An evolutionary game theory application in a smart grid context.

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Report CW 681, February 2015

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Abstract

This technical report describes first experiences in applying evolutionary game theory to a case of smart grid coordination. The use of evolutionary game theory to model population dynamics has been effectively used in domains such as biology and economics but applications to cases of smart grid coordination have not been widespread. In this work, we explicitly model a population consisting of clients of demandside flexibility aggregators and demonstrate a proof of concept analysis of the relationship between an aggregator's market share, its efficiency and the payment rate toward its self-interested clients. Simulations are used to provide expected payoff values as input for the evolutionary game theoretic analysis. Preliminary results show that the payment rate towards clients has a significant influence on the ability to maintain a market share when competing aggregators are present and that evolutionary game theory provides interesting tools for analyzing such cases of strategic interaction between self-interested clients.

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Technical report

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Abstract. This technical report describes first experiences in applying evolutionary game theory to a case of smart grid coordination. The use of evolutionary game theory to model population dynamics has been effectively used in domains such as biology and economics but applications to cases of smart grid coordination have not been widespread. In this work, we explicitly model a population consisting of clients of demandside flexibility aggregators and demonstrate a proof of concept analysis of the relationship between an aggregator's market share, its efficiency and the payment rate toward its self-interested clients. Simulations are used to provide expected payoff values as input for the evolutionary game theoretic analysis. Preliminary results show that the payment rate towards clients has a significant influence on the ability to maintain a market share when competing aggregators are present and that evolutionary game theory provides interesting tools for analyzing such cases of strategic interaction between self-interested clients.

1 Introduction

Demand-side flexibility is a commodity that transmission system operators (TSO) are willing to pay for in order to maintain production-consumption balances in the electrical grid. This flexibility can be found in industrial sites capable of fine-tuning their processes in order to consume less or more power when asked for [7]. There are a number of ways these sites can be coordinated to help balance the electrical grid [16]. Sites can sign strict contracts with the TSO guaranteeing at all times, the availability of this power flexibility. These guarantees are often hard to provide and as such, numerous sites with less predictable power flexibility are dissuaded from signing such contracts with the TSO directly and can benefit more from doing business with demand-side aggregators. These aggregators spread the risk of non-compliance to a TSO's request for curtailment or increased uptake of energy over a wide portfolio of customers. In turn, these customers get a portion of the payments to the aggregator redistributed to them. How much of these payments are redistributed from aggregator to client depends on the

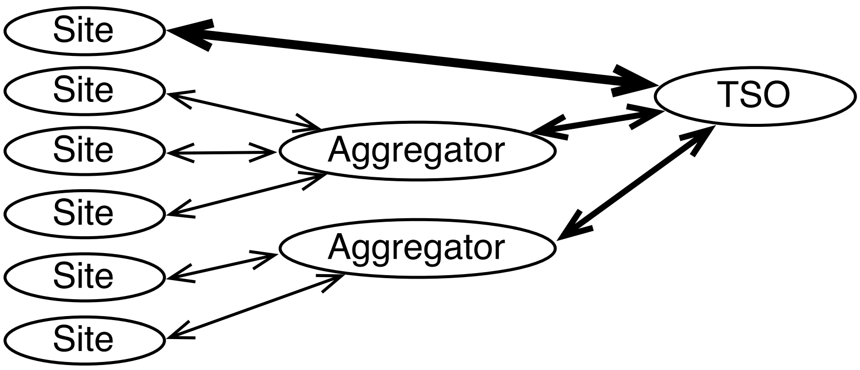

Fig. 1. The hierarchical relationship between sites, aggregators and the TSO. Connections represent bilateral agreements about on-demand power regulation and the boldness of the connection represents the volume of power under regulation in the agreement.

aggregator's intended profit margin and the possible diseconomy of scale from operational costs.

Assuming that participating clients are capable of rationally choosing the aggregator that offers them the most for their flexibility and that a deregulated market mechanisms allows a client to switch between aggregators with relative ease, an aggregator faces a tradeoff between market share and profit margin. The size of an aggregator's portfolio determines the balancing capacity it can offer to the TSO and the compensation payment it can receive, while the size of the portfolio also influences the computational cost needed to determine the optimal use of the resources in the portfolio. This case of grid balancing is just one example of a smart grid scenario where coordination plays a crucial role. Sites prefer consuming energy at a level that is optimal in terms of their own cost-benefit analyzes while the TSO might prefer other consumption levels that balance the energy production. In literature, smart-grid coordination is often centered around the application of centralized or decentralized optimization techniques [5]. In this case, sites prefer consuming energy at a level that is optimal in terms of their own cost-benefit analyzes while the TSO might prefer other consumption levels that balance the energy production. Demand-side flexibility aggregators need to take into account preferences and goals of both the TSO and the sites but also have to think about their own profit margins. Thorough analysis of cases of strategic interactions between agents requires an adequate framework for modeling these interactions. This work proposes the use of Evolution game theory to perform these analyzes.

In this report we analyze how the cost redistribution rate might influence the share of clients willing to do business with the aggregator in question in stead of with a competing aggregator. We use simulations to heuristically estimate payoffs and then use evolutionary game theory to model single population dynamics and to find stable population states in a symmetric game of perfect information between two competing aggregators. We also illustrate how replicator dynamics equations can be derived from heuristic payoff-tables for N agents in this two-action game. The work described in this report is ongoing and focuses primarily on first experiences.

The remainder of this report is structured as follows: After a description of the scenario under analysis in section 2, section 3 will explain the approach we take to analyze this scenario. The actual analysis will be done in section 4. In section 5, some relevant related work and scientific background will be discussed while section 6 provides a conclusion and ideas for future work.

2 Scenario description

The scenario discussed in this work is one of smart grid coordination. More specifically it is a case of grid balancing. Balancing the production and consumption of energy in an electrical grid for a certain region is the responsibility of the transmission system operator (TSO). The TSO agent is in charge of correcting a balancing signal using registered balance responsible parties (BRPs). In this case, BRPs are instantiated by demand-side aggregators.

Demand-side flexibility aggregators are companies that offer energy consumption flexibility through managing a portfolio of clients with flexibility and presenting the aggregated flexibility to the TSO. Aggregators advertise the size of their portfolio's to the TSO to indicate how much of the imbalance they can handle. The TSO then dispatches requests to counter a portion of the imbalance relative to the advertised portfolio sizes and financially compensates the aggregator for it. Aggregators then dispatch requests to their clients in a way that best compensates the portion of the grid imbalance that the aggregator is tasked to handle. The aggregator financially compensates clients for every flexibility activation request sent.

In this scenario the task of the aggregator's clients are fulfilled by industrial sites such as production/processing factories consuming energy at a financial cost and gaining income from produced/processed goods. We assume that these clients are willing to sign up with an aggregator because of the increased financial gains as a reward for offering flexibility and that clients are able to switch services to competing aggregators without any problems.

In this scenario, two different aggregators are present for clients to choose from. Both aggregators are using two different aggregation algorithms for matching and combining client side flexibility to a target amount at a given time. As input the algorithms take the flexibility profiles available from each client and a target value representing the aggregator's portion of the imbalance to correct. A flexibility profiles has the form of a tuple $(id, \delta P, \delta T)$ where δP represents the amount of power it can curtail or consume additionally and δT for the duration this power is available. The target imbalance is a portion of the imbalance the TSO needs to correct proportional to the size of the aggregator's portfolio such that for every aggregator $a_j \in A$ and the total imbalance I: $I = \sum_{a_j \in A} i_{aj}$. As output, the aggregator chooses a combination of flexibility profiles (denoted by the set F) $min(|\sum_{f \in F} \delta P(f) - i_{aj}|)$ under the assumption that for each client only one flexibility profile can be active.

We define two different aggregator implementations agg_{brute} and agg_{heur} as follows. The first algorithm consists of a brute force approach considering all possible combinations and activating the first best fitting one. The search space for this approach increases in size very quickly as the number of client participants grows in relation to the search space size $SP = l^c$with l the amount of flexibility profiles and c the amount of clients per aggregator. The second algorithm consist of the same brute force approach but with a reduced search space from first filtering the client participants and limiting the possible combinations to enumerate. In this approach, only flexibility profiles with a sign opposed to i_{aj} are considered. The main difference between the two algorithms is the computation time and the quality of the solutions found. These implementations are by no means state of the art but in the context of a proof of concept for the analysis discussed in the next section, they suffice.

We formalize this scenario as a game with the clients as players and the aggregators to choose from to do business with, as actions. The player's payoffs are determined by their individual profits from dealings with the respective aggregators. Clients are paid according to an activation payment rate set at 48.8EUR per KWh of power increase or decrease, for each activation. The amount of profiles and the values for δP and δT are all drawn from the same random distributions

3 Approach

Where classical game theory [15] provides tools for static analysis of games focusing purely on the payoffs for individual actions, evolutionary game theory focuses on the payoffs for an action in combination with the amount of agents playing the action [6]. So while classical game theory offers ways of eliciting the possible Nash Equilibria [12], Evolutionary game theory can offer insight into which Nash Equilibria are more likely to occur in practice. EGT can mainly be categorized into two approaches. There is the static approach proposed by Maynard Smith where the solution concept of an Evolutionary Stable Strategy (ESS) is introduced for populations wherein all agents play the same mixed strategy [21], extending the Nash Equilibrium solution concept from classical game theory with a notion of robustness to a small invading sub-population of agents playing a different strategy. The second approach describes modeling biological evolution or in this case the agent's rational choice more explicitly by a system of differential equations. This system regards agents in a population as playing only pure strategies while population states are described by vectors akin to mixed strategies. Using explicit dynamic system modeling allows for the use of the wide range of tools for analyzing dynamic systems for analyzing the population evolution [1]. We follow the second approach in this work.

As input for defining these systems, we follow the approach taken in [26] and treat heuristic strategies as primitive actions for a game theoretic analysis, meaning that an action is this game formulation represents choosing an algorithm implementation or in this case choosing an aggregator executing a specific algorithm. For the entities involved in this coordination case, empirical expected values are estimated using simulation and the resulting heuristic payoff table will be used as a starting point for the analysis. Similar to the approach described in [18], agent's choices are assumed to be independent of their types which allows for a compact representation for the payoff table. In a game of n players and k heuristic strategies to choose from, this payoff table will contain entries of the form

$$p = (p_1, ..., p_k) \tag{1}$$

with p_i representing the number of players bound to action i. The function f maps a vector $p \in P$ onto a vector $q \in Q$ of the form

$$q = (q_1, ..., q_k) \tag{2}$$

representing the expected payoff for agents bound to action i. This expected payoff is an average over all players playing this strategy. The total number of entries in this payoff table is given by

$$s = \frac{(n + k - 1)!}{n!(k - 1)!} \tag{3}$$

. This corresponds to one entry per possible population state, taking into account agent symmetry. In a setting with two agents choosing between two options, there are three population states and therefore entries in the payoff table. Assigning parameters a, b, c and d to the possible payoffs produces the set $\mathcal{Q} = (a, 0), (b, c), (0, d)$ from applying f to set $\mathcal{P} = (2, 0), (1, 1), (0, 2)$ For brevity, the payoff matrix can be written as

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{4}$$

Applied to the case of smart grid coordination where clients help balance electrical grids by choosing an aggregator, selection dynamics are used to model the rational choice between two aggregators in repeated pairwise interaction of clients comparing their choice of aggregator. These clients base their choice on their fitness (ie. the payoffs they receive from the aggregator). The selection dynamics used in this work is the replicator dynamics [23]. The replicator dynamics are a set of ordinary differential equations describing the population dynamics in terms of the fitness of agents choosing the strategy compared to the overall average fitness of the whole population. In this context, fitness is described by an agent's financial costs and rewards arising from the choice of aggregator. The replicator dynamics equations describe how a population share of agents following a specific aggregator will increase/decrease in size as the payoffs for those agents are better/worse than the average payoff of the whole population. Consider a mixed strategy profile x for the population as a population state where each component x_i represents the population share choosing aggregator i in stead of one agent's randomization over the choice of aggregators. The general replicator equations are then given in (5) following the notation used in [27].

$$\dot{x_i} = x_i[u(e_i, x) - u(x, x)] \tag{5}$$

with $u(e_i, x)$ representing the average expected payoff of an agent choosing aggregator i when the population is in state x and $u(x, x)$ representing the overall average expected payoff for an agent from a population in state x. We can also represent the dynamics from (5) in terms of the 2×2 payoff matrix from (4) as

$$\dot{x_1} = x_1[(Ax)_1 - x^T Ax)] \tag{6}$$

with $\dot{x_2} = -\dot{x_1}$ because we assume a constant population size.

From (6) we generalize the replicator dynamics for N agents and 2 actions in terms of the heuristic payoff table entries represented by the function f by defining $u(e_i, x)$ in (7) and $u(x, x)$ in (8).

$$u(e_1, x) = \sum_{i=0}^{N-1} x_1^{N-i-1} x_2^i f(N-i, i)_1 \binom{N-1}{i} \tag{7}$$

$$\begin{aligned} u(x, x) = &\sum_{i=1}^{2} x_i^N f(N - N(i-1), N(i-1))_i + \\ &\sum_{j=1}^{N-2} x_1^{N-j} x_2^j \sum_{k=0}^{S-1} \binom{N-2}{j-k} f(N-j, j)_k \end{aligned} \tag{8}$$

with N specifying the number of agents and S the number of actions to choose from (in this case 2) Finally, we analyze the critical points of these equations in terms of stability.

Simulations are performed for each population distribution of the game with 2, 3, 4 and 5 agents respectively. Each simulation run is repeated 30 times with different randomization seed averaging over the results. The sites each have 8 non-equal flexibility profiles to offer to their respective aggregators and the simulations using these agents drive the evaluation explained in the next section.

4 Analysis

The situation of two agents participating in the game is analyzed first for simplicity and afterwards, an empirical analysis with more agents is done to better estimate the performance of the aggregation algorithms and the influence of the redistribution factor.

4.1 Two Agents

Simulation results provide a normalized payoff matrix A similar to (4). Payoff table A can be normalized further to A' by subtracting c from the first column and b from the second column without loss of generality because the set of NE and the set of ESS are invariant under local payoff shifts in symmetric games when $a1, a2 \neq 0$ with $a1$ and $a2$ being the resulting table entries [27]. The numerical results from simulations are given in (9).

$$A = \begin{pmatrix} 298 & 146 \\ 133 & 197 \end{pmatrix} \rightarrow A' = \begin{pmatrix} 165 & 0 \\ 0 & 51 \end{pmatrix} \quad (9)$$

Based on the signs of $a1$ and $a2$, Weibull [27] categorizes 3 different cases, corresponding respectively to variants of the Prisoner's Dilemma [2] if $a1 * a2 < 0$, a coordination game [4] if $a1, a2 > 0$ and the Hawk-Dove game [21] if $a1, a2 < 0$. The simulation results for the case of equal activation payment rates result in a payoff matrix that corresponds to the coordination game. In the coordination game there are two stable strategies, namely the two pure strategies present (choose agg_{brute} and choose agg_{heur} respectively). The two basins of attraction in the mixed strategy space are separated by the symmetric mixed Nash equilibrium $x = (\lambda * agg_{brute} + (1 - \lambda) * agg_{heur})$ with $\lambda \approx 0.236$ meaning that an initial population distribution $x' = (p * agg_{brute} + (1 - p) * agg_{heur})$ with $p > \lambda$ will move towards a distribution with solely players choosing agg_{brute} respectively agg_{heur} if $p < \lambda$. Together with the two pure strategies, x is also a NE of the game, but in this case x is not an ESS because for every other x', x' is a best reply to x. This shows that although agg_{brute} is not completely dominant (in some cases choosing agg_{heur} is still a best response), the number of players should at least equal five before a population share of λ agents encompasses at least one agent. Drawing conclusions for populations of five agents based on simulations of only two agents is quite insubstantial. Therefore, we show how results from simulations with more agents can provide insight and a more accurate estimation on the probability of each aggregator to become the dominant population share holder. Figure 2 graphically shows a one-dimensional phase plot representing the evolution of the population share playing agg_{brute}.

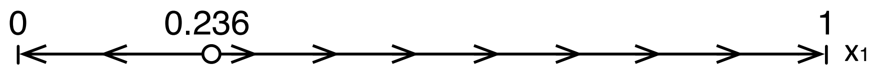

Fig. 2. Phase plot showing the replicator dynamics for the game played by two clients.

4.2 N Agents

Where games concerning 2 agents are analyzed using a two-dimensional payoff table, games with N agents require an N-dimensional payoff table which is impractical to conveniently display. In this section we omit payoff tables and proceed with an empirical evaluation of the dynamic systems constructed from the simulation results. The dynamics equations from (7) are used to construct the replicator dynamics from simulation results with N agents for N ranging from 1 to 4 agents. Results shows that an initial populations x' moves toward the stable point in the dynamics for increasing amounts of participating agents indicating that with more accurate estimated payoffs from simulations with more agents, agg_{brute} clearly dominates the agg_{heur} strategy. Because of the dynamics equation defined in (7), the differences between the case with two participants is that more symmetric equilibria are possible. In the dynamics equation, all points for which $\dot{x_i} = 0$ are considered as fixed points. The significance of the other fixed points will be addressed in the next section.

The next section will discuss how much the agg_{brute} implementation can reduce the activation payment rate while still remaining dominant and how further decreasing these payment rates will influence population dynamics.

4.3 Influence of Retribution Factor

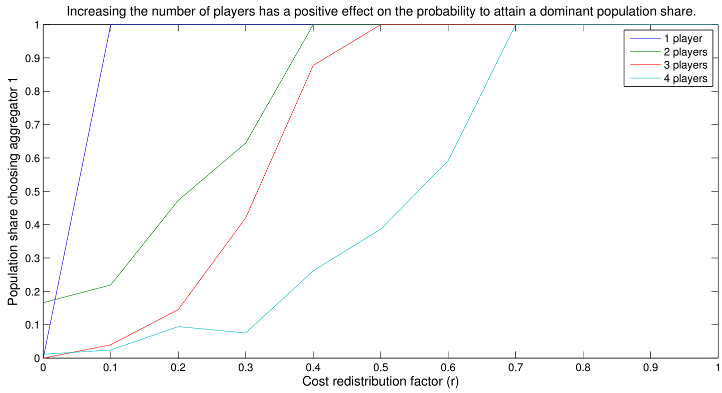

Fig. 3. Plot showing the initial population share needed to maintain a market share in the long run for agg_{brute} (lower is better), for an increasing amount of agents.

Using simulation results for cases with 2,3,4 and 5 agents, the relation between the cost redistribution factor and the repulsors in the dynamics is shown in Figure 3. The cost redistribution factor c determines the fraction of the operational cost that is redistributed to the clients, subtracting from their activation payment rates. Assuming that the total operational costs equals the activation payment rate, $c \in [0, 1]$ determines the effective payment rate $r \in [48.8, 0]$. The results show that increasing the cost redistribution factor for agg_{brute} and therefore decreasing the rewards clients receive for their participation has a significant influence on the probability the agg_{brute} implementation will be able to gather a dominant market share or even maintain a market share in the long run. Figure 3 shows that the ability to maintain a market share decreases if the cost redistribution factor increases. Increasing the number of clients participating does, however, have a positive influence on maintaining a market share. Increasing the number of clients participating allows the aggregator to utilize a wider profit margin but at the risk of losing any probability of attaining a dominant market share. Figure 3 only shows fixed points that are repulsors in the replicator dynamics and do not show the phase plots or other fixed points that might be of interest. For example, Figure 4 shows all fixed points for the case of five clients and a redistribution factor $r = 0.6$ and shows that there are two critical points of which one is an attractor. In this case, there will always remain a portion of the population that chooses agg_{heur} because these clients do not suffer from the payment penalty that clients choosing agg_{brute} do and the average expected payoff for a whole population choosing agg_{brute} will be lower than when there are clients choosing agg_{heur} present. The phase plot displayed in Figure 4 shows this effect of there being no probability of agg_{brute} attaining a dominant market share.

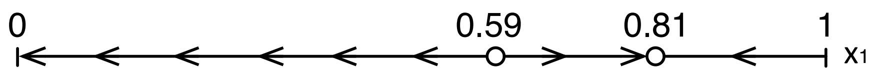

Fig. 4. Phase plot showing the replicator dynamics for the game played by 5 clients.

5 Related Work

The smart grids domain by now has been a well established field for coordination research ranging from demandside management [16] to coordinated charging of electric vehicles [10] [3]. A popular approach towards smart grid coordination in literature is the application of optimization algorithms in coordination protocols [25]. Designing coordination protocols starts from the assumption of willing participants that can be governed easily. When dealing with self-interested agents that are outside the scope of control from the perspective of the protocol designer, a framework capable of modeling strategic interaction is required. Game theory [15] and more specifically inverse game theory, otherwise known as mechanism design (MD) [13], allows for taking into account multiple individual goals for designing incentive compatible coordination mechanisms while accounting for strategic agents [14]. One example of such mechanisms is proposed in [22] where the authors propose a mechanism for scheduling uncertain demand given uncertain supply while dealing with strategic agents.

This work focuses on the addition of using techniques from evolutionary biology in combination with game theory as proposed by John Maynard Smith [21] to analyze demand-side aggregation algorithms. There is a significant theoretical body of work on explaining evolution in games of strategic interactions [20] and in evolution applied to mechanism design (also known as evolutionary mechanism design) [17] in some influencial survey papers. Applications of EGT has gained the most momentum in economical research. For example Walsh et al. offer an analysis of strategic interaction between price bots in different market settings [26] while Phelps et al. compare two double-auction market designs using evolutionary game theory [18] but applications are not limited to the economics domain alone. Sandholm offers an extensive overview of applications in social sciences next to applications in economics [20]. To the best of our knowledge, the application of EGT in the context of smart grids has been limited. This work attempts to address that.

This work proposes the Replicator Dynamics for modeling the population dynamics [23]. The replicator dynamics is in itself a special case of a class of dynamical systems that has been analyzed in terms of limit behavior and stability in general [11]. When it comes to deterministic selection dynamics, other examples and studies of using differential equations for modeling evolutionary dynamics are also available. Hoffbauer and Sigmund [9] and Sandholm [19] have provided excellent surveys on this topic. The evolution aspect in EGT does not necessarily have to be biologically inspired. Other selection dynamics based on learning [24] and imitation [8] are also candidates for modeling evolutionary dynamics but not all of these selection dynamics bare the strong link to Maynards Smith's definition of evolutionary stable states and classical game theoretic notions of the Nash equilibrium [12] that the replicator dynamics do.

6 Conclusion and future work

In this report we show a proof of concept application of evolutionary game theory to a case of coordinated grid balancing. This case is modeled as a game between strategic self interested players and simulations provide expected payoffs for the players. The players represent flexible energy consumers who have a choice between different aggregators to do business with. These aggregators employ aggregation algorithms that differ in efficiency and computational complexity. We show how evolutionary game theory can help analyze the effect cost redistribution from aggregators to their clients (the players) can have on the market share of the aggregator and the simulation results show how the probability of an outperforming aggregator maintaining a market share decreases as their cost redistribution factor increases, when competing with another less efficient aggregator. This report documents first experiences and ongoing work in applying evolutionary game theory in a smart grid context by analyzing client population dynamics.

Future work includes analyzing the effect cost redistribution has on the effectiveness of the actual grid balancing actions and finding the optimal cost redistribution factor that still allows the outperforming aggregator to attain a dominant market share. Furthermore, extending this work to include more participating aggregators and other selection dynamics that model possible real world interaction more closely are interesting research topics that can be visited. The algorithms chosen in this work are by no means state of the art but they illustrate how the analysis can provide insight on the client population dynamics in a flexibility market environment. In the future, more state of the art aggregation mechanisms will be used. Finally, this work considers strategic, self-interested agents but does not include clients capable of handling uncertain information about their own available flexibility nor can they provide false flexibility information to their aggregator. Future work plans to address this.

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