François Beets, Michel Dupuis et Michel Weber (éditeurs)

La science et le monde moderne d'Alfred North Whitehead

Alfred North Whitehead's Science and the Modern World

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Introduction
A. N. Whitehead Natural Philosopher

Jacques Riche (Leuven)

Maxime Bôcher once said to the writer,
"What man would be a philosopher
who might be a mathematician?"
One feels that Mr. Whitehead deserves both titles.[1]

0. Introduction

The title of Whitehead's book, *Science and the Modern World*,[2] could apply to any period of publication. Here, at the end of the first quarter of the 20th century, Whitehead offers an overview and an analysis of scientific materialism that he sees permeating the sciences of his time. As answer and way out of materialism, he proposes a holistic philosophy of organism. Being the first main publication of Whitehead after settling in Harvard, SMW can also be seen as a retrospective look on a scientific period that had just finished and, before starting a new life as a philosopher, a consideration of what had been achieved in his career of applied mathematician, logician, natural philosopher and teacher, a career that would have come to its end had he stayed in Britain.

Wondering whether the orientations of scientific thought that are criticized in SMW have changed and considering Whitehead's prophetic allusion to a sort of ecological and environmental attitude on which it concludes, one could think that the book has kept all its actuality. Nevertheless, in most discussions and debates over the interpretation of current physical theories and the philosophical questions they raise, any reference to Whitehead and his thought are almost consistently absent. This is also the case in the new philosophy of mind. Considering the various domains of the sciences that Whitehead investigated and the world view he tried to found, this absence from the contemporary debates about the world, nature and the mind that inhabits and interprets it, gives the impression that, except for the community of Whiteheadians,[3] his thought has been generally disqualified.

Consider a simple but somewhat sophisticated example. The physicist J. Bell has often insisted on the fact that some models of quantum mechanics allow to give a density rather than a probabilistic interpretation to the wave function. That density is not a density of mass or charge but the density of the real stuff of which the world is made.[4] It follows that, in some models, the usual notion of observable which is problematic in probabilistic contexts and in statements depending on measurement could be replaced by that of '*be-able*', that is, what exists really. This suggestion of Bell is no less unusual than some of Whitehead's metaphysically justified suggestions in his philosophy of nature about what exists really in the world.

Would it be possible to understand or to interpret and present Whitehead's philosophy of nature so that it would no longer be considered as exotic, marginal, typically foreign or restricted to small circles of devotees, but that it would find its proper place on the philosophical scene? The question can receive a positive answer if Whitehead's philosophy is situated and presented in its own historical context first; if it is shown that it does not conflict with the results and interpretations of the sciences of his and our time and that it contributes to their philosophical understanding. This can be performed through a reassessment of his early career and writings as a natural philosopher.

Seeing Whitehead as a natural philosopher is an ambitious program that would require to look at his works of the London period from the perspective of a working scientist, following him from his early mathematical papers, his *Treatise on Universal Algebra*, originally conceived as a synthesis of H. Grassmann's and W. R. Hamilton's mathematical ideas, the *Mathematical Concepts of the Material World* where he proposes various logico-mathematical ways of capturing and interpreting the physical world, and, finally, the trilogy, *Enquiry Concerning the Principles of Natural Knowledge* and *The concept of Nature*, mainly concerned with a dualist view of the world as object of science and the world as object of perception and sensations and the *Principle of Relativity* where Whitehead completes and spells out his own interpretation of Relativity Theory. A comprehensive and unified view of the evolution of Whitehead's ideas with respect to mathematics and its application to the physical world, his own Natural Philosophy that would culminate in *Process and Reality*, would then be obtained.

The working hypothesis, here and elsewhere, is that in the works of his first period, that is, up to SMW, the development of Whitehead's ideas was mainly influenced by Grassmann in logic and mathematics and by Maxwell in natural philosophy.[5] Before being distracted by B. Russell and the elaboration of the *Principia Mathematica* for more than a decade, Whitehead had an agenda, a grand plan of devising an abstract and theoretical framework for natural philosophy that was first influenced by his thorough study of Grassmann's theory of extension and by the more or less explicit philosophy of his masters in Natural Philosophy. The *Treatise on Universal Algebra* constitutes its incomplete premises and the *Mathematical Concepts* gives an example of what his plan could have been. The fourth Volume of the *Principia* should have contained a fuller account of Whitehead's conceptions had it been published. But it is mainly from the *Enquiry* that his system of Natural Philosophy takes its definite orientation.

In this paper, we will consider the very early work of Whitehead in Natural Philosophy and, particularly, its relation with Maxwell's physical theories and philosophy. Indeed, Maxwell was also a philosopher. In the course of investigating Maxwell's works, a question occurred: Would it be possible that what Maxwell had done with Faraday's intuitions about electricity, giving these intuitions a precise mathematical formulation, Whitehead did the same, giving 19th century Natural Science and Maxwell's philosophical ideas a systematic and elaborated philosophical formulation. In other words, could it be a philosophical system of Natural Philosophy that Maxwell and others had in some way anticipated that Whitehead has developed?

Following a short overview of SMW, we will consider Whitehead's curriculum at Trinity, what his dissertation could have been, and his first papers in applied mathematics before considering some aspects of Maxwell's ideas and theories. This will provide a general view on several major issues discussed in Natural Philosophy of that time, the context in which Whitehead started to develop his own answers.

1. A Farewell to 19th Century?

The classical Greek conception of an order of nature in which Western science originated held until the time of Hume when the question arose of justifying a science based on this rational order of nature. In a large overview of the history of Western science, Whitehead shows where this faith in an order in nature comes from. The Scholastics who "*look*(ed) *for an exact point*" and believed that every occurrence had to be correlated to its antecedent opened the possibility of modern science.[6] The reaction against the rigid medieval rationality opened the door to the Reformation in religion, to induction against syllogistic and to efficient causes against final causes in science. Logic and mathematical methods did not prevent this anti-rationalism from anchoring within the sciences whose philosophical assumptions were still embedded in scientific materialism. Newtonian mechanics, for example, only sees the world as matter spread in space and submitted to the laws of motion.

According to Whitehead, the key to induction in science and everyday life is not in inductive logic but in the understanding of the immediate experience of nature.[7] This experience discloses its inclusion in a larger system of relationships and, from this apprehension and systematic character of our experience, our faith in an order of nature arises. A fundamental assumption of scientific materialism is that matter has the property of simple location, i.e., it is situated in space at a definite place and time, independently of other regions and times. Another usual assumption is that common sense gives us the most natural ideas of substance, qualities, location, and it is in these terms that we come to the ordering of the common sense world.[8] Under these assumptions, nature is represented through simplified editions of matter of fact that are elaborate constructions. It is the fallacy of misplaced concreteness, the misplaced concreteness that 17th century ascribed to the scientific schemes.[9]

Incoherence in European thought appeared in the 18th century when scientific realism based on mechanism met an unwavering belief in the world of human beings and higher animals as being made of self-determining organisms. The discrepancies between the materialist mechanism of science and the moral intuitions presupposed in concrete life appeared slowly but it is mainly in literature, for example, with Wordsworth, that the concrete outlook of humanity received its expression.[10] "*The nature-poetry of the romantic revival was a protest on behalf of the organic view of nature against the exclusion of value from the essence of matter of fact*".

From 19th century, mathematical abstraction allowed science to unveil a world of continuity, atomicity, conservation of energy and evolution.[11] In particular, the theory of evolution started the undermining of Materialism. The object of science was now "*the evolution of the complex organisms from antecedent less complex organisms. The doctrine thus cries aloud for a conception of organism as fundamental for nature*."[12] A philosophy of organism, that is, of systems having historicity, is thus required to replace materialism. The mind, no longer separated from the bodily event, is back into nature. The cognitive experience is now an 'ego-object', consciousness here and now, within the world of realities.[13] In a word, Whitehead's position in SMW consists in the analysis of the background ideas that the sciences constituted and that permeated the philosophical thought over time.[14]

2. Whitehead and 19th Century Natural Philosophy

In the *Concept of Nature*, Whitehead reacts against what he calls the bifurcation of nature, the splitting of reality into two systems. One system is that of the natural sciences in which reality is never known because what is known is the second system, the other sort of reality, the byplay of the mind. In other words, this bifurcation of nature is nature apprehended in awareness and nature as the cause of awareness.[15] Four issues related to the bifurcation theory are discussed by Whitehead: causality, time, space and delusion. There is an intermediate form of that theory in which the nature that is discussed is the nature that is directly known, a position that "*lands us to 18th and 19th century materialism, namely, the belief that what is real in nature is matter, in time and in space, and with inertia.*"[16]

According to Whitehead, among "*the necessary prolegomena for philosophy and for natural science is a thorough understanding of the type of entities, and types of relations among entities which are disclosed to us in our perceptions of nature*." This means that "*The primary task of philosophy of natural science is to elucidate the concept of nature* […] *to exhibit the fundamental entities and the fundamental relations between entities in terms of which all laws of nature have to be stated and to secure that the entities and relations are adequate*."[17]

However, the data of science, matter, space, time and their relations do not cover all the data of perception. What would then be an adequate concept? And, in enunciating such concepts, wouldn't it be doing metaphysics? No, because Whitehead's perspective is not that of Schelling who relies on an intellectual intuition of nature, nor that of the empiricists who construct nature, nor that of the *Natur-philosophen* who see nature as independent and self-constructing.[18]

Distinguishing clearly his position from these last metaphysical conceptions with which it is easily confused, Whitehead considers himself more as a mathematician than as a 19th century Natural Philosopher. It is true that the influence of Kant and that of *Natur-Philosophie* was more or less explicitly acknowledged by scientists like W. R. Hamilton, Wewhell, Herschel, Faraday, Davy and others, and, although fading, it was still alive in Maxwell's early philosophical ideas. A reason is that the ideas of vitalism, energetism and dynamism found in that philosophy were apparently supported by the scientific discoveries.

Following D. Emmet, Whitehead's philosophy can be articulated around the two notions of *form* and *facts*. Two essential aspects in his philosophical and mathematical thoughts can be distinguished: First, his interest for abstraction and form, "*the formal schemes of logical relations*", second, his interest in the multiple and concrete aspects of existence, "*the form in facts"*.[19] A characterization of Whitehead's Natural Philosophy amounts thus to setting up and solving some equation involving the world experienced, mathematics, physics and metaphysics.

British Victorian man, genuine product of the end of 19th century Cambridge School of Natural Philosophy in which he received his education and training, Whitehead was involved in (or, at least, was a privileged witness of) the developments of Natural Philosophy at the end of the century. He not only witnessed the scientific developments, he also captured and interpreted the main streams of ideas that had filled up most of the century, starting with H. Grassmann and W. R. Hamilton's neo-kantism, the influence of the German Natur-Philosophie, the debates on the new geometries, Maxwell's physics and philosophy, and the views of Darwin on evolution, to mention the main ones.

As he recognized himself, with no other formal training in Philosophy than what he had learned as a student through the meetings of the 'Select Essay Club' known as the 'Apostles' (because the number of elected members was limited to 12), he nevertheless gathered all the elements that would serve as a basis on which to set up a metaphysical position and on which he would build up his natural philosophy. His early philosophy is first that of an applied mathematician and logician directed towards the natural sciences. And it is the legacy of the great scientists of 19th century that Whitehead brings into the 20th century.

Ideas, intuitions, mathematical tools, sets of equations at the basis of their scientific and philosophical theories hold today as they held in their days. No so-called scientific revolution made them wrong. They are what they were, evolving but provisional theories. Seen in a historical and genetic perspective, mathematics and physics do not show any such drastic changes as Kuhn's 'scientific revolutions'. Even if it is true that the meaning of some concepts had to be adapted, talking of a 'change of paradigm' would require some qualification. We will consider the evolution of science as a continuous process and side with Poincaré to admit that science is what is left after scientific theories are gone, replaced by other theories that better fit the facts, observational and experimental. Relativity theories were just the expected solutions of specific problems. They relied on new developments in mathematics like group theory, invariant theory, vector and tensor calculus that had opened new perspectives to answer open questions. Rival theories coexist and Whitehead's natural philosophy has still to be fully assessed with respect to the leading paradigms of his and of our time. But this is not the place to do it here. We will just consider that he was in tune with the normal evolution of ideas and theories to which he contributed more than ordinarily thought, that had started long before and were still far from having solved many basic problems and answered fundamental questions, in particular, the essential question for Whitehead, of an account of the world as it is actually perceived.

3. Whitehead's Background in Applied Mathematics

George Gabriel Stokes, Peter Guthrie Tait, William Thomson, later known as Lord Kelvin, and James Clerk Maxwell, the leaders of the Cambridge school of Natural Philosophy, are probably the natural philosophers who most influenced Whitehead.

W. Thomson and P. G. Tait begin their famous *Treatise of Natural Philosophy* with a quotation of Fourier: "*Les causes primordiales ne nous sont point connues; mais elles sont assujetties à des lois simples et constantes, que l'on peut découvrir par l'observation, et dont l'étude est l'objet de la philosophie naturelle*." They continue: "*The term Natural Philosophy was used by Newton and is still used in British Universities, to denote the investigation of laws in the material world, and deduction of results not directly observed*."[20]

In the introduction of his *Treatise on Electricity and Magnetism*,[21] Maxwell writes that electricity is not only another branch of physics but "*an aid to the interpretation of nature* […] *promoting the progress of science*". And, according to W. Thomson, "*There is no branch of physical science which affords a surer foundation, or more definite objects for the application of mathematical reasoning, than the theory of electricity*."[22]

At the time of Whitehead's studies in Cambridge, the much needed *Treatise* of Tait and Thomson who started writing it in 1861 and published the first part of Volume I in 1867, was certainly the book of reference on the subject. Originally intended to contain at least four volumes, it was written in a language adapted to a non-mathematician reader, with two sizes of typesets, the larger one for non-mathematicians. The mathematician could also benefit from reading these parts and be "*thus forced to think out for himself what he has been too often accustomed to reach by a mere mechanical application of analysis*." Indeed, "*Nothing can be more fatal to progress than a too confident reliance on mathematical symbols; for the student is only too apt to take the easier course, and consider the formula and not the fact as the physical reality*."[23]

A reviewer wrote that "*The world of which they give the Natural Philosophy is not the abstract world of Cambridge examination papers*

[…] *but it is the concrete world of the senses, which approximates to, but always falls short alike of the ideal of the mathematical as of the poetic imagination*."[24] And von Helmholtz, in the introduction of his German translation, underlined that actuality and consistency to physical facts was there preferred to elegance of mathematical method, adding that "*Perhaps, when science is perfected, physical and mathematical order may coincide*."[25]

Collaborating slowly and mainly through mail, Tait being busy with W. R. Hamilton's Quaternions[26] and Thomson occupied with his own research and industrial applications of physics, the volume on the '*Properties of Matter*', the most expected one, was never published. In spite of their importance and indetermination, topics like '*What is matter?*', the '*subjectivity of force*' and other important themes that had been planned for discussion never saw publication.[27] A shorter version, "*The Elements of Natural Philosophy*" appeared in 1873 and, with the publication of the second part of Volume I of the "*Treatise*" in 1883, the project came to an end. Tait will eventually publish alone a treatise on *The Properties of Matter* in 1885.

4. The role of mathematics

W. Thomson never favored superfluous use of mathematical formalism. For example, all his life he objected to the use of Quaternions that was advocated by Tait. He did not want the physical aspects of a question to be shadowed by the mathematical equations. His mathematics was known as 'instinctive' and was only used to support physical insight. He had the habit of telling his students that "*Mathematics is the only true metaphysics*"; they were a tool to be mastered and kept simple, "*merely the etherealization of common sense*". Always striving to stay close to the physical interpretation of his equations, he developed new mathematical methods getting him new results and suggesting new solutions to old problems, new models and interpretations. His work, with that of Maxwell, who largely shared his views[28], did really set up the standard: "*the concrete interpretation of the details of abstract formulae, of which type of interpretation this was one of the first examples, has become the ideal of mathematical physics*."[29] Analytical results and numerical computation do not suffice; every step in the process must be associated with some intuition and the whole argument must be capable of being conducted in concrete physical terms.

Whitehead the mathematician sees "*modern pure Mathematics* [as] *the most original creation of the human spirit* […] *the pursuit of mathematics is a divine madness of the human spirit, a refuge from the goading urgency* *of contingent happenings*."[30] But, as Russell said, Whitehead was a mathematician of applied mathematics. His goal in IM was to show "*what the science* [of mathematics] *is about, and why it is necessarily the foundation of exact thought as applied to natural phenomena.*"[31] Mathematics will thus pervade all his philosophical work. For example, the mathematical objects, algebraic and geometric, that he started to study in his original and ambitious *Treatise of Universal Algebra* will end up as the eternal objects of SMW.

These eternal objects are individual or relational abstract objects or entities. They can reappear on various occasions in our various different experiences and they can be grasped in thought without any actual exemplification in fact.[32] Symbols of mathematics have their referent in the real objects of the world, in actuality, but they also refer to mathematical eternal objects having their own relationships with other such objects in their own domain, giving to entities or objects of the world the occasion to show the sort of relationship they entertain. This is the ingression of eternal objects in particular actual occasions.[33]

The process of abstraction and application is at the origin of natural laws: "*The progress of science* […] (in) *observing the interconnections and in showing* […] *that the events* […] *are but examples of a few general connections or relations called laws*." Science amounts to see what is general in particulars, permanent in transitory.[34] That means that it is only through experimentation, abstraction, and verification that a law can be decided to be true. With respect to reality, there will always be some approximation, that which is good enough, as long as everybody agrees. Application of mathematics to the external world is thus only performed through the original process and its verification through mathematics. If mathematics seems predictive, it is because we have just discovered regularities, patterns and structures.[35]

5. Whitehead's dissertation and J. Clerk Maxwell

In SMW, Whitehead tells us that from 1880 and for the next 20 years not much happened. How should we interpret this statement which is opposed to the actual state of affairs? Did Whitehead consider Maxwell's *Treatise* published in 1873, second edition in 1881, as the culmination of 19th century physics? Many important developments occurred after Maxwell's death, though. Among them, one may mention the verification and completion of his Electromagnetic Theory of Light following Heaviside's and Hertz's work; the unification of Thomson's Vortex Ether Theory with Maxwell's theory; the works of von Helmholtz, Lorentz and Fitzgerald that would lead to a solution to the problem of the ether etc. In retrospect, it is also true that, at the beginning of the new century, Planck's model of the atom and Einstein's Special Theory of Relativity, had eventually cleaned up and put some order in the field of physics, disposing of many by then irrelevant anterior speculations.

James Clerk Maxwell (1831–1879) is mainly known for his Electromagnetic Theory of Light. He was a mathematician and an experimentalist. He was also a philosopher whose influence on his contemporaries is more important than usually thought. His researches that started with optics and the perception of colours, followed by the elastic solids, the rings of Saturn, Geometry and Mechanics, Faraday's theories of Electricity, Molecular Physics and Thermodynamics, make of him one of the main contributors to almost every fields of physics in the second half of the 19th century and the greatest scientist ever.

On several occasions, Whitehead makes us think or he admits that he was influenced by the work of Clerck Maxwell.[36] According to B. Russell[37], Whitehead chose to write his dissertation of 1884 on Maxwell's *Theory of Electricity and Magnetism*. Following the undergraduate examination in January, he prepared the Fellowship examination that he passed in September while submitting his dissertation.

There are several reasons to believe Russell about the topic of the dissertation. Whitehead probably attended Niven's lectures on Maxwell's *Treatise*. The physicist J. J. Thomson who had already published some works on electricity and who was working at the Cavendish laboratory that Maxwell had headed from 1871 until his dead in 1879, was one of the examiners of Whitehead's dissertation along with R. Forsyth. The later had also attended the lectures of W. D. Niven, one of Maxwell's pupils who published the second edition of his *Treatise*. In addition, E. J. Routh, former student of De Morgan, student at Trinity at the time of Maxwell and a famous coach at Cambridge had been Whitehead's coach.

V. Lowe reports extensively on the studies and the preparation for the examinations of the mathematical Tripos at Cambridge. It was as much a physical as an intellectual test for which the students had to receive a specific preparation. In a rather short period of very intensive studies requiring expert tutoring, the successful students managed to acquire the necessary basis to start teaching or research.

Lowe also regrets that Whitehead's dissertation was not preserved. Whitehead himself never mentioned it. But we may wonder and speculate on what Whitehead's dissertation could have been. The dissertation itself was not necessarily a very important piece of work, certainly not what a dissertation is supposed to mean today given the short period of time allowed to prepare it. The account that J. J. Thomson gives of his course of studies is a typical example of the curriculum of bright mathematics students at Trinity. It gives some clues on what Whitehead's piece of work could have been.

J. J. Thomson[38] who would later discover the electron and receive the 1906 Nobel Prize came up to Trinity in October 1876, four years before Whitehead. He attended the lectures of Cayley, Adams and Stokes while undergraduate. In his *Recollections*, he tells us what the exams were and which qualities were required when he took the Tripos exam in 1880: speed and accuracy. In January 1880, having passed the Tripos, ending second, like Thomson, Maxwell and Clifford before him, and behind Larmor, he ceased to be an undergraduate. He then submitted to the Smith prize, apparently unsuccessfully. Soon afterwards he started teaching nine hours a week while preparing a dissertation to be sent in for the Fellowship examination. His dissertation used the Lagrangian and Hamiltonian treatment of kinetic energy to deal with one of his old idea, that all forms of energy should be considered as kinetic energy. It was published in an expanded form in two papers in the *Transactions of the Royal Society* and, later, it constituted the basis of his book on *Applications of Dynamics to Physics and Chemistry*. The all process of the Fellowship examination was over by the end of the long summer vacations. In September, candidates had to take the examination and to submit two mathematical papers and a third paper on a subject they chose themselves. Kant was the subject J. J. Thomson selected. To his own surprise, he succeeded on his first attempt, as Whitehead did, but unlike Maxwell who succeeded on his second try. He was then elected Fellow for seven years, the custom at the time, Assistant lecturer two years after his degree and Lecturer after three years in 1883, all positions including individual supervision of students.

On the basis of this example, we may speculate and suppose that something similar happened to Whitehead's first two publications in Mathematics[39]. As it is often the case, they may have been inspired by or extracted from his dissertation or his submission to the Smith Price.[40] Except for an article on Geometry prior to his *Treatise on Universal Algebra* and an occasional paper during the war, these two papers were his only publications in Applied Mathematics. Maxwell who had been a candidate in 1854 shared that Smith price with E. J. Routh. As we will see, one of the examination questions concerned a theorem that would play an important role in the development of Hydrodynamics and of Physics in general. It is not irrelevant to note that in 1885, Whitehead proposed an essay on Hydrodynamics for the same Smith Prize.

6. Whitehead Papers on Hydrodynamics

At first, one could consider these articles of Whitehead as a fine exercise in advanced Calculus applied to Hydrodynamics. Commenting on these papers, Lowe insists on their importance for the field, Whitehead's result having a paradoxical consequence. Lowe[41] also tells us that the topic of the papers, the motion of incompressible fluids, had nothing to do with Maxwell's theory or with the problem of the ether. As we will see, this opinion is obviously wrong considering the corpus of research and the history of the theories of electricity and magnetism. These papers are an investigation of a problem in hydrodynamics taking some specific considerations into account but they belong to a larger field of research. It can even be argued in details that Whitehead's topic is related to some much earlier investigation of the ether by Stokes, ether that was not ruled out by the early and still inconclusive results of experiments by Michelson and Morley at the time of the writing of these papers.

Interesting in itself as Whitehead's result may be, apparently, it was not considered nor used by any of the main contemporary workers. For example, neither W. Thomson nor Stokes mention it in their later works on the same topic. Nevertheless, as we will see, replaced in its context, what makes the value of this work is its possible contribution to the then hot research topics in electromagnetic theory, ether theory and W. Thomson's theory of vortex atom.

According to Maxwell who criticizes a substantial interpretation of electricity as a fluid, "*…the use of the word Fluid has been apt to mislead the vulgar, including many men of science who are not natural philosophers…*"[42] And W. Thomson writes that "*In respect to electro-magnetic theory, we have a very fine analogy with viscous fluid motion, which has been obvious, more or less, from the time* […] (of) *Maxwell…*".[43] He was certainly in a position to know about this analogy, having been involved with it and with Stokes' work since the early 1840's.

Indeed, the idea of electricity flowing like an incompressible fluid is as old as Cavendish. In physics, a body is a solid or a fluid depending on its elasticity. Elasticity is the property of matter of requiring force to change its shape, to retain its new shape and to return to its original state when force is released. All bodies have that property to some degree. If the degree of elasticity of a body is non null, it is a solid; otherwise, it is a fluid.[44] The viscosity of a body depends on a resistance to change of shape varying with the rapidity of the change of state, for example, it can be molecular friction.[45]

In physics, the so-called Navier-Stokes equations concern the motion of fluids whose viscosity is taken into account. In establishing the laws of viscosity and building on Poisson’s and Fourier’s earlier work, G. Stokes founded Hydrodynamics. He relied on ideas of J. Green who, in 1828, had applied mathematical analysis to electricity and magnetism. In the Preliminaries of the first volume of his *Treatise*, Maxwell proves —without naming it— a theorem and mentions that it had been proposed by Stokes as a problem for the Smith's Prize Examination in 1854. It is the so-called ‘Stokes theorem’.[46] Indeed, Stokes proposed a generalization of a theorem of Green, the ‘general’ theorem of the paper of 1828 on which he had established the ‘Green’s theorem’.[47] This theorem concerns the potential of a distribution of matter, electricity or magnetism. The function Green calls ‘potential’ had been introduced by Laplace in his study of the earth attraction; it was also used by Gauss, and Tait notes that Ampère had already an analogous process. It represents the sum of all particles acting on a point divided by their respective distance from that point and gives the forces acting on that point from the whole mass of matter.

In his investigations of light and ether, Stokes had reconciled the phenomenon of aberration with the undulatory theory of light in supposing the ether at rest close to the surface of the earth.[48] He had to assume the properties of such an ether. For example, if it behaved like an ordinary fluid, it would be unstable. He concluded with a plausible way of conceiving the opposite properties that have to be attributed to ether, solidity and fluidity; that is, the ether must be viscous. To obtain his result, Stokes assumed a sphere moving uniformly in a fluid, the motion being such that the square of the velocity may be neglected and he supposed that the equation of motion of the particles of ether is an exact differential. What Whitehead did in his papers was to take into account the square and products of the velocities and to develop a method of approximation of the differential.

What matters in these papers of Whitehead is not so much the apparently paradoxical result that Lowe mentions but, rather, that further approximations lead to this result. For example, if one thinks of mathematics as providing a precise and definite answer to questions related to the magnitude of phenomena, it is always an approximation of the true value which, at some point, may even loose any sort of meaning. Knowledge by successive approximations will never reach the object itself (assuming this being possible). Moreover, as Whitehead remarks, the result of this work suggests some explanation without proving it. This may have raised questions related to the experimental and theoretical approaches of natural sciences, questions based on the fundamental principles used in physics, like Lagrange equation, the variational principle and Hamilton's principle of least action.

In his fundamental paper, Green used Cartesian coordinates. In his *Treatise*, Maxwell first compares the use of Cartesian coordinates with that of Quaternions and then proves Green's theorem in terms of Quaternions,[49] relying on the earlier proof of it by Tait.[50] The later tells us that it is Hamilton's 'nabla' operator, ∇, that attracted him to Quaternions because W. R. Hamilton had claimed that it would be useful in physical applications.[51] In his investigation of a process of integration for Quaternions, he managed to prove Green's theorem using Quaternions, the advantage of ∇ being to avoid the use of ordinary Cartesian coordinates. According to Maxwell, a reason to use Quaternions is that in order to reason in physics, contrary to calculate, it is best to introduce first Cartesian coordinates in order to fix the mind on a point in space rather than on its three coordinates, and to fix the mind on the quantity and direction of the forces rather than on their three components. This more natural way of considering geometrical and physical quantities developed by W. R. Hamilton is a method that allows for a direct mathematical representation of physical entities.[52]

This was the context of Whitehead's papers on Hydrodynamics and their relation to Electricity. Although one can speculate on the content and motivations of the dissertation and Smith prize submission of Whitehead, they may have influenced his program of research in applied mathematics. At the time of the publication of his two mathematical papers, he was already teaching on Grassmann who is mentioned in Maxwell's work and he could not ignore the heated polemics about vectors and quaternions that had erupted between the followers of P. G. Tait who supported Hamilton's Quaternions and those who advocated the use of Grassmann and Gibbs vectors, a polemics that lasted for about two decades. This may explain why Whitehead devoted the first volume of his *Treatise of Universal Algebra* to Grassmann and intended to write the second volume on W. R. Hamilton.

7. James Clerk Maxwell

We have a precise idea of Maxwell's curriculum of studies in Edinburgh: classics, mathematics, natural philosophy (i.e., mechanics, optics and elasticity), metaphysics, moral philosophy, Kant, Hobbes, poetry… Very early in his letters we see him commenting on these topics. For example, he appreciated Herschel who experimented on electricity and commented on

Ampère's theory. In Cambridge, student at Trinity College, Maxwell was elected a member of the 'Apostles' in 1853. Many of his contributions to the discussions were preserved, contrary to those of Whitehead. Lewis Campbell and William Garnett write that his contributions there show somebody not overfed by mathematics, but "*taking a survey of the universe*".[53] While continuing his investigation in optics and on vision, he read Berkeley's theory of Vision, Mill's Logic but could not find there the last word on the relation of sense to knowledge.[54] In addition to his investigation on color, he carried on with his study of Faraday's work that had started early in 1849 and, as he says, working into the "*views of heavy German writers*", he eventually completed his work on Faraday's lines of force. In addition to his own researches, he was lecturing, attending lectures of others and became interested in higher education of working men after reading F. D. Maurice, a social Christian.

Maxwell was not ignorant of metaphysics. As a young Fellow, he wrote "*I find I get fonder of metaphysics and less of calculation continually, and my metaphysics are fast settling into the rigid high style, that is about ten times as far above Whewell as Mill is below him, or Comte or Macaulay below Mill*".[55] Later, in 1868, he said that physics and metaphysics have very strong and close relationship and that from one period to the next, metaphysicians can be distinguished according to their knowledge of the physical doctrines of their time and the knowledge they had of them. Although Maxwell had been strongly influenced by the philosophy of W. Hamilton of Edinburgh, his former teacher, on many points he had the opportunity to disagree with him, even to find him wrong or empty. "*Taking metaphysicians singly, we find again that as is their physics, so is their metaphysics.*"[56]

Nevertheless, he found W. R. Hamilton of Dublin more interested by the metaphysical aspects of his method than by its mathematical aspects or its physical applications that were not always clear to the point that one could easily be impressed by the metaphysical depth of his doctrine rather than by its mathematical simplicity. Having studied Quaternions, Maxwell concluded that it was a "*method of thinking and not, at least for the present generation, a method of saving thought* […] (because) *it calls upon us at every step to form a mental image of the geometrical features represented by the symbols, so that in studying geometry by this method we have our minds engaged with geometrical ideas, and are not permitted to fancy ourselves geometers when we are only mathematicians*."[57]

This was the opinion of William Rankine who considered Herschel's *Outline of Astronomy* as an example of a scientist who manages to explain in simple terms a domain that requires the use of higher mathematics.[58]

When some proposition that requires complex algebraic difficulties has to be put to practical use, it is necessary to avoid useless mathematical complexities. The symbols of algebra are a sort of mechanism of economy of thought. In *his Introduction to Mathematics*, Whitehead notes that the mathematical symbols are used in order to relieve the work of the brain and make things easy, help in reasoning almost mechanically by the eye and to perform operations without thinking about them.[59] Similar ideas, although of a more fundamental nature, on the use of symbols in algebra were also stressed by Grassmann and, following him, by Whitehead in his *Treatise on Universal Algebra.*

8. Maxwell's Early Electromagnetic Theory

Maxwell's discoveries in electricity and magnetism and his electromagnetic theory of light, that had to wait for the end of the years 1880 and the results of the German physicist P. Hertz to be recognized and accepted, originate in the works of M. Faraday and W. Thomson. Early in his career, as we have seen, Thomson had proposed an analogy between electricity and heat, between electrical phenomena and the elastic deformation of some material under stress or constraints. The main objective of W. Thomson's research, his obsession, was the discovery of the still mysterious properties of matter that were explained by the properties or structure of the molecules and their relation to ether. Motion was considered as the essence of matter issuing mechanically from light and heat, all originating in gravitation.

On the basis of the analogy between elastic solids and the distribution of electricity in a conductor suggested by Faraday, he produced a mechanical and kinematical representation of electric, magnetic and galvanic forces formulated in terms of the equations of equilibrium of incompressible elastic solids that had been given by Stokes. And it is one of W. Thomson's early publications[60] that convinced Maxwell of the possibility of a mathematical treatment of electricity.

Following his discovery of electromagnetic induction in 1831, Faraday had suggested that the current generated in a primary circuit provokes a state of electric tension in the particles of the iron ring of the secondary circuit.[61] He described that state as a state of polarization of the molecules of matter and this tension state induced a current in the secondary circuit. He then showed that induction took place along curved lines, insisting on the spatial distribution of electrical forces. In order to represent the lines of polarized particles he used the geometric image of lines of forces, reactivating old ideas of Boscovich[62], and he started to speculate on matter and forces. It is important to note that Faraday proposed to see the lines of forces first as a representation of propagation of action and later as a geometrical representation of a physical reality.

In 1856, Maxwell published his paper on '*Faraday's lines of force*' in which he gave Faraday's ideas their mathematical expression. Later on, reviving W. Thomson's analogy and interpreting electrostatic forces in a body as a flux of heat in an infinite body, Maxwell based his theory on the mathematics developed by Stokes and proposed a mechanical and dynamical explanatory model. As we have seen earlier, Thomson, before Maxwell and after Green, had given Faraday's theories a mathematical interpretation. But, rather than following Maxwell, he devised his own theory based on the vortex model of the atom. At the time of Whitehead the student and later on, this vortex atom theory and its relations with Boscovich theory was still fashionable. This gives us an additional clue on the preference of Whitehead for Maxwell's theory. Indeed, in his *Intellectual Development*, Russell notes that Whitehead had suggested him to prefer Maxwell's views to those of Boscovich, a suggestion that Russell followed.[63]

As a young student, Whitehead might have attended some lectures of Stokes who still occupied the Chair of Newton in Cambridge. In these lectures, Stokes developed some of the philosophical ideas of Maxwell and others. For example, about the systematic classification of mathematical quantities that Maxwell considered of great help to the physicist. Because in the various sciences, one finds systems of various nature but whose mathematical form of their mutual relationships could be the same.[64] An example of this is W. Thomson's heat analogy mentioned above that relies on this principle of classification of the mathematical representation of physical entities. But a typical example, according to Maxwell, was probably that of W. R. Hamilton. His quaternions make a distinction between scalar quantities, represented by a numerical quantity and vectorial quantities that require three numerical quantities.

In his mathematical study of the analogy between thermal and electrical phenomena, W. Thomson had assimilated the flow of electrical force to a flow of heat. He noticed that this mathematical analogy suggested a physical analogy, the action of contiguous particles on some intervening medium. Maxwell took on the problem. He was interested in the geometrical idea of Faraday and proposed a geometrical representation of the lines of forces in a framework '*intended as a collection of purely geometrical truths*'. The problem consisted of a mathematical and a physical representation and the interactions of both representations. And the question was "*how to represent the intensity of the force?*". Maxwell supposed an incompressible fluid moving in tiny tubes formed by lines of force. This was not a physical representation of the electromagnetic field, "*not even a hypothetical fluid*", but "*merely a collection of imaginary properties*". He called that geometrical representation a "*physical analogy*" that presents "*the mathematical ideas to the mind in an embodied form*". The analogy between heat, electricity and fluid flow implied mathematical resemblance but not physical similarity. The similarity was between mathematical relations and not between the related phenomena. For Maxwell, the lines of forces express the geometrical structure and intensity of the field. In the *Treatise*, he writes that Faraday should have said, "*the field of space is full of force, whose arrangement depends on that of the bodies in the field, and that the mechanical and electrical action on each body is determined by the lines which abut on it.*"[65]

M. Crowe[66] thinks that it is probably the success of Faraday that led Maxwell to stress the virtues of quaternions because Faraday did not use formal mathematics but geometrical intuition. As Heaviside wrote, ignorant people naturally think in terms of vectors and, actually, quaternions are quantities directed in space. Maxwell even added that it was fortunate that Faraday did not use a mathematical formalism to express his ideas, his constant appeal to experiment to test their truth being probably the key to his success. "*The way in which Faraday did use his lines of force in co-ordinating the phenomena of magneto-electric induction shews him to have been in reality a mathematician of a very high order —one from whom mathematicians of the future may derive valuable and fertile methods.*"[67]

Comparing Faraday's method to that of the mathematicians, Maxwell admitted that his hope in writing the *Treatise* was to make Faraday's ideas the basis of a mathematical method. Nevertheless, with respect to the underlying philosophical ideas, Faraday belonged to the old school. In a letter to Maxwell, he notes that he does not use the word 'force' as Maxwell defines it, "*the tendency of a body to pass from one place to another*" but to him, 'force' means "*the source or sources of all possible actions of the particles or the materials of the universe; these being called the powers of nature…*"[68]

His early achievements in his "*conquest of truth*" made Maxwell proud and happy. In a letter to Litchfield, he writes that "*With respect to the* '*material sciences*', *they appear to me to be the appointed road to all scientific truth, whether metaphysical, mental, or social. The knowledge which exits on these subjects derives a great part of its value from ideas suggested by analogies from the material sciences, and the remaining part, though valuable and important to mankind is not scientific but aphoristic. The chief philosophical value of physics is that it gives the mind something* *distinct to lay hold of…*" And this conquest of truth furnishes the materials for the investigation of *the* great question, "*How does Knowledge come?*" [69]

9. Aspects of Maxwell's philosophy: Matter, Materialism and Evolution

A fair account of Maxwell's philosophy cannot be given in a short space, we thus only sketch a few topics and no attempt is made here to relate them explicitly and precisely to Whitehead's thought.

Claiming to have an *a priori* proof of the laws of motion, H. Spencer had started a controversy with Tait in which Maxwell was involved. According to Spencer, the laws of motion are true *a priori* and do not require experimental proof which is impossible anyway. His claim relied on Thomson's and Tait's *Treatise* where, in the second chapter, they study the dynamical principles and laws. Motion is studied independently of matter and forces that it exerts and, assuming the existence of motion, they come to its actual causes. "*The axiom of the present chapter must therefore be considered to be due to actual experience, in the shape either of observation or experiment*."[70]

To some extent, one could say that Thomson's and Tait's *Treatise* follows Newton's *Principia*, adding just details. Nevertheless they remark that they cannot "*give a definition of matter which will satisfy the metaphysician,* […but] *the naturalist may be content to know matter as that which can be perceived by the senses, or as that which can be acted upon by, or can exert, force*."[71] Later on, they write, "*An axiom is a proposition, the truth of which must be admitted as soon as the terms in which it is expressed are clearly understood. But* […] *physical axioms are axiomatic to those only who have sufficient knowledge of the action of physical causes to enable them to see their truth*."[72] Since the properties of matter might have been different, the axioms could have been different. Hence, the laws of motion "*must be considered as resting on convictions drawn from observation and experiment, not on intuitive perception*."[73]

Although Tait replied that no physical truth could be proved by *a priori* reasoning, Spencer resisted and maintained that the laws of motion were a priori and could not be proved experimentally. Maxwell could not accept the claim that matter was that which could be perceived by the senses. This was metaphysical reasoning foreign to Dynamics. Although he admitted that if the phenomenon of motion, kinematics, had not been given its proper place in education, it is because "*we have been relying too much on symbols and diagrams, to the neglect of the vital process of sensation and* *thought*"[74], 'matter' does not belong to a treatise on Dynamics. Dynamics is concerned first by the distribution of mass in the bodies, then, by the momentum and kinetic energy of the bodies depending on their mass and motion and, finally, by force, the rate of change of momentum. When one speaks of matter in Dynamics, it is not the substratum criticized by Berkeley but it is something as intelligible as a straight line or a sphere. What is relevant in the science of Dynamics is not that matter constitutes the real bodies but that the later "do *behave in a manner strikingly analogous to that in which we have proved that the mass-systems of abstract dynamics* must *behave*."[75] It is thus mass and not matter that is considered in Dynamics.

In order to explain matter in a complete theory, Maxwell states that one must explain mass and gravitation.[76] One theory of atoms that fulfilled all desiderata of such a theory was that of the vortex atoms of Thomson based on a result of von Helmholtz who had shown that in a perfect fluid whose motion is studied by pure mathematical analysis, vortex filaments would stay stable, not subject to dissipation or destruction. Matter being the receptacle of energy and momentum and the vortex rings having always a definite energy and momentum, it is clear that they could explain mass. However, no available theory could explain the mass of large bodies.

In these days, there was another and much important public debate about evolution on which Maxwell commented. One issue was creationism where, according to him, the limits of scientific investigations were reached. Indeed, science is not concerned by the origin or creation of atoms but by the form in which they exist. A burning question was that of Evolution versus Design. Although W. Thomson accepted an historical view of earth, he claimed that, at a time, there was no life and it could only happen by creation or have an extra terrestrial origin followed by evolution, but not evolution through natural selection because Darwin's theory had lost sight of the argument of Design.[77] These claims were contested by partisans of each side.[78] Much earlier, in his essay on "*Design*" (1853), Maxwell had written: "*Design, the word that disturbs our quiet discussions about how things happen with restless questionings about the why of them all. We have recklessly abandoned the railroad of phenomenology*". Having started with a discussion of "*the belief in design* [as] *a necessary consequence of the laws of thought acting on the phenomena of perception*", he concluded with a refutation of W. Hamilton, who claimed that perception is the ultimate consciousness of self and thing together. Indeed, that view would impose to turn the attention away from perception since all perceptions would be particular.[79]

Maxwell was reluctant to accept Darwin's theory of evolution[80] and he expressed clearly his doubts in his 1873 "*Discourse on Molecules*" at the British Association. Light that comes from the stars is the only evidence of their existence. Spectral analysis shows that each star is built out of the same elements that are found on earth. "*No theory of evolution can account for the similarity of molecules, for evolution necessarily implies continuous change and the molecule is incapable of growth or decay, of generation or destruction*." Since no natural process ever produced any difference in the properties of any molecule, their existence or properties cannot be ascribed to any natural cause. Moreover, since each molecule shares the same quality as all others of the same kind, it has the essential character of a manufactured article precluding the idea of being eternal and self existent, that is, it has been created. Maxwell then concluded that, following "*a strictly scientific path,* [we are led] *very near the point at which Science must stop.*"[81]

Similarly, in his paper on "*Atom*", he remarks that molecular science forbids the physiologist from considering infinitely small structural details that can explain the infinite variety of properties and functions of the smallest organisms. A microscopic germ can develop into a highly organized animal and another into an animal of a totally different kind. One can admit that the differences arise from differences in the structure of the respective germs. But the Pangenesists require more. The germ is a representative body containing all the possible hereditary characteristics of every organ and more. Galton had invented the expression "*structureless germs*" to avoid the difficulty involved. But, from the point of view of natural philosophy, material systems can only differ in the configuration and motion which they have at a given instant. Therefore, to explain differences of function and development of a germ without assuming differences of structure is to admit that its properties are not those of a purely material system.[82] Moreover, even if a germ contained in itself a purely physical power of development into some distinct thing arising from the configuration and motion of its parts, the gemmules, it would be non-sense to call them structureless because the microscope does not show the structure. Potentiality is non-sense in materialism except if expressed as configuration and motion.[83]

The following year, at the next meeting of the Association, J. Tyndall replied that believing in the continuity of nature, he cannot stop where the microscope ceases to be of use. There, the vision of the mind replaces that of the eye. That is, "*by an intellectual necessity, we cross the boundary of the experimental evidence in that matter is the promise and potency of all terrestrial life.*"[84] Tyndall was a main representative of scientific materialism, a doctrine that had evolved alongside naturalism and benefited from the accomplishments of the sciences. The materialist scientists relied on molecular physics and thermodynamics, another contribution of Maxwell, to claim that all phenomena were subjected to causality and determinism. But a worldview dominated by matter was not what Maxwell expected. Indeed, materialism was founded on mechanics, implying eternity and infinity, while the second principle of thermodynamics implies beginning and end.[85]

As a response to materialism, Stewart and Tait published their book, *The Unseen Universe*; *or Physical Speculations on a Future State*, 1875, but Maxwell did not appreciate their use of physical principles to refute materialism. Although he agreed with Helmholtz who used the principle of conservation of energy to refute vitalism, still, this did not prevent the existence of a soul that does not depend on physical principles. Christian materialists who defended a dynamical theory of life and mind were struggling with the material organisms and their functions such as their being which were not material while having a mind, material with a soul substantial. But science and religion are well separated domains and religion is a personal question. None of these debates and reflections seemed to convince Maxwell. All he had learned was that science and philosophy are about objects capable of being perceived and if our thought pretends to deal with the Subject, it is only an Object under a false name. 'I am' cannot be a science.[86]

10. Conclusion

In the obituary notice of Maxwell, P. G. Tait regrets "*…the loss which his early death has inflicted not merely on his personal friends, on this Society, on the University of Cambridge, on the whole scientific world, but also, and most especially, on the cause of common sense, of true science, and of religion itself, in these days of much vain-babbling, pseudo-science, and materialism. But men of his stamp never live in vain; and in one sense at least they cannot die. The spirit of Clerk Maxwell still lives with us in his imperishable writings, and will speak to the next generation by the lips of those who have caught inspiration from his teachings and example*".[87] No doubt that A. N. Whitehead caught this inspiration and found there parts of his program.

In the introduction of *Matter and Motion*, 1877, a book intended for the working men, Maxwell the physicist writes that "*Physical Science is that department of knowledge which relates to the order of nature, or, in other words, to the regular succession of events.*" But "*who will lead me in that* *still more hidden and dimmer region where thought weds fact, where the mental action of the mathematician and the physical action of the molecules are seen in their true relation? Does not the way to it pass through the very den of the metaphysician, strewed with the remains of former explorers and abhorred by every man of science?*"[88] Physics and metaphysics share some relation as the names indicate. While in his laboratory the physicist measures time, space and mass, what is the occupation of a metaphysician? "*He speculates on the modes of difference of co-existent things, on invariable sequences and on the existence of matter. He is just a physicist deprived from all his weapons, a disembodied spirit.*"[89]

Although Maxwell the philosopher confesses that he had still a long way ahead of him, that, up to that time, he had "*remained in ignorance of how* [he] *came to be, or, in the Spencerian language, how consciousness must arise*",[90] the physicist and the philosopher was also a man. Referring to the dead of a friend, he wrote, "[…] *it is in personal union with my friends that I hope to escape the despair which belongs to the contemplation of the outward aspects of things with human eyes. Either be a machine and see nothing but "phenomena", or else try to be a man, feeling your life interwoven, as it is, with many others, and strengthened by them whether in life or in death.*"[91] And he was a poet. "*The habit of recognizing principles* [of physical phenomena] *amid the endless variety of their action can never degrade our sense of the sublimity of nature, or mar our enjoyment of its beauty. On the contrary…*"[92]

> Is our algebra the measure
> Of that unexhausted treasure
> That affords the purest pleasure,
>   Ever found when it is sought?
> Let us rather, realising
> The conclusions thence arising
> Nature more than symbols prizing,
>   Learn to worship as we ought.[93]

Notes

[1] G. D. Birkhoff, "Books on Relativity", *Bull. Am. Math. Soc.*, Vol. 22, 4, (1922), pp. 215-221; p. 220.

[2] A. N. Whitehead, *Science and the Modern World*, Cambridge, Cambridge University Press, 1926.

[3] See, for example, M. Weber and A. Weekes (eds.), *Primary Glimmerings. Consciousness Studies from a Whiteheadian Process Perspective.* Whitehead Psychology Nexus Studies II, forthcoming.

[4] J. S. Bell, “Against 'Measurement'”, in *Sixty-Two Years of Uncertainty*: *Historical, Philosophical and Physical Inquiries into the Foundations of Quantum Mechanics*, ed. A. Miller, New York, Plenum, 1990, pp. 17-31.

[5] This is elaborated in a forthcoming book, *A. N. Whitehead, mathematician, logician, natural philosopher*.

[6] SMW, pp. 23-24.

[7] SMW, p. 58.

[8] SMW, p. 64, 65.

[9] SMW, pp. 68-72.

[10] SMW, p. 115.

[11] SMW, p. 31.

[12] SMW, p. 130.

[13] SMW, pp. 173-4; p. 177.

[14] SMW, p. 224.

[15] A. N. Whitehead, *Concept of Nature*, Cambridge, Cambridge University Press, 1920; repr., 1971, pp. 30-31.

[16] CN, p. 42-43.

[17] CN, pp. 48; 46.

[18] CN, pp. 48; 46-47.

[19] D. Emmet, “A. N. Whitehead: The Last Phase”, *Mind*, LVII, n° 227, 1948, pp. 265-274; p. 266.

[20] W. Thomson and P. G. Tait, *Treatise on Natural Philosophy*, (Principles of Mechanics and Dynamics), Cambridge University Press, 1879, 1st ed. Oxford, Clarendon, 1867, p. v.

[21] J. Clerk Maxwell, *Treatise on Electricity and Magnetism,* Oxford, Clarendon Press, 1873; 2nd ed. 1881.

[22] W. Thomson, *Lectures on the mathematical theory of electricity*, 1847-8; repr. as “On the mathematical theory of electricity in equilibrium”, in *Papers on Electrostatics and Magnetism*, London, MacMillan, 1872, pp. 53-85.

[23] W. Thomson and P. G. Tait, *Treatise on Natural Philosophy*, p. viii.

[24] Review in *The Scotsman*, Nov. 6, 1868, quoted in S. P. Thompson, *The Life of Lord Kelvin*, 1910, Vol. 1, New York, Chelsea Publishing Company, 1976, p. 471.

[25] S. P. Thompson, Ibidem, p. 472. Thomson and Tait intended to retain the copyright and expected the publisher to do whatever required preventing piracy in America and translation abroad. Although the book sold rapidly and did not cover the costs, the authors wanted a cheap edition for students. In a letter, to Thomson, Tait suggested to drop a fine book that would only satisfy the publisher's pride. "*Cambridge books are ridiculously dear, and I think he may learn a useful lesson on that point by being made to do the affair cheap for once*." p. 462.

[26] Quaternions are hypercomplex numbers invented by W. R. Hamilton. They are explained in Note 51.

[27] See P. G. Tait, *Dynamics*, London, Adam and Charles Black, 1895. Chap. XII, "General considerations".

[28] Tait tells us that "*in early life he had with great wisdom chosen* [W. Thomson] *as a model*" and J. J. Thomson adds that W. Thomson considered Stokes as his teacher. P. G. Tait, 'Obituary Notice of J. Clerck-Maxwell', in, *Proc. Roy. Soc. Edinburgh*, Vol. X, Dec. 1, 1879; J.J. Thomson, *op. cit.*, note 16, p. 50.

[29] A. E. H. Love, *Memorial Address to the London Mathematical Society*, 1908, quoted in S. P. Thompson, *op. cit.*, Vol. 2, p. 1142. The quotations of W. Thomson come from *ibidem* Chapter XXV.

[30] SMW, pp. 32-33.

[31] IM, p. 2.

[32] SMW, p. 185.

[33] SMW, p. 229.

[34] SMW, p. 4.

[35] SMW, p. 16.

[36] Whitehead's knowledge of this work and its developments makes no doubt, of course. For example, on some occasion, he mentioned *Poynting theorem* as an important result. John Poynting (1852–1914), a former student of Maxwell, did not use an ether model but replaced it by a homogeneous distribution of lines of force in space

in order to represent the electromagnetic field. Electromagnetic phenomena were explained through location and dispositions of lines of force. He specified the path, a vector called "Poynting vector", followed by the energy radiating in an electromagnetic field. Before becoming vital to mobile phone addicts, this theorem may have interested Whitehead for a specific interpretation of the electromagnetic field, at each point the flux of energy; an idea that will somehow reappear in his interpretation of the General Relativity Theory.

[37] V. Lowe, *Alfred North Whitehead. The Man and his Work*, Vol. I, The Johns Hopkins University Press, Baltimore, 1985, p. 107.

[38] J. J. Thomson, *Recollections and Reflections*, London, G. Bell and Sons, Ltd., 1936; repr., New York, Arno Press, 1975.

[39] A. N. Whitehead, "On the Motion of Viscous Incompressible Fluids: A method of Approximation", *Quart. J. of Pure and Appl. Math*., 23, (1889), 78-93; "Second Approximation to Viscous Fluid Motion: A Sphere Moving Steadily in a Straight Line", *Ibid*., pp. 143-152.

[40] In any case, the result was considered worth of publication. As Maxwell once said to Arthur Schuster, "*The question whether a piece of work is worth publishing or not depends on the ratio of the ingenuity displayed in the work to the total ingenuity of the author.*" Quoted in J.G. Crowther, *The Cavendish Laboratory, 1874-1974*, New York, Science History Publications, 1974, p. 67.

[41] *Op. cit*., Chapter VIII, iv, p. 156.

[42] J. Clerck Maxwell, *Op. cit*., Vol. I, 1873, p. 38.

[43] W. Thomson, "Ether, Electricity, and Ponderable Matter", 1889; rep. in *Mathematical and PhysicalPapers*, Vol. III, 1890, pp. 484-515, p. 495.

[44] W. Thomson, *Elasticity and Heat*, Enc. Brit., 9th ed., 1878; repr. in *Mathematical and Physical Papers*, Vol. III, London, C. J. Clay and Sons, CUP, 1890, pp. 1-260; p. 3.

[45] In fluids dynamics, Stokes' law expresses the viscosity, i.e. the resistance of molecules to the motion of a body in a fluid. *Ibidem*, p. 21. In "Motion of a Viscous liquid; Equilibrium or Motion of an Elastic Solid; Equilibrium or motion of an Ideal Substance Called for Brevity Ether; Mechanical Representation of Magnetic Force", 1st publ. 1890, *Ibidem*., pp. 436-465, W. Thomson writes that "*The viscosity of a liquid* […] *is defined as the tangential force per unit*

of area […] *of a simple distortion required to produce change of shape at unit speed*" and he adds that "*Stokes assumed the stress to be in simple proportion to the speed of the change of shape, as basis of his mathematical theory*." (pp. 436-7) This assumption verified for water needs some correction to take into account residual effects of previous conditions in the case of viscous fluids. Nevertheless, in his purely mathematical analogy, Thomson, as Stokes did, uses simple proportionality. Further down in *ibidem*, W. Thomson replaced "viscous fluid" by "elastic solids", then "quasi-elastic" or "quasi-rigid" ideal substance, that he calls 'ether' —that was assumed to correspond to the (still thought of as) 'real' ether— and that proved to be useful to extend his article of 1847, "Mechanical representation of electric, magnetic, and galvanic forces" where he starts from Stokes paper, "On friction of fluids in motion and the equilibrium and motion of elastic solids", 1845, a paper Whitehead also refers to.

[46] To make M. J. Crowe's note on p. 147 of *A History of Vector Analysis*, Notre Dame, University of Notre Dame Press, 1967, clear and accurate, we add that the version of Green's theorem proposed by Stokes at the 1854 Examination as well as other versions and extensions of the same are discussed with the help of Quaternions in P. G. Tait, "On Green's and other Allied Theorems", *Trans. Roy. Soc. of Edinburgh*, Vol. XXVI, 1870. Rep. with addenda in *Scientific Papers*, Cambridge, Cambridge University Press, 1898, Vol. I, pp. 136-150. It is interesting to remark that Tait gives there an application and a development (that he wrongly believed W. R. Hamilton had no time to develop himself): an integration process applicable to the whole Quaternion, not only to the scalar variables. Moreover, he remarks there that, given the simplicity and expressiveness of Quaternions, this little advance allowed him "*to see, with a thoroughness of comprehension* […] *the mutual relationships of the many singular properties of the great class of analytical and physical magnitudes which satisfy what is usually known as Laplace's equation*" (p. 137). It should be noted that Tait was taken to this sort of investigation while considering applications of Maxwell's idea of representing currents by the motion of an imaginary fluid. ("On the steady motion of an Incompressible fluid in two dimensions", *Proc. Roy. Soc. of Edinburgh*, 1870, Rep. ibid., 1898, Vol. I, pp. 132-133).

[47] G. Green, *An essay on the application of mathematical analysis to the theory of electricity and magnetism*, Nothingam, 1828; repr. in

Mathematical Papers of the Late George Green, ed. N. M. Ferrers, 1870, Paris, Herman, 1903, pp. 1-115.

[48] G. G. Stokes, "On the Aberration of Light", 1845 ; "On the Constitution of the Luminiferous Ether, viewed with Reference to the Phenomenon of the Aberration of Light", 1846; repr. in *Mathematical and Physical Papers*, Vol. I, Cambridge, Cambridge University Press, 1880, pp. 134-140, 153-156.

[49] J. Clerk Maxwell, *Treatise*, Vol. I, 1873, p. 25.

[50] P. G. Tait, *loc. cit.* in Note 46.

[51] A problem of 18th century mathematics, that of making sense of irrational numbers like $\sqrt{-1}$ and other negative and imaginary quantities, was solved by W.R. Hamilton who created new systems of numbers, the complex numbers conceived as pairs of real numbers of the form $a + bi$, where a is the real part and bi the imaginary part. $\sqrt{-1}$ denotes the couple of real numbers $(-1,0)$, and for all couples (a,b), $(a,b)=a+b\sqrt{-1}$. The idea amounts to considering $\sqrt{-1}$ as a geometrical object that can represent any direction in space. Trying to extend his theory to triplets and even to polyplets, Hamilton discovered the quaternions, hypercomplex numbers of the form $w + ix + jy + kz$, where the w, x, y, z are real numbers and the i, j, k unit vectors directed along the geometrical axis x, y, z, the coordinates in space. These numbers follow the rules $ij = k; jk = i; ki = j; ji = -k; kj = -i; ik = -j$ and $i^2 = j^2 = k^2 = ijk = -1$; they do not follow the rule of commutativity. w is the real algebraic part, a scalar (its value can run on a scale from minus the infinite to plus the infinite) and the imaginary algebraic part represented by a segment with a definite orientation in space is the vectorial part. For completeness, $\nabla = i\, d/dx + j\, d/dy + k\, d/dz$, where d denotes the derivative. Considering the coordinates of a point p, $p = ix + jy + kz$, and the rules of Quaternions calculus, $\nabla^2 = -d^2/dx^2 - d^2/dy^2 - d^2/dz^2$.

[52] J. Clerk Maxwell, *Treatise*, Vol. I, 1873, pp. 8-10.

[53] Lewis Campbell and William Garnett, *The Life of James Clerk Maxwell*, London, Macmillan, 1882, p. 165. We will heavily rely on Maxwell's letters published in this book (henceforth abbreviated 'Life' in the notes). According to Lytton Strachey quoted by M. Singer, editor of Sidgwick's *Essays on Ethics and Method*, "*The most lasting utterances of a man are his studied writing; the least are his conversations. His letters hover midway between these two*

extremes; and the fate which is reserved for them is capable of infinite gradations, from instant annihilation up to immortality".

[54] *Life*, p. 198.

[55] *Life*, p. 261.

[56] *Life*, p. 209.

[57] J. Maxwell, "On Quaternions", *Nature*, 9 (1873), pp. 137-138. (This anonymous article attributed to J. C. Maxwell is a review of the book of P. G. Tait and Kelland, *Introduction to Quaternions*, 1873. He starts discussing the mathematician as a calculator, much of his routine work being calculation, but his proper work that makes him a mathematician is the invention of methods. It is important for the physicist to attain a direct mathematical representation of physical entities and be thus aided in seeing the physics involved in the mathematics.

[58] W. J. Rankine, p. xv, *Preliminary Dissertation* that contains most of a Lecture on "*Concordia inter scientia machinalium contemplationem et usum*" (The harmony of theory and practice in mecanichs) at the University of Glasgow Senate, 1855. W. J. Rankine, *Manuel de Mécanique Appliquée*, 1858, transl. 7th ed. by A. Vialay, Paris, Dunod, 1876.

[59] IM, passim, pp. 39-42.

[60] "*On a Mechanical Representation of Electric, Magnetic and Galvanic Forces*", 1847; repr. in *Mathematical and Physical Papers*, Vol. I, Cambridge, Cambridge University Press, 1882, pp. 76-80.

[61] The experiment consists in circulating a current in an electrical circuit, a solenoid around an iron ring or bar. This current induces another current in a second similar circuit placed nearby.

[62] Faraday had claimed that his theory of matter that defined matter in terms of inherent powers resembled that of the 18th century Boscovich who had proposed a theory that preserved the Newtonian dualism between force and matter. However, Faraday's theory was closer to that of Priestley who rejected Newtonian dualism and claimed that the defining characters of matter were extension and inherent powers of attraction and repulsion. Priestley having said that his theory was similar to that of Boscovich, it was confused with it and called Boscovich's theory. Boscovich's ideas are in the tradition of Kant's *Metaphysical Foundation of Science*, 1786, the foundation of science on a priori principles. This is also the

tradition of Newton and his legacy of dynamism, mechanism, laws of motion, action and reaction, gravitation and action at a distance. In Boscovich's *Theoria Philosophiae Naturalis*, 1763, "*natural philosophy deduced by a single law of powers*", matter consists in geometrical points whose spatial relations produce a system of powers or tendencies to motion. These powers being functions of the distances between the points, matter does not reduce to a random aggregate of forces but can be represented geometrically by a continuous function with force in ordinate and distance in abscissa. The two constitutive forces have disappeared, replaced by powers that vary continuously with the distance between points that can constitute a stable system in equilibrium, the physical world.

[63] B. Russell, *My Philosophical Development*, London, Allen and Unwin, 1969, p. 43.

[64] M. Crowe, *op. cit.*, 1985, p. 130. J. Clerk Maxwell, "On the Mathematical Classification of Physical Quantities", repr. in *The Scientific Papers of James Clerk Maxwell*, 2 Vols, ed. W. D. Niven, Cambridge, Cambridge University Press; repr., Paris, Librairie Scientifique J. Herman, 1927, Vol. II, pp. 257-266.

[65] *Treatise*, Vol. II, 1873, p. 164.

[66] *Op. cit.*, p. 134.

[67] J. Clerk Maxwell, "Faraday", in *The Scientific Papers*, Vol. II, pp. 355-360, p. 360.

[68] *Life*, pp. 289-290.

[69] *Life*, p. 304-305.

[70] W. Thomson and P. G. Tait, *Treatise on Natural Philosophy*, p. 219.

[71] *Ibidem*.

[72] W. Thomson and P. G. Tait, *Treatise on Natural Philosophy*, p. 240.

[73] *Ibidem*, p. 241.

[74] J. Clerk Maxwell, "Thomson and Tait's Natural Philosophy", repr. in *The Scientific Papers*, Vol. II, in 776-785. p. 778.

[75] *Ibid.*, p. 781.

[76] J. Clerk Maxwell, "Atom", *ibid.*, pp. 445-484. p. 472.

[77] Presidential Address to the British Association, 1871, in S. Thompson, *Op. cit.*, Vol. II, Ch. V.

[78] For example, Chapter 2 of K. Pearson's *The Ethic of Freethought*, London, T. Fisher Unwin, 1888, pp. 33-53, in which he criticizes Stokes' positions in favor of Design is entitled "Prostitution of science".

[79] *Life*, p. 227.

[80] *Life*, p. 573. See also J. Clerk Maxwell, "Molecules", repr. in *The Scientific Papers*, Vol. II, pp. 361-377.

[81] *Life*, p. 359-361.

[82] J. Clerk Maxwell, *op. cit.*, pp. 445-484.

[83] *Life*, p. 390.

[84] Quoted in A. Macfarlane, *Lectures on Ten British Physicists of the 19th Century*, New York, John Wiley & Sons, 1919, pp. 19-20.

[85] *Last Essays in Cambridge*, I, "Does the progress of Physical Science tend to give any advantage to the opinion of Necessity (or Determinism) over that of the Contingency of Events and the Freedom of the Will?", in, Life, pp. 434-444.

[86] *Last Essays in Cambridge,* III, "On Psychophysics", Life, pp. 452-463.

[87] P. G. Tait, Obituary Notice of J. Clerk Maxwell, in *Proc. Roy. Soc. Edinburgh*, Vol. 10, Dec. 1, 1879.

[88] "Presidential Address to the British Association", 1870. Quoted in *Life*, p. 326.

[89] *Life*, p. 436.

[90] *Life*, p. 460.

[91] *Life*, p. 280.

[92] From Maxwell's Inaugural Cambridge Lecture as Cavendish professor, Oct 1871. *Life*, p. 355.

[93] From "A vision, Of a Wrangler, of a University, of Pedantry and of Philosophy", Nov 10th 1852. *Life*, p. 617.