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## **Improved human-computer interaction for mechanical systems design through augmented strain/stress visualization.**

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**Abstract:** Strain/stress evaluation is a crucial operation performed during several stages in the typical mechanical product development/life cycle. However it is often difficult to obtain an appropriate estimation of the strain and stress distribution due to the difficult to model operational conditions, unknown input forces and/or parameters. This makes it particularly difficult for a designer to evaluate whether the final product under testing meets all the operational design specifications. This work presents an extended Kalman filtering approach to obtain accurate strain and stress estimates of a structure under operational loading. This information is exploited in an augmented reality application to visualize strains and corresponding stresses on a real component. This provides a very efficient human-computer interface to evaluate strains and stresses on a physical prototype. This approach is therefore very suitable to improve the design of components due to the good overview of the performance. In order to obtain an efficient formulation, the developed approach is based on the exploitation of reduced order mechanical models based on high fidelity finite-element design models. By including a parameterization in the reduced model, the approach can be made robust with respect to unknown operational parameters, like boundary conditions. The obtained paradigm is validated on a flexible beam with unknown input forces and length. The proposed approach permits a more natural visualization and interpretation of operational conditions. Our results encourage the adoption of the proposed approach not only for design validation but also on-line monitoring of structural components, opening new possibility in the field of Augmented Reality for Maintenance.

**Keywords:** Augmented reality, mechanical system, strain/stress estimation, kalman filter.

**Reference** to this paper should be made as follows: Naets, F. and Cosco, F. and Desmet, W. (xxxx) 'Improved human-computer interaction for mechanical systems design through augmented strain/stress visualization.', *International Journal of Intelligent Engineering Informatics*, Vol. x, No. x, pp.xxx-xxx.

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## 1 Introduction

Strain/stress evaluation is a crucial operation performed during several stages in the typical mechanical product development/life cycle. In particular, in the early stages when computer aided (CAx) tools are used, offline finite-element simulations allow a designer to evaluate strains and stresses even before the actual manufacturing took place. By visualizing the simulation results pattern over the component structure, designers have an immediate overview of the results with spatially localized information regarding eventual structural failures, which in turns they may try to solve with focused design modifications. However, at the later stages of the product development cycle (i.e. system level validation, then operational monitoring), it is often difficult to obtain an appropriate estimation of the strain and stress distribution due to difficult to model operational conditions, unknown input forces and/or parameters. This makes it particularly difficult for a designer to evaluate whether the final product under testing meets all the operational design specifications.

On the one hand, the designer can obtain a wealth of useful information (such as stress/strain distributions, etc.) by means of computer simulations, where usually loads and boundary conditions partially represent reality. On the other hand, during testing of the physical prototype only a (very) limited number of measurement points can be obtained, whose proper selection demand for expertise. Further difficulties arise from the efforts required to effectively compare the two sets of results coming from numerical and experimental test.

This paper proposes a suitable approach to avoid these challenges. The proposed solution involves two consecutive stages: first, estimating the strain distribution from the model and the tests, then visualizing the estimated quantities with some advanced interface able to mimic the numerical simulations results.

In particular, a reduced finite element model with an Extended Kalman Filter (EKF) is adopted for on-line estimation, which allows the recovery of the full field deformation of the product under test, even with the limited number of measurement points. By also estimating parameters, besides the states, the methodology can be made robust with respect to changes in the operational conditions (e.g. mounting stiffness). Finally, the operational strains and stresses are presented to the designer by adopting the Augmented Reality (AR) paradigm: 3d full strain/stress field results, semantically similar to simulation results, are augmented over the structural components.

To ensure real-time performance, the mechanical model is reduced in two stages:

- reduction of the linear dynamics through a projection reduction,
- reduction of the nonlinear parameter dependence through an interpolation based method.

This approach couples well with an extended Kalman filter (EKF) because the interpolation with respect to the parameters enables an easy analytical evaluation of the Jacobian of the equations of motion.

Because the original model for the filter is a finite-element model, it can be exploited for computing the strains in the component. The states which are estimated are used in conjunction with the shape function of the elements to compute the strain distribution. This information is then forwarded to a screen-based video see-through display which allows the visualization of the strains on top of the actual component.

The proposed approach is validated experimentally on a laboratory setup. This setup consists of a cantilever beam with adjustable length. The states and inputs should be estimated for this setup and the unknown parameter is the length of the beam. For the measurements an optical tracking system is employed and a strain gauge is mounted for obtaining a reference value. In the future the optical tracking system should be integrated with the AR camera information which would lead to a very compact and portable system.

## **2 Related work**

This paper brings together a range of different methods presented in literature, from efficient mechanical modeling to Kalman filtering and augmented reality, in order to provide a new approach to evaluate prototypes and series mechanical products. The proposed approach allows a user to get information that was previously only available through offline model evaluation from operational measurements. This can greatly increase the understanding of the performance of a given system for a designer.

In order to recover the strain distribution, the deformation field of a component has to be estimated, which depends on the input forces and parameters. Unfortunately there is no straightforward approach to measure input forces on a general structure because the introduction of (expensive) dedicated force cells typically requires alterations to the structure for locating the sensor in the force path. Obtaining force values from indirect measurements, like calibrated strain gauges, requires a good knowledge of the system parameters, which

might be unknown as well. The same issue appears when forces are estimated through a Kalman filtering approach with a constant model and strain or acceleration measurements [1, 2]. Typical techniques for estimating physical parameters in structural dynamics, on the other hand, require a known input [3]. Many techniques for parameter estimation are frequency-based, but this also implies that the model is excited over a sufficiently broad frequency band. Several authors have proposed time-domain state estimation approaches, such as Kalman filtering, for parameter estimation in operational conditions [4, 5]. As mentioned before the measurement of inputs might not be feasible in operational conditions, especially when online monitoring is the goal. In this work, an augmented discrete extended Kalman filter (A-DEKF) approach is adopted, coupled to a first principles physical model in order to meet the requirements of coupled state/input/parameter estimation with online applicability [6].

The extended Kalman filter is a nonlinear variation on the regular Kalman filter, based on a linearization for a given configuration [7]. The unknown inputs and parameters are estimated by adding them as augmented states to the state vector. This allows a simultaneous estimation of both the inputs and the parameters, such that no particular approximation needs to be assumed with respect to the relation between these variables. In the past this augmented approach has been adopted for either state/input [2, 8] or state/parameter [4] estimation, and has only recently been exploited in a fully coupled sense [6]. This work focuses on exploiting physical models rather than data-driven models. This allows a much closer connection between the different quantities which have to be estimated.

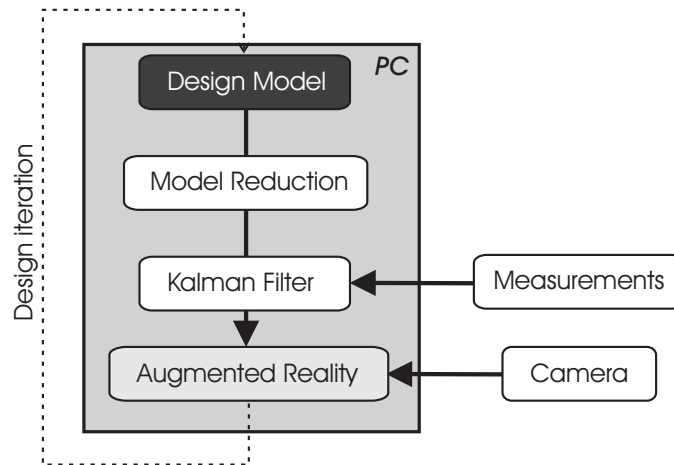
Milgram and Kishino [9], introduced the concept of a virtuality continuum in order to classify Mixed Reality applications. According to their definition, AR refers to any case in which an otherwise real environment is augmented by means of virtual objects. Later on this definition was extended [10] with the following aspects:

- virtual artifacts are spatially registered in three dimensions;
- AR is interactive in Real-time.

Interested readers may find useful information about recent advances of AR in [11, 12], while [13] gives an overview of Industrially relevant applications of Augmented Reality. In the past, the advantages of Augmented Visualization was already exploited for industrial applications, like early stage design usability validation [14, 15], also related to Virtual or Mixed Prototyping techniques.

In this paper, the results coming from system validation tests and/or live monitoring operations are presented to the designers/engineers through the AR interface, which is recognized to be a non-traditional but revolutionizing human-computer interface (HCI) for several application domains. From a user perspective [16], the main advantages are related to the possibility of gaining immediate access to a wide range of location-specific information [17]. In higher education, recent studies demonstrated that students felt more motivated, impacting positively on their learning performance [18, 19]. With a similar success, adoption of AR is under investigation in several industrial fields like logistics [20], manufacturing [21, 22] and monitoring of assembly lines [23].

In [24, 25], an AR framework for inspection and visualization during fatigue tests was successfully adopted. By sharing the same advantages deriving from the use of AR as the HCI, this paper proposes an efficient scheme for simulating the deformation of structural components in real-time, thus enabling the inclusion of the structural behavior in the virtual prototyping experience, for the ultimate purpose of design validation. At the same time,



**Figure 1** Augmented reality for design of mechanical systems process overview

the proposed approach could boost the adoption of AR for a wide range of industrial applications, in particular in the field of inspection and maintenance [13]. In fact, recent work showed that AR-based maintenance operations are performed up to fifty percent faster than in the traditional way [26], or can be realized remotely, with a drastic reduction in terms of after-sale support related costs [27].

For obtaining strain distributions Digital Image Correlation (DIC) has been used and this technique enables the full-field sensing of the strain distribution. However, the methodology proposed in this paper is more widely applicable than the DIC by overcoming two main drawbacks: the requirements of a specific surface finish; and the expensive instrumentation.

### 3 Approach

This section discusses the exploitation of advanced design models for the augmented reality visualization of applied strains and stresses, thus reducing the costs related to the corresponding testing efforts. This is in service to an improved human-computer interaction for the design process, where the actual applied loads to a physical prototype can help to get a better understanding of the possible hotspots in the design. This process is summarized in Fig. 1.

In the proposed approach a detailed mechanical design model (typically a finite-element model) is available. However, typically it is difficult to apply realistic loads in simulation, like in the case of dynamic interaction forces in machinery or road loads in vehicles. Therefore, it is beneficial if a physical prototype can be analyzed with the same possibilities for visualization (stress and strain visualization) as a numerical model. To this end, the proposed approach consists of several steps:

- First, the model is reduced in order to enable an efficient evaluation, and then is also parameterized in order to take varying boundary and environmental conditions into account for the physical prototype.
- After the reductions the model can be coupled to acquired measurements through a Kalman filtering approach.

- Finally, the information from the Kalman filter can be visualized as a digital overlay over the real structure, by means of AR.

The user can then use this operational information in order to iterate on the design of the system.

In the coming subsection we will focus on the different operations which are performed in the computer, a pc in this case. The measurement and camera acquisitions are not treated in detail here as many commercial systems are available for these purposes.

### 3.1 *Design model and model reduction*

A key point in evaluating strains in a mechanical component is the use of appropriate models. In this work a two step modeling approach is adopted:

- *Full model*: a high fidelity finite element (FE) model is constructed which offers good accuracy and a parameterized description for the unknown parameters.
- *Reduced model*: the original model is too computationally expensive to be properly exploited in a Kalman filter setting. In order to obtain a more suitable description a model order reduction has to be performed on the FE model which allows faster evaluation and accurate strain descriptions.

The following two subsections discuss these two modeling steps in more detail.

#### 3.1.1 *Full model*

The equations of motion for a linear finite element model with  $n$  degrees-of-freedom (DOFs) are:

$$M(p)\ddot{q} + C(p)\dot{q} + K(p)q = B(p)F. \quad (1)$$

In this equation  $q$  are the DOFs of the system and  $M(p)$ ,  $C(p)$  and  $K(p)$  are respectively the mass-, damping- and stiffness matrix as a function of the parameters  $p$ , and  $B(p)$  is the loading matrix for the external forces  $F$ . In this work all variables (DOFs, inputs and parameters) are assumed as possibly time varying, so the time-dependency is not denoted explicitly. From a FE method it is relatively straightforward to obtain equations for the strains. The coordinates  $y$  of a point on a body can be obtained as a function of the FE coordinates  $q$  through a shape function  $N$  which is a function of the position  $s$  on the body:

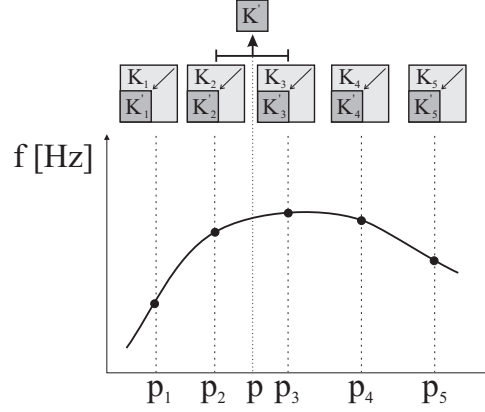
$$y = N(s)q. \quad (2)$$

The strains  $\epsilon$  in different directions can then be computed as the derivatives of these coordinates with respect to the desired direction:

$$\epsilon = \frac{\partial N(s)}{\partial s}q. \quad (3)$$

This projection matrix can be precomputed for the positions of interest and allows a fast evaluation of the strains.

Unfortunately it is not practically feasible to use the full FE model in real-time applications due to the large number of DOFs and the high frequency content in the



**Figure 2** Parameterized model order reduction scheme

model. Over the past decades a wide variety of model reduction techniques for second order systems, such as structural dynamics systems, have been proposed in literature [28]. However, an additional issue is considered in this work. In practice, one or more parameters of a system are typically unknown. Material properties, mounting conditions, etc. can be strongly varying depending on the condition of the structure and cannot always be measured a-priori or might change over time. These parameter variations have a strong impact on the strain distribution and should also be taken into account in the estimator. It is relatively straightforward to get the parameter dependency between the original finite element model and several parameters, but this becomes difficult in the case of a reduced model. For this reason, specific parametric model order reduction techniques are developed [29]. This is discussed in the next subsection.

### 3.1.2 Reduced model

The most general class of parametric model order reduction techniques is based on a sampling of the parametric space, performing a linear model reduction on each sample and interpolating between the samples during simulation [29]. This process is illustrated in Figure 2. The main differences between approaches in this category are in the way the linear reduction and interpolation are performed. In this work the aim is on structural dynamics systems, which are second order systems. In order to perform the local linear reduction, a free-free modal approach is adopted. The original system has  $n$  degrees-of-freedom (DOFs) and through the reduction process only  $n_{red}$  DOFs are retained. The reduced model is obtained through a projection on a truncated modal space [6]:

$$q = \Psi q' \quad (4)$$

By mass-orthonormalizing the modes, the reduced system matrices become:

$$M = I, \quad (5)$$

$$K' = \Psi^T K \Psi, \quad (6)$$

$$C' = \Psi^T C \Psi, \quad (7)$$

$$B' = \Psi^T B. \quad (8)$$

With these matrices, the equations of motion can be evaluated for the reduced system, which drastically reduces the computational load. This reduction is twofold:

- less DOFs leading to faster evaluation of the equations of motion;
- lower frequency content, allowing larger timesteps for the integrator.

Due to this reduction, the equations of motion can be evaluated faster than real-time, such that they can be used for online estimation [30]. This reduction scheme can also be applied for computing the strains in the body:

$$\epsilon = \frac{\partial N(s)}{\partial s} \Psi q'. \quad (9)$$

This allows for a faster reduced strain evaluation.

To evaluate the system for Kalman filtering, it has to be transformed into first order form. With state-space vector  $x$  for this system, the first order equations are:

$$\dot{x} = \begin{bmatrix} \dot{q}' \\ \ddot{q}' \end{bmatrix} = \begin{bmatrix} 0 & I \\ -K' & -C' \end{bmatrix} x + \begin{bmatrix} 0 \\ B' \end{bmatrix} F \quad (10)$$

At this point the reduction is only considered for one certain parameter. For the parameter dependency, a sampling with interpolation approach is employed. The system matrices are evaluated for a grid of possible parameters and for each parameter  $p_i$  a modal reduction space  $\Psi_i$  is computed such that each local model can be reduced. During the simulation an interpolation is performed between the reduced models for the different parameters. In this work linear interpolation is used on the reduced matrices in order to minimize the computational load [6]. Moreover, in the case of mass-orthonormal modes this also leads to a linear interpolation of the reduced eigenvalues of the system. For a given parameter  $p$  between  $p_i$  and  $p_{i+1}$  the interpolated reduced system matrix is:

$$A = A_i(p_{i+1} - p) + A_{i+1}(p - p_i)/(p_{i+1} - p_i). \quad (11)$$

For the use in the extended Kalman filter, also the derivatives of the equations of motion are required. By using the linear interpolation, these derivatives can be readily obtained. With these equations the parameterized reduced model can be evaluated and the Kalman filtering can be performed. The effect of the model reduction schemes employed on the accuracy of the estimates is part of future research. As will be discussed in the next section, the equations of motion need to be time discretized in order to perform Kalman filtering.

### 3.2 *Extended Kalman filter*

In this work the extended Kalman filter (EKF) [7] is selected for the joint estimation problem of the states, inputs and parameters [6]. The EKF is an extension of the regular linear KF by considering a linearization of the nonlinear system around the current configuration. For the propagation of the states the nonlinear equations are used but the propagation of the covariance is linearized. The definition of the Kalman gain originates from the minimization of the trace of the state covariance matrix. In the equations the measurements are taken into account. The measurement noise is considered stationary in this work. Several other Kalman based filters exist which could be exploited in this application, but the EKF provides



good performance in this application at a minimal cost because the required Jacobians are readily available.

In this work the unknown input forces and unknown parameters are added to the state vector in order to be estimated. This leads to the augmented state vector  $x^*$ :

$$x^* = \begin{bmatrix} x \\ F \\ p \end{bmatrix}. \quad (12)$$

With this state vector, a model is required for the augmented states  $F$  and  $p$ . A random walk model is employed for both states [6]:

$$\dot{F} = 0x + r_F, \dot{p} = 0x + r_p. \quad (13)$$

For the random walk model there are two uncertain noise inputs  $r_F$  and  $r_p$  with which respectively the force and the parameter can vary. A high uncertainty means that a state can show strong fluctuations while a low uncertainty creates a rather constant state. This uncertainty also returns in the Kalman filter, in the form of the covariance, as will be discussed later, and serves as a tuning parameter. Since the external forces can show strong variations,  $r_F$  is typically high. The unknown parameters however typically have a relatively slow variation and therefore  $r_p$  should be chosen relatively small. In the experimental validation of the proposed approach, it will be demonstrated that the results from the filter are rather insensitive to the exact values of these uncertainties as long as  $r_F$  is chosen sufficiently large and  $r_p$  sufficiently small. The full augmented system equations can now be used in the Kalman filter in the form:

$$\dot{x}^* = A(x^*)x^* + r. \quad (14)$$

It is important to notice that these are nonlinear equations because the matrix  $A$  is now dependent on the augmented state vector  $x^*$  through parameter  $p$ .

### 3.3 Augmented Reality

This section details the algorithm adopted for implementing the proof-of-concept AR system, which was used to demonstrate the benefits of using AR for visualizing operational strain conditions of structural components. The hardware needed for the AR system consists of a fixed video camera, a commodity PC and a monitor.

The whole AR pipeline is divided in two phases: pre-operational and operational. The pre-operational stage is used to initialize the system. In this stage, a camera calibration is executed for each new installation of the system (i.e. for every new position of the camera) which consists of the following steps:

- store a snapshot of the video depicting the components as the calibration picture;
- choose the global frame of reference;
- define the mesh of the component(s), i.e. the list of the nodes, the position of each node vertex with respect to the global frame of reference, the list of the elements, and the type of each elements to be drawn (triangles, quad, etc.);

- define a set of fiducials, i.e. an ordered set of geometry points (i.e. vertex of the structure, and/or the scene);
- for each fiducial the user must define the 3d position w.r.t the global frame of reference;
- pick the position of each fiducial on the calibration image stored at step 1: this permits extracting the coordinates of each fiducial expressed in the camera frame.
- a nonlinear fitting procedure is executed to recover the *extrinsic* parameters of the camera, which define the relative displacement between the camera and the global frame of reference and the components, and the *intrinsic* parameters of the camera, which control the projection from the camera frame of reference to the image space.

The operational stage adopts a screen-based video see-through display pipeline, which can be outlined as follows:

- a video stream of the real structure is captured by means of the fixed camera;
- current image is extracted from the stream and rendered on the background;
- update the estimation based on the EKF, as described in the previous sections;
- set the camera position transformation;
- augment the estimated quantities by doing the following for each element of the mesh:
  - retrieve updated position of each node of the element
  - retrieve the magnitude of the strain for each node
  - render the element assigning to each node the corresponding nodal quantities: updated position and color corresponding to desired colorspace meaning;
- the obtained augmented image is swapped to be visualized on the screen display.

By using a fixed camera, the proof-of-concept implementation does not require a tracking module. However, the described pipeline can be extended in order to allow motion of the camera, by inserting a tracking update step just above the setting of the camera position transformations.

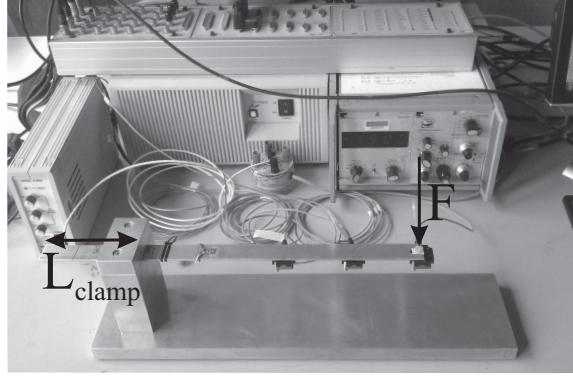
The described algorithm was implemented as a *c++* application, relying on OpenGL programming. Kalman Filter processing was embedded in the same application exporting the Matlab code as a *c++* library by exploiting the Matlab Compiler Toolbox.

#### 4 Example

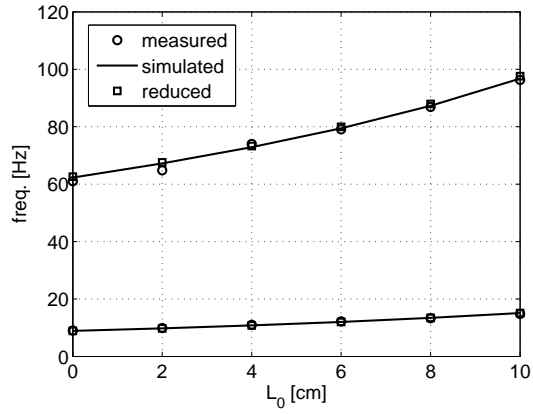
For validating the presented method an experimental validation is performed. The validation is performed on a cantilever beam with adjustable length [6], as shown in Figure 3. In this setup the variable clamping length is used as the parameter for estimation. Furthermore, an unknown tip load is applied by pushing manually.

The model for the Kalman filter is based on a finite element mesh with Euler-Bernouilli beam elements of length  $L_e = 0.005m$ . For this beam model, the bending strain for an element  $\epsilon_e$  can be computed as a function of the nodal displacements of that element  $q_e$ :

$$\epsilon_e = \left[ -6/L_e^2 \quad -4/L_e \quad 6/L_e^2 \quad -2/L_e \right] q_e. \quad (15)$$



**Figure 3** Cantilever beam for validation



**Figure 4** Measured and model (unreduced and reduced) eigenfrequencies

As discussed before, the model is reduced through a parametric modal reduction. In this case the clamping length, which can be easily adjusted on the setup, is used as a parameter. The discretization step is  $\Delta L_{clamp} = 0.005m$ , which corresponds to a discretization where each node is clamped consecutively. The linearized model for a given parameter is reduced with three eigenmodes. The correspondence for the first three eigenfrequencies of the model and the actual test setup is given in Figure 4. This figure shows that the model behavior is close to the measured system behavior. The damping behavior is modeled as stiffness proportional modal damping. With the choice of three reduction modes, the model complies with the requirement on the minimal number of degrees-of-freedom for the model for observability [6]. The estimation augmented state vector is:

$$x^* = \begin{bmatrix} x \\ F \\ L_{clamp} \end{bmatrix}, \quad (16)$$

and the model uncertainty covariance for the extended model is chosen as:

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1e2 & 0 \\ 0 & 0 & 1e-6 \end{bmatrix}. \quad (17)$$

In this example all uncertainty is assumed to be on the external force and the clamping length of the beam. Apart from these unknowns the model is well tuned, such that other state uncertainties are negligible with respect to the extended states. In general however this might not be the case and further tuning of the covariance matrix might be required. It is necessary to choose the force variance sufficiently high. If this value is chosen too low, the algorithm will try to shift the effect of fast force inputs to a change in parameter, which is clearly not the desired behavior. Correspondingly, a sufficiently low variance for the change in parameter has to be selected in order to counteract this effect. In most cases where actual parameter variation takes place this is a reasonable assumption because this variation is typically rather slow. In cases where fast parameter changes are expected, an alternative modeling approach should anyhow be adopted in order to take the proper dynamic coupling between the parameters and the motion of the system into account.

Three position measurements are obtained from an optical tracking system. The optical measurement system is a Nikon Metrology K600 ([www.nikonmetrology.com](http://www.nikonmetrology.com)) system. The K600 uses three linear cameras and synchronized LEDs as markers. The sample time for the position measurements due to the measurement devaluations is limited at  $\Delta t = 1.4ms$ . This also shows the need for stable time-integration methods for the model because it might need to run at relatively large time steps. This system has an error covariance of  $0.256e-9m^2$ . In order to also obtain a reference for the strain in the system, a strain gauge is added. The strain gauges are placed in a full Wheatstone bridge over the beam and the signals are conditioned through an analog conditioner before being fed to a dSPACE system. For the AR system, a Sanyo Xacti VPC-HD2 is used to capture the image in a resolution of  $1280 \times 720$  pixels at 30fps. For this proof of concept the image is transferred posteriori to the processing PC for augmenting the image. In future work a high-speed camera which streams the image directly will be employed.

For a dynamic excitation, the estimated and reference forces are given in Figure 5. This figure clearly shows the good response of the filter with respect to parameter differences. The forces are also reconstructed very accurately. Figure 6 shows the comparison of the strain registered by the strain gauge and that obtained through the estimator. The strains obtained from the estimator with the optical tracking system are very close to the measured strain with the strain gauges. It is important to mention that instrumenting a body with strain gauges is not straightforward and the proposed approach thus provides a viable alternative.

The estimator can now be used to compute the strains of the full body and this can be used in an AR setting to obtain the strain distribution at different time instants, as shown in Figure 7. In the AR case, the position of the beam is adjusted based on the length estimate, the input force is visualized with an arrow and the strain is projected with a color-scale, according to the scheme discussed in Sec. 3.3. This evaluation can be performed in continuous fashion, which leads to a dynamic visualization of the strains in the system during operation ([www.youtube.com/watch?v=VeBYQBy8K5o](http://www.youtube.com/watch?v=VeBYQBy8K5o)).

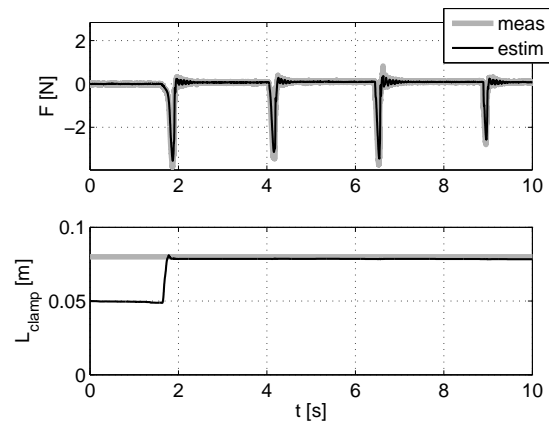


Figure 5 Measured and estimated force and length

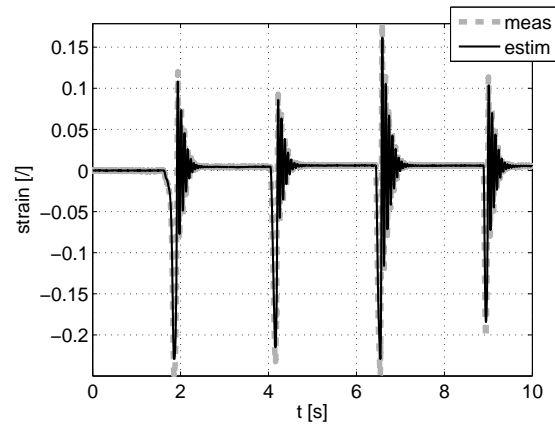


Figure 6 Measured and estimated strain

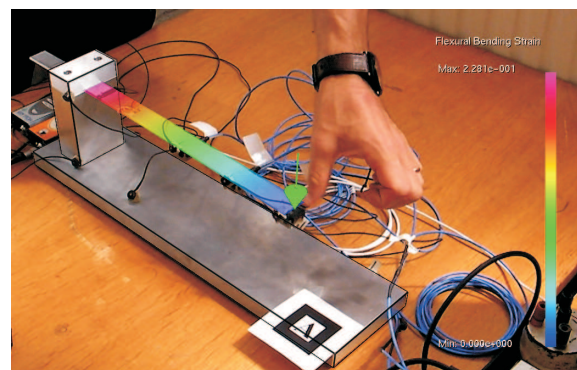


Figure 7 View of the beam with estimated strains augmented onto the view.

## 5 Conclusion

This work shows that it is possible to extract detailed operational information on mechanical systems by exploiting reduced models in a Kalman filtering approach where an optical tracking unit provides measurement data. This work proposes to exploit this information in an augmented reality application to visualize strains and corresponding stresses on a real component. This approach leads to a highly intuitive and efficient human-computer interface to evaluate stresses in an operational context on mechanical components. This technique can have a very beneficial impact on the design cycle where stress-concentrations can easily be found on physical prototypes such that the design can be adjusted accordingly. The use of physics based models allows a straightforward tuning and reliable results from the Kalman filter and enables the computation of derived variables such as strain from the displacement estimates. In order to obtain good efficiency, an appropriate model reduction technique is crucial. In order to obtain robustness with respect to uncertain parameters and unknown inputs, the Kalman filter also estimates some parameters and input forces. This approach is validated experimentally on a flexible beam example, and the strain estimates are shown to provide good accuracy.

## Acknowledgment

This work benefits from the Belgian Programme on Interuniversity Attraction Poles, initiated by the Belgian Federal Science Policy Office (DYSCO). The IWT Flanders within the MODRIO project is also gratefully acknowledged for their support. The authors also acknowledge the support of ITEA2 through the MODRIO project. The research work of Frank Naets is financed by a grant of the Research Foundation - Flanders (FWO). We gratefully acknowledge the European Commission for their support of the Marie Curie IAPP project "DEMETRA" (Design of mechanical transmissions: efficiency, noise and durability optimizations), with contract number 324336; see <http://www.fp7demetra.eu>. The authors also acknowledge the support of the Flanders Make innovation network.

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