

# Identification of Post-Necking Hardening Behaviour of Sheet Metal: Influence of the Yield Function.

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## 1. INTRODUCTION

Pioneering work of Bridgman [1] resulted in a solution for the problem of diffuse necking. Bridgman considered his method as a second level of approximation because the method is only concerned with the distribution of stress and strain across the diffuse neck. He also envisioned a third level of approximation, also referred to as the “complete solution” of the general problem, which takes the material state and the shape of the whole deforming specimen into account. Several researchers arrived at such complete solutions using finite-element based inverse methods. From a practical point of view, however, the coupling between the experimentally measured quantities and the numerically computed response can be a burden. Moreover, the iterative FE simulations to predict the plastic instability are usually very time-consuming. In order to avoid the shortcomings of the FE-based inverse method, a new method based on the complete solution to retrieve the strain hardening behavior hidden in the diffuse necking regime was presented in [2]. The key point in this method is that the strain hardening behavior can be identified by minimizing the discrepancy between the internal and the external work in the region where the diffuse neck develops. This method has been recognized as a special case of the Virtual Fields Method (VFM) where actual fields instead of virtual fields are used. A similar approach to identify post-necking strain hardening behavior of sheet metal using the VFM was recently published [3]. Clearly, both methods [2, 3] rely on the computation of the internal work, and, consequently, need access to the stresses associated with the experimentally measured strains. The latter requires a phenomenological material model and an appropriate stress updating algorithm. In [3] the potential plastic anisotropy of the sheet metal was ignored allowing the use of radial return mapping. In [2], however, plastic anisotropy was taken into account by adopting the r-based Hill48 yield function. In addition, it was assumed that an increase of the cost function caused by an error in the work hardening law is significantly larger than an increase caused by an error in the anisotropic yield criterion. The latter assumption is scrutinized in this contribution.

## 2. Method

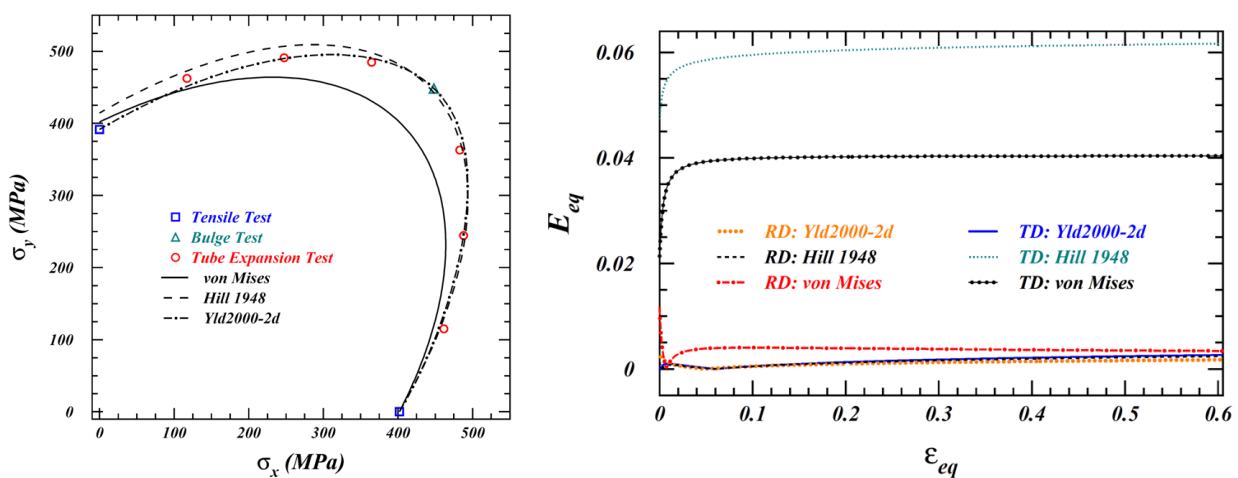
The identification methods [2,3] both require a phenomenological yield function to identify the parameters of the chosen hardening law. In order to investigate the sensitivity of the method presented in [2] with respect to the adopted phenomenological yield function, the Post-Necking Tensile Experiment (PNTE) is simulated using Abaqus/Standard. As such, the computed surface strains, elongation of the monitored diffuse necking zone and the tensile force are used as input for the identification method. This approach excludes all experimental errors and allows studying the impact of the yield function on the identified parameters. In order to have a realistic situation, the simulation of the PNTE was performed using an appropriate material model for a mild steel sheet with an initial thickness of 0.65 mm. This material was characterized in advance and the identified yield loci are shown in the left panel of Figure 1 for a reference plastic strain of  $\varepsilon_0^p=0.29$ . The contour of plastic work at this strain level was obtained through tensile tests, biaxial tensile tests, tube expansion tests [4] and a bulge test. It must be noted that potential differential work hardening is ignored in the simulation. It can be inferred from the left panel of Figure 1 that the Yld2000-2d yield function [5] is the most appropriate material model for the mild steel sheet used in this study. As such, this material model along with the extrapolated strain hardening behavior in the rolling direction (RD) is used to simulate the PNTE. The results of this simulation are then treated as experimental data and the post-necking hardening behavior is identified through the method presented in [2] using the selected yield loci shown in the left panel of figure 1. Although it is obvious to conduct the PNTE in the RD, it has been shown in [2] that it is also possible to retrieve the reference flow curve in the post-necking regime in the RD from a PNTE in the Transverse Direction (TD). To study the impact of the yield function in the latter case, the PNTE was simulated in the RD and the TD.

## 3. Results

The right panel of Figure 1 shows the relative equivalent stress error  $E_{eq}$  as a function of the plastic equivalent strain:

$$E_{eq}(\epsilon_{eq}) = \frac{|\sigma_{eq} - \sigma_{eq}^{ID}|}{\sigma_{eq}} \quad (1)$$

where  $\sigma_{eq} = 541(0.0036 + \epsilon_{eq})^{0.249}$  is the equivalent stress used in the simulation and  $\epsilon_{eq}$  the plastic equivalent strain.  $\sigma_{eq}^{ID} = K(\epsilon_0 + \epsilon_{eq})^n$  is the calculated equivalent stress using the identified hardening parameters  $K$ ,  $\epsilon_0$  and  $n$ . It can be inferred from this figure that  $E_{eq}$  for the PNTE in the RD is quite low for all material models. This is not surprising since all yield functions describe the material response fairly accurate in the vicinity of the stress ratio  $(\sigma_x:\sigma_y)=(1:0)$ . Theoretically, however, the strain state during diffuse necking approaches plane strain. In our simulations, however, the maximum stress ratio is about  $(\sigma_x:\sigma_y)=(5:1)$  and this gives rise to a small stress error  $E_{eq}=0.5\%$  if a von Mises material is assumed. The PNTE in TD, however, shows a large sensitivity with respect to the adopted the yield function. The right panel shows that if the correct material model is used (Yld2000-2d), the hardening behavior can be identified with great accuracy. However, if the yield surface cannot accurately describe the stress state in the vicinity of the stress state  $(\sigma_x:\sigma_y)=(0:1)$ , the identified strain hardening behavior will be incorrect. In this case, stress errors  $E_{eq}$  of the order of 4% and 6% are found if the von Mises and the Hill48 model are used, respectively. Indeed, the von Mises model shows a better prediction in the vicinity of the stress state  $(\sigma_x:\sigma_y)=(0:1)$  than the Hill model which results in a smaller  $E_{eq}$ .



**Figure 1. Left panel: identified yield loci. Right panel: Relative equivalent stress error  $E_{eq}$ .**

#### 4. CONCLUSION

The accuracy of the method presented in [2] clearly depends on the accuracy of the adopted yield surface. To be specific, the yield surface should be able to describe the material response in the vicinity of the stress state during diffuse necking. In this study, the stress ratio during diffuse necking was limited to approximately  $(\sigma_x:\sigma_y)=(5:1)$ . Between the stress ratios  $(\sigma_x:\sigma_y)=(1:0)$  and  $(\sigma_x:\sigma_y)=(5:1)$  the sheet metal in this study can be described by the von Mises yield function with sufficient accuracy. As such, a PNTE in the RD is not very sensitive to the selected yield loci. This is valid for the material under consideration in this study, but this is certainly not conclusive for other materials which can exhibit a stronger plastic anisotropy. Moreover, if biaxial stress states further away from the stress ratio  $(\sigma_x:\sigma_y)=(1:0)$  are probed in the diffuse neck, the accuracy of the yield function will gain importance. For the material under investigation the von Mises and the Hill48 yield cannot accurately describe the material response in the vicinity of the stress ratio  $(\sigma_x:\sigma_y)=(0:1)$  and therefore the PNTE in TD is very sensitive with respect to the adopted yield surface.

#### References

- [1] P.W. Bridgman, *Studies in Large Plastic Flow and Fracture*, McGraw-Hill, NY, 1952.
- [2] S. Coppieeters, S. Cooreman, H. Sol, P. Van Houtte, D. Debruyne, Identification of the post-necking hardening behavior of sheet metal by comparison of the internal and external work in the necking zone, *Journal of Materials Processing Technology*, 211 (3), pp. 545-552, 2011.
- [3] J.-H. Kim, A. Serpantié, F. Barlat, F. Pierron, M.-G. Lee, Characterization of the post-necking hardening behaviour using the virtual fields method, *International Journal of Solids and Structures*, 50, pp. 3829-3842, 2013.
- [4] T. Kuwabara and F. Sugawara., Multiaxial tube expansion test method for measurement of sheet metal deformation behavior under biaxial tension for large strain range, *International Journal of Plasticity* 45, pp. 103-118, 2013.
- [5] F. Barlat, Brem, J.C., Yoon, J.W., Chung, K., Dick, R.E., Lege, D.J., Pourboghra, F., S.H., Choi, E., Ch, Plane stress yield function for aluminum alloy sheets-part 1: theory, *International Journal of Plasticity* 19, pp. 1297-1319, 2003.