Comparison of optimal control formulations for stratified sensible thermal energy storage in space heating applications

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Abstract: In space heating systems equipped with thermal energy storage, model predictive control can be used to support the electrical grid through active demand response. There are however several formulations possible for the underlying optimal control problem, the most accurate being non-convex and thus more difficult to solve as the control horizon lengthens. The current paper presents a non-convex and several simplified formulations for the general optimal control problem which are compared towards attained energy cost, constraint violation and computation time. A novel iterative linear program is proposed to increase controller robustness and to be used in dynamic simulations. The novel formulation results in equal overall energy costs while computation time is reduced by a factor 10 compared to the best performing non-convex problem. This allows simulations for energy performance assessment including model predictive control to be executed on a modern personal computer.

Keywords: demand response, space heating, thermal energy storage, sensible energy, stratification, optimal control

1. Introduction

Active demand response (ADR) facilitates the management of intermittent renewables in the generation mix (Bruninx et al., 2013) and can be achieved by residential space heating using electricity as energy source (Hughes, 2010; Fitzgerald et al., 2012). Heat pumps and thermal energy storage are considered important technologies to further reduce greenhouse gas emissions attributed to space heating systems (Hewitt, 2012).

However, the control of such systems is not straightforward. As the decisions of storage charging are based on future heat loads and energy prices, the use of model predictive control (MPC) seems self-evident. MPC generates control inputs by minimizing an objective function subject to constraints over a finite time horizon and implementing only the first sample of the resulting optimal control signals (Camacho and Bordons, 2007).

When optimizing a system including thermal energy storage, the most common approach is to model the storage energy content as a single state (Henze et al., 2004; Ren et al., 2008; Kriett and Salani, 2012; Fux et al., 2013). However, when considering an actual hot water storage system connected to finite size heat exchangers, the temperature distribution inside the storage becomes important. Due to the non-linear nature of this system, the use of linear constrained optimal control formulations might be insufficient to obtain good control performance (Allgöwer et al., 2004).

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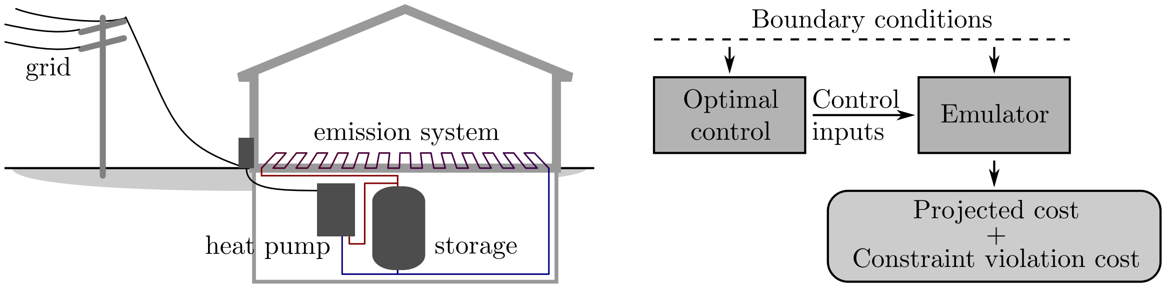
This paper investigates whether a slow, non-convex optimal control formulation of residential space heating including a heat pump and hot water storage tank is necessary and compares several simplified linear alternatives.

2. Materials and methods

2.1. System and approach overview

A schematic representation of the complete system is shown in figure 1(a). This study focusses on the heating system, consisting of an air-source heat pump and storage tank. The interaction of this system with the building is provided by the heat emission system.



(a) Schematic system (b) Schematic approach

Figure 1: Schematic representation of the assumed system and the comparison approach

To compare the different optimization problem formulations, the optimization problems are solved over a finite time horizon assuming perfect predictions of all disturbances. The resulting control profiles serve as input for a system emulator using a detailed model of the storage tank. From this emulation the cost of heating is calculated and compared (Figure 1(b)).

2.2. System emulator model

In the emulator model a storage tank of 4 m^3 is modelled using a 1 dimensional finite volume discretization with 50 nodes and QUICK convection terms (Leonard, 1979). The storage tank is connected directly to the heat pump and emission system. At the inlet of the storage tank, buoyancy effects and mixing of the incoming flow with water near the inlet are modelled by equally dividing the inlet flow rate over the nodes between the inlet and the node with matching temperature and by mixing the volumes near the inlet.

The heat pump is modelled as a heat supply with a fixed supply water temperature ($T_{gen,out}$) and limited power ($\dot{Q}_{gen,max}$). The emission system is modelled as an ideal heat exchanger with uniform secondary temperature (T_{em}).

As inputs, the emulator receives the values of the desired heat pump power ($\dot{Q}_{gen}$), as calculated by the optimization, and the emission power ($\dot{Q}_{em}$). These are converted to mass flow rate values through both systems based on in and outlet temperatures. As the emission power is considered a boundary condition, the building dynamics can not be adressed by this optimization, thus focussing on the storage tank.

The electricity consumption of the heat pump is calculated in a post processing step using an ambient temperature dependent coefficient of performance (COP) fitted on manufacturer data. When the desired emission power can't be delivered by the heating system an electrical backup heater is assumed. The total emulated cost of electricity used by the heat pump and the backup heater will be used to compare different optimal control formulations.

2.3. Optimal control formulations

In the following subsections the investigated optimal control formulations will be explained.

2.3.1. Non-linear program

The most common way of modelling a stratified hot water storage tank is to divide the storage volume in N equal volumes and assuming a uniform temperature throughout each volume. In an optimization context, both the node temperatures (T^i) and capacity flow rates to and from the storage are regarded as variables. The differential equations that govern the changes in node temperatures are implemented as state constraints.

As the upwind convection equations change with the direction of flow, the capacity flow rate inside the storage tank is split into a downwards, $\dot{C}_d$, and an upwards component, $\dot{C}_u$. Evidently only one of both can be non-zero at any time step. Heat losses are added for each node using a fixed UA value and the temperature to which the heat is lost (T_{hl}).

The control horizon is split up in intervals with length Δt. Using a second order collocation scheme, the state constraint for node i at time step k can be written as:

$$\begin{aligned} C^i_{sto}\frac{T^i(k+1)-T^i(k)}{\Delta t} &= \dot{C}_d(k)\left(\frac{T^{i-1}(k)+T^{i-1}(k+1)}{2}-\frac{T^i(k)+T^i(k+1)}{2}\right) \\ &+ \dot{C}_u(k)\left(\frac{T^{i+1}(k)+T^{i+1}(k+1)}{2}-\frac{T^i(k)+T^i(k+1)}{2}\right) \\ &+ UA^i\left(\frac{T_{hl}(k)+T_{hl}(k+1)}{2}-\frac{T^i(k)+T^i(k+1)}{2}\right) \end{aligned} \tag{1}$$

At node 1 and N the temperatures with index 0 and $N+1$ must be replaced with the generation and emission outlet temperatures respectively.

The cost function (J) is formulated as:

$$J = \sum_k \dot{Q}_{gen}(k)\frac{p(k)}{COP(k)}\Delta t \tag{2}$$

Where $\dot{Q}_{gen}$ is the heat added to the storage tank by the heat pump, p, the price of electricity and COP the heat pump coefficient of performance.

As in the state constraints the node temperature, a state variable, is multiplied by the capacity flow rate, a decision variable, this results in a bilinear, non-convex optimization problem (nlp). It is important to notice that no mixing or buoyancy is implemented in this formulation. Incorporating buoyancy or mixing would result in non-smoothness, rendering the model unsuitable for gradient based optimization methods.

2.3.2. Temperature discretization

Instead of a spatial discretization of the storage tank, one could also consider to split the storage in parts with different, constant temperature and varying volume (lp_td). Each part i has a variable heat capacity C^i and a fixed temperature T^i which drops with rising index i. The sum of all heat capacities must always be equal to the total capacity of the storage tank. The state constraint for node i at time k can be written as:

$$\frac{C^i(k+1)-C^i(k)}{\Delta t} = \sum_{j>i}\dot{C}^{j,i}_{gen}(k) - \sum_{j<i}\dot{C}^{i,j}_{gen}(k) + \sum_{j<i}\dot{C}^{j,i}_{em}(k) - \sum_{j>i}\dot{C}^{i,j}_{em}(k) + \dot{C}^{i-1}_{hl}(k) - \dot{C}^{i}_{hl}(k) \tag{3}$$

Here $\dot{C}^{j,i}$ represents the generation or emission capacity flow rate from node j to node i. This assumes that, through buoyancy effects, injected fluid is transported to the location where it's temperature matches without mixing. The heat lost to the ambient by node i can be translated into a flow rate $\dot{C}^{i}_{hl}$ to node $i+1$ which is at a lower temperature.

The cost function can now be determined as:

$$J = \sum_{k} \sum_{i,j} \dot{C}^{j,i}_{gen}(k)(T^j - T^i)\frac{p(k)}{COP(i,k)}\Delta t \tag{4}$$

The main advantage of this formulation is that it is a linear program where the generation temperature is allowed to vary. The main disadvantage is that multiple simultaneous charging or discharging flow rates can be possible which is unrealistic.

This problem can be resolved by including binary variables for both generation and emission flow rate, and an additional constraint stating that the sum of binaries must be less or equal to one at each time step. The resulting formulation (milp_td) is however a Mixed Integer Linear Program which is much more difficult to solve.

Another issue is the assumption that any entering water is injected in the tank layer with the same temperature. In an actual storage tank this can partially be obtained through the use of inlet stratifiers (Furbo et al., 2005; Shah et al., 2005; Brown and Lai, 2011) or multiple controlled inlet valves but as these devices would increase the investment costs, they are omitted.

2.3.3. Ideal linear program

Two extreme storage tank configurations can be considered: ideal stratified tanks and ideal mixed tanks. Reducing the above problem to a situation with only 2 nodes results in ideal stratification (lp_strat), where one part of the fluid is always at a high temperature T_h and the rest is at a low temperature T_l. This formulation is equivalent to the energy content formulation most commonly found in literature (Henze et al., 2004; Kriett and Salani, 2012; Fux et al., 2013) and is described in detail in Baeten et al. (2014).

An ideal mixed storage tank (lp_mix) can also be modelled as a linear program by introducing constraints which limit the generation and emission heat flows according to the storage temperature, maximum flow rates and heat exchanger effectiveness.

The main disadvantage of the ideal linear programs is that the assumptions of the water temperatures exiting the storage tank do not always hold. In an attempt to resolve this issue, an iterative linear program was developed.

2.3.4. Iterative linear program

The description of the storage tank as an energy buffer with heat entering from the generation side and heat exiting to the emission system results in the above stratified linear program. However, this approach holds an implicit assumption concerning the tank outlet temperature. As the heat supplied to the emission system is delivered by a heat exchanger with a slow varying temperature at the emission side, the flow rate must rise when the storage exit temperature drops. The possible heat flow rate is thus limited by the maximum flow rate and dependent on the storage outlet temperature. In the ideal stratified model this storage outlet temperature is fixed at T_h, while in a real storage tank the outlet temperature will vary. On the generation side a similar reasoning holds.

To accurately represent a real storage tank we need to add constraints limiting the maximum heat flows with respect to the maximum flow rates and storage outlet temperatures:

$$\dot{Q}_{gen} \leq \dot{C}_{gen,max}(T_{gen,out} - T_{gen,in}) \quad (5)$$
$$\dot{Q}_{em} \leq (\varepsilon\dot{C}_{em})_{max}(T_{em,in} - T_{em}) \quad (6)$$

In the former constraint, the required temperature ($T_{gen,in}$) is however unknown, and adding it as a function of the average temperature would render the program non-convex. It can however be obtained from a dynamic simulation of the system.

When an actual energy storage tank is nearly empty, the outlet temperature approaches the average temperature. Thus, the latter of the above constraints results in an increased minimum average storage temperature:

$$T \geq T_{em} + \frac{\dot{Q}_{em}}{(\varepsilon\dot{C}_{em})_{max}} \quad (7)$$

This results in an iterative solution of the linear program (lp_iter):

1. estimate the storage outlet temperature
2. solve the linear program
3. perform a simple simulation using the found control inputs
4. calculate the generation inlet temperature and update the minimum storage temperature
5. if the simulation satisfies the constraints finish, else return to step 2

2.3.5. Optimization algorithms

The described non-linear programs were solved using a primal-dual interior point optimization algorithm implemented in IPOPT (Wächter and Biegler, 2006), the linear and mixed integer linear programs were solved using a simplex algorithm with branch and cut strategy if required implemented in IBM ILOG CPLEX.

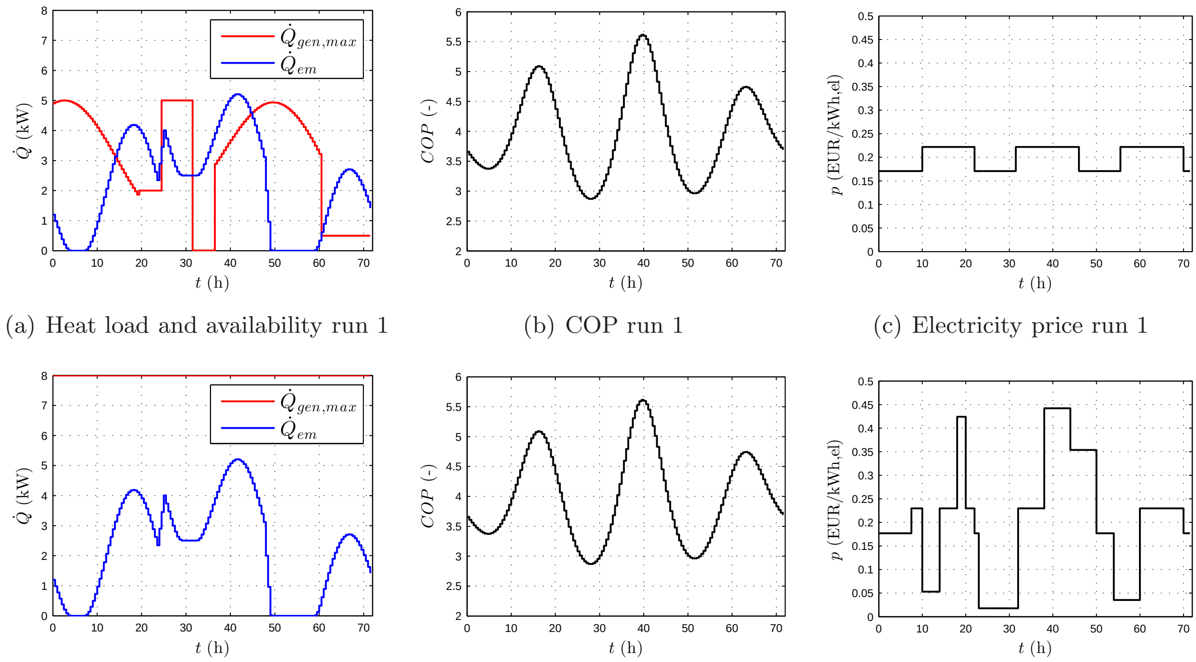
The non-linear program was solved with 20,10 and 5 nodes. An initial guess was obtained by first running the stratified linear program, adding small sinusoidal disturbances to the control signals and removing the final sample. This represents a long running moving horizon MPC.

The linear temperature discretization program was solved using 20 and 8 nodes, for the mixed integer program it was only possible to solve the problem for 8 nodes or less due to computer memory limitations.

2.4. Boundary conditions

Two reference cases were chosen as two distinct control situations can arise in a smart grid context. On the one hand we can assume the control has a strong power limitation. This results in run 1 (Figure 2(a) to 2(c)). On the other hand we can assume that strong variations in energy price may exist as would be the result of a very open electricity market. This results in run 2 (Figure 2(d) to 2(f)).

For both cases the demanded emission heat load is equal and varies in time as depicted in Figure 2(a) and 2(d). The heat pump COP with a supply water temperature of 35°C ($T_{gen,out}$) is only dependent on the ambient temperature and is shown in Figure 2(b) and 2(e). The emission system has a sinusoidally varying temperature between 25°C and 27°C (T_{em}). The storage tank is assumed to be located in an unheated technical room at a constant temperature of 15°C.



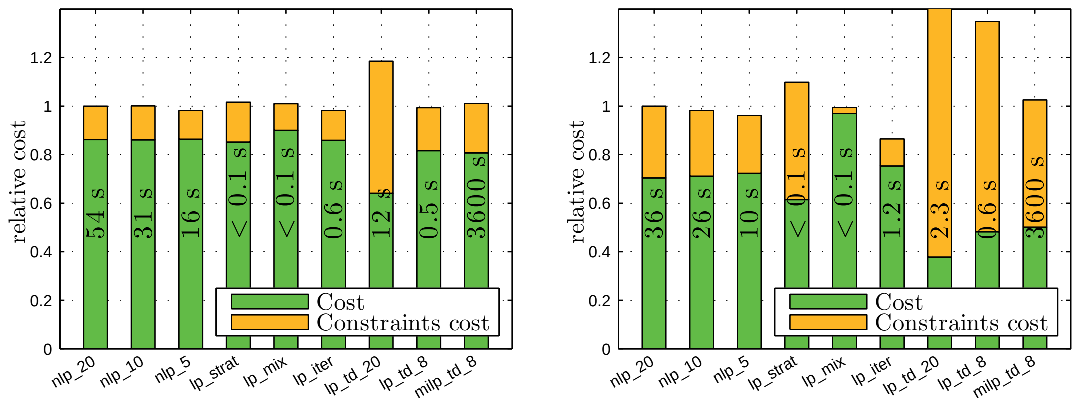
(a) Heat load and availability run 1 (b) COP run 1 (c) Electricity price run 1

(d) Heat load and availability run 2 (e) COP run 2 (f) Electricity price run 2

Figure 2: Reference conditions for all optimal control formulations

3. Results

Figure 3 shows the comparison of actual cost, cost related to constraint violations and average computation time for all calculated problem formulations. In all non-linear formulations the constraint violation cost, due to model mismatch, varies from 15% to 30% of the total cost while there are no significant differences in generation heat flow signals.



(a) Run 1 (b) Run 2

Figure 3: Comparison of energy cost, constraint violation cost and computation time for the different optimal control formulations

The rough approximation of the storage outlet temperature in the stratified linear program causes an increase in constraint violation. The mixed linear program on the other hand has much less constraint violation but the storage is not used to its full extent resulting in an overall cost similar to the non-linear programs. The iterative approach combines the full storage use of the stratified linear program with a better estimate of the outlet temperature resulting in a performance even better than the non-linear formulations.

Although the increased degree of freedom of the heat pump outlet temperature in the temperature discretization formulations results in a significantly lower energy cost, especially in run 2 with highly variable energy prices, this reduction is offset by the increase in constraint costs. Due to the possibility of multiple inlets, there is a large mismatch between control and simulation models. This also causes the model with less nodes to perform better as there are less inlets. The mixed integer formulation further reduces this mismatch but the assumption of a perfect inlet stratifier still causes a large constraint violation.

The computation times presented in Figure 3 are calculated as the 5 run average required cpu time to solve the optimal control problems on a 64bit 2.56Ghz Quad-Core machine. Although the computation times of the non-linear programs are acceptable, the presented times are strongly influenced by the initial guess supplied to the optimization solver. When due to an update in predictions this initial guess significantly differs from the optimum, the solution will require significantly more time. Thus from a robustness point of view, these formulations are depreciated.

The computation time of the mixed integer linear program was limited by the maximum allowed computation time. This problem takes much longer to solve due to the combinatorial nature of the used branch and cut algorithm.

4. Discussion

To explain the performance difference between stratified, mixed and iterative linear program, Figure 4 shows a schematic representation of the assumed storage outlet temperature profiles for discharging with constant heat flow rate. The limiting constraint when discharging the storage is the emission system inlet temperature which cannot drop below a certain value ($T_{em,in,min}$) as the heat must be delivered with a finite mass flow rate. When looking at a realistic outlet temperature profile, the constraint is activated at time t_2. Under the mixed linear program assumption the constraint is activated earlier (t_1) and under the stratified linear program assumption the constraint is activated too late (t_3). The iterative linear program tries to solve this by updating the minimum average storage temperature to a higher value when the heat flow to the emission system can't be obtained, thus reducing the gap between t_3 and t_2. This is not done a priori as the minimum emission inlet temperature is dependent on the emission heat flow rate which in practice is also part of the optimisation problem.

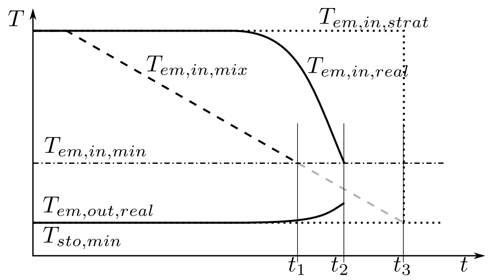


Figure 4: Schematic representation of the active temperature constraint on the emission side for the stratified and mixed linear program and the real temperature profile.

When implementing a model predictive control strategy, both the mixed and stratified formulation will result in a less optimal control and thus an increase in energy costs. In an MPC using the stratified formulation the state variables are updated each control step and the optimization will adapt to an imminent constraint violation leading to control actions at times when they are expensive, hence reducing the cost reduction attained through MPC. On the other hand, when using the mixed formulation, the storage is not used at its full capacity again leading to increased operational costs or an increased investment cost when an oversized storage tank is chosen. However, due to the moving horizon approach in MPC it is possible that the difference in costs between the ideal formulations and the iterative approach is less pronounced.

When considering the dynamic simulation of a building including model predictive control, the computation time of the stratified, mixed and iterative optimization problems are acceptable. With a control time step of 1 hour, all optimizations for a full year simulation would take around 2.5 hours on a modern personal computer in contrast to around 30 hours when using the simplest non-linear formulation. However, due to the increased complexity when including the building states and the emission heat flow as variables, this time will probably increase.

5. Conclusions

A comparative analysis between several optimal control formulations for use in model predictive control of buildings equipped with a heat pump and thermal energy storage tank shows a gap between slow but accurate non-convex formulations and fast but suboptimal or inaccurate linear formulations. A novel iterative linear program is proposed to bridge this gap and to include non-smooth phenomena, giving accurate, close to optimal results with a minimal additional computation time. This facilitates the use of MPC in systems using thermal energy storage in real-world demand response applications or in dynamic simulations during the system design phase.

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