwenberg, E. L. J. (1968). *Structural information of visual patterns: an efficient coding system in perception.* The Hague-Paris: Mouton & Co.
Leeuwenberg, E. L. J. (1969). Quantitative specification of information in sequential patterns. *Psychological Review, 76,* 216-220.
Leeuwenberg, E. L. J. (1971). A perceptual coding language for visual and auditory patterns. *American Journal of Psychology, 84,* 307-349.

Leeuwenberg, E. L. J., & Buffart, H. F. J. M. (1984). The perception of foreground and background as derived from structural information theory. *Acta Psychologica, 55,* 249-272.

Leeuwenberg, E. L. J., & van der Helm, P. A. (2013). *Structural information theory: The simplicity of visual form.* Cambridge, UK: Cambridge University Press.

Leeuwenberg, E. L. J., van der Helm, P. A., & van Lier, R. J. (1994). From geons to structure: A note on object classification. *Perception, 23,* 505-515.

Locher, P., & Smets, G. (1992). The influence of stimulus dimensionality and viewing orientation on detection of symmetry in dot patterns. *Bulletin of the Psychonomic Society, 30,* 43-46.

Mach, E. (1886). *Beiträge zur Analyse der Empfindungen* [Contributions to the analysis of sensations]. Jena, Germany: Gustav Fisher.

Machilsen, B., Pauwels, M., & Wagemans, J. (2009). The role of vertical mirror symmetry in visual shape detection. *Journal of Vision, 9,* 1-11.

MacKay, D. (1969). *Information, mechanism and meaning.* Boston: MIT Press.

Malach, R., Reppas, J. B., Benson, R. R., Kwong, K. K., Jiang, H., Kennedy, W. A., Ledden, P. J., Brady, T. J., Rosen, B. R., & Tootell, R. B. H. (1995). Object-related activity revealed by functional magnetic resonance imaging in human occipital cortex. *Proceedings of the National Academy of Sciences USA, 92,* 8135-8139.

Maloney, R. K., Mitchison, G. J., & Barlow, H. B. (1987). Limit to the detection of Glass patterns in the presence of noise. *Journal of the Optical Society of America A, 4,* 2336-2341.

Mancini, S., Sally, S. L., & Gurnsey, R. (2005). Detection of symmetry and anti-symmetry. *Vision Research, 45,* 2145-2160.

Masame, K. (1986). Rating of symmetry as continuum. *Tohoku Psychologica Folia, 45,* 17-27.

Masame, K. (1987). Judgment of degree of symmetry in block patterns. *Tohoku Psychologica Folia, 46,* 43-50.

Møller, A. P. (1990). Fluctuating asymmetry in male sexual ornaments may reliably reveal male quality. *Animal Behaviour, 40,* 1185-1187.

Møller, A. P. (1992). Female swallow preference for symmetrical male sexual ornaments. *Nature, 357,* 238-240.

Møller, A. P. (1995). Bumblebee preference for symmetrical flowers. *Proceedings of the National Academy of Science USA, 92,* 2288-2292.

Morales, D., & Pashler, H. (1999). No role for colour in symmetry perception. *Nature, 399,* 115-116.

Morris, M. R. (1998). Female preference for trait symmetry in addition to trait size in swordtail fish. *Proceedings of the Royal Society of London B, 265,* 907-911.

Nucci, M., & Wagemans, J. (2007). Goodness of regularity in dot patterns: global symmetry, local symmetry, and their interactions. *Perception, 36,* 1305-1319.

Olivers, C. N. L., Chater, N., & Watson, D. G. (2004). Holography does

not account for goodness: A critique of van der Helm and Leeuwenberg (1996). *Psychological Review, 111,* 261-273.
Olivers, C. N. L., & van der Helm, P. A. (1998). Symmetry and selective attention: A dissociation between effortless perception and serial search. *Perception & Psychophysics, 60,* 1101-1116.
Osorio, D. (1996). Symmetry detection by categorization of spatial phase, a model. *Proceedings of the Royal Society of London B, 263,* 105-110.
Palmer, S. E. (1983). The psychology of perceptual organization: A transformational approach. In J. Beck, B. Hope, & A. Rosenfeld (Eds.), *Human and machine vision* (pp. 269-339). New York: Academic Press.
Palmer, S. E., Brooks, J. L., & Nelson, R. (2003). When does grouping happen? *Acta Psychologica, 114,* 311-330.
Palmer, S. E., & Hemenway, K. (1978). Orientation and symmetry: Effects of multiple, rotational, and near symmetries. *Journal of Experimental Psychology: Human Perception and Performance, 4,* 691-702.
Palmer, S. E., & Rock, I. (1994). Rethinking perceptual organization: The role of uniform connectedness. *Psychonomic Bulletin & Review, 1,* 29-55.
Pashler, H. (1990). Coordinate frame for symmetry detection and object recognition. *Journal of Experimental Psychology: Human Perception and Performance, 16,* 150-163.
Poirier, F. J. A. M. & Wilson, H. R. (2010). A biologically plausible model of human shape symmetry perception. *Journal of Vision, 10,* 1-16.
Rainville, S. J. M., & Kingdom, F. A. A. (2000). The functional role of oriented spatial filters in the perception of mirror symmetry-psychophysics and modeling. *Vision Research, 40,* 2621-2644.
Rainville, S. J. M., & Kingdom, F. A. A. (2002). Scale invariance is driven by stimulus density. *Vision Research, 42,* 351-367.
Rappaport, M. (1957). The role of redundancy in the discrimination of visual forms. *Journal of Experimental Psychology, 53,* 3-10.
Rock, I., & Leaman, R. (1963). An experimental analysis of visual symmetry. *Acta Psychologica, 21,* 171-183.
Roddy, G., & Gurnsey, R. (2011). Mirror symmetry is subject to crowding. *Symmetry, 3,* 457-471.
Saarinen, J. (1988). Detection of mirror symmetry in random dot patterns at different eccentricities. *Vision Research, 28,* 755-759.
Saarinen, J., & Levi, D. M. (2000). Perception or mirror symmetry reveals long-range interactions between orientation-selective cortical filters. *Neuroreport, 11,* 2133-2138.
Sally, S., & Gurnsey, R. (2001). Symmetry detection across the visual field. *Spatial Vision, 14,* 217-234.
Sasaki, Y., Vanduffel, W., Knutsen, T., Tyler, C., & Tootell, R. B. H. (2005). Symmetry activates extrastriate visual cortex in human and nonhuman primates. *Proceedings of the National Academy of Sciences USA, 102,* 3159-3163.
Sawada, T., Li, Y., & Pizlo, Z. (2011). Any pair of 2D curves is consistent

with a 3D symmetric interpretation. *Symmetry, 3,* 365-388.
Schmidt, F., & Schmidt, T. (2013). Rapid processing of closure and viewpoint-invariant symmetry: behavioral criteria for feedforward processing. *Psychological Research.* DOI 10.1007/s00426-013-0478-8
Scognamillo, R., Rhodes, G., Morrone, C., & Burr, D. (2003). A feature-based model of symmetry detection. *Proceedings of the Royal Society B: Biological Sciences, 270,* 1727-1733.
Shubnikov, A. V., & Koptsik, V. A. (1974). *Symmetry in science and art.* New York: Plenum.
Sun, G., Dilcher, D. L., Wang, H., & Chen, Z. (2011). A eudicot from the Early Cretaceous of China. *Nature, 471,* 625-628.
Swaddle, J., & Cuthill, I. C. (1993). Preference for symmetric males by female zebra finches. *Nature, 367,* 165-166.
Szlyk, J. P., Rock, I., & Fisher, C. B. (1995). Level of processing in the perception of symmetrical forms viewed from different angles. *Spatial Vision, 9,* 139-150.
Tapiovaara, M. (1990). Ideal observer and absolute efficiency of detecting mirror symmetry in random images. *Journal of the Optical Society of America A, 7,* 2245-2253.
Tjan, B. S., & Liu, Z. (2005). Symmetry impedes symmetry discrimination. *Journal of Vision, 5,* 888-900.
Treder, M. S. (2010). Behind the looking-glass: a review on human symmetry perception. *Symmetry, 2,* 1510-1543.
Treder, M. S., & van der Helm, P. A. (2007). Symmetry versus repetition in cyclopean vision: A microgenetic analysis. *Vision Research, 47,* 2956-2967.
Treder, M. S., van der Vloed, G., & van der Helm, P. A. (2011). Interactions between constituent single symmetries in multiple symmetry. *Attention, Perception & Psychophysics, 73,* 1487-1502.
Troscianko, T. (1987). Perception of random-dot symmetry and apparent movement at and near isoluminance. *Vision Research, 27,* 547-554.
Tyler, C. W. (1996). Human symmetry perception. In C. W. Tyler (Ed.), *Human symmetry perception and its computational analysis* (pp. 3-22). Zeist, The Netherlands: VSP.
Tyler, C. W. (1999). Human symmetry detection exhibits reverse eccentricity scaling. *Visual Neuroscience, 16,* 919-922.
Tyler, C. W., & Baseler, H. A. (1998). fMRI signals from a cortical region specific for multiple pattern symmetries. *Investigative Ophthalmology & Visual Science, 39 (Suppl.),* 169.
Tyler, C. W., Baseler, H. A., Kontsevich, L. L., Likova, L. T., Wade, A. R., & Wandell, B. A. (2005). Predominantly extra-retinotopic cortical response to pattern symmetry. *NeuroImage, 24,* 306-314.
Tyler, C. W., & Hardage, L. (1996). Mirror symmetry detection: Predominance of second-order pattern processing throughout the visual field. In C. W. Tyler (Ed.), *Human symmetry perception and its computational analysis* (pp. 157-172). Zeist, The Netherlands: VSP.
van der Helm, P. A. (2010). Weber-Fechner behaviour in symmetry

perception? *Attention, Perception & Psychophysics, 72,* 1854-1864.

van der Helm, P. A. (2011). The influence of perception on the distribution of multiple symmetries in nature and art. *Symmetry, 3,* 54-71.

van der Helm, P. A. (2013). *Simplicity in vision: A multidisciplinary account of perceptual organization.* Cambridge, UK: Cambridge University Press.

van der Helm, P. A., & Leeuwenberg, E. L. J. (1991). Accessibility, a criterion for regularity and hierarchy in visual pattern codes. *Journal of Mathematical Psychology, 35,* 151-213.

van der Helm, P. A., & Leeuwenberg, E. L. J. (1996). Goodness of visual regularities: A nontransformational approach. *Psychological Review, 103,* 429-456.

van der Helm, P. A., & Leeuwenberg, E. L. J. (1999). A better approach to goodness: Reply to Wagemans (1999). *Psychological Review, 106,* 622-630.

van der Helm, P. A., & Leeuwenberg, E. L. J. (2004). Holographic goodness is not that bad: Reply to Olivers, Chater, and Watson (2004). *Psychological Review, 111,* 261-273.

van der Helm, P. A., & Treder, M. S. (2009). Detection of (anti)symmetry and (anti)repetition: Perceptual mechanisms versus cognitive strategies. *Vision Research, 49,* 2754-2763.

van der Vloed, G., Csathó, Á., & van der Helm, P. A. (2005). Symmetry and repetition in perspective. *Acta Psychologica, 120,* 74-92.

van der Zwan, R., Leo, E., Joung, W., Latimer, C. R., & Wenderoth, P. (1998). Evidence that both area V1 and extrastriate visual cortex contribute to symmetry perception. *Current Biology, 8,* 889-892.

van Lier, R. J., van der Helm, P. A., & Leeuwenberg, E. L. J. (1995). Competing global and local completions in visual occlusion. *Journal of Experimental Psychology: Human Perception and Performance, 21,* 571-583.

Vetter, T., & Poggio, T. (1994). Symmetric 3D objects are an easy case for 2D object recognition. *Spatial Vision, 8,* 443-453.

Wagemans, J. (1993). Skewed symmetry: A nonaccidental property used to perceive visual forms. *Journal of Experimental Psychology: Human Perception and Performance, 19,* 364-380.

Wagemans, J. (1997). Characteristics and models of human symmetry detection. *Trends in Cognitive Science, 1,* 346-352.

Wagemans, J. (1999). Toward a better approach to goodness: Comments on van der Helm and Leeuwenberg (1996). *Psychological Review, 106,* 610-621.

Wagemans, J., van Gool, L., & d'Ydewalle, G. (1991). Detection of symmetry in tachistoscopically presented dot patterns: Effects of multiple axes and skewing. *Perception & Psychophysics, 50,* 413-427.

Wagemans, J., van Gool, L., & d'Ydewalle, G. (1992). Orientational effects and component processes in symmetry detection. *The Quarterly Journal of Experimental Psychology, 44A,* 475-508.

Wagemans, J., van Gool, L., Swinnen, V., & van Horebeek, J. (1993). Higher-order structure in regularity detection. *Vision Research, 33,* 1067-1088.

Washburn, D. K., & Crowe, D. W. (1988). *Symmetries of culture: Theory and practice of plane pattern analysis.* University of Washington Press.

Weber, E. H. (1834). *De tactu* [Concerning touch]. New York: Academic Press.

Wenderoth, P. (1994). The salience of vertical symmetry. *Perception, 23,* 221-236.

Wenderoth, P. (1995). The role of pattern outline in bilateral symmetry detection with briefly flashed dot patterns. *Spatial Vision, 9,* 57-77.

Wenderoth, P. (1996a). The effects of dot pattern parameters and constraints on the relative salience of vertical bilateral symmetry. *Vision Research, 36,* 2311-2320.

Wenderoth, P. (1996b). The effects of the contrast polarity of dot-pair partners on the detection of bilateral symmetry. *Perception, 25,* 757-771.

Wenderoth, P., & Welsh, S. (1998). Effects of pattern orientation and number of symmetry axes on the detection of mirror symmetry in dot and solid patterns. *Perception, 27,* 965-976.

Wertheimer, M. (1912). Experimentelle Studien über das Sehen von Bewegung. *Zeitschrift für Psychologie, 12,* 161-265.

Wertheimer, M. (1923). Untersuchungen zur Lehre von der Gestalt [On Gestalt theory]. *Psychologische Forschung, 4,* 301-350.

Weyl, H. (1952). *Symmetry.* Princeton, NJ: Princeton University Press.

Wynn, T. (2002). Archaeology and cognitive evolution. *Behavioral and Brain Sciences, 25,* 389-402, 432-438.

Yodogawa, E. (1982). Symmetropy, an entropy-like measure of visual symmetry. *Perception & Psychophysics, 32,* 230-240.

Zanker, J. M. (1995). Does motion perception follow Weber's law? *Perception, 24,* 363-372.

Zhang, L., & Gerbino, W. (1992). Symmetry in opposite-contrast dot patterns. *Perception, 21* (Supp. 2), 95a.

Zimmer, A. C. (1984). Foundations for the measurement of phenomenal symmetry. *Gestalt Theory, 6,* 118-157.