Automatic air cargo selection and weight balancing: a mixed integer programming approach

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Abstract

The present contribution introduces a mixed integer linear programming model as a decision support tool for air cargo load planning. The main objective for the model is to find the most profitable selection from a set of cargo to be loaded on an aircraft. The secondary objective is to minimize the deviation between the aircraft's centre of gravity, and a known target value so as to reduce fuel consumption and improve stability. The model is subject to a large number of constraints that ensure structural integrity and stability of the aircraft, as well as the safety of the cargo and crew. A set of additional constraints guarantees safe and efficient loading and unloading. Experimental results on real-life data show that the model outperforms human expert planners on both objectives, while remaining computationally fast enough for interactive use. This advocates the use of such a decision support model for all air cargo load planning.

Keywords: Integer programming, Aircraft load planning, Weight and balance

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1. Introduction

For many airlines cargo transport constitutes a major source of income; for cargo carriers it is a core business. Logically, both will try to perform their operations as safely and as efficiently as possible. In the very process, one cargo operation is of strategic importance, namely the selection, loading and positioning of cargo on an individual aircraft. The type of cargo and its positioning on board of the plane is liable to a high number of operational and safety constraints complicating decision making during cargo load planning. When too much weight is loaded at the rear, the aircraft will tip on its tail. Loading too heavy cargo at the front, makes the aircraft nose-heavy, preventing take-off. Other safety constraints are imposed to avoid anomalous stresses on the aircraft's structure. All these constraints prevent that flying characteristics differ from the safety norms which, in a worst-case scenario, might result in a crash.

In the present paper the Aircraft Weight and Balance Optimization Problem (AWBP) has been introduced as characterized by two different, be it independent, goals: to maximize revenue obtained from transporting a set of cargo items and, secondary, to optimize the centre of gravity (CG) of the loaded aircraft in both the lateral and longitudinal direction. Reducing the difference between the actual CG and a given target value (defined by the manufacturer) improves stability of the aircraft and indirectly, also decreases fuel consumption during the flight.

The AWBP can be summarized as follows: select the most profitable combination of cargo units from a given set and load them on an aircraft while satisfying a large set of linear and non-linear (safety) constraints, and minimizing the deviation from the aircraft's target CG. In practice, the complex problem just described is being solved about half an hour before flight departure, leaving little time, or none, for finetuning. Charts and software tools are available to the expert planners for assisting in the CG calculation and for checking the constraints. It is, however, a time consuming manual process which oftentimes lacks quality. Therefore, the present paper proposes a mixed integer linear programming model for solving the AWBP.

The model presented was designed to be part of a decision support tool developed by our software partner, B. Rekencentra NV[1]. The first responsibility of this tool is the issuing of safe and legal flight documentation. As an

[1]http://www.rekencentra.be

add-on, the flight performance is improved by means of optimizing the CG. The goal of the mixed integer programming model is to automate this aircraft loading in such a way that the cargo profit is maximized and that the target CG is approached in both the longitudinal and lateral dimension, without violating any (safety) constraints. The model is unique in that it combines characteristics of a knapsack problem, load balancing and the addition of several important real-life features such as overlapping loading configurations, oversized containers and cargo priority. Given that our model definition is flexible, we can state that the W&B as considered in this paper, is NP-Hard. For example, Knapsack instances can be reduced to W&B instances by considering a simple aircraft layout with only a single *'Bulk' position slot* (explained in Section 3.1) and a weight constraint on that position. Simply stated, a bulk position slot is an area on an aircraft where several bulk cargo items can be stored. Letting knapsack items correspond to bulk cargo items, with item value corresponding to cargo profit and item weight corresponding to cargo weight, it is easy to see that solving such a W&B instance corresponds to solving the knapsack problem (i.e. selecting the most profitable set of cargo items subject to a weight constraint).

The paper is organised in such a way that first, in Section 2, a literature review of related work is given. The AWBP as considered for the present purpose is discussed in detail in Section 3. Section 4 shows the details of the proposed mixed integer model for the AWBP. In Section 5 the proposed model is assessed using real-life data. Finally, Section 6 concludes the paper while indicating possible directions for further research.

2. Literature review

2.1. Related work

The earliest discussions on the load balancing problem for aircraft were reviewed by Martin-Vega (1985), who observed that, up till that point, most research had focussed on computer assistance rather than on computer generation of load plans. Noteworthy are the early academic contributions by Larsen and Mikkelsen (1980) and Brosh (1981). Larsen and Mikkelsen developed an interactive system for cargo load planning on the Boeing 747 combi aircraft, integrating two heuristics for the generation of an initial load plan and respecting a wide range of structural and safety constraints. Like in the present contribution, they specifically considered loading standardized containers/pallets (Unit Loading Devices, or ULDs, see Section 3). Their goal was to minimize unloading and reloading operations at intermediate airports in case of multi-leg flights. These operations are required when cargo to be offloaded is positioned after other destined cargo in the offloading sequence. As a secondary goal, their system aimed at obtaining a CG at the centre of its feasible interval, to allow for more flexibility in last minute changes. However, no details were given on how these heuristics achieved these goals or how well they performed except for the average CPU time.

Brosh (1981) formulated a fractional programming model for finding an optimal load layout for a particular case study, considering volume and weight constraints as well as constraints on the CG. The aim of the research was to determine the maximum amount of load that could be transported in different bays of an aircraft without violating any constraints. However, the author did not focus on where to actually place specific units of load (pallets, containers, etc) in an aircraft, which is the main focus of our contribution.

More recently, Amiouny et al. (1992) focused solely on the one-dimensional balancing problem (applicable to general vehicle/aircraft loading). It is shown that the decision version of the one-dimensional balancing problem is NP-Complete. Several heuristics were developed for packing weighted items into a bin such that the CG achieves a certain target value. However, the approach is limited in that items must be packed contiguously, one after another, without items being placeable side-by-side.

Heidelberg et al. (1998) and Kaluzny and Shaw (2009) discussed the AWBP in a military context, dealing with non-standardized cargo. Kaluzny and Shaw presented a mixed integer linear model for positioning non-standard cargo (modelled as rectangles of variable size), suitable for either minimizing deviation from optimal CG or maximizing cargo value. However, apart from item spacing and CG envelope constraints (which define limits for the CG during different stages of flight, see Section 3), the problem description did not consider structural and safety constraints such as those related to floor strength, which play a key role in safe and legal cargo transportation.

The contributions of Mongeau and Bès (2003) and Limbourg et al. (2011) have the strongest relation to this paper. Both consider the placement of standardized containers (denoted ULDs) to predefined positions and the impact thereof on the aircraft's centre of gravity. Mongeau and Bès developed a linear integer programming model that maximizes the total loaded cargo onto an aircraft, and ensures that the CG is within a certain ϵ from a target value. Furthermore, the authors consider important structural integrity constraints such as those on the maximum load as well as volume capacity constraints.

Limbourg et al. (2011) on the other hand, assume that the most interesting selection of containers has already been made. Their main objective is to minimize the moment of inertia of the cargo in order to improve stability, to reduce stress on the aircraft's structure and to decrease fuel consumption. Moreover, they consider many real-world constraints to ensure the structural integrity of the aircraft. However, the assumption of being able to load all containers is limiting when the number of available containers is high.

Other related work focuses on problems that precede the AWBP. In aircraft cargo loading, it is important to pack as much cargo as possible either into a container or onto a pallet. Chan and Kumar (2006) present a three-dimensional bin packing approach for pallet loading. Li et al. (2009) propose a large-scale neighborhood which is used within a simulated annealing heuristic for assigning cargo boxes to cargo containers. Furthermore, the weight distribution in each container also plays an important role in container loading, as it is often assumed that containers are packed homogeneously (and thus the CG is in the center of the container). In this context, Davies and Bischoff (1999) developed a new container loading heuristic. They demonstrated that a high space utilisation can be obtained with an even weight distribution.

At a higher decision level, freight carriers, typically operating a hub-and-spoke type network, need to decide on an overall container loading plan that determines the percentages of *pure* (filled with single destination packets) and *mixed* (filled with multiple destination packets) containers to be used between each origin-destination pair in the network. At the downstream operational decision level, this container loading plan determines the characteristics of the payloads considered for the aircraft loading problem. Yan et al. (2008) and Tang (2011) discuss different approaches for determining such container loading plans under stochastic demands.

2.2. Our contribution

The present contribution can be considered an extension to the papers of both Mongeau and Bès (2003) and Limbourg et al. (2011). As in these studies, the focus is on loading standardized containers (ULDs) onto a discrete set of positions. The issue of non-standardized cargo as discussed by e.g. Heidelberg et al. (1998) and Kaluzny and Shaw (2009) is ignored in this paper as our model is motivated from a commercial application where ULDs are the de facto standard means of cargo transport.

The present approach combines both the objective of maximizing profit (which is strongly related to packing as much load as possible) and minimizing the deviation of the CG from a target value. This allows for a more flexible model that always returns the most profitable, feasible selection of containers to be transported, and that also finds the optimal placement of ULDs in terms of CG deviation from a target value. Furthermore, the model improves on previous work by incorporating relevant new real-world features (e.g. overlapping loading configurations and oversized ULDs) and real-world constraints, related to the safety and (structural) stability of the aircraft. Furthermore, an insight into more commercially oriented constraints is given as well.

3. Problem description

An optimal selection from a set of available Unit Loading Devices (ULDs) has to be loaded into an aircraft. Given a set of operational and safety constraints, the AWBP aims at optimizing the total cargo value as well as the position of the centre of gravity of the loaded aircraft. In a previous phase, the set of available ULDs has already been determined by the commercial department of the airliner as the cargo to be transported on the considered flight. Ideally, all ULDs should be loaded into the aircraft to satisfy the commercial department and the customers. However, given various constraints on the positions of these ULDs, it may not be possible to safely load the entire set. The present paper aims at determining the most profitable selection of ULDs to be loaded in a feasible manner.

ULDs are standardised units for air transportation of cargo. They can be containers or pallets and are characterised by their type, size, weight, and height. The content of a ULD determines its possible profit. As opposed to pallets, containers all have a standard height. An aircraft consists of one or more decks, where ULDs can be loaded in various configurations. A configuration defines a set of *position slots* on an aircraft where ULDs can be loaded safely. A common configuration is a *30" pallet* configuration (shown in Figure 1) in which ULDs are positioned in two rows along the structure (or *fuselage*) of the aircraft. ULDs, however, cannot be placed at any position slot, as position slots are restricted to certain types of ULDs. Therefore, different configurations for different types of ULDs can be 'mixed' in the final load plan (see e.g. Figure 2). It is noteworthy that most positions overlap with positions from other configurations. Therefore, assigning a ULD to one position in the final load plan may prevent several other positions from being used. This can be observed when comparing Figures 1 and 2: A ULD

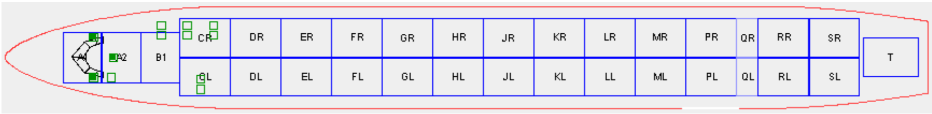

Figure 1: A common loading configuration for 30" pallets.

is placed on position 'DE', preventing placement of ULDs on the four 30″ positions 'DL', 'DR', 'EL' and 'ER'.

Position slots are defined by their longitudinal and lateral coordinates w.r.t. a reference *datum*. This reference datum is commonly defined as a fictitious point in the nose of the aircraft. For the purpose of the present research, the coordinate of interest is the Balance Arm (BA) of a position. The BA corresponds with the point at which a uniformly loaded ULD would exert its gravitational force on the position slot. The longitudinal BA is the longitudinal coordinate referenced to the datum (BA zero), the lateral BA is the lateral coordinate referenced to the centre line of the aircraft. For convenience' sake and from now on we refer to the longitudinal BA, unless stated otherwise.

The centre of gravity (CG) of an aircraft plays an important role in aviation. It is the fictional point in space where gravity would exert its force on the entire weight of the aircraft. Alternatively, it also represents the fictional point on which the entire mass of the aircraft could be perfectly balanced. The position of the CG largely determines the stability and manoeuvrability of an aircraft during flight. Indirectly, it also influences the fuel consumption, as flying with a 'perfect' CG minimizes drag during flight (and thus less engine power and fuel is required to maintain speed) due to an ideal setting of the horizontal stabilizer.

Basic principles of rotational mechanics allow determining the CG. The sum of all rotational moments (referenced to the datum) of gravitational forces (exerted by ULDs, crew, fuel tanks, etc.) divided by the sum of all gravitational forces, determines the position of the CG (see Figure 3). The rotational moment of a mass w.r.t. a datum is calculated by the vector product of the distance vector to the datum, and the gravitational force vector. The position of the CG is thus expressed by the following expression:

$$\vec{CG} = \frac{\sum_i \vec{M_i}}{\sum_i \vec{F_i}} = \frac{\sum_i W_i \, \vec{g} \times \vec{r_i}}{\sum_i W_i \, \vec{g}} = \frac{\sum_i W_i \, \vec{r_i}}{\sum_i W_i} \tag{1}$$

In this context, it is safe to assume that the gravity acceleration $\vec{g}$ is constant

Figure 2: Different loading configurations can be mixed to construct the final load plan.

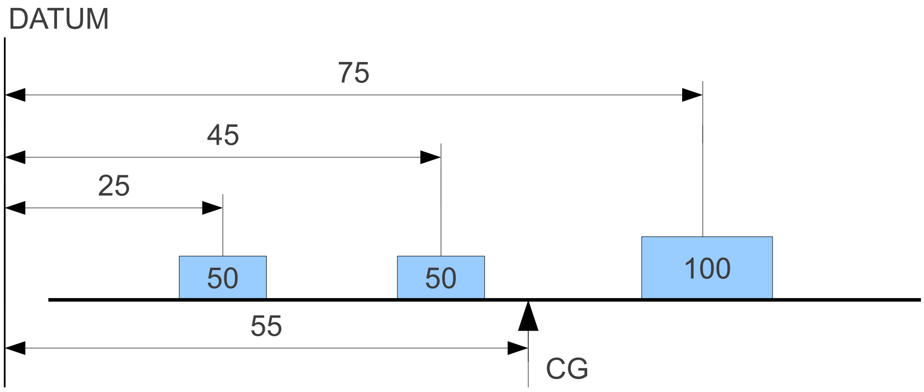

Figure 3: Graphical representation of a CG calculation. The three loads of 50, 50 and 100 exert a gravitational force on the surface, which can be virtually replaced by a fictitious load of 200 positioned at the CG $\left(= 55 = \frac{25\cdot 50+45\cdot 50+75\cdot 100}{50+50+100}\right)$. Alternatively, the CG also represents the position on the surface where the three loads would be perfectly balanced.

and that height differences between decks are negligible. When calculating the CG of a loaded aircraft, it is thus sufficient to divide the sum of the rotational moments of all loaded ULDs (the weight of the ULD multiplied by the BA of the centre of its allocated position slot), and the rotational moment of the aircraft itself, by the corresponding sum of forces (the weights of the ULDs and the aircraft). The basic weight of the aircraft and the BA of its unloaded CG are determined and provided by the manufacturer after physically weighing the aircraft.

To fly and manoeuvre safely, the CG and the weight of an aircraft must be between allowable limits, defined by the manufacturer. The longitudinal coordinate of the CG (referenced to the datum) is of main importance since it influences the pilot's control on the pitch of the aircraft: if it is too far forward the aircraft will be *nose heavy*, if it is too far aft the aircraft will be *tail heavy*. As the CG approaches a certain target position, an optimum balance between lift and drag produced by the wings can also be achieved, allowing for an optimal setting of the horizontal stabilizer. To give an idea of the impact, Mongeau and Bès (2003) report that "...a displacement of the centre of gravity of less than 75 cm in a long-range aircraft yields, over a 10,000 km flight, a saving of 4,000 kg of fuel". It must be noted that some aircraft are equipped with systems that can shift the CG of an aircraft by transferring fuel between tanks. However, a CG close to the target value is still preferable as the effectiveness of these systems is limited. The secondary goal for this work is therefore to minimize the deviation of the CG from this target value to achieve an optimal balance (also denoted as *trim*).

For a decision support tool to be of practical value it must adhere to all safety and commercial constraints imposed by the aircraft's constructor and by the airline operator. The main contribution of the present approach is a very tight representation of the physical conditions, enabling it to be used *as it is* with little to no interaction with a human planner. The following subsections go into further detail on these constraints, which are all applicable to the Boeing 747-400 series, the case study of this paper. Please note that the selected case does not restrict the generality of the model.

3.1. Structural and stability constraints

The main objective of expert weight and balancing engineers when generating a load plan, is that the aircraft can takeoff, fly and land safely. They therefore need to consider the limits on the CG as previously discussed. However, the aircraft's weight also has to be considered at different stages of the flight. Indeed, as fuel is consumed during flight, the weight in the fuel tanks decreases and the aircraft's CG shifts.

The Operational Empty Weight (OEW) is defined as the weight of the empty aircraft including the staff, while the Zero Fuel Weight (ZFW) equals the OEW plus the weight of the cargo (i.e. in our case, all loaded ULDs). Equally important are the Taxi Weight (TW), which is obtained by adding the fuel weight to the ZFW; the Take Off Weight (TOW), which equals the TW, minus the fuel needed to reach the runway; the Landing Weight (LW), which equals the TOW minus the fuel consumed during the flight. All CG calculations from the TW to the LW have to take into account the shape of the fuel tanks and the fuel distribution over these tanks. The CG limits defined by the manufacturer are expressed using a graphical representation, called the graphical envelope (see Figure 4). It defines the limits for the weight and the CG of the aircraft at ZFW, TW, TOW and LW.

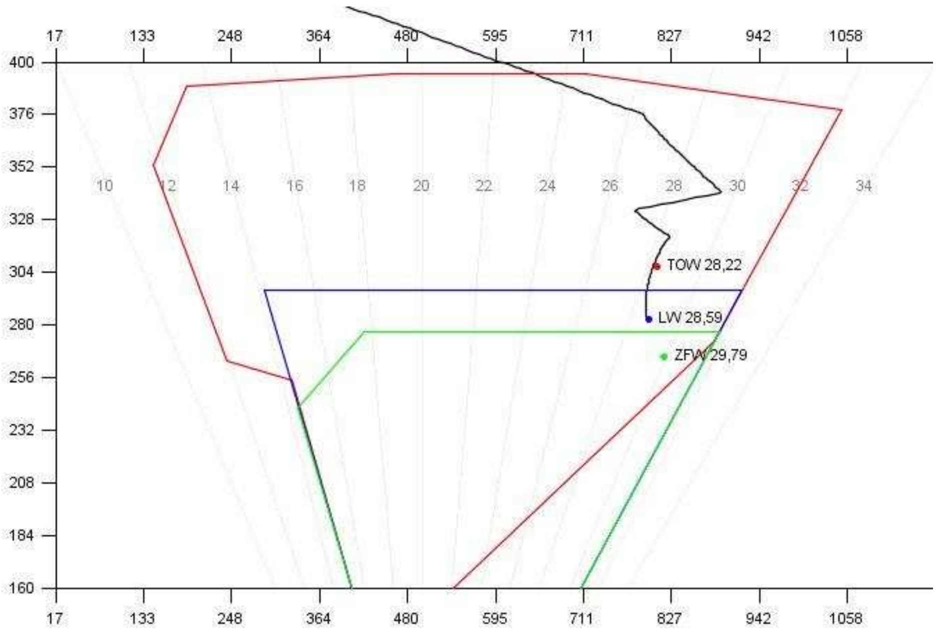

Figure 4: Graphical representation of the CG limits. The horizontal axis shows the longitudinal coordinate of the CG (in inches from the datum), the vertical axis shows the weight of the aircraft (in lbs.). The inner, outer and middle regions show the limits for the CG and the weight of the aircraft during loading (ZFW), take off (TOW) and landing (LW). The black line indicates the change of the aircraft's weight and CG during the different stages of flight.

Planners also need to ensure that high or heavy ULDs are not placed on position slots that do not support such height or load. Concerning the load of a position slot, the situation is even more complex. A distinction must be made between regular position slots, *bulk* position slots allowing multiple cargo items, and *multiweight* position slots which have load limits that are dependent on the aircraft's payload. For *bulk* positions, planners need to check whether the combined load does not exceed the maximum load of the position slot. In case of *multiweight* position slots the load limits depend on the actual payload, in a piece-wise constant manner. For example, Figure 5 shows the limits for a multiweight position slot, where the limit is 3 metric tons if the payload stays below 75 tons, but changes to 4.5 tons when the payload exceeds 75 tons.

The aircraft manufacturer, however, also defines more complex constraints

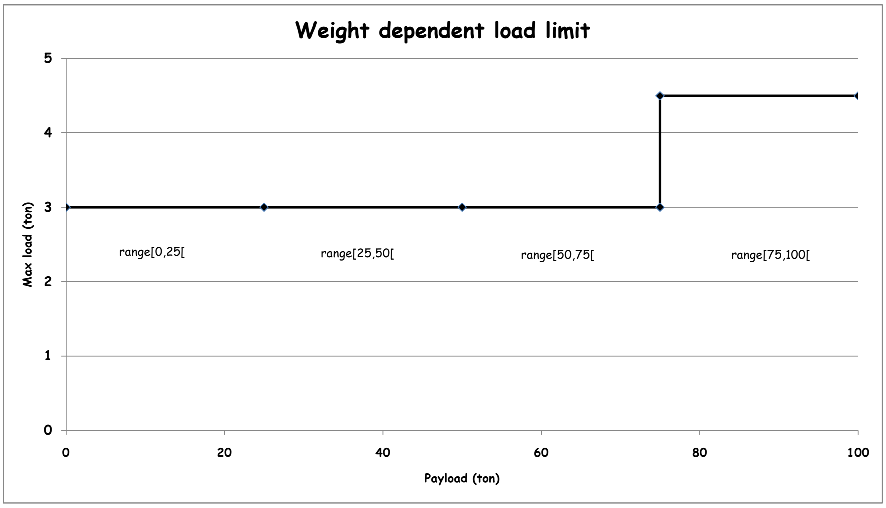

Figure 5: Position slot with multiple load limits, depending on the payload of the aircraft (*multiweight* position slot).

in order to guarantee that the load does not impose all too large forces on the fuselage of the aircraft. For each deck individually, a number of load limits can be defined for certain areas of the deck in terms of BA boundaries.

- Cumulative load constraints limit the total weight in a certain area to be below a threshold, which can be a function of the ZFW of the aircraft.
- Linear load constraints limit the total weight per unit length on a series of neighbouring positions.
- Similarly, Floor load constraints limit the total pressure (weight per unit surface area) in a certain area.
- Counter balance constraints define minimum weights per area in order to maintain the structural integrity of the aircraft.
- Unsymmetrical load constraints define the maximum differences in weight loaded onto the left and right positions per area.

3.2. Safety constraints

Besides the safety and stability of the aircraft's structure and flying characteristics, the experts also need to consider both the crew's and cargo's safety. Dangerous goods (DRGs) should not be placed near the cockpit. *Crushable* cargo, on the other hand, is preferably placed near the cockpit so as to reduce the impact of cargo should a locking mechanism fail and cargo starts sliding. Therefore, ULDs can only be placed at position slots that are suitable for the ULD type and the goods it contains.

Some ULDs also cannot be placed in each other's vicinity, e.g. a ULD with radioactive contents cannot be placed near ULDs containing food. Thus, it is possible to define pairs of ULDs which cannot be placed close to each other.

3.3. Commercial and operational constraints

A completely different category encompasses constraints that need to be imposed to ensure stability and operational efficiency while (un)loading the aircraft. For example, when a flight consists of multiple legs, ULDs for the first destination should be placed closer (as defined by an *offload sequence*) to the cargo doors than containers for later destinations. Otherwise, ULDs would need to be unloaded and loaded again to allow access to a certain ULD, increasing the handling time.

ULDs can also be assigned priorities, to prioritize certain ULDs over others even though they may be less profitable. In the case of an excess number of ULDs, high priority ULDs should be selected first. For example, in multi-leg flights, cargo for the furthest destination often has priority over cargo for a nearer destination . This prevents the aircraft from flying almost empty in the last leg (as it is often uncertain what cargo can be picked up at in-between locations). Other factors that determine priorities are e.g. load factors on consecutive sectors, client satisfaction, contracts, etc.

4. Mixed integer programming model

The aircraft weight and balance problem described above is modelled as a mixed integer linear programming model. A basic model is introduced first. The subsequent sections build upon this model and describe the structural and safety constraints, as well as the commercial constraints. The notation and variables will be gradually introduced as the model is described. Furthermore, Table 1, 2 and 3 also provide an overview of the constants, parameters and variables of the model. Note that upper case symbols (W_i, N_{ULD}, etc.) define constants/parameters known a priori, while lower case symbols (x_{ij}, a_{ZFW}, etc.) define variables.

Constant	Explanation
N_{ULD}	Total number of ULDs available for loading
i	Index for ULDs
S_i	Floor surface area of ULD i
T_i	Type of ULD i
W_i	Weight of ULD i
H_i	Height of ULD i
G_i	Profit gain for transporting ULD i
$\{DGR\}_i$	The set of dangerous goods (DGRs) transported in ULD i
I_p	The set of all ULDs with priority p
$Separation$	The set of pairs of ULDs (i_1, i_2) that may not be placed in each other's vicinity.
N_{POS}	Total number of (possibly overlapping) position slots on the aircraft
$Bulk$	The set of positions that can handle multipe ULDs (bulk positions)
$MultiWeight$	The set of positions that have a maximum weight dependent on the payload range
j	Index for position slots
$Limits_j$	The set of limits defined for multi weight position j
Seg_{jl}	The set of range variables for which limit l for position j must be used
BA_j	The longitudinal balance arm of position j
$BA_{LAT,j}$	The lateral balance arm of position j
$MaxW_j$	The maximum weight capacity of position j
$MaxW_{jl}$	The maximum weight capacity of position j for limit l
$MaxH_j$	The maximum height of position j
$\{T\}_j$	The set of allowable ULD types for position j
$\{DGR\}_j$	The set of allowable DGRs for position j

Table 1: Overview table of the constants and parameters of the model.

Constant	Explanation
A_{OEW}	The Operational Empty Weight (OEW) of the aircraft
N_{FUEL}	The total number of fuel tanks of the aircraft
w	Index for specifying weight ($w \in \{ZFW, TW, TOW, LW\}$)
$FW_{t,w}$	The (estimated) weight of fuel t at weight w
$FBA_{t,w}$	The longitudinal balance arm of tank t at weight w
BA_{OEW}	The longitudinal balance arm (or CG) of the aircraft at Operational Empty Weight (OEW)
BA_{OPT}	The target balance arm (as defined by the manufacturer) of the aircraft at ZFW
$MaxW_w$	The maximum weight of the aircraft for weight w
$MinCG_w, MaxCG_w$	The minimum and maximum CG limits for the aircraft at weight w
$WeightRanges$	The set of ranges r for the payload
r	Index for ranges
$startWeight_r, endWeight_r$	The payload interval for range r
$N_{LLC}, N_{CLC}, N_{FLC}, N_{CBC}$ N_{ULC}	The total number of linear load, cumulative load, floor load, counter balance constraints, and unsymmetrical load constraints.
k	Index for linear, cumulative, floor and counter balance constraints
P_{jk}	The percentage for which position j plays a role in constraint k
$MaxDelta_k$	The maximum load difference between left and right position slots in constraint k
$\#Prio_p$	The number of ULDs with priority p
N_{PRIO}	The number of priorities for ULDs
K_1, K_2	Parameters to denote the relative importance of maximizing total profit, and deviation of resp. longitudinal and lateral CG

Table 2: Overview table of the constants and parameters used within the model (contd.).

Variable	Explanation
$x_{ij} \in \{0,1\}$	Binary variable indicating if ULD i assigned to position slot j
$a_w \geq 0$	The weight of the aircraft at weight w
$PAYLOAD \geq 0$	The weight of all loaded ULDs
$m_w \in \mathbb{R}$	The rotational moment of the aircraft at weight w
$m_{long}d^-, m_{long}d^+ \geq 0$	The negative resp. positive deviation of the rotational moment from its target value at ZFW
$m_{lat}d^-, m_{lat}d^+ \geq 0$	The negative resp. positive deviation of lateral rotational moment from its target value (= 0) at ZFW
$range_r \in \{0,1\}$	Binary variable indicating if the payload is within range r

Table 3: Overview table of the variables used within the model.

4.1. Basic formulation

The model considers the assignment of N_{ULD} ULDs to N_{POS} position slots (a composite set over all configurations), which is expressed with binary decision variables.

$$x_{ij} = \begin{cases} 1 & \text{if ULD } i \text{ is placed on position slot } j, \\ 0 & \text{otherwise.} \end{cases}$$
$$\forall i = 1, \ldots, N_{ULD}, j = 1, \ldots, N_{POS} \tag{2}$$

Each ULD can be assigned to at most one position slot:

$$\sum_{j=1}^{N_{POS}} x_{ij} \leq 1; \qquad \forall i = 1, \ldots, N_{ULD} \tag{3}$$

Furthermore, each position slot can hold at most one ULD, except for *bulk* positions where several ULDs (see further) can be loaded:

$$\sum_{i=1}^{N_{ULD}} x_{ij} \leq 1; \qquad \forall j = 1, \ldots, N_{POS} : j \notin Bulk \tag{4}$$

with $Bulk$ a subset of the composite position slot set. Different configurations define different position slots that can be combined to construct the final load plan. However, positions slots from different configurations may spatially overlap. The assignment of ULDs to overlapping position slots must therefore be prohibited:

$$\sum_{i=1}^{N_{ULD}} (x_{ij} + x_{ik}) \leq 1; \qquad \forall j, k = 1, \ldots, N_{POS} \mid j < k,\ k \text{ overlaps } j \quad (5)$$

The primary objective O_1 of this work is to maximize the total cargo value. Thus, the first objective of the model is given by:

$$O_1 = \text{Maximize} \sum_{i=1}^{N_{ULD}} \sum_{j=1}^{N_{POS}} G_i \cdot x_{ij} \quad (6)$$

with G_i the profit corresponding to transportation of ULD i.

The weight of the loaded aircraft without fuel, the Zero Fuel Weight (ZFW), is an important factor in the stability of the aircraft. It is denoted as a_{ZFW} and is defined as:

$$a_{ZFW} = A_{OEW} + \sum_{i=1}^{N_{ULD}} \sum_{j=1}^{N_{POS}} W_i \cdot x_{ij} \quad (7)$$

with W_i the weight of ULD i and A_{OEW} the Operational Empty Weight (aircraft + crew). Similarly, the rotational moment m_{ZFW} (w.r.t the reference datum) of the aircraft at ZFW is modelled as:

$$m_{ZFW} = BA_{OEW} \cdot A_{OEW} + \sum_{i=1}^{N_{ULD}} \sum_{j=1}^{N_{POS}} W_i \cdot BA_j \cdot x_{ij} \quad (8)$$

with BA_j the balance arm of position j and BA_{OEW} the CG of the aircraft at OEW.

The deviation between the target position of the CG, BA_{OPT}, and the CG of the loaded aircraft is modelled as:

$$BA_{OPT} \cdot a_{ZFW} = m_{ZFW} + m_{long}d^- - m_{long}d^+ \quad (9)$$

with $m_{long}d^- \geq 0$ and $m_{long}d^+ \geq 0$ denoting the negative resp. positive deviation of the rotational moment from its target. Similarly, the deviation between the lateral CG and the center line of the aircraft is expressed as:

$$\sum_{i=1}^{N_{ULD}} \sum_{j=1}^{N_{POS}} W_i \cdot BA_{lat,j} \cdot x_{ij} = m_{lat}d^+ - m_{lat}d^-; \quad (10)$$

The secondary objective O_2 in this work is to minimize both the longitudinal and lateral deviation of the CG in order to reduce fuel consumption and to improve stability. It is expressed as follows:

$$O_2 = \text{Minimize } \frac{(m_{long}d^- + m_{long}d^+)}{K_1} + \frac{(m_{lat}d^- + m_{lat}d^+)}{K_2} \tag{11}$$

with $K_2 >> K_1$, to denote the relativly higher importance of the longitudinal CG to the lateral CG.

Both objectives are combined as a weighted sum, with appropriate weights to denote the higher importance of the primary objective, i.e. maximizing profit (Equation 6). The complete objective function of the MIP model is defined as:

$$\text{Maximize } O_1 - O_2 \tag{12}$$

with the values K_1 and K_2 in O_2 sufficiently large to account for the higher priority of maximizing the total cargo value (O_1).

4.2. Structural, stability and safety constraints

4.2.1. Safety and stability

Some of the manufacturers' safety constraints are rather simple while many other ones are very complex. The constraints on the total load and the CG of the aircraft ensure safety and manoeuvrability during flight. It is important to note that these constraints must be satisfied at all times (loading, taxiing, take off, flying, landing) while the CG and weight of the aircraft change during the flight. It is, however, very complex to model this change as it depends on various parameters and flight characteristics. Therefore, the constraints were approximated such that the weight and CG lie between the defined limits at several critical discrete moments: at ZFW, TW, TOW and LW. Note that the model minimizes the deviation from the target CG. This makes it very unlikely for the CG to exit the safe region of the envelope during flight, even without the aforementioned discrete CG checks.

Using estimates of the fuel volumes and consumption, the weight and rotational moment of the aircraft at TW, TOW and LOW are defined as:

$$a_w = a_{ZFW} + \sum_{t=1}^{N_{FUEL}} FW_{t,w} \qquad w \in \{TW, TOW, LW\} \tag{13}$$

$$m_w = m_{ZFW} + \sum_{t=1}^{N_{FUEL}} FW_{t,w} FBA_{t,w} \qquad w \in \{TW, TOW, LW\} \tag{14}$$

with N_{FUEL} denoting the number of fuel tanks, and $FW_{t,w}$ and $FBA_{t,w}$ denoting the estimated weight and balance arm resp. of fuel tank t at weight w corresponding to one of the reference weights.

Consequently, the weight and the CG of the aircraft are limited by the maximum weight of the aircraft, $MaxW_w$, and the minimum and maximum value of the (longitudinal) CG, $MinCG_w$ and $MaxCG_w$, at weight w:

$$a_w \leq MaxW_w; \qquad \forall w \in \{ZFW, TW, TOW, LW\} \tag{15}$$

$$MinCG_w a_w \leq m_w \leq MaxCG_w a_w; \qquad \forall w \in \{ZFW, TW, TOW, LW\} \tag{16}$$

Besides the safety constraints on the weight and the CG of the aircraft, the ULDs' contents also need to be considered. Specifically, dangerous goods (DGRs) are of importance in this context. Position slots are restricted to a specific set of DGRs, and ULDs may contain several DGRs. With $\{DGR\}_j$ denoting the allowable set of DGRs for position j, and $\{DGR\}_i$ denoting the DGRs contained in ULD i, the restriction is:

$$x_{ij} = 0; \qquad \forall i = 1, \ldots, N_{ULD},\ j = 1, \ldots, N_{POS} : \{DGR\}_i \cap \{DGR\}_j \neq \emptyset \tag{17}$$

Given that these constraints put restrictions on individual variables, our implementation prohibits the assignment by not introducing the corresponding variables.

Another constraint related to DGRs, is that some ULDs may not be placed next to each other, in order to avoid contamination of their contents. To reprise the example from the problem description, a ULD with e.g. radioactive contents may not be placed near ULDs containing food. With *Separation* denoting a set of pairs of ULDs which may not be placed in each others vicinity, this is expressed as:

$$x_{i_1 j_1} + x_{i_2 j_2} \leq 1 \qquad \forall \quad (i_1, i_2) \in Separation,\ j_1, j_2 = 1, \ldots, N_{POS} : j_1 \text{ close to } j_2 \tag{18}$$

4.2.2. Structural integrity

In the aircraft manual, the manufacturer defines constraints to ensure the structural integrity of the fuselage. Those constraints are either defined per position slot or per zone.

Position slots are restricted to certain types of ULDs, as well as to ULDs of a certain maximum height and weight. The type and height restrictions can be modelled as:

$$H_i x_{ij} \leq MaxH_j; \qquad \forall i = 1, \ldots, N_{ULD}, \forall j = 1, \ldots, N_{POS} \tag{19}$$

$$x_{ij} = 0; \qquad \forall i = 1, \ldots, N_{ULD},\ j = 1, \ldots, N_{POS} : T_i \notin \{T\}_j; \tag{20}$$

with H_i and T_i denoting the height and type of ULD i, and $\{T\}_j$ denoting the allowable ULD types for position j. The same remark as for Equation 17 holds: the implementation does not introduce the corresponding variables if these constraints are violated.

For the maximum load of a position slot, three distinct cases are considered: regular, bulk and multiweight position slots. Regular position slots can simply exclude all ULDs that are too heavy:

$$\begin{aligned} W_i x_{ij} \leq MaxW_j; \qquad & \forall i = 1, \ldots, N_{ULD}, \\ & j = 1, \ldots, N_{POS} : j \notin Bulk, j \notin MultiWeight \end{aligned} \tag{21}$$

with $MaxW_j$ denoting the maximum weight on position slot j, and $Bulk$ and $MultiWeight$ subsets from the entire set of position slots denoting the bulk and multiweight positions slots, respectively.

For *bulk* positions, the total weight of all cargo assigned to the position should be below the maximum load:

$$\sum_{i=1}^{N_{ULD}} W_i x_{ij} \leq MaxW_j; \qquad \forall j \in Bulk \tag{22}$$

Finally, *multiweight* positions are restricted to a maximum load that is dependent on the payload of the aircraft, defined as a piece-wise constant function. To model this limit, the payload range of the aircraft is discretized in a set of smaller ranges (e.g. the range from 0 to 100 tons is discretized into 0-25 tons, 25-50 tons, etc.). These ranges are visualised by the markers in Figure

5. For each range, a binary variable $range_r$ is defined indicating whether the aircraft's payload is in the corresponding range. The entire set of $range_r$ variables, covering the entire payload range, is denoted as $WeightRanges$.

The payload of the aircraft is defined as:

$$Payload = \sum_{i=1}^{N_{ULD}} \sum_{j=1}^{N_{POS}} W_i x_{ij} \tag{23}$$

Let $startWeight_r$ and $endWeight_r$ denote the limit values of range r. The indicator variables $range_r$ take on the correct value depending on the payload of the aircraft, which is modelled with the following big-M formulation:

$$startWeight_r \cdot range_r \leq Payload, \qquad \forall r \in WeightRanges \tag{24}$$

$$Payload + (range_r - 1) \sum_{i=1}^{N_{ULD}} W_i \leq endWeight_r, \qquad \forall r \in WeightRanges \tag{25}$$

$$range_r \in \{0, 1\}; \qquad \forall r \in WeightRanges \tag{26}$$

Furthermore, the payload can only lie within exactly one of the smaller ranges:

$$\sum_{r \in WeightRanges} range_r = 1 \tag{27}$$

Multiweight positions are then modelled as follows. For each segment l of the piece-wise constant function, a limit value $MaxW_{jl}$ is defined, as well as a set Seg_{jl} containing the $range_r$ variables for which the corresponding range lies in the considered segment. The entire set of segments for position j is denoted as $Limits_j$. To illustrate this, consider again the example of Figure 5. The piece-wise function defines two segments (indexed by l_1 and l_2), 0-75 tons, and 75-100 tons, with corresponding limit values $MaxW_{jl_1} = 3$ tons and $Max_{jl_2} = 4.5$ tons. The sets Seg_{jl_1} and Seg_{jl_2} are then defined as:

$$Seg_{jl_1} = \{range_{[0,25[}, range_{[25,50[}, range_{[50,75[}\} \tag{28}$$

$$Seg_{jl_2} = \{range_{[75,100[}\} \tag{29}$$

The constraint on the load of multiweight positions is thus expressed as follows:

$$\sum_{i=1}^{N_{ULD}} W_i \cdot x_{ij} + \leq MaxW_{jl} + \sum_{i=1}^{N_{ULD}} W_i(1 - \sum_{r \in Seg_{jl}} range_r), \qquad \forall j \in MultiWeight, \forall l \in Limits_j \tag{30}$$

The other sets of structural constraints considered in this work are defined over certain areas of the decks of the aircraft. The areas are modelled in terms of position slots by defining a parameter P_{jk}, which represents the percentage of the surface area of the position slot j that overlaps with the considered area in constraint k. It is assumed that ULDs are uniformly loaded such that the percentage surface overlap is an accurate measure of the weight in the considered area. Four sets of constraints are considered:

- Cumulative load constraints specify that the total accumulated load over a specific area has to be below a certain threshold value, which can be a linear function of the ZFW of the aircraft.

  $$\sum_{i=1}^{N_{ULD}} \sum_{j=1}^{N_{POS}} W_i P_{jk} x_{ij} \leq C_k \cdot a_{ZFW} + D_k, \qquad \forall k = 1, \ldots, N_{CLC} \tag{31}$$

  with C_k and D_k the coefficients of the linear function. In the simplest case, $C_k = 0$ and D_k is equal to the maximum accumulated load in the area. Linear load constraints are similar but they specify that the total load per inch (along the longitudinal axis) over a specific area has to be below a certain threshold value, possibly a function of the ZFW of the aircraft. This type of constraint can again be modelled as a cumulative load limit:

  - by dividing the weight of the ULD by the longitudinal dimension of the position slot, the load per inch for the position slot is obtained. Note that this assumes uniformly loaded ULDs.
  - by sorting all position slots in the considered area by their longitudinal boundaries, and splitting the area in smaller regions at

the boundaries of the position slots for which a cumulative load constraint is then defined:

$$\sum_{i=1}^{N_{ULD}} \sum_{j=1}^{N_{POS}} \frac{W_i}{L_j} P_{jk} x_{ij} \leq C_k \cdot a_{ZFW} + D_k, \qquad \forall k = 1, \ldots, N_{LLC} \quad (32)$$

with L_j the longitudinal dimension of position slot j.

For example, consider a linear load constraint for the area in Figure 6a with four position slots. This linear load constraint can be converted to three cumulative load constraints for the sub-areas shown in Figure 6b, as the load per inch can be considered 'constant' over the sub-area.

- Floor load constraints define that the surface load per square-inch over a specific area k has to be below a certain threshold $MaxFL_k$. With S_j denoting the surface area of position j, this is expressed as:

$$\sum_{i=1}^{N_{ULD}} \sum_{j=1}^{N_{POS}} W_i \cdot P_{jk} \cdot x_{ij} \leq \sum_{i=1}^{N_{ULD}} \sum_{j=1}^{N_{POS}} S_j \cdot P_{jk} \cdot x_{ij}, \qquad \forall k = 1, \ldots, N_{FLC} \quad (33)$$

- A peculiar set of constraints in this context comprises the counter balance constraints, which specify that the load in a certain area k has to be *above* a certain threshold $MinW_k$:

$$\sum_{i=1}^{N_{ULD}} \sum_{j=1}^{N_{POS}} W_i \cdot P_{jk} \cdot x_{ij} \geq MinW_k, \qquad \forall k = 1, \ldots, N_{CBC} \quad (34)$$

- Finally, unsymmetrical load constraints are defined for positions that are laterally adjacent (i.e. as in the 30" pallet configuration in Figure 1. This constraint expresses that the difference between the load on a position slot left of the center line and the load right of the center line should not exceed a certain threshold. With $MaxDelta_k$ denoting the

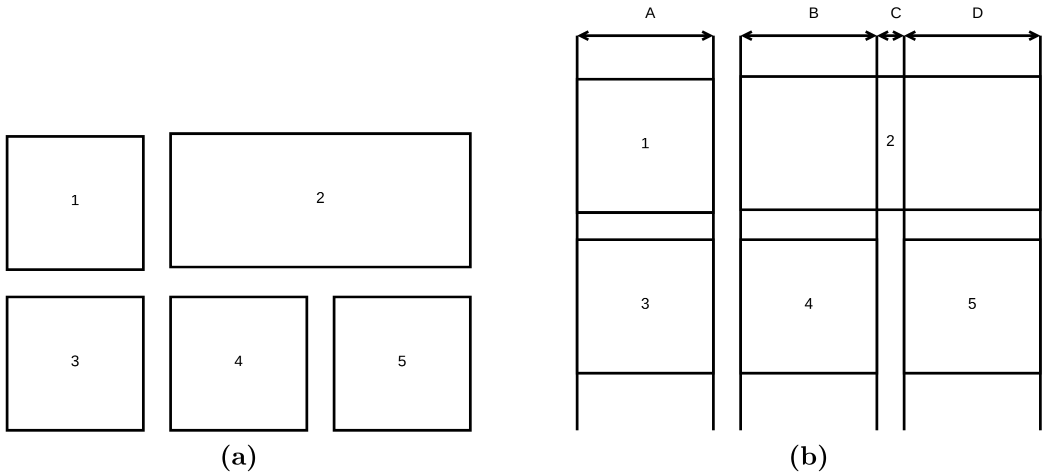

Figure 6: Converting linear load constraints over positions 1-5 (a) to cumulative load constraints by considering uniform sub-areas A,B,C and D (b).

maximum load difference, this is expressed as:

$$-MaxDelta_k \leq \sum_{i=1}^{N_{ULD}} \sum_{l} W_i \cdot P_{lk} \cdot x_{il} - \sum_{i=1}^{N_{ULD}} \sum_{r} W_i \cdot P_{rk} \cdot x_{ir} \leq MaxDelta_k, \quad (35)$$

$$\forall k = 1, \ldots, N_{ULC},$$

$$with\ l = 1, \ldots, N_{POS} | l \text{ is left position slot },$$

$$and\ r = 1, \ldots, N_{POS} | r \text{ is right position slot}$$

4.3. Commercial constraints

The primary objective of the model is that the most profitable selection of ULDs is transported, given the constraints discussed above. However, sometimes it is necessary to prioritize some ULDs over others. Therefore, ULDs are also assigned a priority, ranging from 1 (highest) to N_{PRIO} (lowest). The priority constraint that is implemented in our model ensures that ULDs of lower priority are only loaded if all ULDs of higher priority are loaded. In case some ULDs with priority p cannot be loaded, the model will only assign all ULDs of higher priority ($< p$), and the most profitable selection of ULDs of priority p, while leaving behind all remaining ULDs of priority p and lower ($\geq p$). The model is a literal interpretation of how the constraint was explained to the authors. In future work, a relaxation of the constraint may be taken into consideration, thus allowing to compensate for not taking a high priority ULD by collecting a number of lower priority ULDs with a large composite value.

The constraint is expressed as follows:

$$\#Prio_{p-1} \sum_{j=1}^{N_{POS}} x_{ij} \leq \sum_{k \in Prio_{p-1}} \sum_{j=1}^{N_{POS}} x_{kj}, \qquad (36)$$
$$\forall p = 2, \ldots, N_{PRIO}, \forall i \in Prio_p$$

with $Prio_p$ denoting the set of ULDs of priority p, and $\#Prio_p$ denoting the number of ULDs with priority p. This constraint assumes, without loss of generality, at least one ULD per priority class p.

5. Experiments

5.1. Experimental setup

The model presented has been integrated in the Sable software suite for aircraft weight and balancing, developed by B. Rekencentra N.V[2]. The MIP model was implemented in Java and solved using the Gurobi 4.6.1 solver under an academic license. Cooperation with the software company was essential for understanding the details of the AWB problem. It also enabled access to real data for validation of the model.

We obtained data from 51 flights operated by one commercial cargo carrier. The flights considered here were operated with 3 types of aircraft from the Boeing 747-400 series: the 747-400ERF (Extended Range Freighter), the 747-400F (Freighter) and the 747-400SF (Special Freighter). The 747-400ERF and -400F are freighter versions of the classic 747-400, the 747-400SF is a converted 747-400s (passenger aircraft) from the Boeing Conversion Program. The relevant features of these aircraft are listed in Table 4.

The details of the considered flights are given in Table 5. We report on the number of ULDs from which a selection must be made, the total weight of the ULDs, the number of *oversized ULDs* (which require a special position slot) and the number of bulk items.

[2] `http://www.rekencentra.be`

Aircraft	N_{CLC}	N_{LLC}	N_{FLC}	N_{ULC}	N_{POS}	# flights
747-400F	37	0	0	13	168	24
747-400ERF	21	35	7	13	168	16
747-400SF	41	19	10	13	168	11

Table 4: Number of positions, cumulative load constraints, linear load constraints, floor load constraint and unsymmetrical load limits for the 747-400F, -400ERF and -400SF.

5.2. Discussion

The relevance and the efficiency of the model are determined by comparing the results obtained by the model and those of a human expert planner on each of the 51 obtained flights. The comparison is made on the basis of both objectives: maximizing profit and minimizing CG deviation from the target value.

The primary objective of the MIP model was to maximize the profit gained from transporting ULDs. On most of the considered flights, however, the human expert was able to load all cargo. In total, over all 51 flights, four ULDs were left behind. The model on the other hand, was able to load the entire cargo set on the considered flights, resulting in a considerable gain in profit. These results are summarized in Table 6.

The secondary objective, minimizing the deviation from the target CG, proved to be equally interesting. Both the longitudinal CG and the lateral imbalance obtained by the MIP model are compared with those obtained by the human expert in Table 7.

The longitudinal CG is expressed as $\%MAC$, which denotes the longitudinal position of the CG as a percentage of the *Mean Aerodynamic Chord* (MAC). The MAC represents the average chord ($\sim$ width) of the wing, averaged over its entire span. The *Leading Edge MAC* (LEMAC) is defined as the longitudinal distance from the datum to the MAC (Figure 7). The longitudinal CG is then expressed in $\%MAC$ as distance aft of LEMAC, relative to the MAC:

$$CG_{\%MAC} = \frac{BA_{CG} - LEMAC}{MAC} \tag{37}$$

The results in Table 7 show that for all three types of aircraft, on average, the CG shifts by more than 4 percentage point (%MAC) to the target value of

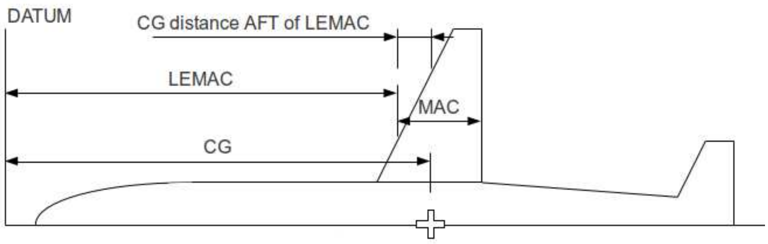

Figure 7: Graphical representation of the Mean Aerodynamic Chord (MAC) and the Leading Edge MAC (LEMAC).

30 $\%MAC$ (for these aircraft). The longitudinal CG obtained by the model is always negligibly close to the target value for an optimal trim, reducing fuel consumption.

For the lateral CG, the lateral imbalance (in kg.) is reported. The lateral imbalance is the difference between the load on the right and the left of the aircraft. Given that the lateral BA of position slots is usually small, this measure is more interesting. The results in Table 7 show that a huge improvement is made on the lateral imbalance: 89%. Although the impact on the fuel saving of this improvement is limited, an increased stability and greater flexibility in fuel distribution over the different tanks is obtained.

Table 7 also reports the time required for solving the model to optimality. For all flights, the computation time remained below 10 seconds, averaging at approximately two seconds, which is fast enough for interactive use. Given that the model is NP-Hard, these low computation times can be explained by considering the characteristics of real-life instances. In practical use cases, the number of ULDs is limited. The real-life instances addressed in this paper have at most 50 ULDs, thus bounding the model's size. In particular, the number of bulk ULDs in the current setup is at most 5.

Considering the time needed by a human expert planner, which is around 10-15 minutes for a full load, the MIP approach offers a drastic improvement. Finally, it must be noted that the model always produced feasible results, strictly respecting all structural, safety and commercial constraints.

6. Conclusion

The paper presented a mixed integer programming model as part of an automated decision support tool for air cargo selection and weight balancing.

This task is of critical importance during flight preparation of any aircraft, as the cargo and its positioning has a strong influence on the aircraft's stability, manoeuvrability and fuel consumption. The model implements most of the loading constraints relevant in practice, allowing it to be used as it is without manual intervention or corrections.

The main objectives of this model are to maximize profit gained from transporting cargo, and to minimize the deviation of the aircraft's centre of gravity from a target value, thus improving stability and reducing fuel consumption. Using real life flight data from a commercial cargo carrier, the designed model was tested extensively. The longitudinal centre of gravity was strongly improved on all considered flights, reducing the average deviation from the target CG by more than 4 percentage point aft from the Mean Aerodynamic Chord (MAC). As the method was always able to take all the cargo, this improvement leads to considerably smaller fuel consumption for transporting the same cargo, thus increasing the profit for the airliner. In cases where the human expert failed to load all cargo, the presented method was able to load additional ULDs without violating any of the safety, stability and structural integrity constraints. Moreover, the average lateral imbalance for the considered flights was reduced from more than 3 to less than 0.3 tonnes.

The maximum calculation time required to obtain these results was less than 10 seconds for the commercial MIP solver Gurobi 4.6.1. Given that a human expert needs at least ten minutes to achieve a suboptimal solution, this is an excellent result. This clearly shows the relevance of such a decision support model for the air traffic business.

Acknowledgement

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	N_{ULD} (incl. bulk)	Payload (kg)	$N_{OVERSIZED}$	N_{BULK}
747-400F				
avg.	40.54	90494.62	1.29	1.29
min.	33	47514	0	0
max.	46	117488	4	5
747-400ERF				
avg.	41.62	88522	1	1.38
min.	37	6148	0	0
max.	50	113220	3	4
747-400SF				
avg.	41.18	91038.73	0.91	1.82
min.	29	66101	0	1
max.	47	108198	2	3

Table 5: Characteristics of the considered flights for the 747-400F, -400ERF and -400SF.

	Expert (#ULD)	MIP model (#ULD)
747-400F	0	0
747-400ERF	2	0
747-400SF	2	0

Table 6: Comparison of total number of ULDs left behind, by the human expert and by the model.

	Longitudinal (%MAC)			Lateral (kg)			
	Expert	MIP	Target	Expert	Mip	Target	Time (s)
747-400F							
avg.	25.30	29.99	30	4540.13	277.44	0	2.61
min.	19.84	29.82	30	1175	4	0	1.15
max.	28.9	30.09	30	10136	938	0	4.51
747-400ERF							
avg.	24.97	29.95	30	4502.79	279.42	0	2.49
min.	22.55	29.73	30	532	2	0	1.09
max.	27.7	30.06	30	19749	858	0	4.04
747-400SF							
avg.	25.36	29.90	30	3325.37	149.27	0	3.13
min.	23.33	29.71	30	172	0	0	1.22
max.	27.97	30.03	30	9384	346	0	7.25

Table 7: Average, minimum and maximum longitudinal and lateral CG obtained by an expert planner vs. the MIP model on 51 different flights.