Economic growth and Business cycles: a pendulum movement between neoclassicals and Keynesians

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Academic Year 2014/2014

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Introduction to this course

The below Figure 1shows the natural logarithm of an industrial production index of the United States from 1919 to 1996. Over these 78 years, this natural log has increased by 2.7, meaning that the index itself has become e2.7 or almost 15 times higher!

Figure 1: logarithm of the industrial production index

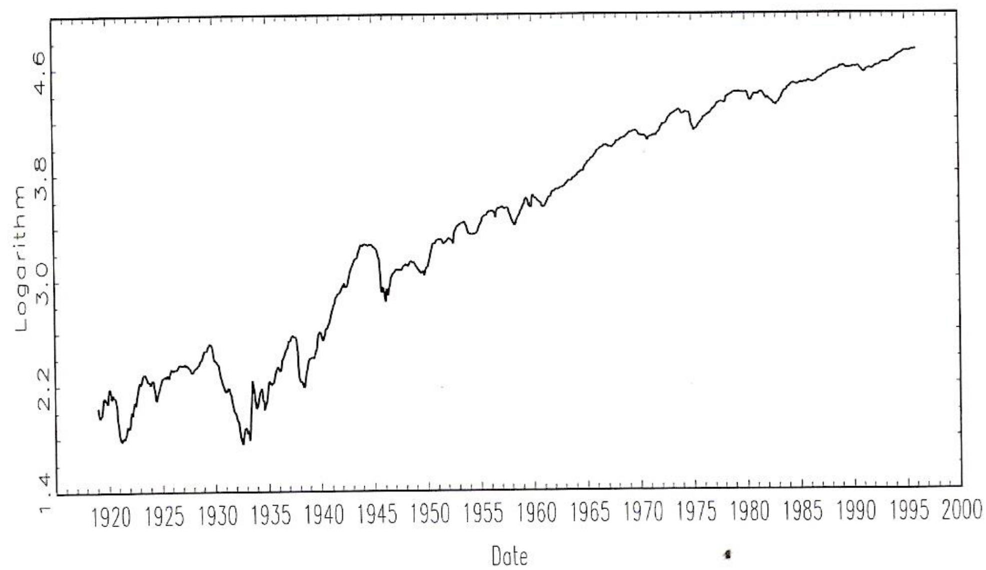

Fig. 1.1. Industrial production index (logarithm of levels).

Source: Stock and Watson, 1999, Figure 1.1., page 5.

The series in this Figure 1 also show the largest periods of increases and declines that the US (and therefore the world) has known. The recession of the 1930’s, the growth during WWII, the sustained growth during the 1960’s (also associated with spending on the Vietnam War), the Oil Crises during 1973-1975, the disinflation recession of the early 1980’s followed by a long period of expansion, the recession of 1990 associated with the invasion of Kuwait by Iraq and the long expansion during the 1990’s (Stock and Watson , 1999). All in all, the index does not grow in a stable fashion, but also contains cyclical fluctuations, also known as business cycles. And these phenomena is what is course is about; we will start our discussion by considering a model of growth. Next various theoretical models of business cycles will be presented and discussed.

But the difference is more fundamental than that. The growth model that we start with, describes long-term growth of an economy in equilibrium. In contrast, discussing business cycles means that we will depart from the previous notion of long-term economic growth, i.e. growth in steady state or in equilibrium. For, in a Walrassian world of equilibrium prices, economic agents possess all information, namely through the price level. They can therefore realize all wanted transactions, and cycles do not occur, other than resulting of random shocks. If and when agents do not have access to all information free of costs, then some agents will find that they cannot realize all wanted transactions, for prices will not be in equilibrium. However, the fact that some agents will be rationed in their transactions is information by itself. As a result, there will be an adjustment process and this might introduce a cyclical process in the economy

So cycles are the results of discrepancies between wished and actual transactions, or –put differently- because the expectations or anticipations of agents do not become true. Now theories differ in to how these discrepancies between wishes and results come about. These theories can be categorized by how they consider the price mechanism. A first broad categorization is the flexprice view and the fixprice view, or equilibrium theory and disequilibrium theory.

The most extreme representative of the flexprice view are the theories of rational expectations and the theory of real business cycles. These theories –which will be explained in more detail further in this reader- assume that prices are fully flexible, and that markets therefore clear. However, this does not necessarily mean that all anticipations will become reality. The hypothesis of rational expectations does stipulate that all agents have all available information. However, information on erratic disturbances is by definition not available a priori and due to these disturbances, differences between anticipations or expectations and realizations will occur, though they will not be systematic. So, cycles in this flexprice view are caused by random and exogenous disturbances, for example unexpected behavioural changes of sellers or buyers, or natural disasters. So business cycles are not a signal of disequilibria, but of an economy adjusting to recent unexpected shocks. If there are no more shocks, then the oscillations will eventually stop and the economy will stabilize on its steady state path.

The fixprice view of disequilibrium theories start from the thesis that prices are not flexible in the short run, and that disequilibria on the goods- and labour markets therefore occur. This is the most common approach to business cycles. As Mankiw (1992, 215) puts it "in the long run, prices are flexible and therefore can respond to changes in supply or demand. In the short run, however, many prices are "stuck" at some predetermined level".

So, contrary to the Walrassian flexprice approach, agents trade with disequilibria prices, which implies that the quantities of goods or labour actually traded are the minimum of supply and demand. So, either suppliers or demanders of a good or labour will not be able to buy or sell the anticipated quantities.

Another important consequence is that disequilibrium (or the difference between anticipations and results) on one market will cause the same agents to reconsider their anticipations on other markets. For example, agents offering labour are the same agents that buy consumer goods. So if there is a demand shortage on the labour market, there will be unemployment. The unemployed will not receive the income that they anticipated, and they will consequently decrease their demand for consumer goods. As a result, there will be a demand shortage on the goods market.

This is one of the basic points of the theory of Keynes, as we will see later on. As a result of these fixed or sticky prices and the non-clearing markets which follow from it, a difference between anticipations and realizations can not only occur due to random shocks, as in the flexprice view, but can also occur systematically.

Another goal of this course is to explain the development that economics as a discipline has gone through. Starting from (neo)classical flexprice model, the crisis saw the start of Keynesian, fixprice, economics. During the 1970s, flexprice models again came on the forefront, cumulating to extreme neoclassical models where economic fluctuations is the result of continuous technical shocks. It is only recently that neo-keynesian models are on the way back. In describing this pendulum movement between flexprice and fixprice approaches, the goal of this course is to familiarize the student with key thoughts in the economics discipline.

We start the course by discussing the Neoclassical line of thought, best represented by the models by Solow and Swan.

Economic growth in the long run: the Solow model of exogenous growth[1].

The goal of the Solow model –or any growth model- is to find out how growth in the capital stock, in the labour force and advances in technology interact, and how they affect output.

The supply of goods

The supply of goods is the production function, relating output (GDP) to the input factors capital and labour. Thus Y=F(K,L). Assume constant returns to scale and write everything in relative size of the working force. So y=f(k,1) where y=Y/L and k=K/L. These are called **per capita values[2]**.

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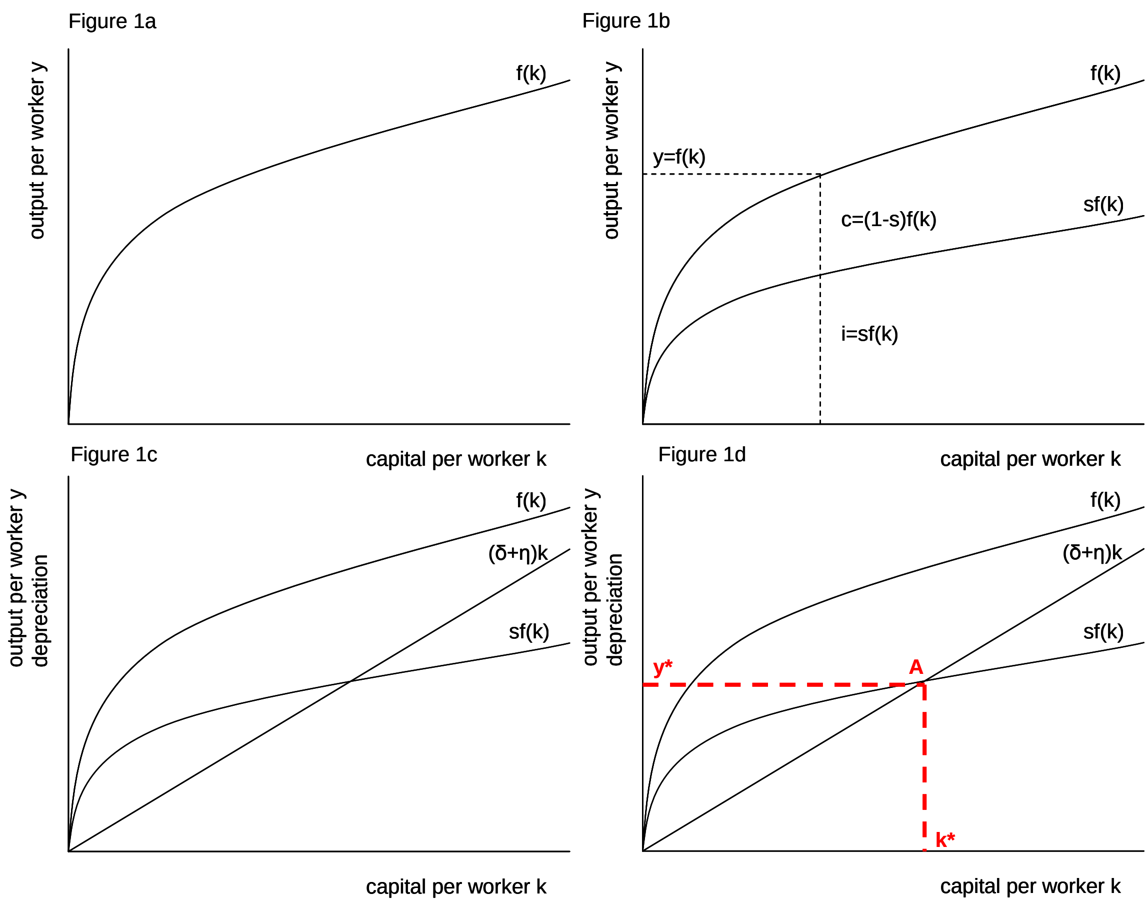
[1] What follows is based on Mankiw, 1992. Note that a neoclassical model was simultaneously developed by the Australian Trevor Swan. Both papers by Solow and Swan were published in 1956, but Swan's was published ten months later –and in an Australian journal. Swan's version of the model however included a more complete description of technical progress, which Solow discussed in a 1957 paper. Dimand and Spencer (2008) argue that it was ultimately by accidental factors that Swan has not been recognized as much as Solow. Many recent papers however refer to the neoclassical model as the "Solow-Swan model".
[2] The assumption of constant returns to scale is crucial. It means that multiplying both input factors by a constant factor x will increase output by that same factor. So if you double capital and labour input, output will double as well. If you do not assume constant returns to scale, you obviously cannot express the model in per capita terms, since dividing the input factors by L (K/L and L/L) will only result in output being equal to Y/L under the assumption of constant returns to scale. This assumption is not undisputed. Many

This production function is presented in Figure 2a below. The slope is the marginal product of capital; which denotes how much extra output per worker we get from one extra unit of capital per worker. This effect of the capital stock on output is given by the Marginal Productivity of Capital

MPC = f(k+1) – f(k)

wich is the *slope* of the production function f(k) or df(k)/k. This MPC follow the so-called Inada conditions (df(k)/k=∞ if k=0; df(k)/k=0 if k=∞) which means that the marginal productivity of capital MPC *decreases* with capital k.

Figure 2: The building blocks of the Solow model



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more recent models assume positive returns to scale, i.e. f(xK,xL)>xY. This assumption reflects the idea that large firms can work more efficiently than small firms, for example because some machines, transport utilities or storage facilities have a minimum size. A real example of this are big oil tankers or bulk carriers that are more cost-efficient per litre of oil transported, than small oil tankers.

This means that, given the number of workers, each extra machine adds less to output. Possible causes for the decreasing marginal productivity of capital are, for example, that supervision, handling or maintenance of the machines becomes more difficult when the number of machines grow. When there are two workers and one computer, then a second computer will have a very large impact on output. A third computer will however have a much smaller effect on output, because our two workers already are working on a computer. If the employer would buy 20 computers for these two workers, then the last computers will not be handled and will thus add nothing to output[3].

This assumption of a decreasing MPC is crucial in the neoclassical (exogenous growth) growth model, because without it, a steady-state equilibrium does not exist. When we come to discuss endogenous growth models, this assumption will be replaced by one where the marginal productivity of capital remains constant, or increases.

The demand for goods

Demand for goods comes from consumption and investment. So,

$y = c + i$

which is the national accounts identity. The consumption function in the Solow model takes the simple form

$c = (1 - s)\,y$

where s is the savings rate, with $0 \leq s \leq 1$. Substitute this in the national accounts identity, to get $y = (1 – s)y + i$ and rearrange to

$i = sy$

So investments and consumption are both proportional to income (or output) as shown in Figure 2b.

The steady state

[3] Note that marginal productivity and returns to scale are *not the same* and the hypothesis of decreasing marginal productivity and constant returns to scale are not in conflict. Returns to scale refer to the situation where *all* inputs K and L are changed by the same amount. This is the case here, because a equal proportional change of K and L will keep k constant. The marginal productivity hypothesis pertains to marginal changes of *one* input factor while the other remains constant (and k therefore changes).

We have seen that changes of output are related to changes of the capital stock. This means that a steady state, a situation were there is no growth of per capita output, is also the situation that the per capita capital stock remains unchanged.

The capital stock per capita k=K/L changes as a result of
- Changes of the capital stock (the numerator K):
  - Increases as a result of investments
  - Decreases as a result of depreciations and
- Changes of the labour force (the denominator L)

Formally: $\Delta k= i$ - depreciations

We know that $i = sy$ and $y=f(k)$. Hence, $i = sf(k)$ and $c = (1 – s)f(k)$. The higher the level of output y, the higher investment i and consumption c. How a change of the capital stock per worker k is 'translated' into investment and consumption is determined by the savings rate s.

$\Delta k=sf(k)$ – depreciations

Next to investments, the capital stock per worker decreases as a result of depreciations and an increase of the labour force. Assume a simple depreciation function where depreciation is a constant fraction δ (delta) of the capital stock. As k is the per capita capital stock, it will also decrease with L. Assume furthermore that the population (and also the labour force) increases by a rate η (eta). Figure 2c shows a straight line with angle *(δ+η)* to represent depreciations.

So, the change of the capital stock in per capita units is the balance of increases resulting from investment per capita and decreases resulting from depreciations and the growth of the labour force[4].

$\Delta k=sf(k) – (\delta + \eta)k$

Think of *(δ+η)k as break even investment*, i.e. the amount of investment necessary to keep the per capita capital stock constant if there are no savings and therefore no investments.

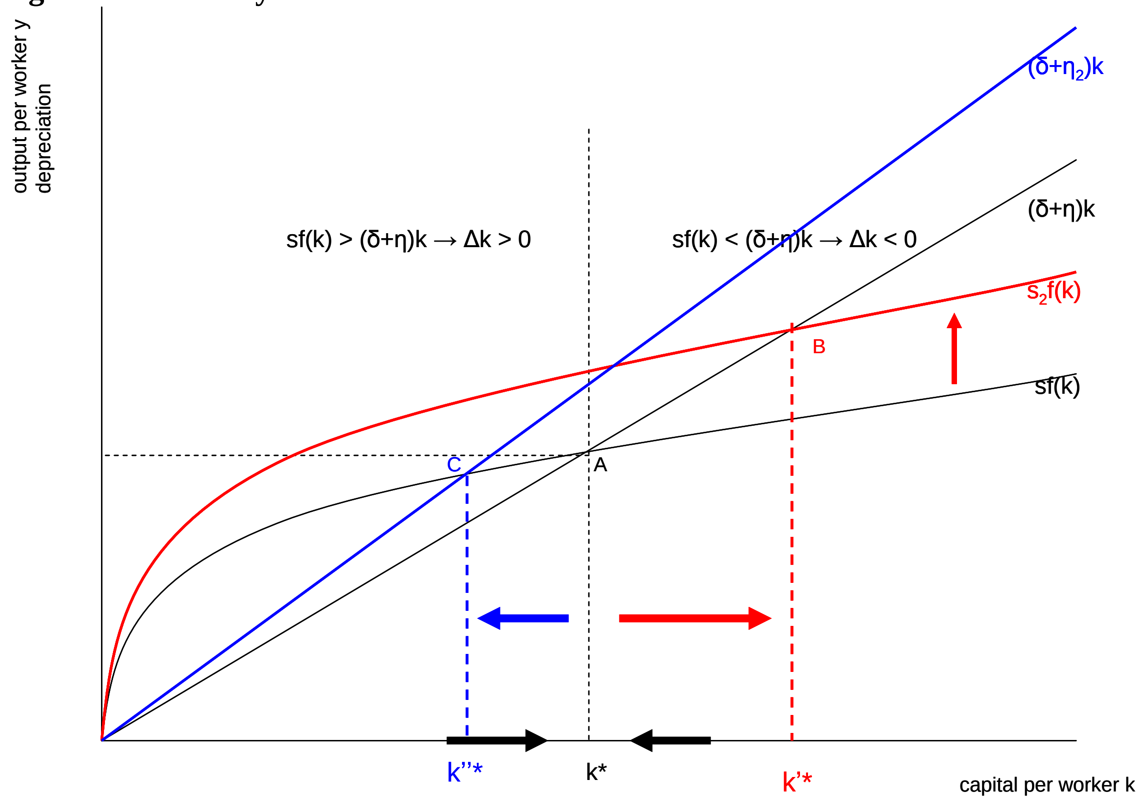
It now becomes clear that there is a **multiplier effect between output and the capital stock**. Output induces investments, investments increase the capital stock, the capital stock increases output by the MPC and this again results in more output. This effect of the capital stock on output is given by the Marginal Productivity of Capital, which is the

[4] To see this, note that the logarithmic differential, or the proportional change over time of k equals (dk/dt)(1/k). Start by (dk/dt)(1/k)= (dk/dt)(1/k). Rewrite the right hand of this equation to ((dK/dt)-(dL/dt))(1/k) or (dK/dt) (1/k) - (dL/dt) (1/k). Now assume no investments, then the (dK/dt) (1/k) equals – δ and the growth rate of labour relative to k (dL/dt) (1/k) equals η. Hence ((dK/dt)-(dL/dt))(1/k) = (dk/dt)(1/k) = – (δ + η). Rewrite this to (dk/dt)= – (δ + η)k or Δk=– (δ + η)k. See the website of Bill Gibson (2014) for a simple overview on the rules in growth rates.

slope of the production function f(k). The magnitude of this multiplier depends on the savings rate s and the MPC. Due to the decreasing marginal productivity of capital MPC, a positive effect of the capital stock on output (and hence investment) eventually comes to a halt. Furthermore, the higher the capital stock, the higher depreciation.

In the long run, the capital stock will reach its steady state value *k**, where Δk=0 because investments sf(k) equal depreciations (δ+η)k. This is in point A in Figure 1c and Figure 3.

Figure 3: the steady state in the Solow model.



The steady state is where Δk=0 and hence Δy=0. So, ***the per capita agregates are constant over time****.*

The Solow model is a static model: If the parameters remain unchanged, the economy will eventually settle down in an equilibrium or steady state where the capital stock and output in per capita terms remain unchanged over time.

In equilibrium or steady state, investment has two purposes:

1. to replace depreciated capital
2. to provide new workers with the steady state level of per capita capital

So, if saving rates and depreciations do not change, output per capita is constant. The population growth rate η (eta) hence implies that output in nominal terms increases, for the capital stock increases in nominal terms to make the per capita capital stock constant.

This is easy to see: $\Delta y=0$ or $\Delta(Y/L)=0$

- ⇨ $\Delta Y - \Delta L=0$
- ⇨ $\Delta Y = \Delta L$

Multiply both parts by (1/k) because the figure 1 is all expressed in terms of k

- ⇨ $\Delta Y(1/k) = \Delta L(1/k)$
- ⇨ $\Delta Y(1/k) = \eta$

As a conclusion: in equilibrium or steady state, output *per capita* (or standard of living) is constant. The only factor explaining persistent output growth is the growth rate of the labour market.

A practical example

Suppose a production function where $y = \sqrt{k}$. So if *k*=9 then *y*=3 or if *k*=16 then *y*=4. Furthermore, suppose that30% of income is saved and depreciations equal to 10%. Hence s=0.3, and δ=0.1. Finally, suppose no population growth (η=0). What will be the steady state values of k and y?

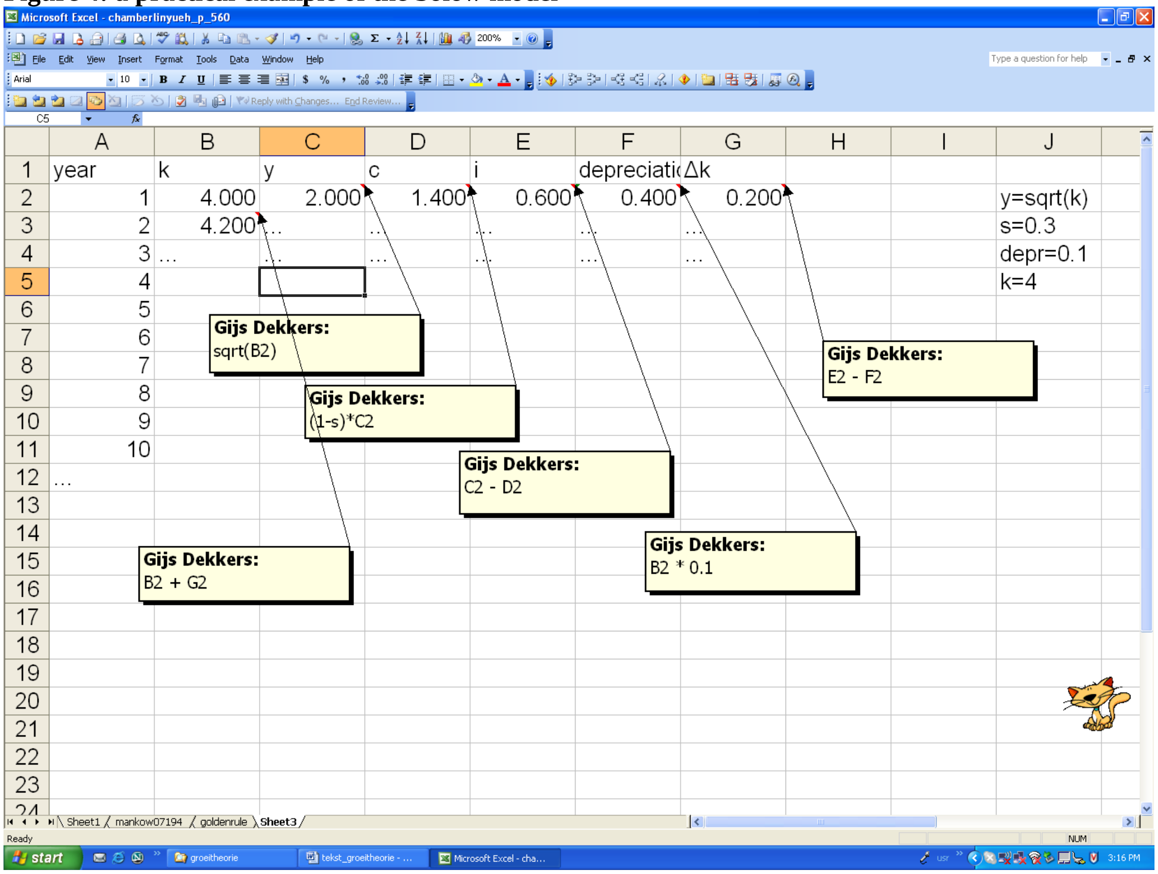
The steady state condition is $\Delta k=0.3*\sqrt{k} - 0.1*k=0$ or $k/\sqrt{k} = 3 \leftrightarrow k^2/k = 9$. So the steady state value of k=9 and that of y is therefore 3.

How does an economy reach its steady state[5]?

To see this, let us go back to the previous practical example, and apply this in Excell, as follows

[5] Formally speaking, if lim(k)↓0 then f(k)>0 because f'(0)=∞, and $(\delta+\eta)k=0$.So $\Delta k>0$. Likewise, if lim(k)↑∞ then f(k)↑k^{max} ($k^{max} <\infty$) because f'(∞)=0. Also $(\delta+\eta)k=\infty$.So $\Delta k<0$. As a result, a point must exist where $\Delta k=0$, and this equilibrium is stable.

Figure 4: a practical example of the Solow model



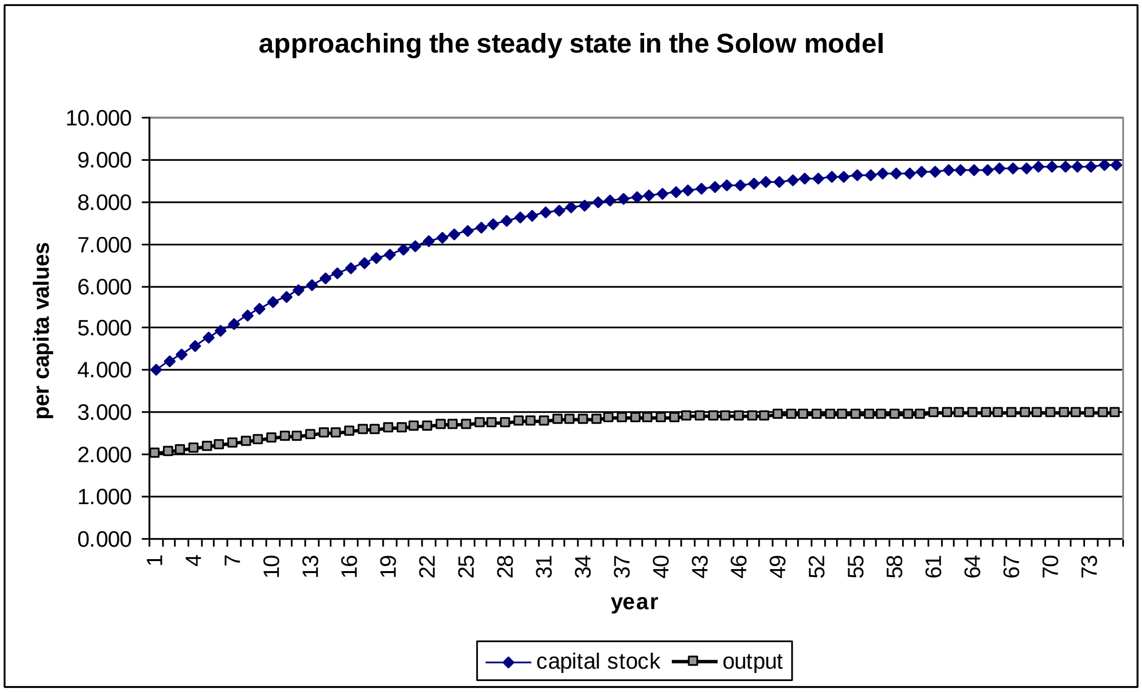
In the above example, the starting value of k is 4, which is lower than the steady state value 9. This is the first of the two possibilities.

1. The starting level of the economy is below the steady state level.

Suppose it starts with less than the steady-state level of capital k<k*. In this case, investments sf(k) exceeds depreciations (δ+η)k and the capital stock will rise. Due to the capital-output multiplier, output will rise as well. Due to the decreasing MPC, the increase of output will however diminish and eventually come to a halt. The increasing k will furthermore induce higher depreciations. The increases of the capital stock k will slow down and continue until it reaches k*. At this point, the multiplier has become so small (due to the decreasing MPC) that any further investments will increase depreciations such that the capital stock will decrease. So k will not grow further than k*.

In short, we should see that the capital stock k and output y increase towards their steady state values. The below Figure 5 shows that this is indeed the case.

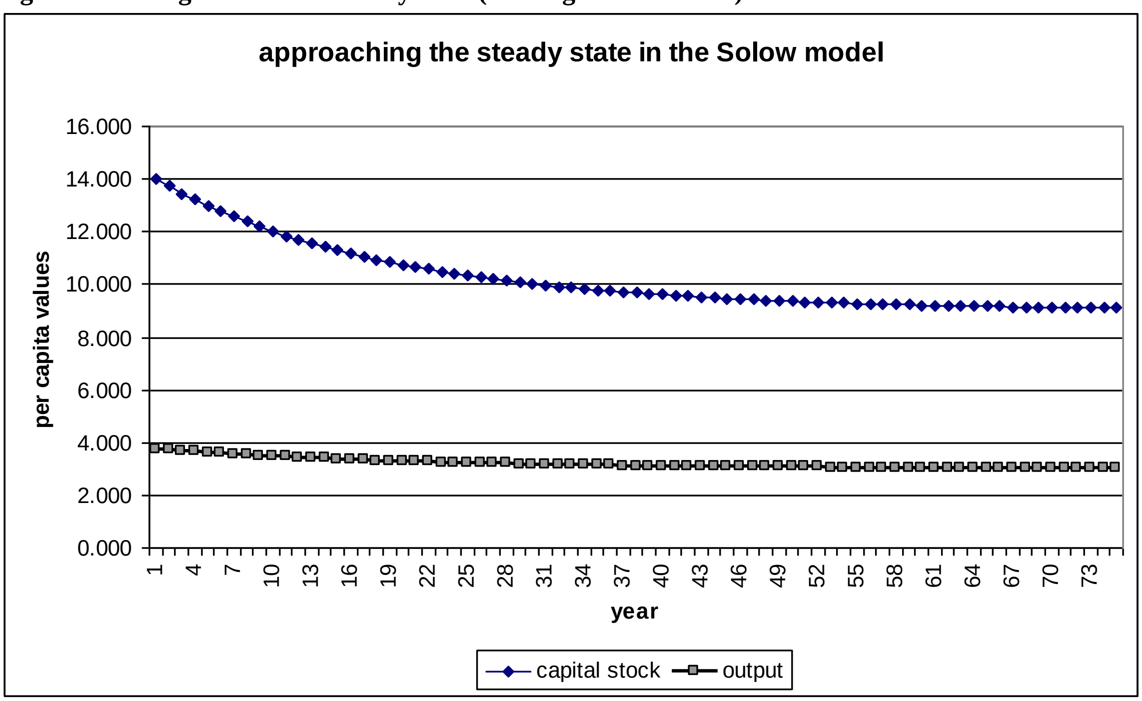
Figure 5: moving towards the steady state (starting value of k=4)



Now what happens if we change the starting value of k in the above model to 14? This second possibility is shown in Figure 6.

2. The starting level of the economy is above the steady state level.

Figure 6: moving towards the steady state (starting value of k=14)



Now the convergence process works in the opposite direction, obviously. In this case, the capital stock is high and depreciations are high as well. Investments sf(k) is less than depreciations (δ+η)k and the capital stock will decline until it reaches k*.

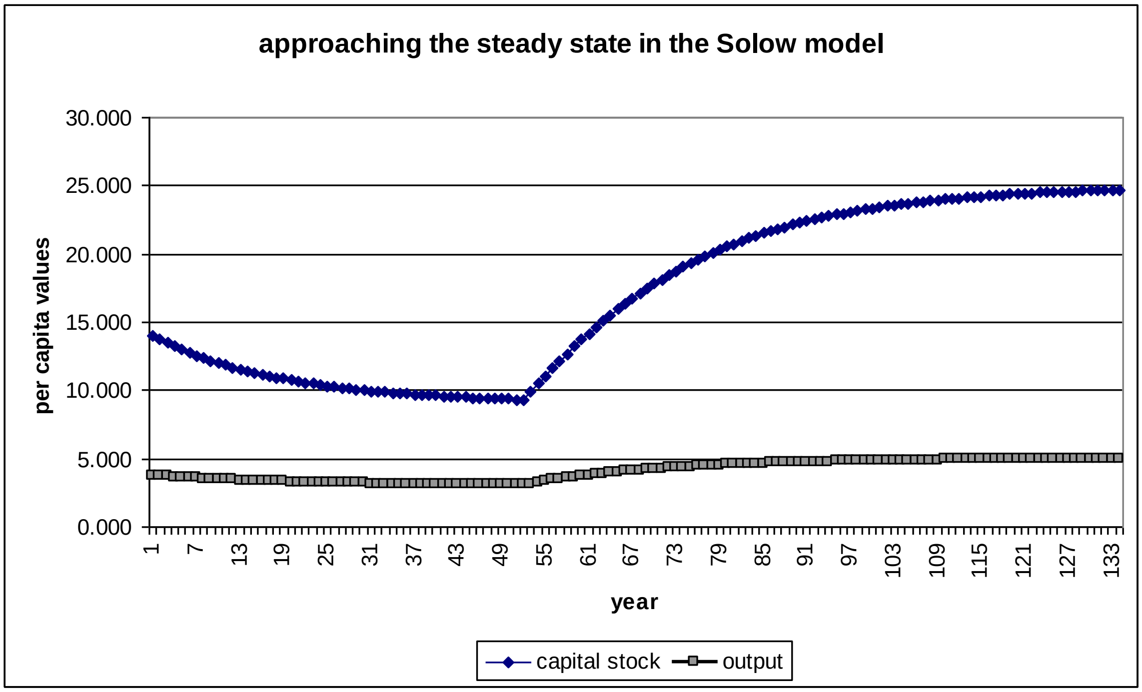
The effect of changes of the savings rate and population growth rate

The fundamental parameters of the Solow model are the savings rate and the growth rate of the population. What is the effect of a change of these fundamental parameters? Or, what is the difference in steady state between two countries with different parameters?

1. What is the difference between two countries with a different savings rate? Or, what will happen if the savings rate increases?
    a. The steady state level of capital per capita k* will increase to k'* in Figure 3. The new investment function intersects with the depreciation line in point B. In the practical example (with the savings rate being 30%), the ss-value of k was equal to 9. Now suppose that the savings rate increases to 50%. The new steady state condition is Δk=0.5* $\sqrt{k}$ – 0.1*k=0 or $k/\sqrt{k} = 5 \leftrightarrow k^2/k = 25$. So the steady state value of k increases from 9 to 25 and that of y therefore increases from 3 to 5.

    b. In the short run, a change of the savings rate will cause the growth rates of the capital stock and output per capita to be positive, until the new steady state levels are reached. In steady state, the *levels* will be higher, but the *growth rates* will be the same (namely zero). For example, suppose that in the above Excell-version of the Solow model, the savings rate increases from 30% to 50% in period 52. The results are seen in Figure 7.

Figure 7: moving towards the steady state (an increase of the savings rate in period 52)



Analogous to this, suppose two countries A and B. Country A has a savings rate of 30% and country B a savings rate of 50%. Furthermore, both countries are in steady state. *The country B with the higher savings rate will have a higher level of output and capital stock per capita, compared to country A, but the growth rates of k and y will be the same in both countries! (namely zero).*

2. What is the difference between two countries with a different population growth rate η? Or, what will happen if the population growth rate increases?

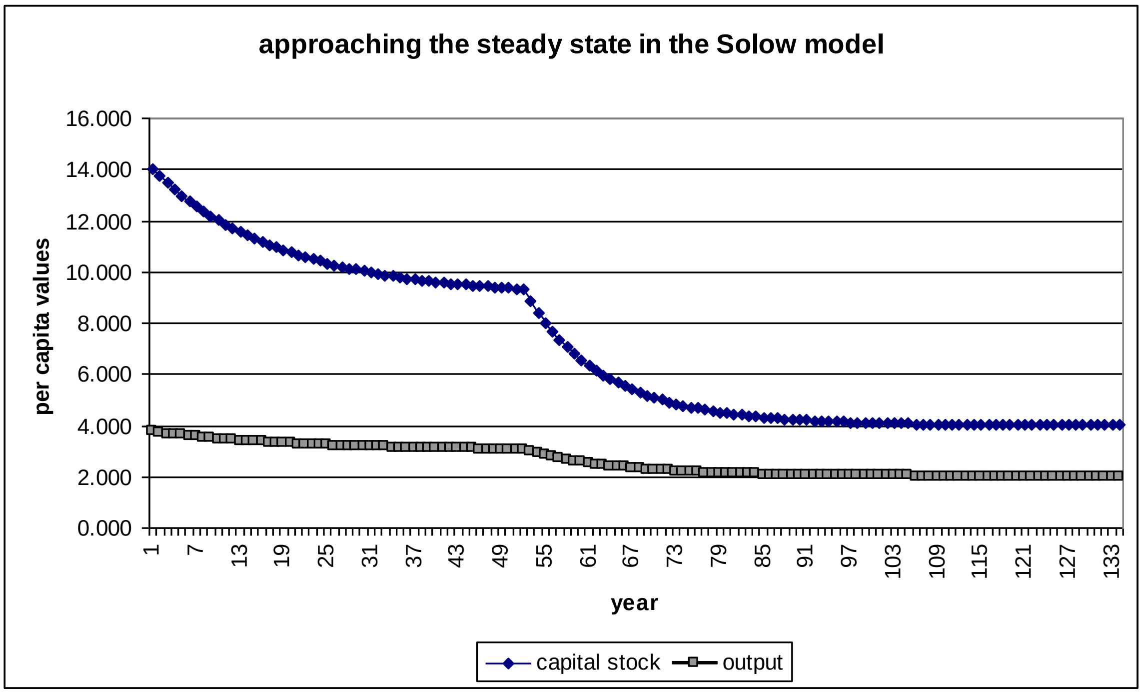
To see what happens, start again from the practical model where $y = \sqrt{k}$, s=30%, δ=10%. The steady state level of k is 9 and of y is 3. Now add population growth (η=5%) to this model. What will be the steady state values of k and y?

In the previous version of the model, break even investments (investments needed to keep k constant, Δk=0) was equal to depreciations. The capital stock is however written in per captita terms, so to keep k=K/L constant, the minimum investments should cover depreciations (10%) and the growth rate of the population (5%). The steady state condition thus becomes Δk=0.3* $\sqrt{k}$ – (0.1+**0.05**)*k=0 or $k/\sqrt{k} = 2 \leftrightarrow k^2/k = 4$. So the steady state value of k decreases from 9 to 4 and that of y therefore decreases from 3 to 2. You can see the same happening in Figure 3: the steady state level of per capita capital k* will decrease to k''* in Figure 3 (the equilibrium shifts from point A to point C), and so will the steady state level of output per worker. *A country with higher population growth will have lower levels of output per person.*

In steady state, the growth rate of y (output per capita, or output per person) remains zero, so the growth rate of **nominal** output Y will increase to compensate for the higher population growth rate. *A country with higher population growth will have higher growth rate of output Y. But the growth rate of y, output per worker or per capita, will remain zero in the steady state.*

The growth rate of the population affects depreciations, so we can show the effect of this increasing population growth rate by increasing depreciations in the above practical Excell model. Suppose that the rate of population growth increases from 0 to 5% in period 52. The results are seen in the below

Figure 8: moving towards the steady state (an increase of the population growth rate in period 52)

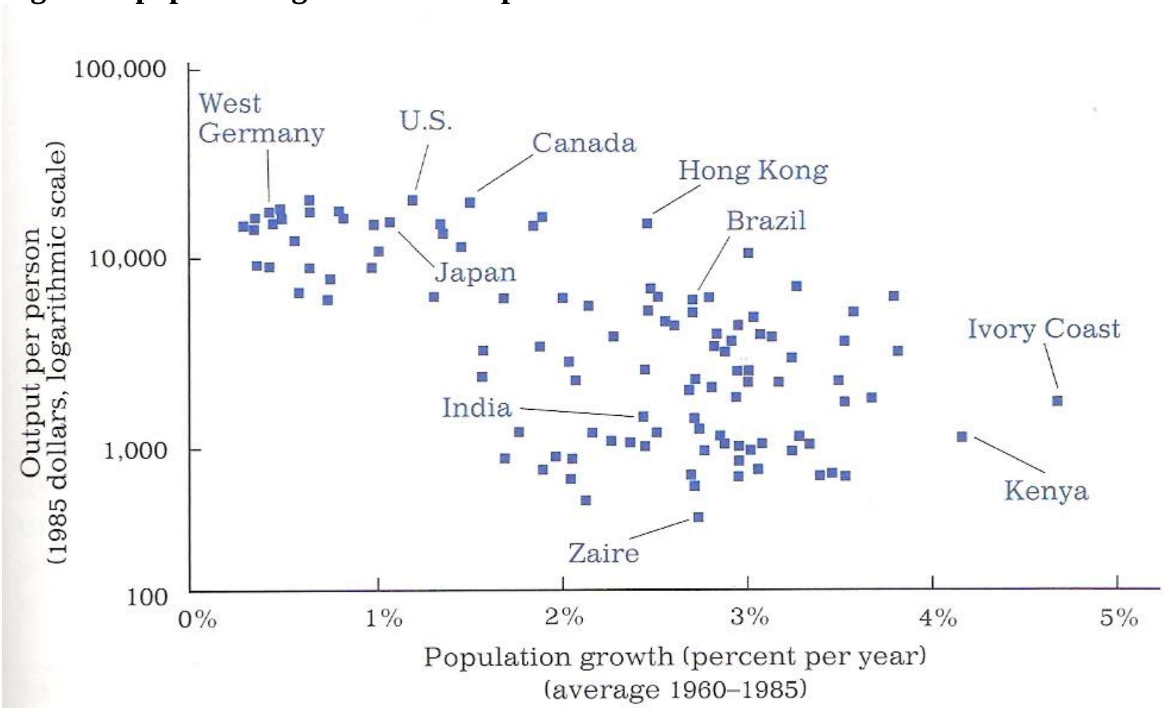


As before, you see that the model was converging towards the 'old' steady state values (k=9, y=3). In period 52, however, population growth increases and the steady state values of k and y therefore decrease. The model converges towards these lower steady state values. In the new steady state, and as before, the growth rates of the per capita values are again zero.

On the whole, the Solow model predicts that a country with a *higher growth rate of the population* than another country will have a *lower level of output per person*, and the same per capita growth rates as before (namely zero). However, this implies that the not-per capita values K and Y have increased to cover for the higher population growth rate.

According to the Solow model, a country with a higher rate of population growth will have a lower level of per capita capital stock and output. In other words, a high population growth will impoverish a country, because it is difficult to maintain a certain level of capital stock per worker if the number of workers is increasing rapidly. Figure 9 shows a scatterplot of 112 countries.

Figure 9: population growth and output



Source: Mankiw, 1992, page 99.

It roughly confirms the Solow model in that countries with a higher population growth rate tend to have a lower standard of living. In a 1990 paper published with the National Bureau of Economic Research, Mankiw, Romer and Weill studied a dataset of in total 195 countries over the years between 1960 and 1985. They find that saving and population growth affect per capita income in the direction that Solow predicts. Moreover, these variables explain more than half of the cross-country variation in income per capita. However, the effect of savings and population growth appears too large in the data, as compared to what the Solow model predicts. The authors conclude that a third variable, human capital, which is positively correlated to physical capital, accounts for this difference. The Solow model does not take human capital into account, so that its effect becomes visible in the estimator of savings and population growth.

Before ending this discussion on the effect of population growth on economic growth, it should be noted that not only changes in the population growth rate, but also of the population structure can affect economic growth. Indeed, demographic ageing not only causes the growth rate of the population to decrease, but also changes the age structure of the population. In 2008 paper in the Review of Income and Wealth, Rafael Gomez and Pablo Hernandez de Cos find that an increase of the proportion of prime age workers (aged 35-54) causes per capita GDP levels to be higher. This is because their productivity is highest (combination of recent schooling and experience). Based on a large international panel dataset over the last four decades, they show that a 5% increase in the ratio of working age persons can account for roughly one-quarter of per capita GDP. Furthermore, they suggest that an optimal ratio of prime age workers exists, since there are positive but diminishing returns of this ratio on per capita GDP. This turning point is where there are roughly 0.95 workers aged between 34 and 54 for every single worker aged 15-34. They call this 'roughly one mentor for every mentored worker'.

The golden rule level of capital[6].

We have seen that an increasing savings rate will in the short run increase the growth rate of per capita output, and will in the long run result in a higher level of output per capita.

You might think that saving everything (s=1) and consuming nothing is optimal, because this would maximize the steady state output per worker, i.e. the *level* of the economy. However, having no consumption at all is not good either. The optimal savings rate is not that which maximizes output per worker, since workers do not care for output, but only for consumption.

Suppose that you have the power to change the savings rate s. We seek that steady state level of capital that maximizes consumption.

Remember that

$$y = c + i$$

So

$$c = y - i$$

In the steady state, $y^*=f(k^*)$ and $\Delta k=0$ so $sf(k^*) - (\delta+\eta)k^*=0$. Since $i=sf(k)$, this steady-state condition becomes $i - (\delta+\eta)k^*=0$ or $i = (\delta+\eta)k^*$.

So

$$c^* = f(k^*) - (\delta+\eta)k^*.$$

So steady-state consumption is what is left of steady-state output after paying for steady-state depreciation. A higher level of capital has two opposing effects on consumption: more capital means more output, so more consumption. But more capital means more depreciation, so a higher part of output must be used to replace old capital, so less consumption. There is only one level of capital – the Golden Rule level of Capital- that maximizes steady-state consumption c*. Call this c**.

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[6] This paragraph is a.o. based on Mankiw, 6th ed., 2007, section 7.2, page 198.

So consumption c** is the maximum of c*. So

$$c^{**}=\text{MAX}\{ f(k^*) - (\delta+\eta)k^*\} \rightarrow c^{**}= f(k^{**}) - (\delta+\eta)k^{**}$$

Or k** is that level of capital k* where the vertical difference between y=f(k**) and (δ+η)k is maximal.

The first-order condition for the maximum is d(c*)/dk*=0

$\rightarrow d(c^*)/dk^* = df(k^*)/dk - (\delta+\eta)=0$

$\rightarrow df(k^*)/dk^* = (\delta+\eta)$

$\rightarrow MPC = (\delta+\eta)$

$\rightarrow MPC- \delta = \eta$

Marginal productivity of capital MPC minus the depreciation rate equals the rate of population growth

This is known as the *golden rule of capital*

As shown in

Figure 10a, the Golden Level k** is where the slope of the production function equals that of the depreciation function.

If k* ≠ k then c* is not at its maximal value, as is the case in**

Figure 10b.

Increased capital has two effects on steady state consumption

1. It causes greater output, and hence higher consumption
2. It causes higher depreciation and hence lower consumption.

If k* < k** then the savings rate s is below the optimal level. An increase of s causes k* to increase. This increase of k* towards k** causes output to increase more than depreciation, so consumption increases.

If k > k** then the savings rate s is above the optimal level. A decrease of s causes k* to decrease. This decrease of k* causes output to decrease less than depreciation, so consumption decreases.

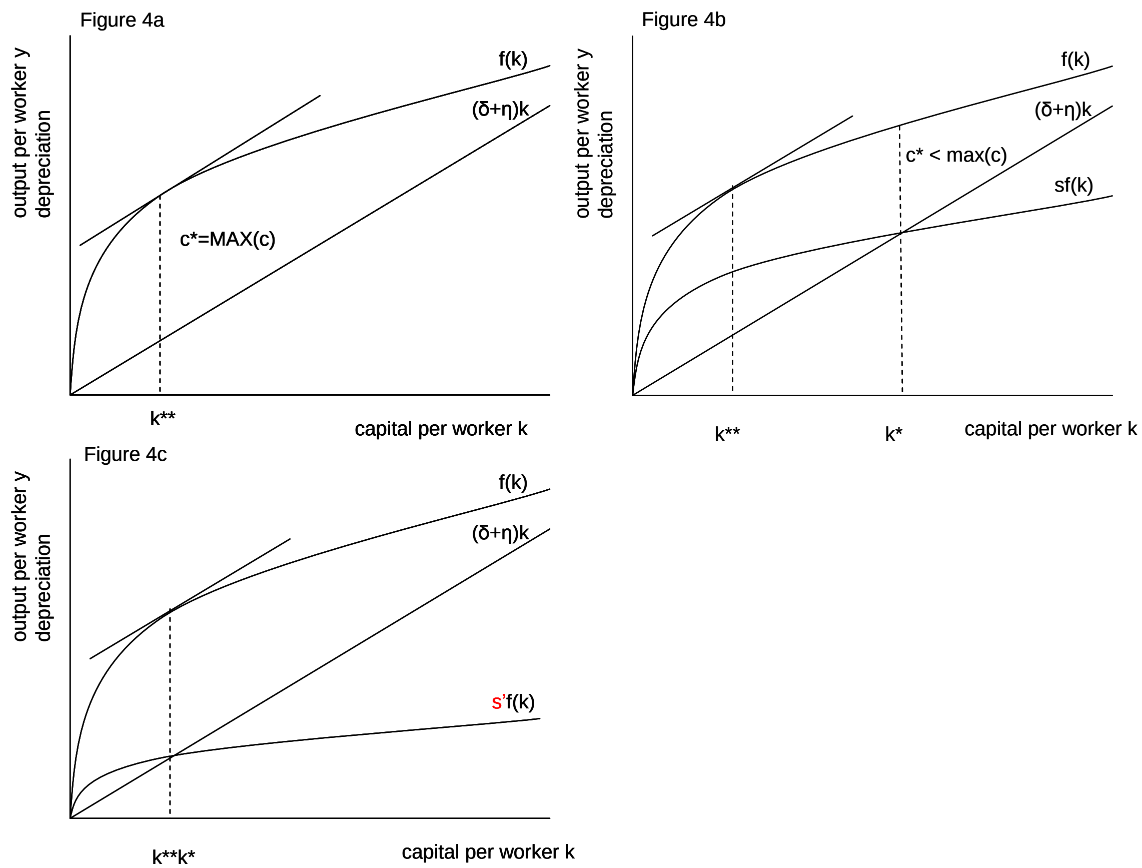
The optimal savings rate is that which guarantees that k = k*. This is the case in**

Figure 10c.

Note that the economy does not automatically more to the Golden Rule steady-state, but that the eventual steady state depends on the savings rate s. So you, as a dictator, needs to set the savings rate s so that the steady-state becomes the Golden Rule steady state. You can do so by changing public saving (budget deficit or surplus) or by trying to change private saving (for example by imposing or removing taxes on capital).

Also note that adjusting the savings rate by policy measures implies that the consumption of current and future wages should in some way weigh the interests of current and future generations. Suppose that k* > k (see**
Figure 10b) then the savings rate s should be decreased. In this case, the extra consumption for current generations would be larger than the loss in consumption of future generations. In this case, the marginal product of capital exceeds the growth rate of the population η and δ. Here we speak of a situation of *dynamic inefficiency* (see later). An economy is said to be dynamically *in*efficient when there is too much capital stock, so that a generation can "devour a portion of the capital stock and then holding constant the consumption of all future generations" (Abel et al., 1989, 1).

Figure 10: optimal consumption and the golden rule of capital



But if k* < k** then there is too little capital stock to maximize consumption. In this situation, the savings rate is below its Golden Rule level and we speak of a situation of *dynamic efficiency*. Thus, the savings rate *s* should increase. In other words, current generations should consume less in order to increase the consumption of future generations. Here, there is a generational conflict. Will they accept increasing their savings? If policy makers give all generations equally weight, then they will try to reach k**. If policy makers give current generations (which, after all, are voters) a higher weight, then they will not. Otherwise, the government can increase public savings (i.e. reduce the public budget deficit), however hoping that the public will not react by dissaving.

The notion of dynamic efficiency or inefficiency is crucial because it links consumption in steady state to the marginal product of capital and the growth rate of the population. Assume for a moment that there are no depreciations, so $\delta=0$. Then there is a situation of dynamic efficiency when the marginal product of capital exceeds the growth rate of the population, i.e. the MPC > η (the North-East quadrant of the above Figure 10). Now we can show that the rate of interest equals the marginal product of capital, hence r = MPC[7]. A situation of dynamic efficiency thus means that the interest rate r exceeds the growth rate of the population η, or $r > \eta$.

Summarizing:

- ✓ Capital stock > golden rule level = $r < \eta$: dynamic INefficiency
- ✓ Capital stock equals golden rule level: $r = \eta$: **consumption path is optimal**
- ✓ Capital stock < golden rule level = $r > \eta$: dynamic efficiency

Now why is this relevant? For many reasons, but one of them is that a situation of dynamic efficiency or inefficiency has important consequences on the comparison between Capital Funding and Pay-As-You-Go pension systems.

Comparisons between funded and unfunded pension systems are often based on the Aaron condition (based on work by Aaron, 1966). In its standard form, this condition is rather straightforward. Define the rate of return of a pension system the proportional difference between the benefit rate (b) and the contribution rate (q) that a system generates. Or, the rate of return of a system is the benefit rate given the contribution rate (b/p).
Suppose that a typical individual lives two periods: in the first, he or she works, and in the second, he or she is retired. In any period t, a generation of Nt young workers contributes W_tN_tq to the pension system (w=earnings, q=contribution rate).
We start by setting the rate of return of a funded pension system. A generation of N_t young workers contributes W_tN_tq to the pension system (w=earnings, q=contribution rate). This total mass generates r times capital, which in the next period results in W_tN_tb (b=benefit rate) pension benefits. Hence

[7] This is not difficult to show. Suppose a simple production function Y=F(K,L) and suppose profit C to be equal to PY-wL-rK (P=goods prices, w=nominal wage, r=nominal interest rate). The firm buys capital stock as to maximizes profit, so it will do so until the last unit of capital bought does not add to profit. So maximize profit with respect to capital K.

$dC/dK = p(dY/dK)-r = 0$ or $(dY/dK) = MPC = (r/p)$

where dY/dK is the marginal productivity of capital MPC. Assuming p = 1 means that

MPC = r.

Note that one can also show that the marginal productivity of labour equals the real wage rate w/p.

$r W_t N_t q = W_t N_t b$

or

$r = b/q$

The rate of return of a funded pension system is obviously the market interest rate r. The rate of return of a PAYG pension system is a bit more complicated. Suppose that a typical individual lives two periods: in the first, he or she works, and in the second, he or she is retired. In any period t, a generation of N_t young workers contributes $W_t N_t q$ to the pension system (w=earnings, q=contribution rate). In the same period, the government pays out $W_{t-1} N_{t-1} b$ in pension benefits to the N_{t-1} people who are then old (b=benefit rate). So, for the government to have a balanced budget, the constraint is

$W_t N_t q = W_{t-1} N_{t-1} b$.

Next, assume that wages remain unchanged over time and assume that the population growth rate equals η, so that Wt=Wt-1 and Nt = η Nt-1 and write the budget constraint as

$W_{t-1} \eta N_{t-1} q = W_{t-1} N_{t-1} b$.

Take away W_{t-1} and $N_{t-1}$from both sides of the equation, and bring q to the other side

$\eta = b/q$.

The return of a PAYG system is the growth rate of the population (assuming no growth rate of wages).

Now we can compare the rates of returns of both the capital funding and PAYG pension systems. These are equal when the growth rate of the population equals the interest rate.

$\eta = r$

which is when the capital stock equals its *golden rule level*! If not, then either one of the two systems has a higher rate of return than the other.

According to the “classical” Aaron condition (Aaron, 1966), a PAYG generates a lower (higher) return than a CF when the growth rates of population is lower (higher) than the gross interest rate. Thus

Summarizing:

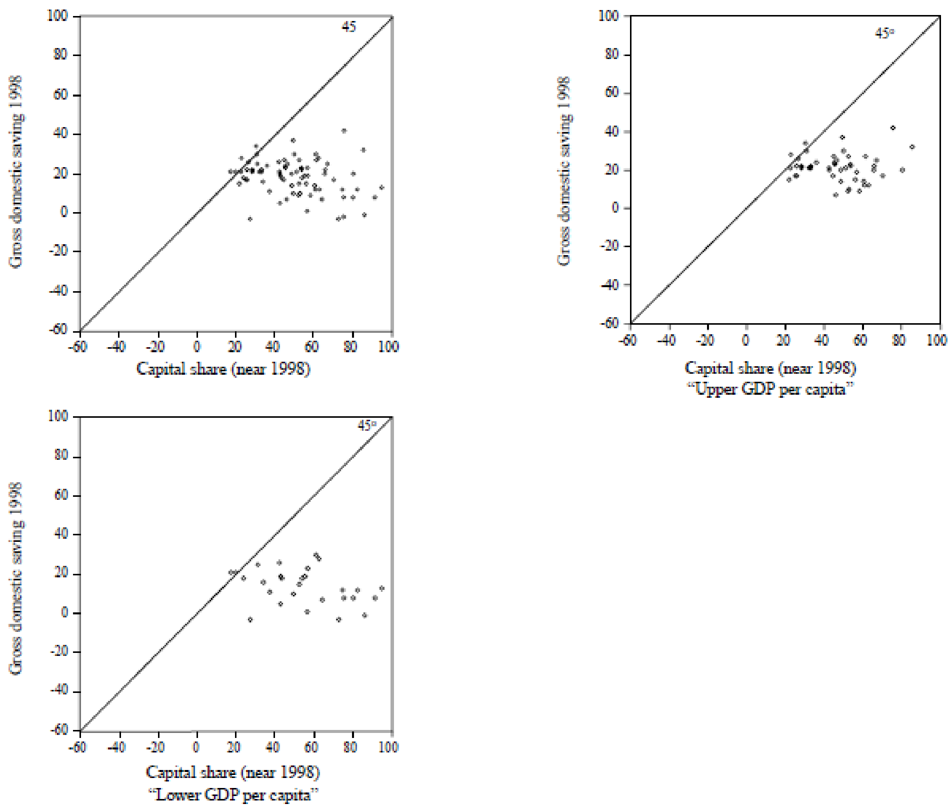
- ✓ Capital stock > golden rule level = $r < \eta$: dynamic INefficiency: PAYG higher return than CF.
- ✓ Capital stock equals golden rule level: $r = \eta$: **consumption path is optimal; return of PAYG and CF are equal.**

- ✓ Capital stock < golden rule level = r > η: dynamic efficiency: CF higher return than PAYG.

The Golden Rule in practice

Now is there any indicator to be found suggesting that countries optimize their consumption in steady state?

Figure 11: some scarce empirical assessment of the Golden Rule (Elias, 2001)



The Figure 11 relates gross domestic savings to the share of capital in the economy for the countries included in the World Developments Indicators' dataset of the World Bank. The capital share of output is (df(k*)/dk*) times k, or MPC times k. Under the Golden Rule, this should equal the savings rate.

Remember that savings is total income minus consumption s = y – c, or s = f(k) - c

In steady state c = f(k) – (δ + η)k

So s = f(k) – [f(k) – (δ + η)k] or s = (δ + η)k

Now we have seen that under the Golden Rule $(\delta + \eta)$ equals $(df(k^*)/dk^*)$, or equivalently MPC = $(\delta + \eta)$.

Thus s = MPC k

Now in Figure 11, the capital share is consistently bigger than the savings rate (savings relative to GDP). This suggests that we save too little and therefore consume too much. We therefore are in a situation of dynamic efficiency, and, the Aaron condition teaches us that a capital funding (CF) pension system in this case generates a higher rate of return than a Pay-As-You-Go (PAYG) pension system.

Abel et al. (1989, 1) develop a more practical definition of dynamic efficiency. If the capital sector is regularly contributing to the level of consumption (if profit exceed investment), then the economy is dynamically efficient. If investments exceed profits, so that the capital sector reduces consumption, then the economy is inefficient. Using this definition, they find that the major OECD countries (U.S., UK, France, Germany, Italy, Canada and Japan) are all dynamically efficient. Thus, following the Aaron condition, the rate of return of Capital Funding schemes should exceed that of PAYG-pension systems.

The below Table 1 from Knell (2010, page 718) confirms this. The geometric mean over the average 20 year period of equity returns was 242.6 for the US with a standard deviation of 263.6%. The mean return in France was considerably lower (93.2%) with a standard deviation that is much higher (337.1%). The return on PAYG (real pc GDP growth: population growth times wage growth, or the growth rate of the wage mass) was 46.8% for the US with a standard deviation equal to 11.6. For France, the return on PAYG was higher than in the US (48.6) but also with a higher standard deviation (30.0).

TABLE 1
20-YEAR EQUITY RETURNS AND PER CAPITA GDP GROWTH FOR SELECTED COUNTRIES

	Real return on equities over 20 years		Real per capita GDP growth over 20 years		
	Mean (%)	Standard deviation (%)	Mean (%)	Standard deviation (%)	Correlation
1900–99					
USA	243.6	263.6	46.8	11.6	0.073
UK	207.8	296.9	33.3	17.2	0.481
France	93.2	337.1	48.6	30.0	− 0.016
Anglo-Saxon	246.9	255.2	40.0	17.2	0.045
Group of Nine (G9)	152.3	386.3	47.3	50.5	0.484
1950–99					
USA	302.9	337.3	56.5	8.6	− 0.751
UK	349.9	374.5	49.6	4.2	− 0.373
France	284.9	427.0	64.9	32.3	− 0.729
Anglo-Saxon	266.7	279.1	53.3	11.3	− 0.222
Group of Nine (G9)	255.9	493.6	74.6	59.6	0.416

Notes
Own calculations based on Dimson *et al.* (2002) and Maddison (2003). The (geometric) means are calculated from 20-year periods, first for periods between 1900 and 1999, and then for periods between 1950 and 1999. The (sample) standard deviations and correlations are based on the same data of 20-year returns. For the summary statistics of the country group 'Anglo-Saxon', the data for four Anglo-Saxon countries (USA, UK, Canada and Australia) are pooled. 'Group of Nine' (G9) also refers to a country group that pools the return data of all large economies (USA, Japan, Germany, France, UK, Italy, Spain, Canada and Australia).

So even though the returns differ considerably between countries, the general conclusion is that the rate of return of Capital Funding (CF) pension systems exceeds that of a PAYG pension system.

This confirms that these economies, including France, are "dynamically efficient". All of these countries have a capital stock which exceeds the level that maximizes consumption (the North East pane of Figure 10), which means that consumption in the steady state is not maximized

Some extensions of the Solow model

This assumption that consumption (and, therefore, savings-investment) is a fixed fraction *c* of income-output Y is of course somewhat simplistic[8]. One earlier extension has been to assume that savings-investment is a given fraction of non-wage income (profits). This model therefore requires that a distinction is made between profits and wages, or non-labour income and labour income. This was the approach developed by Ramsey (1928), Kass (1965) and Koopmans (1965). This approach however still requires that one decides on what fraction of income y or non-wage income is saved and invested. There is, in other words, no micro-foundation to this decision. A second and more advanced extention is to derive the savings-investment decision from the behaviour of an intertemporal utility-maximizing households and competitive profit-maximizing firms

[8] This paragraph is based on Solow, 2003, pages 643-644.

(see Solow, 2003, for a discussion). Obviously, these extentions make the above model much more difficult to solve. But fortunately, it has been shown that at least the steady state properties of these extended types of modes are exactly the same as of the original Solow model: capital and output per head are both constant, so capital and output grow at the same rate as the labour force.

Technological progress in the Solow model

So far, we have seen two sources of economic growth in the Solow model: capital accumulation can only cause growth in the short run. Population growth can explain persistent growth of output. However, there is a third factor explaining growth (and a second factor explaining persistent growth) and that is *technological progress*.

It is not difficult to modify the Solow model to take technological progress into account, and the key is to use **efficiency units** instead of labour units. The new situation is shown in Figure 12. The well-known production function is Y = F(K,L). Now we introduce a variable E which reflects the efficiency of labour. This E depends on the health, education, skills and knowledge of the labour force. The new production function becomes

$Y = F(K,EL)$

This technical progress is *fully exogenous*, labour-augmenting. Suppose that E growths by speed g. So, it is as if labour grows an additional g percent a year. So g is the rate of labour augmenting technological progress. The labour force grows at speed $\eta + g$ per year.

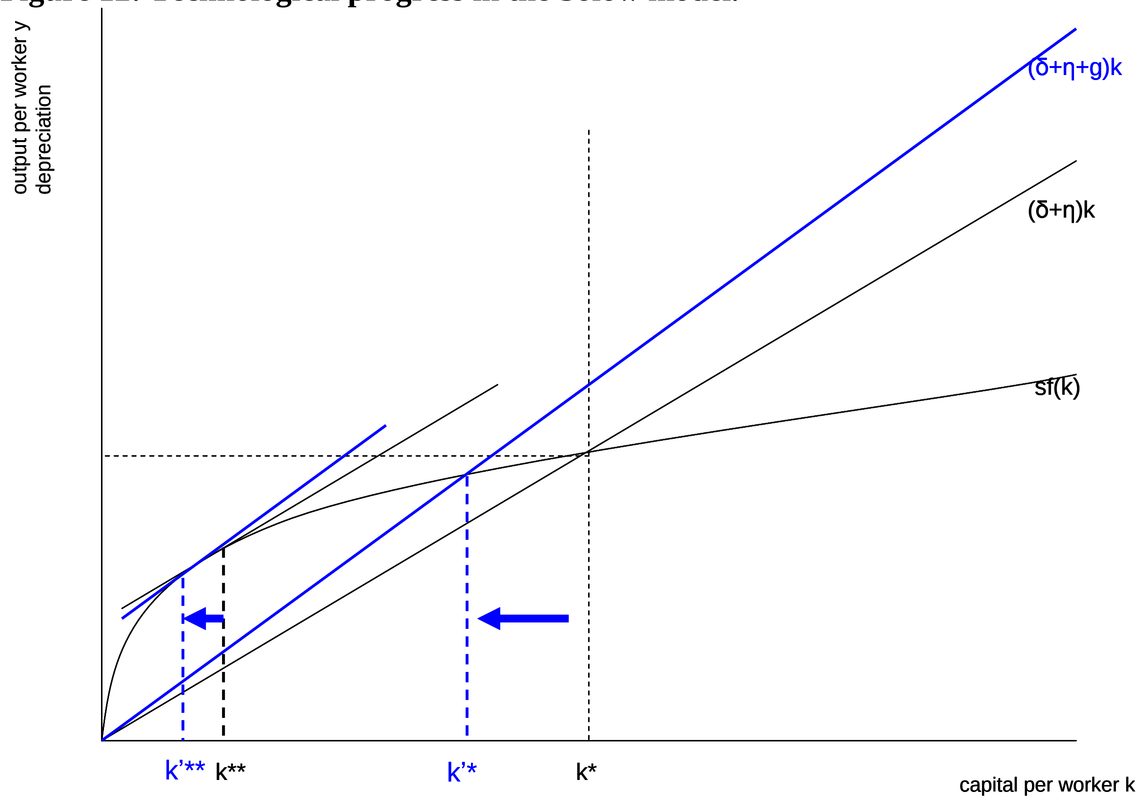
Use again the fact that there are no economies of scale and write output not in terms of the labour force L, but in terms of efficiency units EL.

$y = f(k,1)$ or $y = f(k)$ where $y = Y/EL$ and $k = K/EL$

Now the changes of the capital stock per efficiency unit becomes[9]
$\Delta k = sf(k) - (\delta + \eta + \mathbf{g})k$

[9] This is analogous to footnote 3. The logarithmic differential, or the proportional change over time of k equals (dk/dt)(1/k). However, k is now expressed in efficiency units k=K/EL.
So (dk/dt)(1/k) = ((dK/dt)-(dEL/dt))(1/k) =
((dK/dt)-[(dE/dt) + (dL/dt)])(1/k) =
((dK/dt) (1/k) - (dE/dt) (1/k) - (dL/dt) (1/k)) = – δ – η – g.
Hence ((dK/dt)-(dL/dt))(1/k) = (dk/dt)(1/k) = – (δ + η + g).
Rewrite this to (dk/dt)= – (δ + η +g)k or Δk=– (δ + η + g)k.

Figure 12: Technological progress in the Solow model.



Which means that the marginal product of capital MPC increases as a result of introducing technological progress.

And the steady state is again where $\Delta k=0$ or $sf(y) =(\delta + \eta + g)k$.

So, the inclusion of a 'constant stream' of technological progress in the Solow model causes k* to decrease to k'*. This is because k=(K/L) and k'=(K/AL). In the new equilibrium, however, the growth rate of per capita output will have **increased**.

For the steady state before technological progress implied that $\Delta k= \Delta y=0$ in per *capita units*. Now the steady state growth rate becomes $\Delta k= \Delta y=0$ in *efficiency units*. As the efficiency of labour increases by *g*, this means that *k* and *y* in per capita terms increase by *g* as well!!

$\Delta(Y/EL)=0$ so $\Delta Y-\Delta EL=0$

- ⇨ $\Delta Y = \Delta EL$
- ⇨ $\Delta Y = \Delta E+\Delta L$

Again multiply everything by (1/k) to get to the parameters in Figure 11.

- ⇨ $\Delta Y(1/k) = \Delta E(1/k) + \Delta L(1/k)$
- ⇨ $\Delta Y(1/k) = g + \eta$

By including continuous technological progress in the Solow model, it can be used to explain persistent per capita growth of output (or of living conditions), and the capital stock per worker. Output and the capital stock therefore grow at the rate of the population plus technological growth.

So technological progress –including education, training and the acquirement of skills of the labour force- can cause many variables (including output) to raise over a long period of time (i.e. in steady state). These variables, such as output per worker and the capital stock per worker would in the steady state increase by a rate g, i.e. increase with the speed of technological progress. This is called 'balanced growth'

Note, finally that the introduction of technological progress will also change the golden rule of capital. This is because steady state consumption now becomes

$c^* = f(k^*) - (\delta + \eta + g)k.$

So the golden rule now becomes MPC = $(\delta + \eta + g)$ or

$MPC - \delta = \eta + g$

This means that the net marginal product of capital should equal the rate of growth of total output. It can be seen from the previous figures that k** will decrease to k'** (see Figure 12), which means that optimal consumption will increase. Or, current generations can consume more (i.e. increase their utility) without decreasing the output of future generations.

Before closing the discussion of the Solow model, we should emphasize that one of the most important conclusions which follows from it is that one should not expect convergence if the steady state conditions of two countries differ (notably the savings rate s). Instead, different countries generally reach different steady states. However, convergence can be the result from flows of capital and technology between countries. Countries with a high savings rate can invest in counties with a low savings rate, with the result that the steady state output in the two countries will converge. Furthermore, via direct investments, less developed countries can 'catch up' with more developed countries.

Beyond the Solow model: endogenous growth models.

The Solow model is sometimes referred to as an 'exogenous growth model' because the factors that cause persistent economic growth, population growth and technological progress, are exogenous.

A key element in the Solow model is the condition that the *marginal productivity of capital decreases*. Without this assumption, a steady-state equilibrium would not occur, as we will see now. From the Solow model came a strand of models which take technological change as endogenous, i.e. as something that the government can influence. It is for these reasons that these models are called endogenous growth models.

A very simple example of this is the so-called AK model. This is a model where the capital stock K is interpreted as being not only embodied (such as machines and stuff) but also disembodied (such as knowledge or 'human capital'). In this interpretation of the capital stock, the above assumption of decreasing marginal productivity of capital in the Solow model becomes somewhat strange, and should be replaced by the assumption of *constant marginal productivity of capital*.

Suppose $Y=\alpha K$ or, in efficiency units $y=\alpha k$

The marginal product of capital $f'(k)$ is now not decreasing, but constant and equal to α.

As before, the change of the capital stock per efficiency unit is

$\Delta k=sf(k) - (\delta + \eta + \mathbf{g})k$

Or in this case

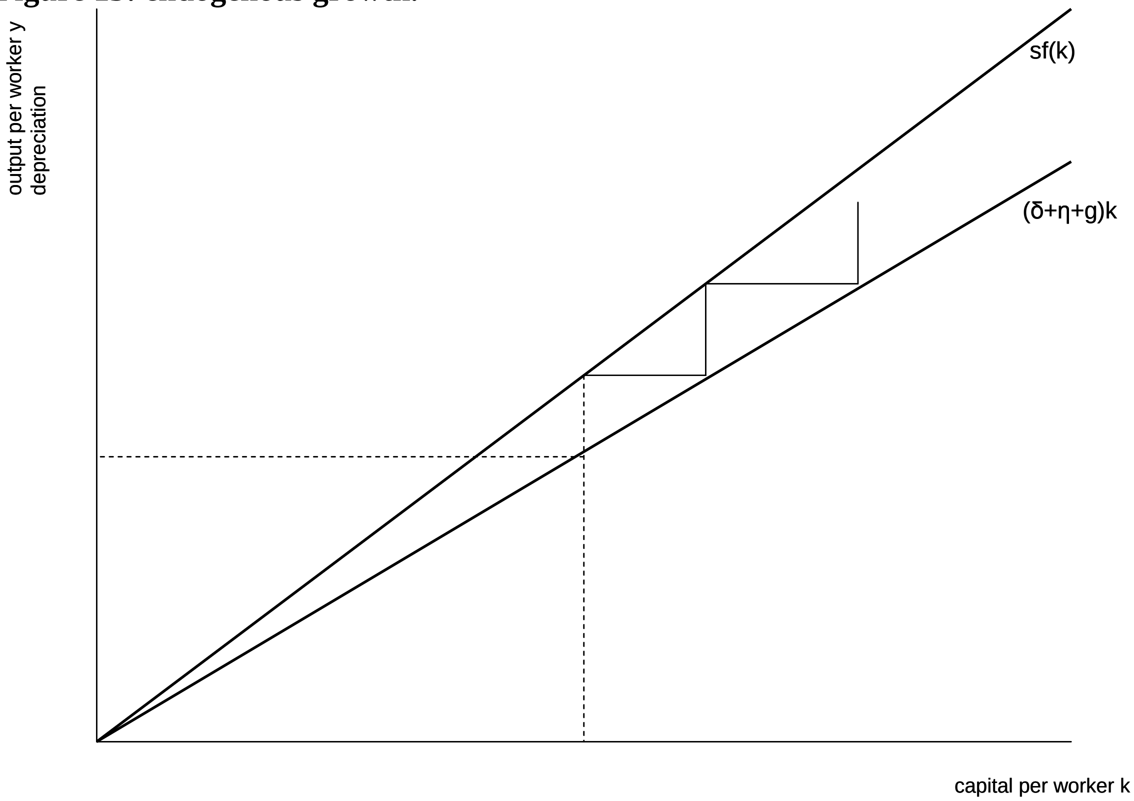
$\Delta k=s\alpha k - (\delta + \eta + \mathbf{g})k$

In the Solow model, an increase in savings results in a temporary increase in economic growth. For it will cause the capital stock to increase. The higher the capital stock k, the higher output and investment. However, due to the decreasing marginal productivity of capital MPC, this positive effect of the capital stock on output (and hence investment) eventually comes to a halt. Furthermore, the higher the capital stock, the higher depreciation. In the long run, the capital stock will reach its steady state value, where $\Delta k=0$.

In the simple AK production function we now consider, the marginal productivity does not decrease, but remains constant. The production function in Figure 13 becomes a straight line. As a result, any increase of the capital stock will go on to increase output at the same non-decreasing speed. Depreciations will never grow fast enough to stop this process. So, as soon as $s\alpha k > (\delta + \eta + \mathbf{g})k$, a steady state ($\Delta y=\Delta k=0$) as defined previously will never be reached. The capital stock will continue to increase forever, even without the assumption of technological progress.

Actually, in this case, the steady state has to be redefined. It is *no longer a situation without growth, but a situation with a stable growth*. The introduction of technological progress in such a model will cause the growth path to increase by itself in the long run.

Figure 13: endogenous growth.



The AK model shows why countries can show diverging grow paths when they have different savings levels (and thus different investments in physical and especially human capital). Hence, richer, more developed countries have an advantage that poorer countries can never catch up. However, the model does not explain why constant returns to scale would be achievable with this definition of capital.

The Romer model (developed by Romer, yes) makes A endogenous, by assuming two types of workers, those who produce and those who work in R&D. The productivity A is thus the number of R&D workers times the number of inventions per year. So technological progress now becomes something that the economy can invest into by shifting workers away from production towards R&D. So the richer country can maintain its advantage over the poorer country by investing more in R&D and hence generating endogenous technological progress.

The Solow model predicts that countries will reach a different steady state, depending on differences in savings rates, capital stocks and different efficiencies in the utilisation of production factors. As a result, there will be no convergence (nor divergence). The endogenous-growth models however predict that these differences will cause different growth paths of different countries, i.e. divergence, unless countries adopt best practices in production, R&D and technology.

From theory to practice I: The Japan experience

In a 2003 paper in the Journal of Economic Education, Benigno Valdés uses the Solow model to explain the development of GDP of Japan after the war. The relevant events are the following:

1. Valdés cites literature suggesting that, during the war, about 25% of the capital stock of Japan was destroyed.
2. After the war, Japan's savings quote increased. A reason for this is that land is scarce, and land and housing therefore is expensive. After the war, Japanese banks were not willing to lend too much to families so dwellings were financed out of savings.
3. After the war, the rate of technological growth increased. For political reasons (the Cold War), the US allowed Japan to copy their technology, which they did without much regard for patent laws. Furthermore, Japan was allowed to export to the US while imposing trade barriers preventing imports. Also, Japan made quite an effort increasing the level of education of the population (human capital). This increased the rate of adaptation of technology.
4. From approximately 1973 on, the rate of technological growth was truncated. At some point, the rate of technological progress slows down if a country does not shift from imitation to invention. And there is empirical evidence that this happened around 1973. All over the world, the growth rate of TFP slowed down and this was especially marked in Japan.

The Figure 14 shows the impact of these events on the steady state development of GDP. First of all, the decrease of the capital stock during the war (point 1) according to the Solow model should result in a lower level of output in steady state. This is the vertical downward movement to β_T from the steady state level SS1 at time T. If the fundamental parameters would not have changed, the economy would gradually have gone back to its original stead state level SS1. However, the fundamental parameters did change: the increase of the savings quote (point 2) increases the level of output in steady state from SS1 to SS2. The slope, however, remains the same i.e. mildly positive, the result of a slow rate of technological growth. At the same time, however, the rate of technological growth increased (point 3). This results in an immediate decrease in the level of output (the vertical difference between SS2 and SS3 in time T) and a higher growth rate of output in steady state. The steady state path is now SS3. The fourth point, the decrease of the rate of technological growth is not depicted in Figure 14.

Figure 14: Explaning Japan using the Solow model (Valdés, 2003)

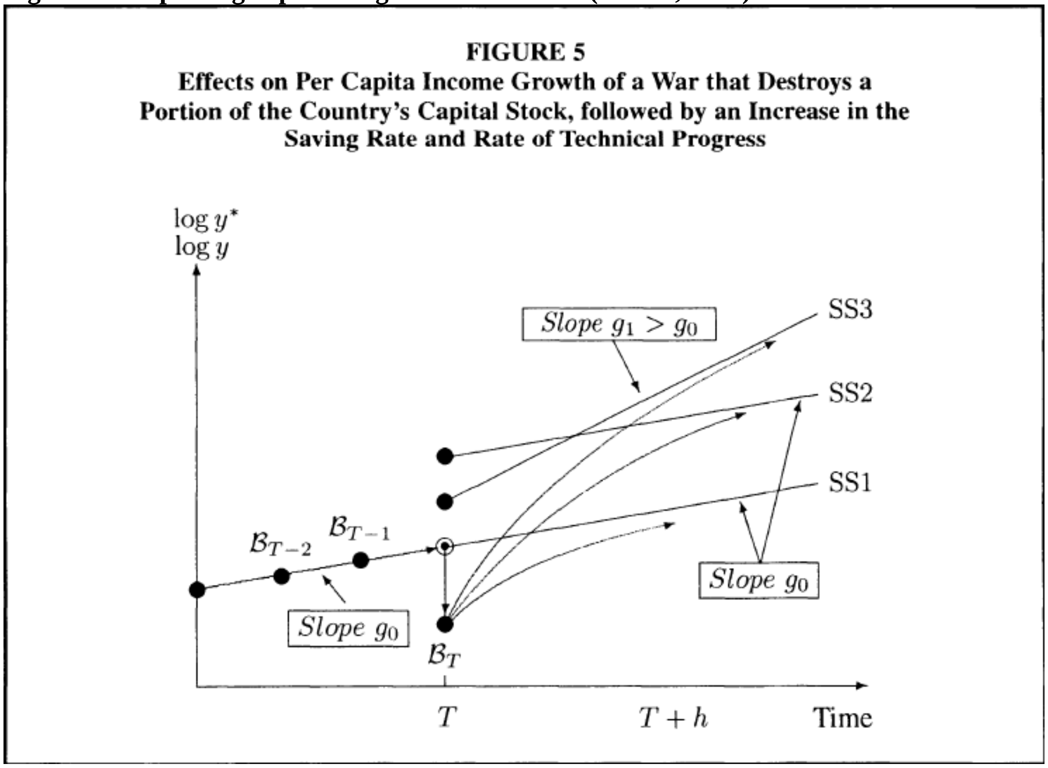
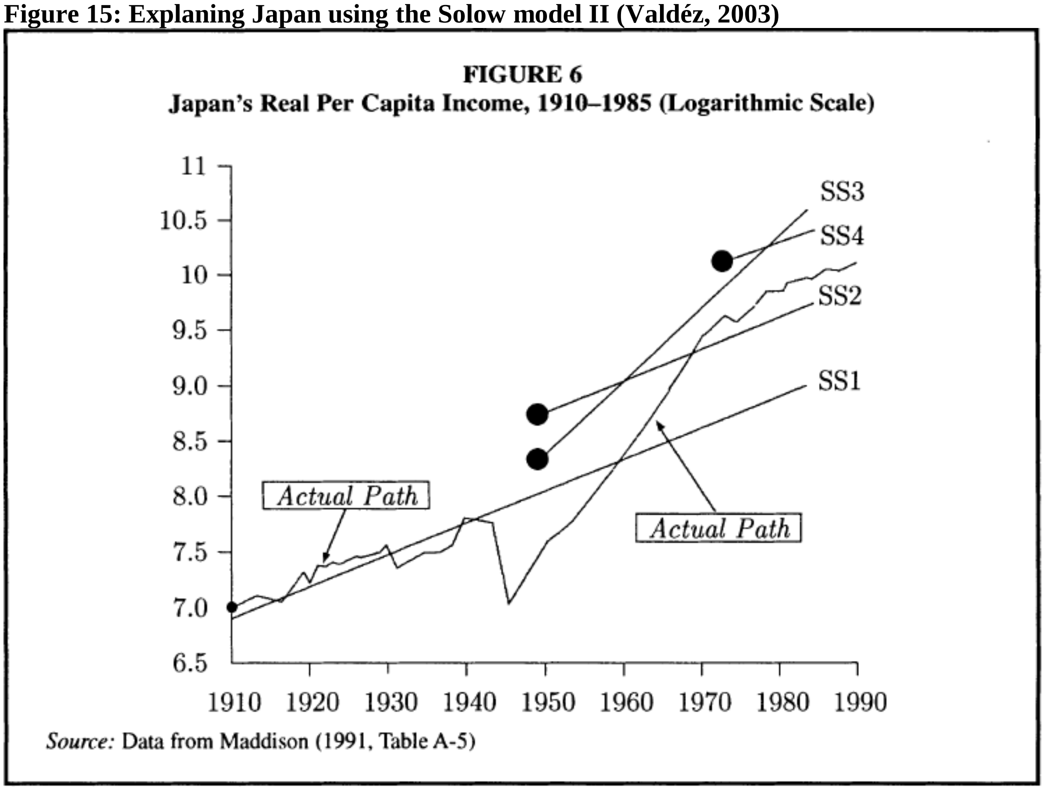


Figure 15: Explaning Japan using the Solow model II (Valdéz, 2003)



Now the next Figure 15 shows whether this development actually explains the development of GDP over time in Japan. The figure suggests that it would. The decrease of the capital stock during the war (point 1) indeed resulted in a lowering of GDP per capita. The other developments (increase of the savings rate, and the higher rate of technological growth – points 2 and 3) coincide with a higher growth rate of the Japanese economy. Finally, the decrease of the rate of technological growth from the mid-70's on (point 4) resulted in a steady state growth path SS4 and –indeed- a lower growth rate of GDP in Japan. The conclusion seems to be that, at least heuristically, the Solow model seems to be an appropriate instrument describing the long-run economic developments of Japan.

From theory to practice II: Convergence in the Solow model? Some (very) cursory evidence.[10]

To end this discussion of the Solow model, let us discuss one of its strongest predictions, which is called the *convergence hypothesis*. Suppose two countries that have the same steady state level of output and share the same population growth and technology. However, neither are currently on this steady state level. In fact, one country starts of at a lower level of output than the other, and both economies move towards the common steady state level. The country with the lowest level of output is the developing country and the one with the higher level of output is the developed country. Now what will be the result? The developing country with the lowest level of output will experience the highest productivity on capital. The investment rate in this country will thus be higher, capital stock and output growth rates will be higher. The economy of the developing country will move towards the steady state faster than the developed country. Or, the output per worker gap between the developing and developed country will become smaller over time, and will become zero when they both reach the same steady state level. This may explain the high rates of growth of Germany and Japan in the post-war period. Indeed, these countries had very low levels of output after the war, but a population with high levels of education and productivity. Furthermore, many factories and industrial machinery were destroyed. This opened the opportunity to adopt quickly the latest technology in the production process, thereby drastically increasing the marginal productivity of capital. Together with the Marshall plan, which resulted in strong public spending and economic aid, this caused the economies of these countries to catch up very quickly with that of the United States, as shown in Table 2.

[10] Based on Chamberlin and Yueh, 2006, par. 16.2.4. The paper by Valdés (2003) discusses and confirms the convergence hypothesis for Japan using the Solow-Swan model.

Table 2: annual growth rates of output per capita and rates of technological progress.

	Growth rates of output p.c. (%)		Rates of technological progress (%)	
Country	**1950-1973**	**1974-2002**	**1950-1973**	**1974-2002**
France	4.0	1.8	4.9	2.3
Germany	4.9	2.1	5.6	1.9
Japan	8.0	3.1	6.4	1.7
UK	2.5	1.8	2.3	1.7
US	2.2	1.6	2.6	0.6

Source: Chamberlin and Yueh, 2006, Table 16.1., page 558.

Indeed, Germany and especially Japan –the two countries that sustained the most damage during the Second World War- experienced the highest growth rates in the post-war period. When the oil crisis broke out in 1974, they had caught up to a large extent, and growth rates thus converged.

Table 2 seems to give cursory evidence to the convergence hypothesis. This however may be subject to the choice of countries. Indeed, the convergence hypothesis however seems in contradiction with the observed trend growth rates of the European Union and the United States, as will be shown later. If the EU and US share the same steady state characteristics, then why the divergence? This convergence hypothesis also seems to ignore the ongoing polarisation between developing and developed countries. and specifically the continuous lagging behind of the African continent. Or why did some 'East Asian Tigers' (Hong Kong, Singapore, South Korea and Taiwan) show such enormously high growth rates from 1965 to 1990? Why is this? The answer is that the conditions of the convergence hypothesis are not met: countries will only converge if their steady state conditions are the same. This result in a weaker form, the *conditional convergence hypothesis*: countries will converge, but only those countries that have the same steady state, i.e. that share the same fundamental characteristics.

Chamberlin and Yueh use data from the World Bank to compare 1997 levels of various economies with their growth rates between 1980 and 1997.

Table 3: per capita GNP and annual growth rates

Non-OECD countries			OECD countries		
	1997-level ($US)	**1980-1997 growth rate (%)**		**1997-level ($US)**	**1980-1997 growth rate (%)**
Mozambique	100	-1.2	Portugal	10500	2.9
Bangladesh	300	2.3	Spain	14500	2.0
Nigeria	300	-1.2	Ireland	18300	4.2
China	800	11	Italy	20100	1.4
Indonesia	1100	5.5	UK	20700	2.0
Philippines	1200	1.1	France	26500	2.0

Turkey	3100	1.7	US	28700	1.7
South Korea	10500	7.8	Switzerland	44300	1.6

Source: World Bank, World Development Report. In Chamberlin and Yueh, 2006, Table 16.4, page 560.

What can we learn from this Table? First of all, that it is very difficult if not impossible to draw any general conclusions on whether the convergence or conditional convergence hypothesis holds. Indeed, some countries such as China, Indonesia and South Korea, that some decades ago were less developed than the OECD-average, show growth rates that are much higher than average. Also, within the OECD countries, the poorer countries tend to have higher growth rates, with Spain and Italy as the exception. This supports the convergence hypothesis. However, many poor countries, such as Mozambique, Nigeria and the Philippines show considerably lower growth rates than OECD-countries. The conditional convergence hypothesis suggests that the steady states in these last poor countries are different from those in the first group. Mankiw (2007; page 248) argues that it is increases in labour-force participation, increases in the capital stock and especially increases in educational attainment that caused these high growth rates. As these increased the marginal productivity of capital, the steady-state conditions of these countries themselves converged, and the economies thus followed. Remember that this is the line of thought followed by endogenous growth theory, especially the Romer model.

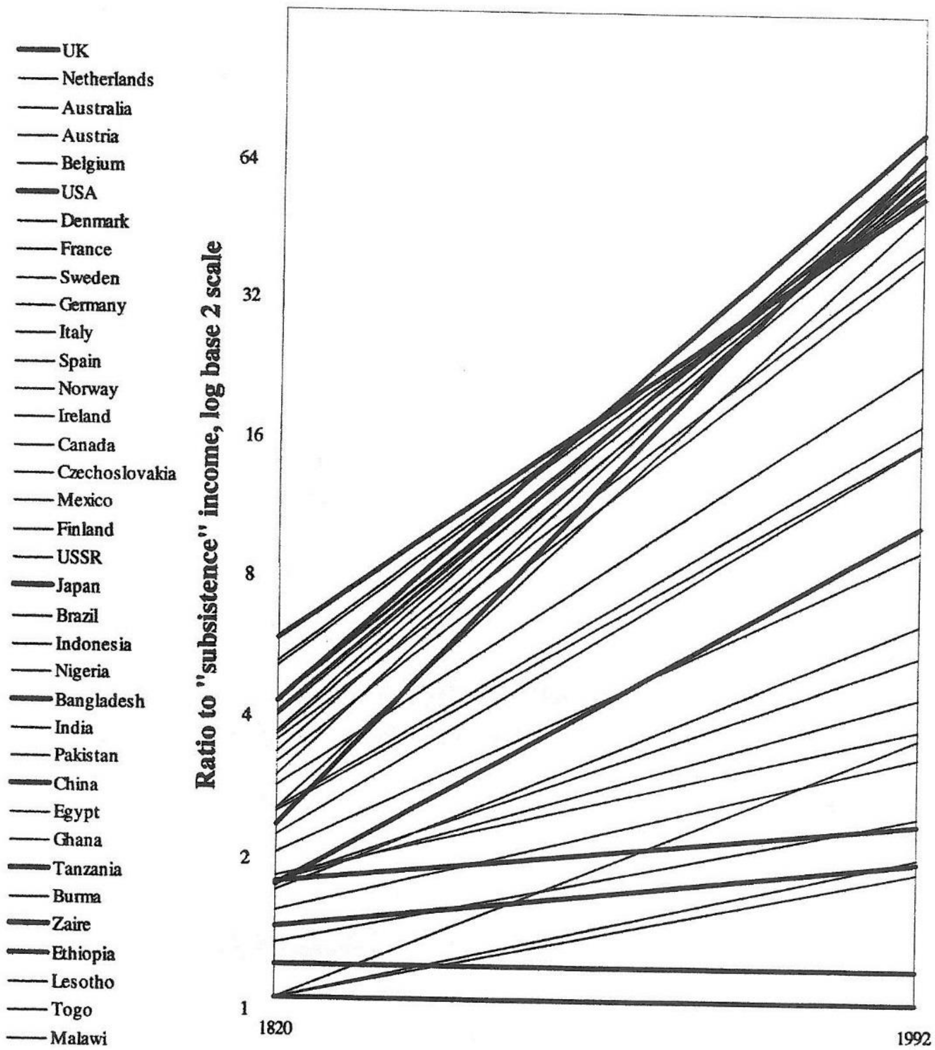
However, the study of economic convergence and divergence and which policy measures may (or may not) influence long-run economic performance, are hampered by methodological problems and results have therefore been limited to date. Indeed, Grattan and Schmitz (1999, page 671) argue that there is virtually no correlation between income levels and subsequent growth rates, and that growth rates show very little persistence. This suggests that the measures and methods we came to discuss should be limited to producing cursory evidence, or they should be limited to countries that are somewhat alike. Indeed, many recent papers shows that there is large heterogeneity among countries (see Alfo et al., 2008, for a discussion).

According to the Solow model, if a country is in steady state, the savings rate and investments (capital accumulation) are independent of growth rates. The hypothesis of convergences however assumes that countries share the same technology and preferences. If this is not the case, then not finding convergence may not be a reason to reject the Solow model. Alfo et al., 2008, take into account these country-specific differences by using a latent variable model to regress on GDP growth rates using an international cross-section dataset from 1960 to 1995. They find that the explanatory power of the Solow growth model is enhanced when cross-country heterogeneity is considered. Indeed, they find that growth rates are not significantly associated with investment rates. Furthermore, they find that they different groups of countries do not converge, suggesting that the unobserved factors are long-lasting characteristics not accounted for by the Solow model.

Next we have the difference between exogenous and endogenous growth models. The Solow model is an exogenous growth model, where technology growth (including the growth of human capital) can only bring the economy to a steady state per capita growth path if it is continuous and never stops. A one-shot increase in technology or human capital will cause the Solow-economy to settle into a higher steady-state level, but – again- with no growth of the per capita variables. By contrast, an one-shot technological shock will result in a higher growth path in endogenous growth models. Arnold et al. (2007) assess the role of human capital accumulation for economic growth in a sample of 21 OECD countries over the 1971-2004 period. They find a positive and significant impact of human capital accumulation on economic growth, as the endogenous growth models suggest. The estimated long-run effect of output of one additional year of education on output is roughly between 6 and 9%. They also find a strong growth effect from the accumulation of physical capital. On the whole, their findings support the endogenous growth model over the exogenous growth model. Furthermore, In a 2001 paper published in the World Bank Review, William Easterly and Ross Levine use data by the famous economic historian Maddison (1995) to show the diverging growth rates of GDP between 1820 and 1992.

Figure 16: Figure 3 from Easterly and Levine (2001, 194)

FIGURE 3. Growth Rates Diverge between Rich and Poor: 1820–1992



Note: Order in 1820 from richest (top) to poorest (bottom).
Source: Maddison 1995.

This figure shows that there has been "divergence, big time" (*op. cit.*, 193). The ratio of richest to poorest went from 6 to 1 in 1820 to 70 to 1 in 1992. It is not so much that the poor countries get poorer, no, it is more that the rich countries get richer faster than the poor countries do. We will get to why this might be later on in this reader, but the suggestion that endogenous growth models may have something to do with it, is obvious.

From theory to practice III: Growth accounting

In his 1957 article "Technical Change and the Aggregate Production Function," Solow observed that about half of economic growth cannot be accounted for by increases in capital and labour. He attributed this unaccounted-for portion—now called the "Solow residual" or Total Factor Productivity (TFP)—to technological innovation[11]. From the 1960s on, Solow's studies helped persuade governments to channel their funds into technological research and development to spur economic growth. We are now going to visualize this, using a well-known technique called "growth accounting".

Write

Y=L(Y/L)

which is that output is expressed as the product of labour and the productivity of one unit of labour.

Total labour L can be expressed as the product of the number of hours worked (HRS) times the employment or number of workers (EMPL), or

L = HRS * EMPL

Hence

Y = HRS*EMPL*(Y/L)

Take the logarithmic differential[12]

$$\left(\frac{dY}{dt}\right)\left(\frac{1}{Y}\right)=\left(\frac{dHRS}{dt}\right)\left(\frac{1}{HRS}\right)+\left(\frac{dEMPL}{dt}\right)\left(\frac{1}{EMPL}\right)+\left(\frac{d(Y/L)}{dt}\right)\left(\frac{1}{(Y/L)}\right)$$

Or a change in GNP is caused by a change in the number of hours worked, plus a change in the employment, plus a change in the productivity of labour.

This leaves the change in the productivity of labour to be explained. First of all, note that the last part of the above equation can be rewritten to (dy/dt)(1/y) with y=Y/L

Next, we turn back to our well-known production function

Y = F(AK,AL)

[11] http://www.britannica.com/EBchecked/topic/553649/Robert-M-Solow
[12] The logarithmic differential, or the proportional change over time, of a equals (da/dt)(1/a).

Note that we assume here at the technical progress, A, affects both labour and capital[13]. Again assuming constant returns to scale, A can be brought outside f(.) and all variables can be expressed in per capita amounts.

y = Af(k)   with y=Y/L, k=K/L

Now to keep things simple, assume that output is the product of technical progress A and capital.

Thus y = Ak

Logarithmic differentiation again results in

$$\left(\frac{dy}{dt}\right)\left(\frac{1}{y}\right)=\left[\left(\frac{dk}{dt}\right)\left(\frac{1}{k}\right)+\left(\frac{dA}{dt}\right)\left(\frac{1}{A}\right)\right]$$

In words, the growth of the productivity of labour is the sum of the growth rates of the per capita capital stock (this is called 'capital deepening') and of total factor productivity or TFP.

Combine both equations into

$$\left(\frac{dY}{dt}\right)\left(\frac{1}{Y}\right)=\left(\frac{dHRS}{dt}\right)\left(\frac{1}{HRS}\right)+\left(\frac{dEMPL}{dt}\right)\left(\frac{1}{EMPL}\right)+\left[\left(\frac{dk}{dt}\right)\left(\frac{1}{k}\right)+\left(\frac{dA}{dt}\right)\left(\frac{1}{A}\right)\right]$$

The conclusion is that a proportional change of output over time can be explained by proportional changes of

- *the number of hours worked*
- *the employment rate*
- *the per capita capital stock ('capital deepening')*
- *total factor productivity or TFP*

Because TPF = ΔA/A is not observed, it is used to balance both sides of the above equation. Or, put differently,

$$\left(\frac{dA}{dt}\right)\left(\frac{1}{A}\right)=\left(\frac{dY}{dt}\right)\left(\frac{1}{Y}\right)-\left(\frac{dHRS}{dt}\right)\left(\frac{1}{HRS}\right)-\left(\frac{dEMPL}{dt}\right)\left(\frac{1}{EMPL}\right)-\left(\frac{dk}{dt}\right)\left(\frac{1}{k}\right)$$

How can TFP be interpreted? We have to be cautious here, because we are essentially interpreting something we cannot see! A natural link is to look at the only unobserved

[13] In the above discussion of the Solow model, we used labour augmenting technical progress E, or growth in labour efficiency. The relation between the two is that (dA/dt)(1/A)=(1-α)(dE/dt)(1/E) where α is the capital's share of output MPC(K/Y). See Solow, 1957, in: Mankiw, 1992, footnote 11, page 115.

variable that makes an economy in steady state – technological progress g. This is the first of two possible interpretations by Daube (2012: 14): TFP captures the overall efficiency at which inputs map into aggregate output, therefore distortions in the allocation of factors across sectors or firms can result in lower levels of output per units of input if resources are reallocated to inefficient sectors or firms. Thus, if the goal is to maximize TFP, then policy makers need to find policies, politics, market failures or structural characteristics that cause these distortions. The second group of interpretations is that measured TFP just captures all measurement and specification errors in the equation explaining GDP. This –again- may include changes in technology, externalities (where some successful sectors increase productivity in other sectors) changes in the sector decomposition of production and the adoption of lower-cost production methods.

Applications

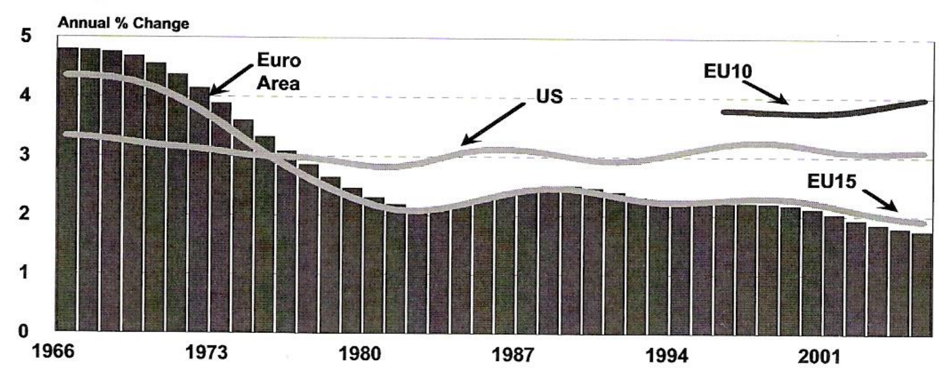
An obvious application of what we have learned so far, is the comparison of the macroeconomic developments in Europe and the US.

The below Figure 17 shows the trend growth rates in Europe (the Euro area, EU15, EU10) and the US between 1966 and 2002.

Figure 17: trend growth GDP in Europe and the US

Graph 2 : Trend Growth and its Labour and Labour Productivity Components (1966-2002)

A. Trend GDP Growth



Source : Ameco and ECFIN calculations

This Figure 17 is part of Graph 2 in EC, 2006, page 5. The main conclusion is that where these growth rates were comparable until the mid-seventies, they diverged afterwards. This divergencing process has been reinforded from the mid-nineties on. It is indeed so that Europe is more subject to demographic ageing than the US, and following the Solow-model could this result in lower growth rates of GDP. But there obviously is more. Table 4 shows the growth rates of Gross Domestic Product GDP and the contribution of labour (the number of hours worked and employment) and labour productivity (TFP and capital deepening).

Table 4 shows that the EU growth rates of GDP were superior to those of the US in the sixties, but consistently inferior afterwards. So, clearly the EU and US economies are diverging and this divergence process has gained momentum since the mid-nineties. According to the Solow model, this divergence should be caused by divergences in labour or in technological change. Let us see whether this is the case.

Until the mid-nineties, labour did not contribute to GDP-growth in Europe. This was caused by a decrease in the hours worked. This went on after 1996, but then did employment increase enough to compensate for it, so that labour eventually contributed to GDP growth. This contribution however remained limited as compared to the contribution of labour productivity.

Table 4: Decomposition of EU and US growth of GDP.

	1966-1970	1971-1980	1981-1990	1991-1995	1996-2000	2001-2005
			EU15			
GDP	**4.7**	**3.1**	**2.4**	**1.6**	**2.9**	**1.6**
Labour	**-0.4**	**-0.7**	**0.0**	**-0.6**	**1.0**	**0.4**
hours worked	*-0.6*	*-1.0*	*-0.7*	*-0.3*	*-0.5*	*-0.4*
Employment	*0.2*	*0.3*	*0.7*	*-0.3*	*1.5*	*0.8*
labour productivity	**5.2**	**3.7**	**2.5**	**2.3**	**1.8**	**1.3**
TFP	*3.4*	*2.2*	*1.7*	*1.3*	*1.4*	*0.7*
capital deepening	*1.8*	*1.5*	*0.8*	*1.0*	*0.4*	*0.6*
			US			
GDP	**3.4**	**3.3**	**3.2**	**2.5**	**4.1**	**2.3**
Labour	**1.5**	**1.8**	**1.7**	**1.3**	**2.0**	**-0.3**
hours worked	*-0.9*	*-0.4*	*0.0*	*0.3*	*-0.1*	*-0.7*
Employment	*2.4*	*2.2*	*1.7*	*1.0*	*2.1*	*0.4*
labour productivity	**1.9**	**1.5**	**1.4**	**1.1**	**2.1**	**2.6**
TFP	*1.3*	*1.1*	*1.1*	*0.8*	*1.7*	*1.5*
capital deepening	*0.6*	*0.4*	*0.3*	*0.3*	*0.4*	*1.1*

Source: EC, 2006, Table 2, page 16.

It is striking that an increase in employment from the mid-nineties on, has been accompanied by a decrease of labour productivity. A first reason is that capital deepening has decreased from 1 to 0.4. So the extra influx of labour has not been accompanied by extra machinery, and this may cause the decreasing labour productivity. Secondly, the decrease of labour productivity associated with the extra labour input might be a selection effect, where those individuals with lower productivity (lower level of education, lower training, and so forth) were not hired and remained out of the labour force at first. As a result of active labour market policies in Europe, these individuals entered the labour market, thereby bringing average productivity down. So, employment increased, but at the 'cost' of a lower productivity. Note, however, that in the last observed period (2001-2005) this was no longer the case; labour as well as TFP dropped considerably.

Over the whole period labour productivity adds to GDP growth in Europe. This contribution was mainly caused by TFP. However, the contribution of labour productivity decreased considerably and consistently over the whole period. This decrease was in broad lines caused by decreases of capital deepening, but mainly of TFP growth.

To summarize the development in Europe: the contribution of labour (notably employment) has risen from the mid-nineties on, and the contribution of productivity has decreased, mainly because of a drop in TPF growth. This diverging process was reinforced in the last observed period (2001-2005) when both labour as well as TFP decreased.

US growth rates of GDP were lower than those of the EU in the second half of the sixties, but consistently higher afterwards. Save in the early 2000's, labour contributed strongly to GDP growth. In fact, save for the sixties and from 1996 on, labour was a more important contributor than labour productivity. This stands in opposition to the EU, where labour productivity was a much stronger contributor (and labour in many periods even decreased). As in the EU, there was a decrease of hours worked, but it was inferior to that in the EU. But more importantly, employment grew much stronger than in the EU (save for the last observed period).

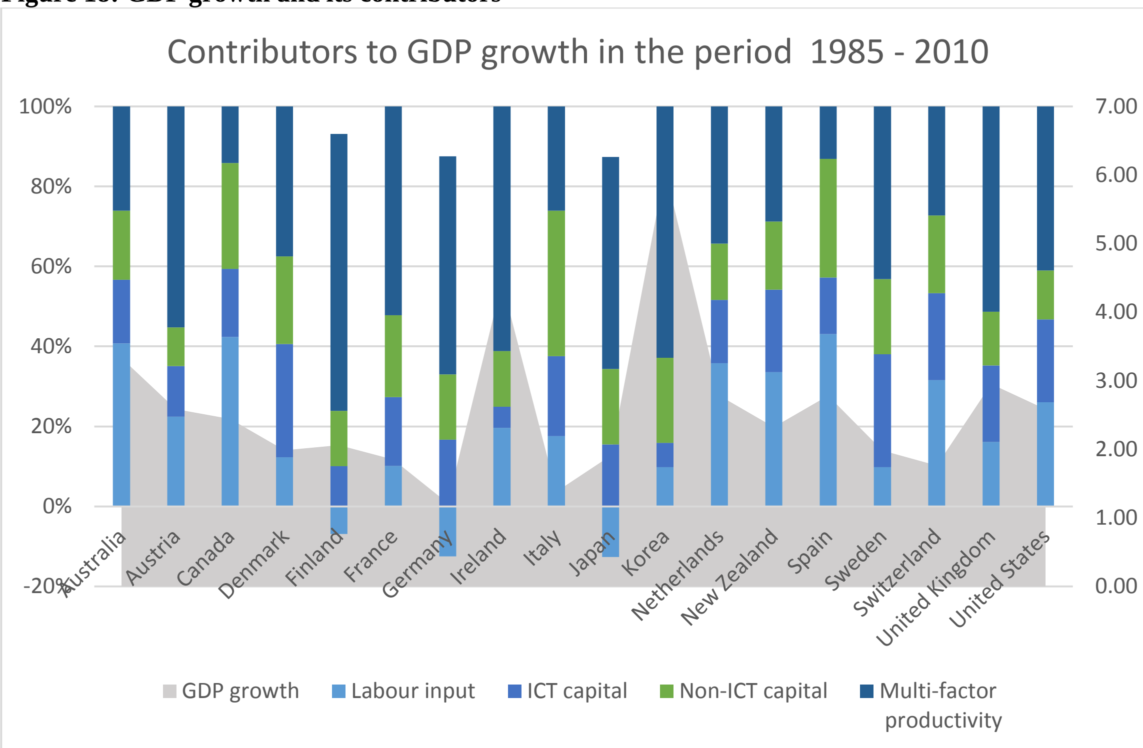
Labour productivity adds consistently to GDP growth in the US, but its contribution is lower than in the EU before 1996. Only from the mid-nineties on is labour productivity in the US higher than in the EU. And this is mainly so because of the higher TFP in the US than in the EU. A possible reason for this difference is that the US is rapidly developing its ICT-production sector, as well as using technology and ICT in other sectors. Furthermore, compared to the EU, the US is investing much more in research and development.

To summarize the situation in the US: the contribution of labour productivity, which was quite low, has increased since the mid-nineties and is now (much) more important than in the EU. This is especially caused by the TFP growth.

Another interesting application of growth accounting is provided in a recent dataset by the OECD(2013). The following figure shows the decomposition of average annual growth of GDP in a number of OECD countries over the whole of the period between 1985 and 2010, and decomposing it into labour inputs and capital inputs. The latter is however broken down into Information and Communication Technologies (ICT) capital (which includes hardware, communication and software) and non-ICT capital (transport equipment and non-residential construction; products of agriculture, metal products and machinery other than hardware and communication equipment; and other products of non-residential gross fixed capital formation). The stacked bars show the proportional contributions of the various components to average annual GDP growth rate (compare with the left vertical axis) wheras the line is the average annual GDP growth rate itself, which should be compared to the right vertical axis.
From 1985 to 2010, GDP growth in most OECD countries was mostly driven by growth in capital and what the authors call Multi Factor Productivity, or TFP in this reader. The contribution of labour input growth is on average 18%. It has the highest contribution of 42% in Canada, but has a negative contribution in Finland (-9%), Germany and Japan (about -17%), not surprizingly countries with considerable demographic ageing.
Next, in 14 out of 18 countries, the Solow Residual is the largest contributing factor to GDP growth. It is only in Austria, Italy and Spain that the contribution of non-ICT capital growth is more important, while in Demark the contribution of ICT capital growth is more important. The contribution of “multi-factor productivity” growth to GDP growth is 44% on average over all countries, and varies between 81% in Finland and more than 70% in Germany and Japan, to a contribution of less than 15% in Canada and Spain.
On average, the contribution of capital (the sum of ICT and other) to GDP growth is about 37% and it is therefore only a bit less important than the contribution of the Solow-

residual. It is more than 50% in Denmark and Italy, countries with relatively high labour costs, while it is quite low (less than 30%) in Austria, Finland, Ireland, Korea and the Netherlands. The differences within the capital (between ICT and other) are however revealing as well. The contribution of ICT capital growth to GDP growth is more important than than of non-ICT capital in only a few countries, of which Demark, Sweden, the US and UK are the most important. But in more countries is the contribution of non-ICT growth more important, including in Ireland, Italy, Japan and Korea.

Figure 18: GDP growth and its contributors



The key lesson of the above figure is that TFP or the Solow Residual is a key contributing factor to GDP growth, together with capital deepening. In a seminal paper of 2001, Easterly and Levine present the growth accounting results for 15 countries in various time periods.

Figure 19: Table from Easterly and Levine (2001, 183)

TABLE 1. Selected Growth Accounting Results for Individual Countries (percent)

Economy	Share of capital in national output	GDP growth	Share contributed by Capital	Labor	TFP
OECD 1947–73					
France	.40	5.40	41	4	55
Germany	.39	6.61	41	3	56
Italy	.39	5.30	34	2	64
Japan	.39	9.50	35	23	42
United Kingdom	.38	3.70	47	1	52
United States	.40	4.00	43	24	33
OECD 1960–90					
France	.42	3.50	58	1	41
Germany	.40	3.20	59	–8	49
Italy	.38	4.10	49	3	48
Japan	.42	6.81	57	14	29
United Kingdom	.39	2.49	52	–4	52
United States	.41	3.10	45	42	13
Latin America 1940–80					
Argentina	.54	3.60	43	26	31
Brazil	.45	6.40	51	20	29
Chile	.52	3.80	34	26	40
Mexico	.69	6.30	40	23	37
Venezuela	.55	5.20	57	34	9
East Asia 1966–90					
Hong Kong, China	.37	7.30	42	28	30
Singapore	.53	8.50	73	32	–5
Korea, Rep. of	.32	10.32	46	42	12
Taiwan, China	0.29	9.10	40	40	20

Source: For OECD, Christenson, Cummings, and Jorgenson (1980) and Dougherty (1991); for Latin America, Elias (1990); for East Asia, Young (1995).

On average over the whole table, the share of GDP growth contributed by FTP is about 31%. On average 47% is contributed by capital deepening, and only 18% to labour. But the numbers under these averages are much more interesting. In OECD countries, TFP growth accounts for about half GDP growth, while the contribution of labour is negligible and in some cases even negative. The key contributing factor to GDP growth in Latin American countries is capital deepening rather than TFP, and the contribution of labour on average explains one-quarter of GDP growth. Finally and somewhat surprizing, the contribution of TFP to GDP in China and other East Asian countries is on average 14%, while the contribution of labour is about 35%. Indeed it would appear that labour and capital accumulation explains the emergence of the so-called Asian tiger economies.

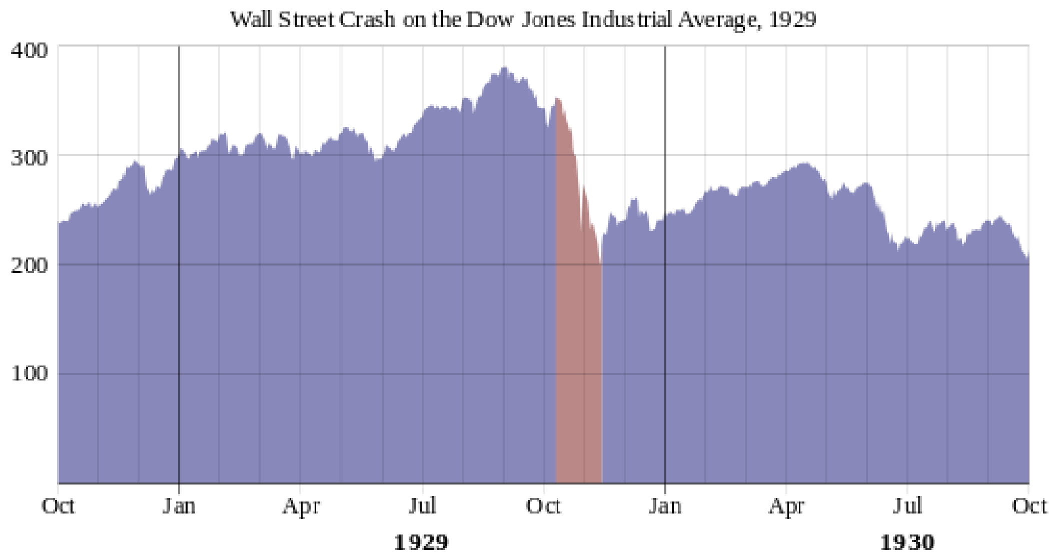
Finally, let us focus the discussion to the Latin American and Caribbean countries (LAC). in a recent IMF paper, Sosa et al. (2013: 7) apply growth accounting to discern the main drivers for GDP growth in LAC; unfortunately without Suriname. They find that, although the recent pickup of GDP growth in the Latin American countries -but not in the Caribbean- was mainly due to higher TFP, factor accumulation and especially labour, rather than TFP growth, on the whole remains the main driver of output growth in LAC. This strong contribution of labour is mostly stemming from declining unemployment. This might be potentially problematic, as a persistent low or negative contribution of TFP, combined with slacking investments in capital, and natural boundaries of labour might in the future result in lower GDP growth rates than currently observed. In that case, the current strong growth momentum is unlikely to be sustainable.

Daude (2012: 14) expresses the results of a growth accounting relative to a benchmark, which is the US. He confirms this low and even decreasing contribution of TFP for the LAC. Between 1960 and 2008, TFP grew in total by 20% in the US. This growth rate was only achieved by 3 LAC countries (Chile, Paraguay and Uruguay), but on average, TFP in the LAC countries was 10% lower in 2008 than in 1960. Especially in Brazil is TFP decreased by about 30%! Hence, one might conclude that 'all' politicians have to do is eliminate frictions and distortions in the economies of the LAC. Unfortunately, that is too simple. Using alternative specifications, and especially when endogeneizing TFP (following an endogenous growth model), Daude shows that the role of the production factors labour and capital become more important. An important policy consequence is therefore that the LAC need to invest in human capital, thereby increasing the productivity of the labour force.

Enter Keynes: Aggregate demand and the business cycle

In August 1929th, production in the US declined by 20% and wholesale prices decrease by 7.5% annual rate. The stock prices started falling around September 4th, and hit the front pages on October 24th, known today as Black Tuesday.

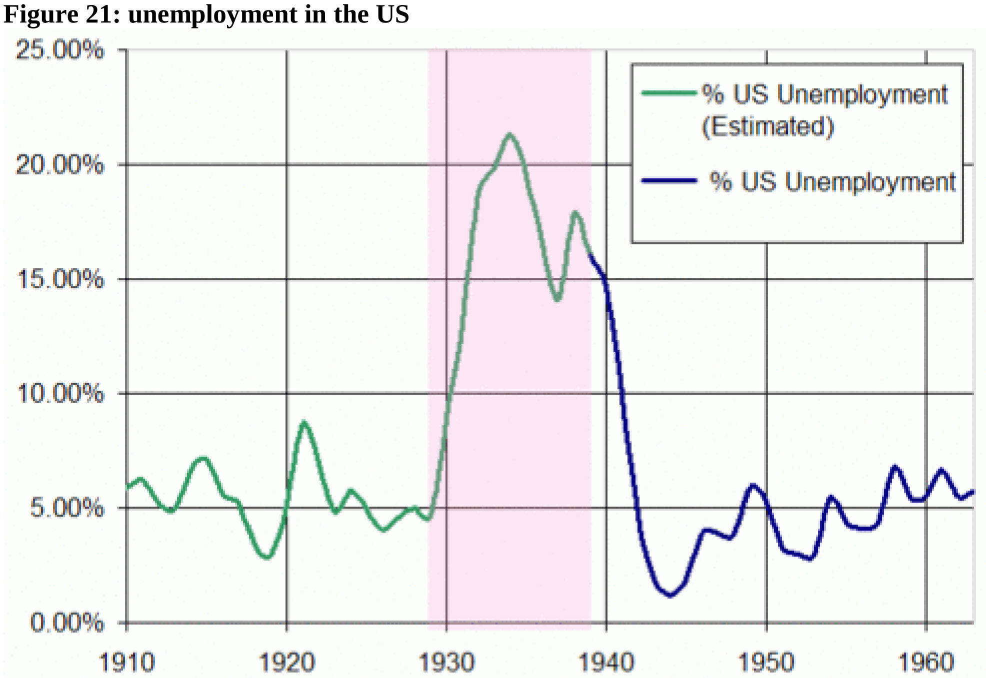
Figure 20: Dow Jones during the 29s and 30s



Source: http://en.wikipedia.org/wiki/Great_Depression [32/12/2014]

The stock market crash, a number of major bank failures and the incompetence of the government to abolish rigidities on some goods markets were generally seen as the reason for the crisis. But another striking consequence of this worldwide economic crisis is that unemployment went through the roof:

Figure 21: unemployment in the US



Source: op. cit.

But since markets always clear, unemployment is impossible in the (Neo)classical way of thinking! How can there be non-voluntary unemployment? In that case, wages would decrease (which they did), and labour demand would therefore go up until full employment was restored. So why didn't it? This is where Keynes came in.

The starting point: a static Keynesian-style model

Remember the good-old Solow model? Let us make the difference with a simple Keynesian-style income-expenditure model

Solow model	**Keynesian-style model**
Y=F(K,L) with L exogenous	Y=F(K,L) with L exogenous
Y=C+I	Y^d=C+I
C=(1-s)Y	C=(1-s)Y
I=sY	I=exogenous

Again, Y can be interpreted as output or disposable income. However, contrary to the neoclassical Solow model, this model now makes a distinction between income Y and expenditures Y^d (hence the name income-expenditure model), which need to be on the same level for the model to be in equilibrium. This equilibrium, however, need not be where the economy is on its Production Possibility Frontier. If it is not, then the equilibrium of the Keynesian model comes with unemployment. This is a short-term characteristic that is a fundamental difference with the Solow model.

It is easy to see that Y^d=(1-s)Y+I. In equilibrium, the goods market clears, so that Y=Y^d=Y*. This implies that Y*=(1-s)Y*+I and thus $Y^* = \frac{1}{s} I$ [14].

The factor 1/s is the Keynesian multiplier. This is the reason the model is also known as the *multiplier model*. An increase of the autonomous expenditure component, in this case investments I, would increase output in equilibrium by a factor which is inversely related to savings rate.
The initial effect ΔI increases Y. A part of this increase, (1-s) ΔI, increases consumption. This again increases Y. A part of this increase increases consumption by $(1-s)^2$ ΔI which again increases Y further. Hence, the total effect of an increase of I on Y is this initial increase of I plus all the subsequent changes in consumption brought about by increasing income

[14] Note that this also implies I=sY, as was the case in the Solow model. However, contrary to the Solow model, investment are now exogenous. Investments therefore determine consumption (and therefore savings), and hence output. In the Solow model, output determines investment.

$\Delta Y=\Delta I+(1-s)\Delta I+(1-s)^2\Delta I+(1-s)^3\Delta I+(1-s)^4\Delta I+(1-s)^5\Delta I+\ldots$

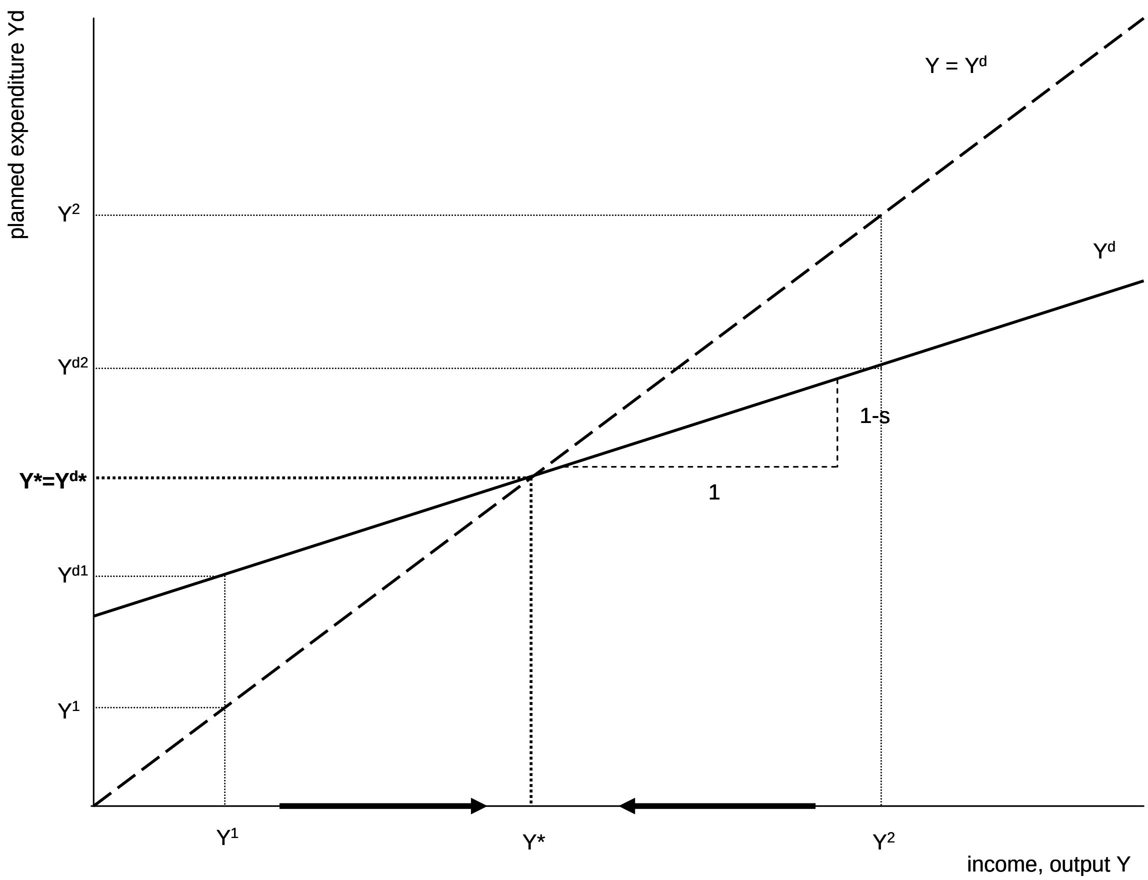
Or

$\Delta Y= \mu\ \Delta I$, with $\mu=1+(1-s)+(1-s)^2+(1-s)^3+(1-s)^4+(1-s)^5+\ldots+(1-s)^\infty$

As $(0\leq c\leq 1)$ this sum to infinity can be written as $\mu=(1/1-(1-s)) = (1/s)$, which is indeed the Keynesian multiplier.

A graphical way of representing the equilibrium is by the so-called **Keynesian Cross**, a graph relating planned expenditure or demand to income. The expenditure curve, $Y^d=(1-s)Y+I$ is shown in Figure 22. This curve starts at I if Y=0 and increases with (1-s). So, the savings rate 'leaks' an increase of spendings Y^d away from consumption, and hence away from output Y. The equilibrium requires that output Y equals demand Yd. Graphically, this equality is represented by a 45-degree line.

Figure 22: The Keynesian cross

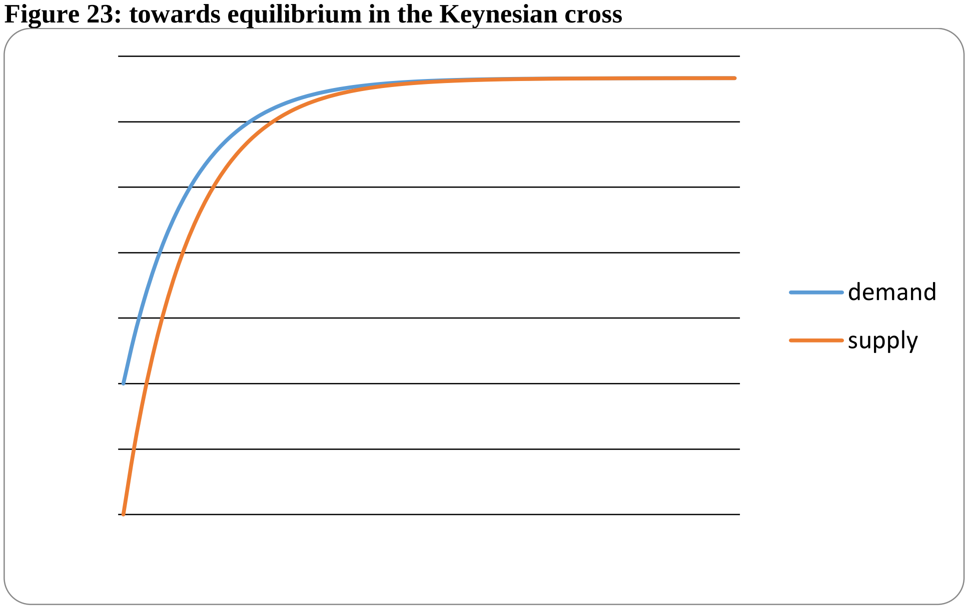


Obviously, equilibrium is where both curves intersect, which is at output $Y^*=Y^{d*}$. Now how does the economy go towards this equilibrium? Suppose that output is at Y^1, i.e.

lower than the equilibrium level. In this case, demand Y^{d1} exceeds Y^1. Firms are selling more than they are producing. Inventories fall and firms will hire more workers and increase production until that level that the equality between demand and production is restored. In the opposite case, if output is at Y^2 and higher than the equilibrium level, firms are producing more than they can sell. Inventories therefore rise and firms will lay off workers and decrease production until the equality between production and demand is restored.

This process can be shown using a practical example. Suppose that exogenous investments are always equal to 1 and suppose a savings rate of 30%. The equilibrium level of output is then 1/0.3=3.33. Suppose that we start with supply being equal to 0 and, finally, suppose that firms cannot fully adapt their output to demand. Instead, assume that supply changes each period with half of the difference between demand and supply. In the first period, supply equals 0 and consumption is therefore 0 as well. Total demand therefore equals investments and has value 1 (in the above Figure 22 we are on the vertical axis). Supply then changes with 0.5*(1-0)=0.5. In the second period, supply is 0.5 and consumption therefore equals (1-0.3)*0.5=0.350. Total demand is therefore 0.350+1=1.350, and so forth. The following Figure 23 shows this process of demand and supply, which converge to their equilibrium level of 3.33.

Figure 23: towards equilibrium in the Keynesian cross



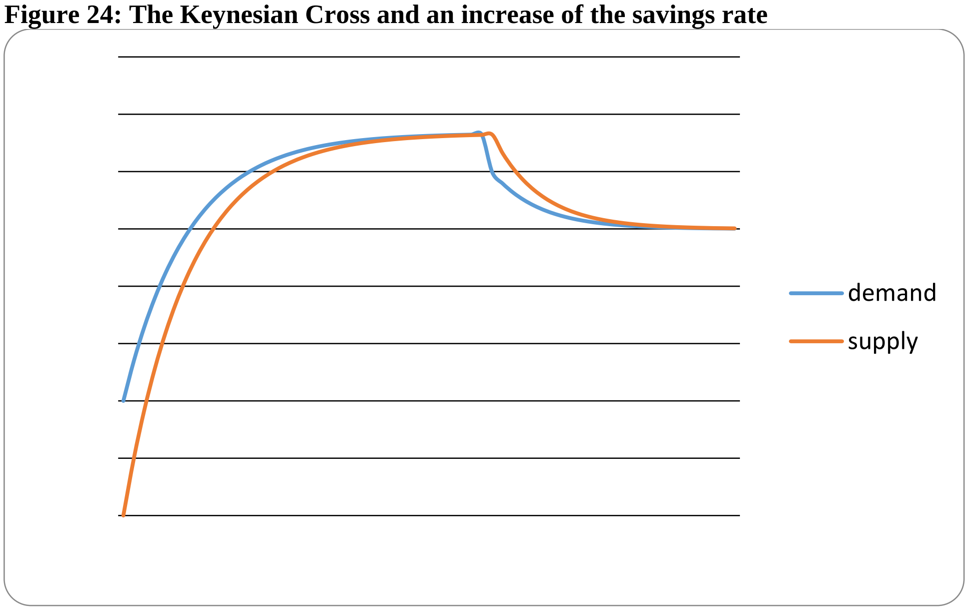
The savings rate has a fundamental role in determining the equilibrium in the Keynesian model as well as in the neoclassical Solow model discussed before. In the Solow model, output (or supply) and demand are always equal. In the Keynesian model, this is not necessarily the case. This discrepancy between output and demand implies that an increase of the savings rate has an inverse effect in both models

Keynes: Y* The Yd curve in the above Keynesian cross becomes flatter (i.e. its angle decreases). This results in a decrease in Y*, due to lower consumption, because investments and hence the capital stock do not react to the higher savings rate.

Solow: Y* increases in steady state, due to a higher equilibrium capital stock.

The Figure 23 shows what happens when the savings rate increases from 30 to 40%. The previous equilibrium was 3.33 and it now becomes 1*(1/0.4)=2.25. As the savings rate increases, consumption and demand decreases and supply will sluggishly follow behind. Demand and supply will again be equal in the new equilibrium.

Figure 24: The Keynesian Cross and an increase of the savings rate



Finally, note that the equilibrium in the Keynesian model will in most cases come with unemployment. To see why this is so, suppose Y^f=F(KL) the full-employment. Then in the Keynesian model $Y^* < Y^f$, so there can be unemployment in equilibrium, due to a shortage of spending (consumption and investment)[15]. Or, as Keynes put it (Keynes, 1936/1937: 27, in Arnold, 2002, 18) "to justify any given amount of employment there must be an amount of current investment sufficient to absorb the excess of total output over what the community chooses to consume". If the level of investment is lower, then unemployment will be the result. Suppose $Y>Y^*=Y^d$. Then the higher disposable income is split in C and S. But since I is exogenous, S>I. Or, supply will not create its own demand.

The Keynesian view on fiscal policy…

Compared to the Keynesian cross model derived when discussing business cycles, we now extend the Keynesian cross with government expenditures G and taxes T. Let us start by extending the model by government expenditures G. So aggregate demand is the

[15] The situation that $y^* > y^f$ cannot occur, for the most restricted market determines the ultimate outcome. So, if the savings rate is so low that equilibrium $y^* > y^f$ then the outcome $y= y^f < y^*$ (Arnold, 2002, 19).

sum of consumption, investment and government expenditures, or Yd=C+I+G. Investments I and government expenditures G are exogenous, and consumption C is again a fixed fraction (1-s) of income or output Y. The equilibrium in the Keynesian model now becomes $Y^* = \frac{1}{s}\left(I + G\right)$.

Hence, a change of I or G have the same effect on output.
But now we extend our model not just with government expenditures G, but also with taxes T. In this case, not all income or output is consumed or saved, but some is taxed and goes to the government in order to pay for government expenditures. So consumption is a function of income minus taxes T. So C=(1-s)(Y-T). Now if demand equals output, so Y=Yd, and substitution results in

$Y=(1/s)[I + G - (1-s)T]$

Where (1/s) is again the Keynesian multiplier. This simple model reflects the Keynesian views on government expenditures. Government expenditures G increase income and output by a proportion determined by the savings rate.

$\Delta Y=(1/s)\Delta G$

Contrary to this, taxes T will decrease output. The overall effect of a tax rise on income is

$\Delta Y=(-(1-s)/s)\ \Delta T$

Where $-(1-s)\ \Delta T$ is the initial fall in expenditure, and (1/s) is again the multiplier.

Now suppose that the government cannot go into debt, then a rise in spending should be accompanied by an increase in taxes.

$\Delta G= \Delta T$

The overall effect of the increase in G and T on income is then

$\Delta Y=(1/s)\ \Delta G + (-(1-s)/s)\ \Delta T$

Substitute ΔT for ΔG and rewrite to

$\Delta Y=\Delta G$

This result is called the **balanced budget multiplier**, and this concept argues that the government fiscal policy can play a significant role in determining aggregate demand and thus national income, *even if the government cannot go into debt*.

…is in sharp contrast to the Neoclassical view: the Ricardian equivalence proposition

A key element in the Keynesian cross is that consumption is based on current income alone. This is a gross simplification, if not completely at odds with reality. The optimal consumption model assumes that consumption depends on expectations about the future. In the optimal consumption model, consumers choose the optimal consumption path over time, subject to recourse constraints in order to maximize life time utility. This is going to be shown in a simple two-period model.

In period 1, the individual receives income Y1 and consumes C1. So savings in the first period S1=Y1-C1. Obviously, savings S1 can also be negative, in which case it is borrowing. Given that the individual must reach the end of period 2 without debts, consumption in period 2 is equals (1+r)S1 + Y2. So

C2 = (1+r)S1 + Y2.

Substitute for S1 results in

C2 = (1+r)[Y1-C1] + Y2.

Divide both sides by (1+r) and rearrange to

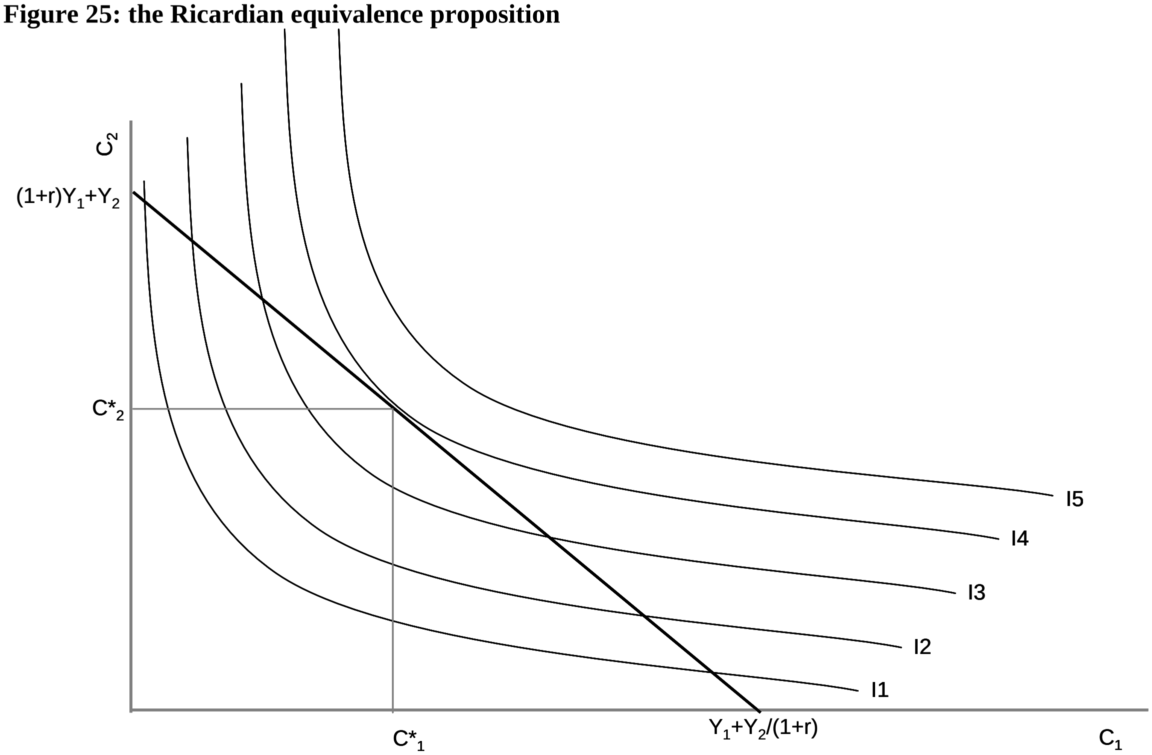
$$C_1 + \frac{C_2}{(1+r)} = Y_1 + \frac{Y_2}{(1+r)}$$

This is the intertemporal budget constraint, which shows that the present value of consumption can never exceed the present value of income. This intertemporal budget constraint contains all the feasible consumption decisions. Now which of these consumption decisions is optimal? To see this, we start from a lifetime utility function.

U=U(C1,C2)

Now the marginal utility of consumption MUc denotes how utility changes with consumption MUc=ΔU/ΔC. Following the law of diminishing marginal utility, the positive effect of utility of one extra unit of consumption decreases with consumption. The rationale behind this is that consumers order the goods they would want to consume so that they first purchase those goods that the want the most, i.e. that give them the highest utility. Once these goods are consumed, they move on to consuming goods that they still want, but not as bad as the first goods. So the utility that they gain with each extra unit of consumption, decreases. The appendix derives the solution to this model. But a more simple way to describe this system is by looking at it in a graph.

Figure 25: the Ricardian equivalence proposition



The horizontal and vertical axis reflects consumption in period 1 and 2, respectively. The straight line reflects the intertemporal budget constraint, i.e. shows all combinations of C1 and C2 that are possible given lifetime recourses Y1 and Y2. If the consumer consumes less in period 1 (i.e. saves more), then he can consume more in period 2. The angle of the budget function is set by the interest rate. The lines I1 to I5 reflect increasing levels of utility. The lines I1 to I5 denote all combinations of C1 and C2 that give equal utility. These are called indifference curves. Given consumption in period 1 C1, the line I2 has a higher consumption in period 2 C2 than I1. In other words, the utility associated with I2 is higher than that with I1, and so forth. The goal of the utility maximizing consumer is to set C1 and C2 so that utility is maximal given the intertemporal budget constraint. The indifference curve associated with the highest utility reachable given the intertemporal budget constraint, is I4 and the optimal consumption path is (C*1 ,C*2). This is where the indifference curve is tangential to the intertemporal budget constraint.

Now we turn back to our discussion of government debt. In our two-period world, the government must pay back its debt at the end of period 2. So it faces a budget constraint, which is

$$(1+r)[G1-T1] + [G2-T2] = 0$$

Now let us extend the model with taxes. Suppose an immediate tax cut in period 1 as ΔT1. The intertemporal budget constraint now becomes

$$C_1 + \frac{C_2}{(1+r)} \leq Y_1 - \Delta T_1 + \frac{Y_2 - \Delta T_2}{(1+r)}$$

The government may not have a deficit or surplus at the end of period 2, so

$(1+r)\Delta T1 + \Delta T2 = 0$ or $(1+r)\Delta T1 = -\Delta T2$

Substitute this in the budget constraint, which results in

$$C_1 + \frac{C_2}{(1+r)} \leq Y_1 - \Delta T_1 + \frac{Y_2 + (1+r)\Delta T_1}{(1+r)}$$

Which can be rewritten to

$$C_1 + \frac{C_2}{(1+r)} \leq Y_1 + \frac{Y_2}{(1+r)}$$

The variable ΔT1 does no longer appear in the budget constraint. In other words, fiscal policy does not change the budget constraint. Consequently, it does not change the optimal consumption path of the utility maximizing individual, and it does not change the economy either. So as long as the government runs a balanced budget, the timing of taxes is irrelevant and will not affect the consumption decision and hence the economy. This is called the *Ricardian Equivalence proposition*.

This point of view is still very often used to argue that the government should not pursue an expansionary policy, since it will be ineffective. The public knows that taxes that go down today (or spending that goes up) will eventually result in higher debt, which has to be serviced. It is however quite seldom that one hears the opposite argument, which is that –according to this view- increasing taxes/reduced public spending would not harm economic growth either.

From Keynes to Solow: butting heads or crossing paths?[16].

The previous chapter discussed extensively the Solow model. The previous section of this chapter introduced the thoughts of Keynes, and more specifically the Keynesian cross. The classical and Keynesian approaches seem to be at odds with each other. Indeed, one may ask "should we save or should be consume?" and both theories would come up with opposite answers. In this section, we hope to argue that "the relationship between Keynes and the classics is not one of butting heads but of crossing paths" (Garrison, 1995, 127). Furthermore, we will make the case that it is either a matter of chance or design whether the economy is in a full-employment equilibrium.

We do this by relating the Keynesian cross to the "Production Possibilities Frontier (PPF)", which is the level of output that comes with full employment. The panel A of the below Figure 26 shows essentially the same Keynesian Cross as in Figure 22. The difference is that the spending function Y^1=C+I is now accompanied by the consumption function C. Both lines are parallel to each other, and the vertical difference is obviously

[16] This section is based on Garrison (1995) and Kwok (2007).

equal to investments I. The equilibrium in this case is where spending equals output, which is where the function Y^1 crosses the 45-degree line. This ‘Keynesian equilibrium’ comes with consumption C^1 and Investments I^1.

The panel B of Figure 26 shows the same income-expenditure relationship. It consists of the same equations that made the Keynesian equilibrium in panel A, but with Y eliminated from the equations. So the starting point is a consumption equation

C=a + (1-s)Y

And a spending equation

Y = C + I

Now substitute the spending equation for Y in the consumption function, like so

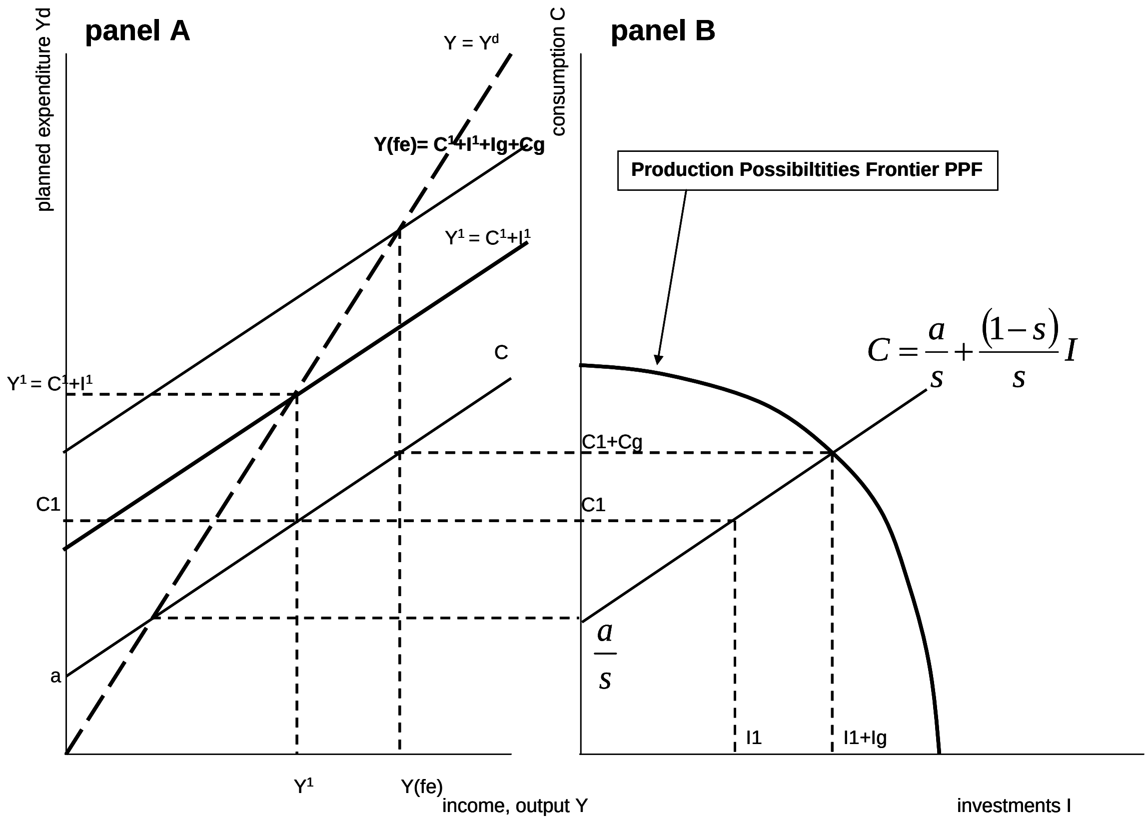
C = a + (1-s)[C + I]

And reshuffle to

$$C = \frac{a}{s} + \frac{(1-s)}{s} I$$

This is the demand function in panel B; the combinations of C and I given Y. It starts off at a/s (when I=0 and thus Y=C) and has a slope (1-s)/s.

Figure 26: linking the Keynesian cross and the PPF



This slope is easy to derive from the multiplier. Indeed, an increase of the exogenous I of one unit will cause Y to increase by (1/s). As a result of an increase of Y by one unit, C will increase by (1-s). So, the effect of one unit of I on C is the effect of I on Y (1/s) times the effect of Y on C (1-s), which is (1-s)/s.

Next, interpret the PPF as showing the various combinations of consumption goods C and investment goods I that characterize an economy in full employment of its factors labour and capital. So given K and L, the economy cannot produce more C+I.

Now we consider the equilibrium in panel A. This is where output is at level Y^1 and consumption is at level C^1. The demand function in panel B shows that investments are at level I^1. It is easy to see that this combination (C^1, I^1) is below the PPF. Put differently, the economy is below its full-employment potential and the equilibrium will come with unemployment.

Now according to Keynes, full employment exists only by accident or design. Indeed, suppose that a government can increase spending (Ig + Cg) by going into debt. Then it can choose to set this extra spending at such a level that the spending function in panel A increases from Y^1 to Y(fe). Total consumption therefore becomes C^1+Cg. We can see from panel B that this implies that investments equal I^1+Ig and the economy then is at full-employment level or PPF.

Now that we understand the relation between the Keynesian cross and the PPF, let us use this to link the Keynesian and classical approach to savings by discussing what is known as the **paradox of thrift** or the **savings paradox**. Suppose that people in a fully employed economy have read about the Solow model, and decide to save more in order to increase future consumption[17]. The result is shown in Figure 27.

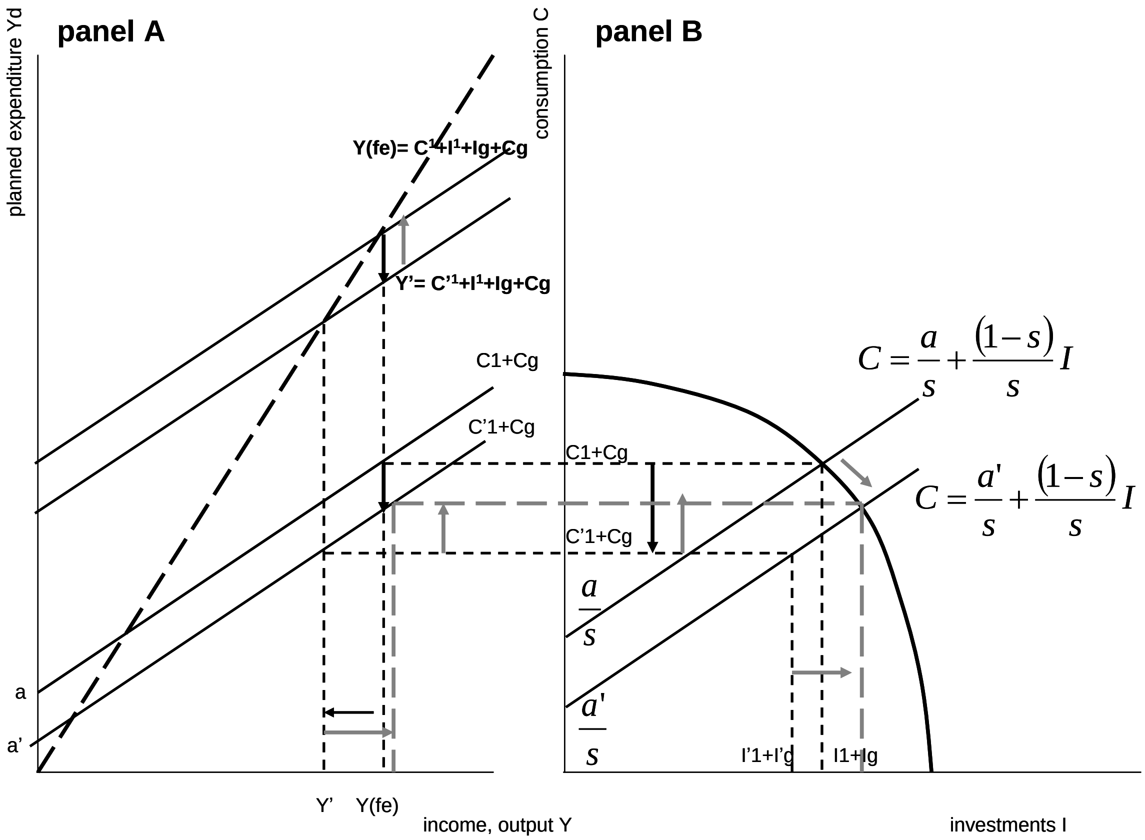
So people save more and hence decrease consumption given output. The intercept of the consumption function decreases from a to a'. Now what happens? The expenditure function lowers from $Y(fe)=C^1+I^1+Cg+Ig$ to $Y=C'^1+I^1+Cg+Ig$. As a result, panel B shows that the new equilibrium has consumption $C'^1+ Cg$ and investment I^1+Ig. This is below the PPF so increasing savings in order to have more output in the future results in output, investment and consumption being *lower*, and the result will therefore be unemployment! This is known as the Keynesian paradox of thrift. Keynes (1936, 84. In Kwok, 2007, 118) described this as that "every such attempt to save more by reducing consumption will so affect incomes that the attempt necessarily defeats itself".

But now we move from the *short* run to the *medium* run[18]. So, time goes by and factor prices (the interest rates for investments and the prices of consumption goods change to adapt to the new situation). Indeed, savings have increased and investments have decreased, so the interest rate will decrease to restore equilibrium between supply and demand of money for investment. The consequences of this are shown in grey in Figure 27. This lower interest rate will partially restore investments. Furthermore, falling prices will (through increased real money demand) increase consumption[19]. These restored investments and lower consumption prices will set the Keynesian multiplier in motion, and the resulting output will in the medium run be higher than the original Y(fe). Hence, decreasing prices and interest rates will bring the economy back to the PPF (the point where the new consumption function crosses the PPF in panel B), and the resulting output then follows from panel A.

So, the paradox of thrift may occur in the short run, as Keynes argues, but in the medium run, prices will change and full-employment equilibrium is restored!

Figure 27: the paradox of thrift? Prices and interest rates adapt in the medium term!

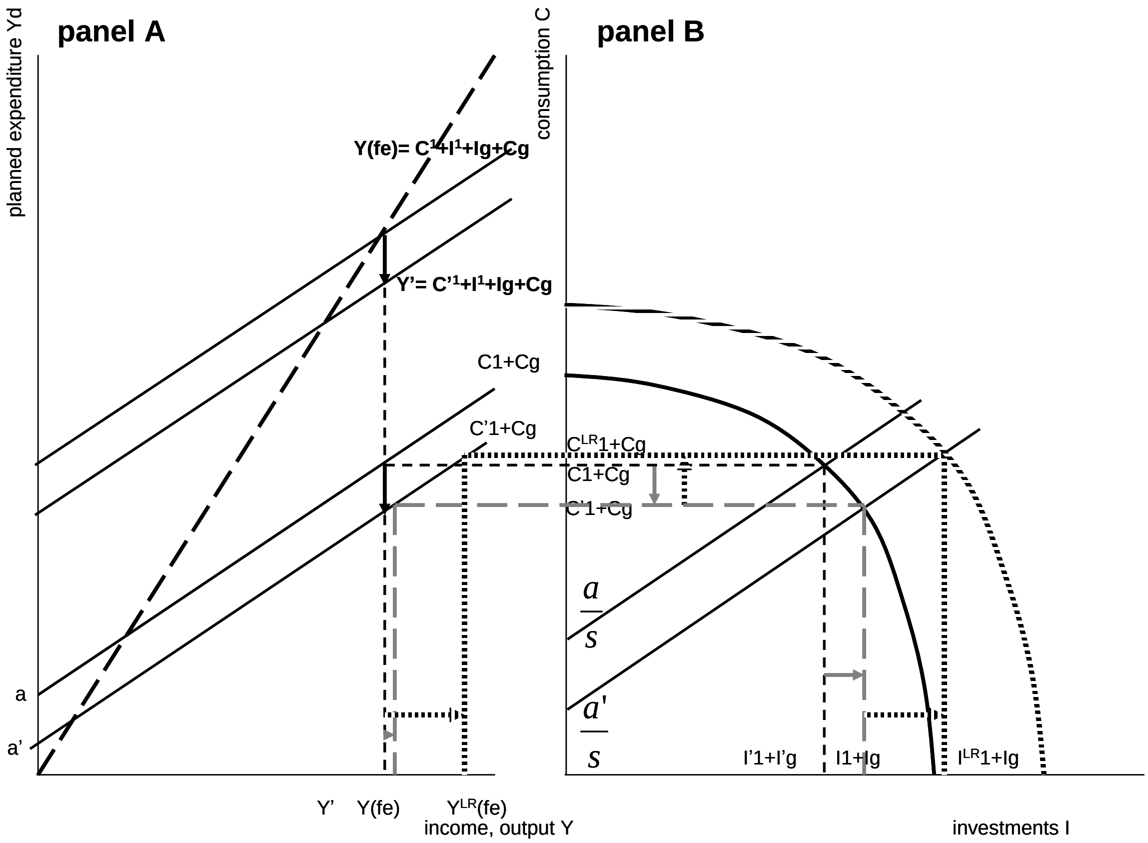
[17] In this particular example, the savings rate remains unchaged (because otherwise the angle of the spending function would change). Instead, the exogenous part of consumption a is decreased.
[18] Here we follow Kwok's (2002, 117) definition of short, medium and long term. The short run is when no prices adjust and therefore quantities adapt to a "shock" or decision. The medium run is the time frame for the adjustment of prices and interest rates. Finally, the long run is defined as the time frame for capital accumulation and convergence to a steady state.
[19] Of course the magnitude of the correction of investments and consumptions depends on their elasticies to the interest rate and prices, respectively. Kwok (2007) shows that this process can be described by the IS/LM model. We do not discuss that here, because it would make things too complicated.



However, Solow taught us that the story does not end here. Indeed, as we move from the medium to the long run, the economy settles into a new steady state. Indeed, in the medium run, the increased savings cause investments to increase, and the capital stock starts to increase. In the long run, as we have seen in the previous chapter, the economy will settle in a new steady state with a higher capital stock and higher output and –again- with full employment. This implies that the level of potential output, the PPF moves to the North-East, as shown in the next Figure 28. To keep this figure clear, the short-run effect of the decrease of a to a' is no longer included in this figure: it only contains the transition from the medium to the long run. As a result of the PPF moving outwards, as we move from the medium to the long run, output increases to Y^{LR}(fe). Consumption hence increases to C^{LR}1+Cg and investments increase to I^{LR}1+Ig[20].

Figure 28: the effect of increased savings in the medium to long run: PPF moves

[20] Remember that investment in steady state are only to cover depreciations, i.e. to keep the capital stock in per capita terms constant. But in the new equilibrium, the capital stock has increased. Depreciations have increased as well, and investments in steady state will therefore be higher.



The conclusion from all this therefore is that Keynes and the classicals do not provide alternative views on the economy, but describe the same economy in different time frames. As a result, short run paradoxes such as the “paradox of thrift” occur in the short run. Indeed, an increase in savings will not increase but decrease effective output and result in unemployment. But in the medium run, prices and interest rates will adjust; the paradox will be resolved and the economy will again move to the full-employment level. And in the long run, the economy will converge to a new steady state, where the higher initial investments will cause a higher capital stock and therefore a higher Production Possibilities Frontier PPF. As a result, consumption and investments will indeed grow even further than to compensate for the initial decrease and the goal of this increased savings will have been achieved.

Having established this link between the classicals and the Keynesians, we now turn to the first subject of this chapter, and that is to give several theoretical underpinnings of business cycles following the fixprice and flexprice approach.

The multiplier-accelerator model

Already in 1939, so shortly after Keynes published his *General Theory of Employment, Interest and Money* (1936), Samuelson discovered that a simple dynamic extension to the income-expenditure model was capable of generating business cycles. The above multiplier model is extended by not assuming investment I to be exogenous, but by partially endogeneizing it with an *accelerator*.

$I_t = I^x + \nu \Delta Y_{t-1} + \varepsilon_t$

Where I^x is the exogenous part of investment, ν>0 (nu) is the accelator and ε (epsilon) is white noise.

Assuming that the goods markets clears ($Y_t^d = Y_t$), the reduced-form of the model becomes

$Yt = (1-s)Yt + I^x + \nu \Delta Y_{t-1} + \varepsilon_t$

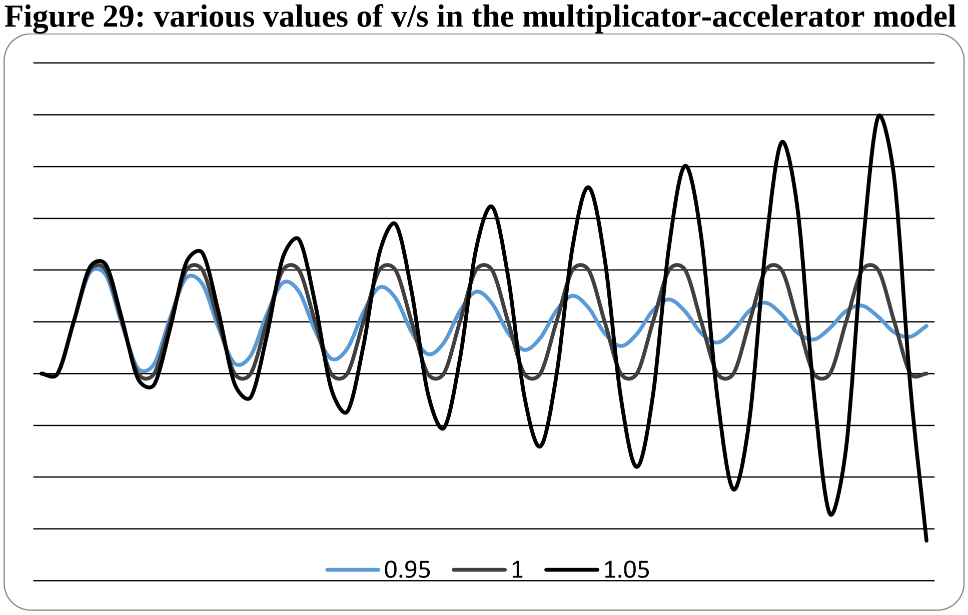
Which can be rewritten to a stochastic second-order difference equation

$$Y_t - \frac{\nu}{s} Y_{t-1} + \frac{\nu}{s} Y_{t-2} = \frac{I^x + \varepsilon_t}{s}$$

It is obvious that, given the multiplicator s, the higher the accelerator ν, the more 'explosive' the model will react to a shock. In fact, it can be shown (which we do not do) that oscillations will only occur if $\nu < 4s$ and that these oscillations are only damped if $\nu < s$. In other cases, i.e if ν is higher compared to s, the model can generate all sorts of movements: monotonous oscillations, exploding oscillations, non-oscillating convergence or finally a non-oscillating explosion.

The below Figure 29 shows the development of output Y in the multiplicator-accelerator model for various values of the ratio of v over s. To make comparison easier, the mean value of y is always 2. This is done by setting the values of Ix and ε such that the right hand side of the stochastic second-order difference equation is always equal to 2.

Figure 29: various values of v/s in the multiplicator-accelerator model



This figure confirms that v<s (v/s=0.95) results in oscillations which are decreasing in size. When v=s (v/s=1) then the oscillations remain of the same magnitude and v/s>1 results in oscillations of increasing magnitude.

Now what is the economic line of reasoning behind Samuelson's multiplier-accelerator model? Suppose an economy that is in steady state, and suppose an positive investment shock $\varepsilon_t > 0$ (t=1,2,3,…). What will be the result?

As investments I increase, aggregate output Y starts to rise. However, output cannot continue increasing forever: due to the decreasing marginal productivity of capital, the growth rate of output ΔY will decline. And once output growth ΔY starts to fall, investments I will start to decline as well. This is the upper point of the business cycle.

As investments decline, ΔY will become more negative, investments thus further decline and the economy decreases below its initial value. As however there is only one shock, and as investment has its exogenous level I^x, at some point, the decrease of ΔY will become smaller ($\Delta^2 Y>0$). This increase in production growth causes an increase in investment one period later ($\Delta I= \Delta^2 Y>0$) and this is where the economy reaches the lower turning point.

From then on, an upswing starts. However, as before, there is no reason why this upswing should halt at the old steady state value, so output 'overshoots' and the whole process starts all over again. Of course, the unemployment rate cycles with output.

Hicks: the theory of bounded waves.

The business cycles that were discussed so far have in common that output or income Y is not restricted. The implicit assumption is that output during recession is never so low that the capital stock would decrease by more than depreciation. Analogous, during the upswing is output never that high that it cannot be produced given the existing labour or capital stock. This is why these waves can be characterized as 'free'. We will in this section briefly discuss an example of a theory were these fluctuations have an upper and lower limit.

Like Samuelson did in the previous model, Hicks makes a distinction between endogenous investment, which follows the acceleration principle explained above, and fixed autonomous investment that are needed to implement technological change in the production process. He furthermore assumes that free waves are possible, but within boundaries. The upper boundary, i.e. the highest level of output achievable is there where the entire professional population (save frictional unemployment) is actually employed. The lower boundary of output is there where investments are so low that not even depreciations are neutralized, meaning that the capital stock in absolute terms decreases. Apart from that, is model is basically the same as the Keynesian multiplier-accelerator model previously discussed.

Now suppose that the economy is in equilibrium when a technology shock occurs. This causes exogenous investments to increase. Due to the multiplicator-accelerator, output increases. However, as we have seen previously, this growth process 'fades out' because a part of the extra income is consumed. But let us assume that the economy reaches its upper boundary of full employment before the multiplicator-accelerator has stopped. Output 'bumps into' its potential level and there will be demand surpluses i.e. too much consumption and investment. As prices are not included in a Keynesian model, or at least are not endogenous, they do not change and the markets do not clear. As actual output is hampered, its growth rate becomes zero, and due to the accelerator the endogenous part of investment hence becomes zero as well. Net investments then are equal to the much lower autonomous investment and the multiplier starts working in the other direction: output starts to decrease. The economy will 'shoot past its equilibrium' and will continue to decrease. However, this decrease becomes smaller because a part of the decrease is reflected by decreasing savings so that the decrease in consumption is smaller than the decrease in output. But before the economy stabilizes again, we assume that it bumps into the lower boundary. As there are no or too little endogenous investments, the capital stock decreases. The decrease of output is slower than the decrease of the capital stock and the utilisation rate of the capital stock therefore increases. As a result, producers will start to replace the oldest parts of the capital stock to increase its productivity, and this will cause output to increase again.

Inflation, unemployment and the Phillips curve[21]

The Phillips curve reflects a tradeoff between inflation and unemployment. As policy makers move the economy up and down the aggregate supply curve, unemployment and inflation move in opposite directions. The Philips curve is considered to be a key element in Keynesianism, together with the Keynesian cross.

The Philips curve, is an alternative way to express aggregate supply, was first established by the British/New Zealand economist W.A. Phillips in a paper that was published in 1958 in economica. The original paper established a link between money wages and unemployment. Paul Samuelson and Robert Solow in 1960 took Phillips' work to set a link between inflation and unemployment. This is the description which is often found in economics textbooks. Of course, the link between inflation and unemployment is indirect, and goes via output. Thus, we need a link between output and unemployment as well. So, what is the link between output and unemployment? This is known as *Okun's Law*. Arthur Okun published a paper in 1962 in which he discussed an observed inverse relation between unemployment and GDP. He found that for every percentage point that unemployment increases, the real GDP decreases by 2 percent. So, according to Okun's Law, if **output is higher (lower) than its natural rate, then unemployment is below (above) its natural rate**.

[21] This section is based on Mankiw, 2007, page 385, and on Mankiw, 1992, 303 and further.

Now what is the line of reasoning behind the Phillips curve? Suppose that inflation for some reason accelerates. Producers observe a higher price level than expected. They misperceive this as a specific price increase and not a general price increase, and therefore expand output.

The derivation of the Phillips curve starts from the aggregate supply function that relates the price level to the output supplied by firms. This aggregate supply curve is

$$y = \bar{y} + \alpha\left(p - p^e\right)$$

If prices are higher than expected, then output will be higher than its natural level. Or, the higher prices, the higher output.

Rewrite this equation to

$$p = (1/\alpha)\left(y - \bar{y}\right) + p^e$$

Next, add to the right hand side of this equation an exogenous supply shock *v* (the greek letter nu). Next, subtract the price level of the previous year *t-1* from the equation.

$$p - p_{-1} = (1/\alpha)\left(y - \bar{y}\right) + \left(p^e - p_{-1}\right) + \nu$$

Now we introduce Okun's Law, which states that when output is higher than its natural level, then unemployment *u* is lower than its natural rate u^n. Write Okun's law as

$$(1/\alpha)\left(y - \bar{y}\right) = -\beta\left(u - u^n\right)$$

Substitute $(1/\alpha)\left(y - \bar{y}\right)$ for $-\beta\left(u - u^n\right)$ in the Phillips curve results in

$$p - p_{-1} = -\beta\left(u - u^n\right) + \left(p^e - p_{-1}\right) + \nu$$

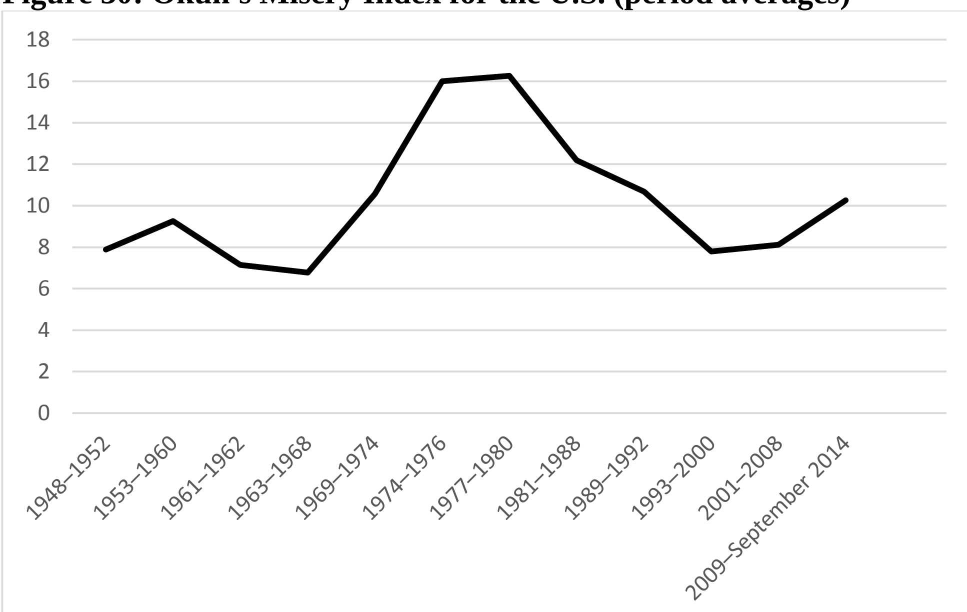
So the aggregate supply equation and the Phillips curve represent essentially the same thing. Unemployment is related to unexpected movements in the inflation rate.

Note that inflation in itself has no real effects as long as this inflation is foreseen. If inflation is foreseen, then the economy 'absorbs' this inflation via de wages and prices that people set. As a result, there is no effect on output and unemployment will be at its minimal level. We will see later that this is a key element in the contribution of Milton Friedman's monetarists. Furthermore, suppose that unemployment is at its natural (minimal level), so $u-u^n=0$, and suppose no exogenous supply shocks $v=0$. Then follows that prices equal expected prices $p-p_{t-1}=p^e-p_{t-1}$. In this case, the rate of inflation does not slow down nor speed up. It is for this reason that the natural level of unemployment u^n is more commonly known as the NAIRU, the **Non Accelerating Rate of Unemployment**.

Now this Phillips curve makes it possible to show two possible forces that changes inflation. First of all, β(u-u^n) shows **demand pull inflation**. The higher unemployment, the lower demand and hence the lower inflation. When unemployment lowers, demand increases, and so does inflation. The Phillips curve thus led to the belief that there was a stable relation between inflation and unemployment. Reaching full employment (i.e. unemployment lower than the NAIRU) through Keynesian policy was only possible at a higher (and increasing) inflation. And indeed, historical evidence as described in Phillips' paper suggested this trade-off between inflation and unemployment/output. In the Keynesian world, a recession (decreasing output and increasing unemployment) therefore can never occur jointly with inflation.

Arthur Okun created the "misery index", which is the sum of the inflation rate and unemployment rate. The following figure shows the development of the misery index in the United States.

Figure 30: Okun's Misery Index for the U.S. (period averages)



Source: data taken from http://en.wikipedia.org/wiki/Misery_index_%28economics%29

Following the Keynesian view, unemployment and inflation are substitutions, and therefore the sum of the two should be stable. And it was…until it was no longer. In the period 1969-1974, the misery index rose from 7.8 to 17. Thereafter it stayed high, on average over 16 between 1974 and 1980. The US, and therefore the rest of the world, was thus confronted with a situation of *both* unemployment and inflation. This is known as **stagflation** and occurs when the economy is in a recession while there is inflation.

But how could this be? Keynesian economists were shocked, because stagflation is impossible in the Keynesian framework described by the Phillips cure. Needless to say that the Keynesian line of thought was fiercely attacked by neo-classical or monetarist economists, like Milton Friedman and others to be discussed later in this text.

Before however turning to the alternative theories of business cycles, let us return to the Phillips curve once more. It is actually not very difficult to include stagflation in the Phillips curve. The second term of the above Phillips equation, the variable ν, denotes *supply shocks*. An adverse supply shock, causes ν to increase and inflation to rise. This is called **cost push inflation**. The most well-known and very disastrous example of cost push inflation are the strong reductions in the supply of oil by the OPEC around 1974 and again around 1979. This resulted in increasing inflation, even with unemployment increasing and output decreasing. Thus, cost-push inflation in a situation of recession is the underlying reason for stagflation. This remains one of the most serious economic problems policy makers can face[22].

Aggregate supply

The Keynesian approach which was explained in the previous sections assigns a key role to aggregate demand. Insufficient demand results in unemployment. Furthermore, prices are fixed and the government can use fiscal (spending) and monetary policies to stabilize the macroeconomy. It is obvious that the Keynesian theories so far are to be categorized as being part of the **fixprice approach**. This fixprice approach sustained heavy academic damage as a result of the observed stagflation, and alternative theories gained ground. These alternatives increasingly went back to the neo-classical line of through, where prices are not fixed but flexible and where markets reach their equilibrium. These several theories to be discussed therefore categorize under the **flexprice approach**.

The monetary approach

In the second half of the sixties, the monetarist approach came as a counterposition of Keynesian economics. This recovery of the neoclassical model was brought back into the spotlight by the failure of large demand-driven macroeconometric models with exogenous expectations to capture the economic situation of the 70's. Moreover, the famous Lucas-critique stressed the need for a more structural approach to modelling the economy.

Monetarists, of whom the Nobel laureate Friedman is the most famous, believe that demand management (spending and tax policy) is of no need, because there are self-corrective forces at work. At any rate, fiscal and monetary policies are ineffective stabilisators, because supply-side factors determine output and employment.

[22] The sum of inflation and the unemployment rate is sometimes called the "misery index", a term introduced by Arthur Okun.

Let us start with what we already know[23]. As with the Keynesian IS-LM model, the equilibrium in the goods market is described by the IS curve

$y_t = \sigma\, r_t$

(σ =sigma; $\sigma<0$) where *y* is the log of output, and *r* is the log of the real interest rate. The equilibrium in the money market is described by the LM curve

$m_t - p_t = \varphi\, y_t$

(φ=phi; $\varphi>0$) where *m* is the log of the money supply, *p* is the log of the price level, sigma is the log of the turnover. So, this is the log version of the *LM curve*.

Contrary to the previous Keynesian models, we now introduce expectations in the model. Assume that agents form expectations adaptively. The inflation in period *t+1* expected as of period *t* is assumed equal to current inflation.

$\Delta p_{t+1}|_t^e = \Delta p_t$

where *p* is the log of the price level.

Next, suppose a negative relation between aggregate output and real wages.

$\gamma y_t = -(w_t - p_t)$

(γ = gamma; $\gamma>0$) where *w* is the log of the nominal wage level. If prices increase relative to wages, profits increase as well. Firms will expand output and hire more labour until the wage increases to the new price level[24].

Furthermore, the nominal wage level is assumed equal to expected prices

$w_t = p_{t+1}|_t^e$

As a result of the last two equations, if expectations are fulfilled (i.e. $p_t = p_{t+1}|_t^e$), then y_t equals its 'natural rate' (in this somewhat theoretical model, the natural rate of output y=0 or Y=1).

As a first step in the solution of the model, impute the wage equation into the relation between output and wages. This results in

$\gamma y_t = -(p_t|^e{}_{t-1} - p_t)$.

[23] What follows is based on Arnold, 2002, chapter 3, pages 32 to 40.
[24] Combine this equation with Okun's Law and you get the Philiips curve again.

So aggregate output exceeds its natural rate if prices are higher than expected, and vice versa.

Next, lag the expectations equation by one period and rewrite to $p_t|^e_{t-1} - p_{t-1} = p_{t-1} - p_{t-2}$. Also write $\Delta^2 p_t$ as $p_t - p_{t-1} - p_{t-1} + p_{t-2}$, rewrite to $p_{t-1} - p_{t-2} = -\Delta^2 p_t + p_t - p_{t-1}$ and substitute this back in the rewritten expectations equation. This results in $\Delta^2 p_t = p_t - p_t|^e_{t-1}$.

This can be substituted into the previous equation, resulting in

$$\gamma y_t = \Delta^2 p_t.$$

This is known as the *accelerationist Phillips curve*. It says that, if expectations are not fulfilled because inflation accelerates, then output increases above its natural rate (the NAIRU). If on the other hand inflation does not accelerate (and is therefore correctly anticipated), then output is at its natural rate, and determined on the economy's supply side.

The monetarists' explanation for the Phillips curve is that workers and firms suffer from 'money illusion'. When an inflationary surprise occurs, workers are fooled into accepting lower pay because they underestimate inflation, and therefore overestimate their future real wages. Firms hire them because they see the inflation as allowing higher profits for given nominal wages. So, firms also suffer from 'money illusion' and hire more labour and expand production. As a result, at least in the short run, accelerating inflation will increase output and decrease unemployment. However, *if no further shocks occur*, workers and firms will adjust their expectations adaptively. So, workers will increase their wage demands, and firms will reduce their output. Output and unemployment will fall back to their original levels.

Now we have a model where the demand and supply side are described by the LM curve and the accelerationist Phillips curve, respectively. How does the interaction between supply and demand result in business cycles? Take differences twice in the LM curve, which results in

$$\varphi \Delta^2 y_t = \Delta^2 m_t - \Delta^2 p_t$$

Finally, substitute the Phillips curve into this equation. This results in the following stochastic second-order difference equation:

$$y_t - \frac{2\varphi}{\varphi+\gamma} y_{t-1} + \frac{\varphi}{\varphi+\gamma} y_{t-2} = \frac{1}{\varphi+\gamma} \Delta^2 m_t$$

if we assume that $\Delta^2 m_t$ is white noise. Now the model has two characteristics that are essential in explaining economic fluctuations:

1. Following the Phillips curve, inflation rises (declines) whenever output is above (below) its natural rate (y>0).
2. According to the twice-differenced LM curve and assuming no monetary shocks, output growth decreases ($\Delta^2 y_t<0$) whenever inflation rises ($\Delta^2 p_t>0$).

Now suppose that the economy is in steady state when a positive money shock occurs, after which money growth is constant for a while. This shock causes output to increase a above its natural level (via the LM curve). Hence, inflation starts to accelerate (1) and so output growth declines (2). The economy will sooner or later reach an upper turning point.

After the turning point, output is still above its natural rate and inflation continues to rise (1) and output continues to decline (2). The downswing hence gains strength and output overshoots its natural rate. When output is below its natural rate, inflation declines (1) and output growth increases (2). This ultimately results in a lower turning point.

Before closing the discussion of the monetary approach to economic fluctuations, let us discuss once more the Phillips curve[25]. The interpretation of this curve was changed by the Monetarist approach of Milton Friedman, who extended it to include expected inflation. If inflation accelerates beyond what is expected, then producers misperceive the higher price level as a specific price increase and not a general price increase, and therefore expand output. If the monetary authorities such as the central bank have an 'information advantage' over economic agents, they can make monetary shocks *m* happen without them being expected. This way, monetary authorities can use the Phillips curve to 'steer' the economy and reduce unemployment. This however assumes that the monetary authorities knows more than everybody else in the economy, because it assumes that the central bank fools producers into believing that a general price increase is not inflation, but a specific price increase for every single producer instead.

Note, finally, that even though the above model setup included the IS curve, this does not enter the solution to the model. In the monetarist view, the economy is a function of the LM curve and the Phillips curve and the IS curve does not play a role. As a result: it is the money market that has a key role in explaining fluctuations and the goods market has no role in this.

In the next model, we assume that there is no information advantage, i.e. we assume that economic agents know when the monetary authorities change *m*.

An application: possible explanations for the Great Depression

In the 2006 edition of his famous textbook on macroeconomics, Gregory Mankiw discusses various possible explanations of the Great Depression. This discussion will be summarized in this section. The Great Depression –indeed written with capitals- was about the largest economic recession that had ever hit the United States (and therefore the

[25] What follows is again based on Mankiw, 1992, 303 and further.

entire world) that started with the 1929 Wall Street crash, and lasted throughout the 30's of the previous century. Until today, the debate continues on what actually caused this Depression, but economists tend to agree that there is no single cause and that economic and financial shocks resulted in changing expectations, a process reinforced by some unfortunate decisions by policy makers. Table 5 below describes the development of several macroeconomic key variables during that unfortunate period.

Table 5: Macroeconomic developments in the US during the Great Depression[26].

year	unemployment rate	real GNP	consumption	investment	Government purchases
1929	3.2	203.6	139.6	40.4	22
1930	8.9	183.5	130.4	27.4	24.3
1931	16.3	169.5	126.1	16.8	25.4
1932	24.1	144.2	114.8	4.7	24.2
1933	25.5	141.5	112.8	5.3	23.3
1934	22	154.3	118.1	9.4	26.6
1935	20.3	169.5	125.5	18	27
1936	17	193.2	138.4	24	31.8
1937	14.3	203.2	143.1	29.9	30.8
1938	19.1	192.9	140.2	17	33.9
1939	17.2	209.4	148.2	24.7	35.2
1940	14.6	227.2	155.7	33	36.4

	Nominal interest rate	money supply	price level	real money balance
1929	5.9	26.6	50.6	52.6
1930	3.6	25.8	49.3	52.3
1931	2.6	24.1	44.8	54.5
1932	2.7	21.1	40.2	52.5
1933	1.7	19.9	39.3	50.7
1934	1	21.9	42.2	51.8
1935	0.8	25.9	42.6	60.8
1936	0.8	29.6	42.7	62.9
1937	0.9	30.9	44.5	69.5
1938	0.8	30.5	43.9	69.5
1939	0.6	34.2	43.2	79.1
1940	0.6	39.7	43.9	90.3

Let us review several hypotheses.

The spending hypothesis

Using the obvious truth that causes generally precede effects, one immediately may think as the Wall Street crash of 1929 as a possible cause of the Great Depression. This may

[26] Source: Mankiw, 6th edition, 2006, Table 11-2, pages 318-319. All figures save the nominal interest rate, have 1958 as the base year.

have had several effects. First of all, firms which have stocks on the stock market run a higher risk of being taken over by other firms. To block possible takeovers, they need to reserve enough resources to eventually buy their own stocks. These reserves cannot be invested. Furthermore, lower prices of stocks and bonds means that the possibilities to gather additional external capital for investments decrease, and this may further hamper investments. Third and maybe the most important, this crash may have resulted in lower expectations of producers, resulting in lower investments as well. Furthermore and fourth, most individuals invest in the stocks and bonds to save for their pension. The crash may therefore have decreased the pension income of the retired, as well as negatively affect the expectations of the group of consumers as a whole. The observed strong decrease of investment in housing may be a signal of that, though there are economists who argue that this decrease was a result of excessive investments in the 1920s.

As a result of all this, demand on the goods market decreases, given the rate of interest. In other words, there might have been a contractionary shift in the IS curve. As a result, output declined and the Depression began. At this stage, there were several important bank failures, due to lack of regulation and a lack of investments. This made it even more difficult for firms to get the funds for investments, and had a further negative effect on the expectations of consumers.

In addition, politicians were only concerned with balancing the budget than with using expansive fiscal policy to counter the economic downswing. They reacted to decreasing tax incomes by decreasing government spending and by implementing the 1932 Revenue Act, which increased various tax rates.

The money hypothesis

Another important development took place in the monetary sphere. The Federal Reserve Bank decreased money supply by no less than 25 percent between 1929 and 1932. In the same time, unemployment increased from 3.2 to 25.2 percent. According to the money hypothesis, defended among others by Milton Friedman, the primary blame for the Great Depression lie with the Federal Reserve Bank. As a result of the decrease of real money balances, the LM curve showed a contractionary shift, thereby causing output to decrease even further.

There are however several possible arguments against this view. First of all, even though money supply fell, prices fell even more and real money supply therefore did not decrease at all, and the LM curve did not shift, or at least not as strong as what one would expect from the decrease of the money supply alone.

Furthermore, if the economic contraction had been the result of mainly a shift in the LM curve, the interest rates should have gone up. Instead, they went down. The only possible explanation could be that the contractionary shift in the IS curve was even bigger than that of the LM curve. This seems somewhat incredible, given the strong decrease in money supply. So, the situation seems to be a bit more complicated, and Mankiw discusses several other devastating effects of the strong deflation.

Unanticipated inflation or deflation and output: the debt-deflation theory

First of all, according to the debt-deflation theory, a situation with unexpectedly falling prices enriches creditors while empoverishing debtors. Suppose that a creditor loans a certain amount *x* at time 1 with price level p_1. At time 2, the debtor has to repay the same debt *x*, but then the price level has unexpectedly decreased to p_2 ($p_1 > p_2$). The real value of the debt has increased from $\frac{x}{p_1}$ to $\frac{x}{p_2}$. Therefore, in real terms, the debt that the debtor has to repay to the creditor has increased by $\frac{p_1}{p_2}$. The creditor therefore gains at the expense of the debtor. If this deflation would have been foreseen, it would have been taken into account when setting the rate of interest. But the deflation was not foreseen, so it has not been taken into account.

Now suppose that debtors have a higher propensity to consume than creditors (which is the reason they are debtors in the first place), then an implicit shift of income from debtors to creditors resulting from deflation will decrease total spending. In the context of the IS-LM framework, this is a contractionary shift of the IS curve, resulting in a further reduction in output.

In conclusion, unanticipated deflation (inflation) enriches creditors (debtors) and empoverishes debtors (creditors). This is equivalent to a contractionary (expansionary) shift of the IS curve and a further economic recession (expansion).

Also, once consumers observe the deflation, they may expect this to continue. This may induce them to postpone consumption, since they believe they can get their consumption goods at a better price in the future. This may in the short run result in a dramatic fall in consumption, further contracting the IS curve.

Next, some more recent approaches will be discussed. These put emphasis on the supply side, but also some New Keynesian approaches will be presented afterwards. This discussion will start with the rational expectations approach, because it is strongly linked to the monetarist approach discussed in the previous section.

New Classical economics: Rational expectations[27]

In the Keynesian model, expectations (of investors) are exogenous and changing expectations therefore shift the demand curve upwards or downwards. In the monetarist model, expectations are endogenous and adaptively. The inflation in period t+1 expected as of period t is assumed equal to current inflation. This is somewhat inconsistent,

[27] This section is based on Arnold, 2002, chapter 4.

because it means that we, who have knowledge of the model, know more than the representative agents who are ‘in’ the model. Why not assume that these representative agents know exactly as much as we do, and therefore know the model they are in? In other words, when forming expectations, representative agents use the corresponding mathematical expectations. This new classical economics approach was introduced by Robert Lucas in a 1972 paper. In 1995, he would be awarded the Nobel Prize in economics. It would take us too far to discuss the Lucas model. However, Arnold (2002) shows that it is not very difficult to change the monetarist model discussed in the previous sector to include rational expectations. This allows drawing the same conclusions as on the basis of the Lucas model.

The first two equations are exactly the same as in the previous section. We start with the LM curve or quantity equation, which describes the equilibrium in the money market.

$m_t - p_t = \varphi \, y_t$

(φ=phi; φ>0) where m is the log of the money supply, p is the log of the price level, sigma is the log of the turnover.

Next, we again suppose a negative relation between aggregate output and real wages.

$\gamma y_t = -(w_t - p_t)$

(γ = gamma; γ>0) where w is the log of the nominal wage level. The equation describing the nominal wage level differs from the model in the previous section. As before, nominal wages are equal to expected prices

$w_t = E_{t-1}p_t$

but the expectations are no longer adaptive, but are rational instead.

Analogous to the previous section, The first step in the solution of the model is to impute the wage equation into the relation between output and wages. This yields

$\gamma y_t = (p_t - E_{t-1}p_t)$

This is a simple version of the so-called Lucas supply function. It is the rational-expecations equivalent of the accelerationists Phillips curve: if expecations are correct, then output is equal to its natural rate of zero. In other words $E_{t-1}y_t=0$, because $E_{t-1}(p_t - E_{t-1}p_t) = E_{t-1}p_t - E_{t-1}p_t = 0$.

Aggregate output is above its natural rate if the aggregate price level exceeds its expected value, and vice versa. If inflation accelerates, producers observe a higher price level than expected. They misperceive this as a specific price increase in their own market, and not a general price increase. They therefore mistakenly expand output above its natural level.

Now how do we find the solution to this model? First, rewrite the LM curve to $p_t = m_t - \varphi y_t$ (i) and take expectations: $E_{t-1}p_t = E_{t-1}m_t - \varphi E_{t-1}y_t$. This last variable is zero, so $E_{t-1}p_t = E_{t-1}m_t$ (ii).

Next, take the Lucas supply function, $\gamma y_t = (p_t - E_{t-1}p_t)$, and substitute for p_t from the LM curve (i). This yields $\gamma y_t = (m_t - \varphi y_t - E_{t-1}p_t)$. After that, substitute for $E_{t-1}p_t$ from (ii) to find $\gamma y_t = (m_t - \varphi y_t$ (i) $- E_{t-1}m_t)$. Finally, rewrite to

$$y_t = \frac{m_t - E_{t-1}m_t}{\gamma + \varphi}$$

In equilibrium, output is positively related to money surprizes. Or, put differently, only unanticipated changes in money supply matter.

A second difference between the previous monetary model and this 'Lucas version' of the model is that the assumption of a random walk of the money supply *m* is replaced by Lucas' description of money supply. Assume that a central bank ties the supply of money to past realizations of money supply, output and prices.

$$m_t = a + \sum_{j=1}^{\infty} b_j m_{t-j} + \sum_{j=1}^{\infty} c_j y_{t-j} + \sum_{j=1}^{\infty} d_j p_{t-j} + \eta_t$$

This is called the 'feedback money supply rule'. The parameters a, b_j, c_j and d_j are parameters that can be set by policy makers. The variable η (eta) is a random component, reflecting that the Central Bank does not have full control over money supply.

And here is where the rational expectations hypothesis comes in: this hypothesis assumes that representative agents in our model have all available information to their disposal. This means that they are aware of the above feedback money supply rule, and of its parameters. The only thing about this rule which they cannot anticipate (simply because the Central Bank has no control over it), is the random component η (eta)! When our representative agents hence form expectations of the money supply, they will take the non-stochastic part of the feedback money supply rule into account. Therefore

$$E_{t-1}m_t = a + \sum_{j=1}^{\infty} b_j m_{t-j} + \sum_{j=1}^{\infty} c_j y_{t-j} + \sum_{j=1}^{\infty} d_j p_{t-j} \quad t$$

So

$m_t - E_{t-1}m_t = \eta_t$

which means that their expectation errors will be random, but on average zero. Substitute this strong result into the above output equilibrium relation to get:

$$y_t = \frac{\eta_t}{\gamma + \varphi}$$

This is a crucial consequence of the hypothesis of ratioal expectations: *the policy parameters a*, b_j, c_j *and* d_j *have no effect on aggregate production! Only unanticipated shocks in the supply of money affect output*. This is Thomas Sargent and Neil Wallace's famous *policy ineffectiveness proposition*, which they presented in a paper in the Journal of Political Economy of 1975.

The policy ineffectiveness proposition is that only unanticipated monetary shocks cause unanticipated inflation, and only unanticipated inflation raises output. So, because economic agents have full available information, including information on the policy measures taken by the government and the central bank, they will take this information into account. Conscious policy measures taken by the government and the central bank will therefore have no real effects on output, but will only change (anticipated) inflation. From the monetary point of view, this means that the economy is best served by a stable and predictable money supply, where the occurrence of unanticipated shocks η_t (eta) is minimized.

Real Business Cycles

The lesson to be learned from the previous model of rational expectations is that money does not matter, insofar that consious monetary shocks are anticipated, so that only random and uncontrolled monetary shocks can affect the economy. This was the predominant view during the 70's. During the eighties, however, things in a way became even simpler. The controversional finding appeared that simple Solow-like equilibrium models, when driven by shifts in Total Factor Productivity as derived using the growth accounting approach, could generate time series with the same complex patterns of persistence, comovement and volatility as observed in reality[28]. So, business cyles were explained by a model involving market clearing, no monetary factors whatsoever and without a rationale for macroeconomic management. This extreme neoclassical approach was developed by Edward Prescott and Finn Kydland and first published in a 1982 paper in econometrica. In 2004 both would share the Nobel price economics for this approach.

Remember the way the Total Factor Productivity of TFP was derived in the context of growth accounting? TFP was the change of output that was not attribuable to changes of the inputs labour (employment and hours worked) and capital (capital deepening). Now we do the same thing, but in a different form, and not in changes but in levels.

Suppose that output Y depends on capital K and labour L, a 'random productivity shock' variabe A_t and a deterministic component of productivity X_t. Furthermore, suppose the well-known Cobb-Douglas production function.

[28] What follows is based on King and Rebelo (1999).

$$Y_t = A_t K_t^{1-\alpha}\left(L_t X_t\right)^{\alpha}$$

With α (0≤α≤1) being the labour's share of national output[29]. We assume this variable known. Write the production function in logs

$\log(Y_t) = \log(A_t) + (1- \alpha)\log(K_t) + \alpha\log(L_t) + \alpha\log(X_t)$

Reshuffle to

$\log(A_t) + \alpha\log(X_t) = \text{Log}(Y_t) -(1- \alpha)\log(K_t) - \alpha\log(L_t)$

Of this equation, the left hand side contains the unknowns, and the right hand side contains the known variables. Write

$\log(SRt) = \log(A_t) + \alpha\log(X_t)$

as the sum of the unknowns. This variable SR is known as the **Solow Residual**, the equivalent of TFP in the text on economic growth.

Now we deal with the unknowns. Suppose that the deterministic component of productivity grows at a constant rate γ (gamma).

$X_{t+1}=\gamma X_t, \qquad \gamma > 1$

Rewritten to logs, this becomes $\log(X_{t+1}) = \log\gamma + \log(X_t)$, which means that

$\log(X_t) = \log\gamma + \log(X_{t-1})$.

Furthermore, suppose that the random component of productivity follows an autoregressive process (AR(1)), as

$\log(A_t) = \rho\log(A_{t-1}) + \varepsilon_t$

Here, ρ(=rho) is the persistence parameter, and ε (epsilon) is the standard deviation of innovations. In order to establish how the economy fluctuates over time, we need to establish the values of γ, ρ and the standard deviation of ε. This is done as follows:

We know that $\log(SR_t) = \log(A_t) + \alpha\log(X_t)$ and $\log(X_t) = \log\gamma + \log(X_{t-1})$

so

[29] To see this, note that labour income is equal to the marginal productivity of labour MPL times the amount of labour. Or labour income equals MPL*L. Now MPL is found by differentiating the production function to labour, which is MPL=α(Y/L). Labour income thus equals α(Y/L)*L= αY.

$\log(SR_t) = \log(A_t) + \alpha \log\gamma + \alpha\log(X_{t-1})$. Now continue to substitute for X_{t-1}..

$\log(SR_t) = \log(A_t) + 2\alpha \log\gamma + \alpha\log(X_{t-2})$

Iteration back to the starting value $X_{t-t}=X_0$ results in

$\log(SR_t) = \log(A_t) + t\alpha \log\gamma + \alpha\log(X_0)$, which can be reshuffled to

$\log(SR_t) = [\log(A_t) + \alpha\log(X_0)] + t\alpha \log\gamma$

from the time points between 0 and t, A_t is exogenous, so the first part of this equation is really an intercept term. In other words, to find $\log\gamma$, we fit a trend to $\log(SR_t)$.

Next, we can use $\log\gamma$ to calculate the whole deterministic component of productivity series $X_0 \rightarrow X_t$.

We now know the development of the Solow Residual $\log(SR_t)$ as well as *X*. Confronting these two variables results in the series of *A*. The value of ρ can be derived from this time series. Finally, the standard deviation of ε is of course equal to the 'misfit' of the estimated trend of $\log(SR_t)$, for this estimation error reflects those changes of $\log(SR_t)$ that are not attributable to γ (the development of X) and they must therefore be attributable to changes of A. Hence, the standard deviation of A, ε, is equal to the standard deviation of the estimation errors of the trend regression.

The basic RBC model is a rather neoclassical model of infinitely-living rational economic agents who maximize current and future utility from leisure (not work) and consumption (work). That they have infinite lives is just a way to ensure that they foresee the future given all current information (rational expectations), that they substitute work today for work tomorrow (or vice versa) and that they take the utility of future generations into account. So in every situation, economic agents choose choose the quantities of leisure (i.e. not work), consumption and investment that maximize utility.

Now what happens when a positive random productivity shock occurs in A_t? First of all, the marginal productivity of labour increases. In a competitive labour market, the real wage equals the productivity of labour, so the real wage increases[30]. The wage one does

[30] This is not difficult to show. Suppose a simple production function Y=F(K,L) and suppose profit C to be equal to PY-wL-rK (P=goods prices, w=nominal wage, r=nominal interest rate). The firm hires labour as to maximizes profit, so it will hire labour until the last unit of labour hired does not add to profit (the marginal effect of labour is zero). So maximize profit with respect to labour L.

$dC/dL = p(dY/dL)-w=0$

or

$(dY/dL)=(w/p)$

not receives if one takes leisure will increase as well, and workers will therefore work more and shift leisure to the future. Hence, output and income will increase as a result of the positive productivity shock.

Furthermore, the marginal productivity of capital increases. In a competitive capital market, this is equal to the (real) interest rate, so the latter increases as well. As a result, most of the extra income made by working harder will be saved and thus invested.

After a while, however, the productivity shock will wear off and productivity of labour increases to its previous level. However, the capital stock will have increased to a higher level due to previous investments, and as a result of the diminishing marginal returns to capital, the productivity of capital will be *below* the original level. In a way, there is excess capital and the interest rate will be below its original level. Therefore, consumption will increase (savings will decrease) and will only return to its original level when this excess capital stock is gone.

So, in short, the early part of the responses to a productivity shock is that work effort, production, investment and consumption increase. The later part is dominated by the reduction of capital towards its stationary level. This is shown in the below Figure 31 and Figure 32.

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where dY/dL is the marginal productivity of labour. This way, one can also show that the marginal productivity of capital equals the real interest rate r/p.

Figure 31: the dynamics of a productivity shock in RBC (source: King and Rebelo, 1999)

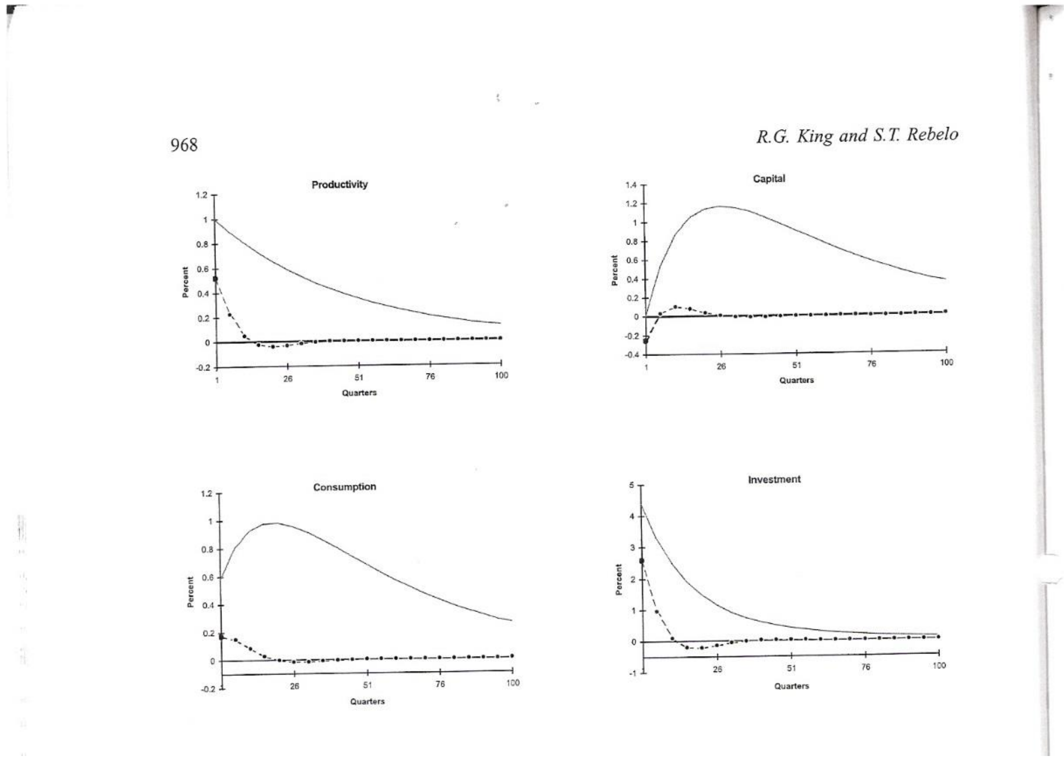

Figure 32: the dynamics of a productivity shock in RBC (source: King and Rebelo, 1999)

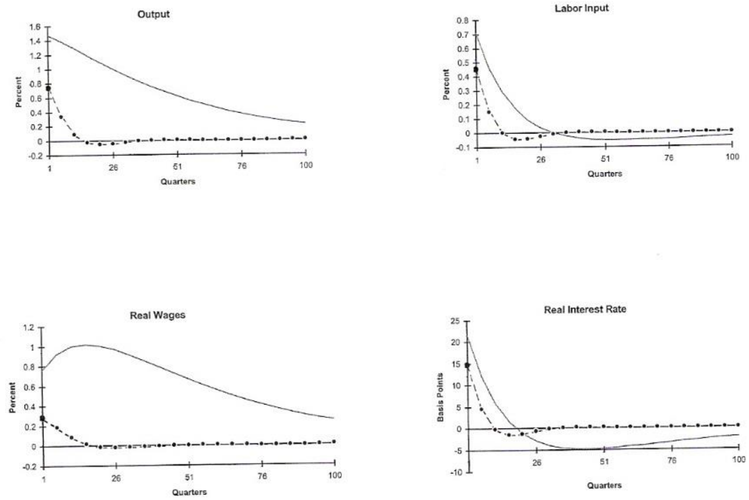

Fig. 10. Comparative dynamics to more persistent productivity shock. Circled lines are impulse responses filtered with the Hodrick–Prescott filter

To summarize, given the development of the stochastic component of productivity A_t, the general equilibrium model adjusts both via the work effort of economic agents (substitution of work for leisure) and their consumption/savings-decision, which results in capital accumulation. Hence, the whole model 'wobbles' as it adjusts itself to a new equilibrium every time and we see the cyclical comovements of several variables in the economy.

Applications to US data show that these random technology shocks (i.e. the RBC model) explain between 70 and 75 percent of business cycle fluctuations. Indeed, the fundamental point of the RBC models is that technology shocks are the dominant source of fluctuations. Furthermore, wages and prices adjust instantaneous and people therefore substitute labour for leisure. Finally, as its name would suspect, real business cycle RBC theory assumes that money does not matter, even in the short run. In opposition to the view of monetarists, monetary changes in RBC models have no effect on output and employment.

The starting point of RBC models that technology shocks are the dominant source of fluctuations remains controversial to this day. Apart from highly technical remarks on what values were used to 'pinpoint' the model to actual economic time series, there were two strands of more fundamental critique. A first critique is that the procyclical movement of (real) wages that follows from RBC models is inconsistent with the findings of many econometrical studies (King & Rebelo, 1999, 961). Secondly, critics claim that the Solow residual is wrong, and leads to excessively volatile productivity shocks. The first criticism was overcome by developing more advanced models allowing for wage-smoothing contracts between (risk averse) firms and workers. However, the second criticism indeed remains "the Achilles heel of the RBC literature" (op. cit., 962). Indeed, the driving force behind fluctuations in the RBC model is stochastic component of productivity $\log(A_t) = \rho\log(A_{t-1}) + \varepsilon_t$. Increases in the parameter ρ and of $var(\varepsilon_t)$ increase the volatility of $\log(A_t)$ and hence of the economy. So, for RBC models to work, productivity shocks must be large and persistent. But if these technology shocks are big enough to rock the entire world economy, why can't we see these technology shocks? Why can we not link a boom or slump in the economy to one or more inventions?

And finally…New Keynesian Economics

As said when introducing the monetary approach, Keynesian economics sustained a heavy blow in the seventies, when large demand-driven models where not able to simulate and project stagflation. The attention of economists then shifted towards neoclassical approaches to the business cycle. However, starting from the two books "New Keynesian Economists", edited by Gregory Mankiw and David Romer in 1991, New Keynesian economists fought back, among other things by developing rigorous microeconomic foundations of these models. Starting from the assumption of rational agents, New Keynesian models have wage and price rigidities as a key element, and these rigidities arise from the microeconomics or wage and price setting. If prices are sticky in a goods or labour market, then the supply curve will be upward sloping rather than vertical. In this case fluctuations in demand can cause fluctuations in output and employment. The New Keynesian School of economics is a diverse (and not always internally consistent) class of models that try to explain sticky prices by studying the microeconomics of short-run price adjustment. In this way, it challenges the (neo)classical Walrassian price-setting mechanism from various angles. Besides Mankiw and Romer, other well-known advocates of this school are Paul Krugman and Joseph Stiglitz.

As said, the New Keynesian School contains a diverse class of explanations of why prices are sticky. However, there are some broad lines of thought that can be discerned.

The costs of price adjustment.

When a firm changes its prices, there are costs involved. They have to let their customers know that prices have changed by sending them new catalogues, by making new price lists and inform their sales staff, and so forth. These costs –called *menu costs*- lead firms to adjust prices intermittently rather than continuously. Note that these menu costs also introduce a bias: firms will be less inclined to decrease prices than to increase prices. In the latter case, they lose menu costs but only to gain more from a higher price. So, prices will generally be too high. This has a negative impact, not only from the point of welfare theory (a smaller consumer surplus), but also macroeconomically. This is called the *aggregate demand externality*.

To explain this, suppose that there are two firms in the economy, and suppose that their prices are too high, as a result of menu costs. Now what would happen if firm 1 would lower his price? Then the macroeconomic price level (the average of the price of both firms) would decrease as well. This lower macroeconomic price increases the real money balance M/P. As a result of this increased real supply of money, the LM curve will shift to the right and output Y will expand. This economic expansion raises the demand for products of *both* firms. This is the aggregate demand externality, and it implies that even it may be optimal for a firm to keep its price fixed, sticky prices are undesirable to the economy as a whole.

The negative impact of coordination failure

In the case of recessions, output is low and there is unemployment, due to a lack in demand. Firms could overcome this by adjusting their prices (to a lower level), thereby including the aggregate demand externality. So why don't they? New Keynesian scholars believe that coordination problems can arise because firms (and unions) who set prices and wages must anticipate the actions of other price and wage setters.

So see why this may be problematic, let us reconsider our example of two firms. Suppose that these firms face the decision to lower their prices or not. This decision has to be based on profit-maximizing considerations. However, profits of each firm also depend on the actions of the other firm. When *both* firms cut their prices, the aggregate demand externality will avoid a further recession. When *just one or neither* firms cut their prices, there will be a recession. So, when firms can coordinate, it would be optimal for both of them to cut their prices. However, suppose that they cannot communicate or coordinate. So each firm has to face in solitude the decision to cut its price or not. If firm 1 cuts is price and firm 2 does not, there will be a recession and the profit of firm 1 will be lower compared to that of firm 2. The same goes for firm 2. So, if each firm expects the other to maintain its price, both will maintain their prices. If, on the other hand, each firm expects the other to lower its price, all prices will be lowered and the aggregate demand externality will avoid a recession. The result is that prices may be sticky because individual firms are risk averse, even if this is not in the interest of the group of firms as a whole.

The staggering of wages and prices

Not everyone sets wages and prices at the same time. Instead, some firms or unions set wages and prices before others. This slows the process of coordination and price adjustment. As a result of staggering, the overall level of wages and prices adjust gradually, even if individual wages and prices change frequently.

Suppose again our two firms, who set the price on exactly the same moment. Suppose that money supply increases, thereby increasing aggregate demand. Both firms will at the same time increase their prices, so that the relative price levels remain unchanged.

Now suppose that firm 1 sets its price before firm 2. What happens? Firm 1 will be the first to react to the increased demand. However, it will not raise its price very much, because this will also increase the price relative to that of firm 2. As a result, firm 2 will get a larger share of the increased demand. Firm 2 will then also introduce a limited increase of prices, because it does not want to lose its beneficiary position of its relative price. On the whole, both firms will want to keep relative prices as close to each other as possible, and they will therefore adjust their prices sluggishly.

To conclude, Neo Keynesian models have wage and price rigidities as a key element, and try to explain these rigidities by a microeconomic analysis of wage and price setting. However, its advocates do not agree

Appendix[31]: the Ricardian equivalence proposition

Suppose that the utility function is additive, then the problem is to maximize

$U=U(C_1,C_2) = U(C_1) + U(C_2)$

under the condition of the budget equation.

$C_2 = (1+r)[Y_1-C_1] + Y_2$.

To find the solution for this optimization problem, substitute the budget equation for C_2 in the utility function results in an unconditional optimization problem

Max U(C1) + U((1+r)[Y1-C1]+Y2)

Maximizing this involves taking the derivative with respect to C_1 (because that is the decision variable) and setting it equal to 0. Do not forget to apply the chain rule when doing this!

$U'(C_1) + U'((1+r)[Y_1-C_1]+Y_2)[-(1+r)] = 0$

Now $(1+r)[Y_1-C_1]+Y_2$ can be substituted back to C_2, resulting in

$U'(C_1) + U'(C_2)[-(1+r)] = 0$

Or

[31] This is based on Obstfeld and Rogoff, 1999, page 1.

$U'(C_1) = (1+r)U'(C_2)$

This is the Euler equation. It simply states that, in utility maximum, one cannot increase utility by shifting consumption between periods. Another way of writing the same Euler equation is

$$\frac{U'(C_2)}{U'(C_1)} = \frac{1}{(1+r)}$$

The left hand side is the marginal rate of substitution of present consumption for future consumption. The right hand side reflects the price of future consumption in terms of current consumption. This is also the angle of the iso-utility function in the graph in the main text, given an optimal consumption path.

The iso-budget equation can be found by deriving

$$\frac{dY_2}{dY_1} = \frac{1}{(1+r)}$$

So in the case of the optimal consumption path, the iso-utility function is equal to the angle of the iso-budget equation, which is exactly what Figure 5 in the main text says.

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