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Calculus from approximation to theory. (English) Zbl 1464.26002

AMS/MAA Textbooks 64. Providence, RI: American Mathematical Society (AMS) (ISBN 978-1-4704-5588-0/pbk; 978-1-4704-6305-2/ebook). ix, 571 p. (2020).

This is a calculus textbook treating all the classical material, including, besides the central topics of differentiation and integration, also series expansions and the standard set of transcendental functions. Somewhat less generally present in an introductory course are the chapters introducing elements from complex analysis and differential equations. The approach is essentially bottom up: intuitive notions and many examples lead eventually to a definition, a property, or a theorem, which is followed by many exercises. This is the opposite of a top-down formally structured texts that typically consists of a sequence of repeated definition-theorem-proof schemes, possibly followed by an application. The innovative aspect of this text is the order in which the material is introduced. The title: Calculus from approximation to theory already gives a hint about the approach taken. The author's basic point of view is that calculus developed historically by trying to solve practical problems though approximation, like the Greek approximated the circumference or area of a circle by considering regular polygons and in Newton's time the derivative emerged as instantaneous changes of motion, which was approximated by changes over short time intervals. A similar historical view was proposed in the book by *D. M. Bressoud* [Calculus reordered. Princeton: Princeton University Press (2019; Zbl 1418.01001)].

Defining derivatives and integrals as limiting processes to approximate tangents and areas explains why the book starts with sequences and series. Successive approximations form a sequence converging to the actual result and truncated series are approximants for the true function. Moreover, series are in a sense discrete integrals of a sequence. The elements of a sequence can satisfy a recurrence relation, so that this chapter also treats some elements of difference equations as a forerunner for the differential equations of the last chapter. The second chapter is about functions, limits and continuity. The concept of a function is intuitive, no formal definition is given, avoiding all kind of exceptional exotic cases. Chapters three and four then introduce derivatives and integrals with the technical calculus rules and some applications. Next, it is observed that the affine approximation that was used to introduce the derivative is like the first two terms in a Taylor series. This is generalized to Taylor polynomials and series expansions in chapter five and these then lead in chapter six to standard transcendental functions like exponential, logarithmic, trigonometric, and hyperbolic functions. The remaining Chapters 7 and 8 are introductions to complex analysis and (ordinary) differential equations with some applications.

It is clear that this practical approach links the material to a basic numerical course. The propositions and theorems are formulated and formatted as is usual in a textbook to make them stand out, but they are never followed by a formal proof. They are preceded by a list of concrete examples that gradually move to a more general derivation of the result that is then summarized in the formal statement. Epsilon-delta definitions and proofs are not made explicit but they are mollified and only implicitly used. With this approach the numerous exercises that follow every first level section are obviously essential to assimilate the material. An appendix gives a list of answers to selected exercises: just the crisp answers, no long derivations.

Reviewer: Adhemar Bultheel (Leuven)

MSC:

26-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to real functions
00-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to mathematics in general
26A06 One-variable calculus

Keywords:

textbook; calculus; derivative; integral; sequence; series; difference equation; ordinary differential equation; complex analysis

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