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Ben-Ari, Mordechai

Mathematical surprises. (Mathematische Überraschungen.) (German) Zbl 07990347

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This is the German translation by the author of the English original [Mathematical surprises. Cham: Springer (2022; Zbl 1507.00009)]. It is a collection of mathematical results that are supposed to surprise the reader who is assumed to have some knowledge of secondary school mathematics (if needed all the necessary material can be found in an appendix). The purpose is to raise the interest of the reader to continue exploring mathematical topics. A typical example of such a result is the Feuerbach circle where 9 points are constructed in an arbitrary triangle that all fall on the same circle. This fact is surprising and needs a proof to believe it. This example is *not* included, but Ben-Ari has collected a number of similar facts, some of which are less known, and/or usually do not fit in a regular curriculum.

Many of the discussions are geometrical like constructions with straightedge and compass, some are known to be impossible like the classical problems of trisecting an angle, squaring the circle or doubling the cube, while (surprise!) an heptadecagon is constructible. Some constructions are using a neusis (a straightedge with two marks) or the quadratrix; while others are more restrictive, using only a compass, or only a straightedge and one circle. It is interesting that some historical false (since only approximate) constructions involving irrational numbers are discussed. In one of the chapters, axioms of origami are given and it is illustrated that it can be used to solve geometric problems and that the system is stronger than what is possible with straightedge and compass since trisecting an angle and doubling a cube become possible in the origami system.

Other problems involve graphs like colouring problems, and the museum guard problem (how many guards does it take to watch all the walls of a polygonal museum hall). Also here, not only proofs for six and five colour theorems, but also a false proof of the four-color problem is analysed to illustrate the flaw.

There are problems related to induction, the solution of quadratic equations, some Ramsey theory, and several less known, but amusing problems (Langford, Van der Waerden, Josephus) and less known but useful methods (Lill's and Po-Shen Loh's methods, SAT-solving, the Beloch fold).

The text is of Martin Gardner flavour and quality. Many proofs are included but there are no exercises. Every chapter ends by explaining what exactly was so surprising about it and why. The source for the origin of the problem is included.

There were some minor flaws in the English version that have been corrected in the German translation, e.g. the typo $(\sqrt{20/2})^2 = 5$ on page 84 of the English version has been corrected as $(\sqrt{20}/2)^2 = 5$ on phage 88 of the German text. Others, are still there like the wrong spelling Poh-Shen Lo, and the notation V being used for both the number of vertices in a graph (as in Euler's formula $V - E + F = 2$) and the set of vertices (Eckpunkt v in V). On the other hand using G for 'graph', V for 'vertices' and E for 'edges' is natural in English, it is less natural in German to use G for 'Karte', while E for 'Kanten' and V for 'Eckpunkten' is even confusing. The LaTeX source of the book is available in several languages via github.

Reviewer: Adhemar Bultheel (Leuven)

MSC:

00A09 Popularization of mathematics
00A05 Mathematics in general

Keywords:

constructions with straightedge and compass; combinatorics; graphs; Ramsey theory; coloring problems; origami; axiomatic system; induction proofs; quadratic equations; cubic equations; Langford problem; Lill's method

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